A REVIEW OF DEFINITIONS AND MEASURES OF SYSTEM RESILIENCE

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ABSTRACT
Modeling and evaluating the resilience of systems, potentially complex and large-scale in nature, has recently raised significant interest among both practitioners and researchers. This recent interest has resulted in several definitions of the concept of resilience and several approaches to measuring this concept, across several application domains. As such, this paper presents a review of recent research articles related to defining and quantifying resilience in various disciplines, with a focus on engineering systems. We provide a classification scheme to the approaches in the literature, focusing on qualitative and quantitative approaches and their subcategories. Addressed in this review are: an extensive coverage of the literature, an exploration of current gaps and challenges, and several directions for future research.

KEYWORDS
Resilience, Engineering systems
1. INTRODUCTION

Historically, the primary questions asked during a risk assessment study are: (i) what can go wrong?, (ii) what is the likelihood of such a disruptive scenario?, and (iii) what are the consequences of such a scenario? [1]. Risk management strategies have traditionally focused on reducing the likelihood of disruptive events and reducing the potential consequences of the event, as well as some synthesis of both. As such, risk management strategies often emphasized mitigation options in the form of prevention and protection: designing systems to avoid or absorb undesired events from occurring. The main objective of protection strategy is to detect the adversary early and defer the adversary long enough for an appropriate respond. While a protection strategy is critical to prevent undesired events or consequences, however recent events suggested that not all undesired events can be prevent. Hurricane Sandy, which devastated NY/NJ in 2012, is among the more recent examples of a disruptive event that adversely impacted multiple networked systems (e.g., months after the storm, power had not been restored to all communities in the NY/NJ area [2], one million cubic yards of debris impeded transportation networks [3]). Plenty of other disruptions have highlighted the resilience, or lack thereof, of networked systems: the August 2003 US blackout that caused transportation and economic network disruptions [4], Hurricane Isabel devastated the transportation system of the Hampton Roads, VA, region in 2003 and overwhelmed emergency response [5], the 2011 9.0 magnitude earthquake and tsunami that struck Japan, causing over 15,000 confirmed deaths and disrupting global supply chain networks [6]. It is because of these recent large-scale events that the Department of Homeland Security, among others, has placed emphasis on resilience through preparedness, response, and recovery [7,8].

The term resilience has increasingly been seen in the research literature [9] and popular science literature [10] due to its role in reducing the risks associated with the inevitable disruption of systems. This paper presents a comprehensive review of resilience in various disciplines, published from 2000 to April 2015. In this paper, we primarily focus on the quantitative perspective of modeling resilience, distinguishing our work from existing excellent review papers [11, 12].

The word resilience has been originally originated from the Latin word “resiliere,” which means to “bounce back.” The common use of resilience word implies the ability of an entity or system to return to normal condition after the occurrence of an event that disrupts its state. Such a broad definition applies to such diverse fields as ecology, materials science, psychology, economics, and engineering. A graphical depiction of the initial impact and subsequent recovery of a six recent U.S. recessions is shown in Fig. 1 [13]. For example, the figure shows that for the 1980s recession, there was a disruption that affected a change roughly equal to -1.2% and that the recovery lasted roughly six months.
Several definitions of resilience have been offered. Many are similar, though many overlap with a number of already existing concepts such as robustness, fault-tolerance, flexibility, survivability, and agility, among others.

Some general definitions of resilience that span multiple disciplines have been offered. For example, Allenby and Fink [53] defined resilience as the “capability of system to maintain its function and structure against internal and external changes and downgrade the performance of system when it must.” Pregenzer [54] defined resilience as the “measure of a system’s ability to absorb continuous and unpredictable change and still maintain its vital functions.” Haimes [55] defined the resilience as the “ability of system to withstand a major disruption within acceptable degradation parameters and to recover with a suitable time and reasonable costs and risks.” Disaster resilience is characterized by Infrastructure Security Partnership [56] as the capability to prevent or protect against significant multi-hazard threats and incidents, including terrorist attacks, and to recover and reconstitute critical services with minimum devastation to public safety and health. Vugrin et al. [57] defined system resilience as: “Given the occurrence of a particular disruptive event (or set of events), the resilience of a system to that event (or events) is that system’s ability to reduce efficiently both the magnitude and duration of deviation from targeted system performance levels.” Two elements of this definition are noted: system impact, the negative impact that a disruption imposes to a system and measured by the difference between targeted and disrupted performance level of system, and total recovery efforts, the amount of resources expended to recover the disrupted system.

The concept of resilience has also been approached from particular disciplinary perspectives and across application domains, including psychology, ecology, and enterprises, among others. A variety of definitions for the notion of resilience have been proposed. We identify four domains of resilience: organizational, social, economic, engineering. Note that this classification may vary depending on researcher’s perspective. We provide a variety of definitions of resilience according to four aforementioned groups.
1.1. Organizational Domain
The concept of organizational resilience has emerged to address the need for enterprises to respond to a rapidly changing business environments. The resilience of an organization is defined by Sheffi [19] as the inherent ability to keep or recover a steady state, thereby allowing it to continue normal operations after a disruptive event or in the presence of continuous stress. Vogus and Sutcliffe [20] defined organizational resilience as “the ability of an organization to absorb strain and improve functioning despite the presence of adversity.” Sheffi [21] defined resilience for companies as “the company’s ability to, and speed at which they can, return to their normal performance level (e.g., inventory, capacity, service rate) following by disruptive event.” McDonald [22] defined resilience in the context of organizations as “the properties of being able to adapt to the requirements of the environment and being able to manage the environments variability.” Patterson et al. [23] highlighted that collaborative cross-checking can greatly enhance the resilience of organizations. Collaborative cross-checking is an enhanced resilience strategy in which at least two groups or individuals with different viewpoints investigate the others’ activations to evaluate accuracy or validity. By implementing collaborative cross-checking, erroneous actions can be detected quickly enough to mitigate adverse consequences. More definitions of resilience in the context of organizational, enterprises and can be found in [24-27].

1.2. Social Domain
The social domain looks at the resilience capacities of individuals, groups, community, and environment. Adger [28] defined social resilience as “ability of groups or communities to cope with external stresses and disturbances as a result of social, political, and environmental change.” The Community and Regional Resilience Institute [29] defined the resilience as the capability to predict risk, restrict adverse consequences, and return rapidly through survival, adaptability, and growth in the face of turbulent changes. Keck and Sakdapolrak [30] defined social resilience as comprised of three dimensions: coping capacities, adaptive capacities, and transformative capacities. The term of community resilience is described by Cohen et al. [31] as ability of community to function properly during disruptions or crises. Pfefferbaum et al. [32] defined community resilience as “the ability of community members to take meaningful, deliberate, collective action to remedy the effect of a problem, including the ability to interpret the environment, intervene, and move on”. The concept of resilience has been well studied in subdomains of the social domain such as ecology [33-35], psychology [36-38], sociology [39-42].

1.3. Economic Domain
Rose and Liao [43] described economic resilience as the “inherent ability and adaptive response that enables firms and regions to avoid maximum potential losses.” Static economic resilience is referred by Rose [44] as the capability of an entity or system to continue its functionality like producing when faces with a severe shock, while dynamic economic is defined as the speed at which a system recovers from a severe shock to achieve a steady state. A more specific definition of economic resilience is presented by Martin [45] as “the capacity to reconfigure, that is adapt, its structure (firms, industries, technologies, institutions) so as to maintain an acceptable growth path in output, employment and wealth over time.”
1.4. Engineering Domain
The concept of resilience in the engineering domain is relatively new in comparison to other domains. The engineering domain includes technical systems designed by engineers that interact with humans and technology, such as power grid electrical systems. Note that Youn et al. [14] defined engineering resilience as the sum of passive survival rate (reliability) and proactive survival rate (restoration) of a system. Another definition of engineering resilience is presented by Hollnagel et al. [15] as the intrinsic ability of a system to adjust its functionality in the presence of disturbance and unpredicted changes. Hollnagel and Prologue [16] pointed out that, for resilience engineering, understanding the normal functioning of a technical system is important as well as understanding how it fails. The American Society of Mechanical Engineers (ASME) [17] defined resilience as the ability of a system to sustain external and internal disruptions without discontinuity of performing the system’s function or, if the function is disconnected, to fully recover the function rapidly. Dinh et al. [18] identified six factors that enhance the resilience engineering of industrial processes, including minimization of failure, limitation of effects, administrative controls/procedures, flexibility, controllability, and early detection.

Infrastructure systems such as water distribution systems, nuclear plants, transportation systems, and locks and dams, among others, can be considered as subdomain of the engineering domain as their construction and restoration require engineering knowledge. National Infrastructure Advisory Council (NIAC) [52] defined the resilience of infrastructure systems as their ability to predict, absorb, adapt, and/or quickly recover from a disruptive event such as natural disasters. Infrastructures are also considered as subdomain of social domain in which the lack of their resilience can lead to adverse impacts on communities. According to Percoco [46], infrastructure systems can greatly improve the economic efficiency of a country. Due to the crucial role of infrastructures on society and economy, research work has recently focused on infrastructure resilience [47-50]. Ouyang and Wang [51] assessed the resilience of interdependent electric power and natural gas infrastructure systems under multiple hazards, noting how interdependent network performance could be measured in physical engineering terms or in terms of societal impact.

1.5. Analysis of Resilience Definitions
The review of resilience definitions indicates that there is no unique insight about how to define the resilience, however several similarities can be observed across these resilience definitions. The main highlights of resilience definitions reviewed above are summarized as follows:

• Some definitions does not specify mechanisms to achieve resilience; however many of them focus on the capability of system to “absorb” and “adapt” to disruptive events, and “recovery” is considered as the critical part of resilience.

• For engineered systems, such as nuclear and power plant systems, reliability is often considered to be an important feature to measure an ability to stave off disruption.

• Some definitions, such as Sheffi [19] and ASME [17], emphasize that returning to steady state performance level is needed for resilience, while other definitions do not impose that the system (e.g., infrastructure, enterprise, community) return to pre-disaster state.
The definition offered by Haimes [55] suggests a multidimensionality to the quantification of resilience, that particular states of a system are inherently more resilient than others. Further, Haimes stresses that the resilience of a system is threat-dependent.

Some definitions such as Allenby and Fink [53], Pregenzer [54], and Adger [28] defined resilience in terms of preparedness (pre-disaster) activities, while the role of recovery (post-disaster) activities are discarded. Definitions presented by organizations such as National Infrastructure Advisory Council (NIAC) [52] emphasized on the role of both preparedness and recovery activities to achieve resilience.

The rest of paper includes the following structure. Section 2 our approach to reviewing the literature, and Section 3 provides a classification methodologies that are used to measure and assess the resilience in various disciplines. Section 4 summarizes important lessons obtained the literature, and Section 5 discusses the existing gaps and restrictions on assessing resilience. Finally, we provide concluding remarks in Section 6.

2. Literature Review Methodology

In this section, we discuss framework we used to identify resilience-related literature. We also report, to the extent that we can, the distribution of literature by domains, years of publication, and journals.

To present a breadth coverage of literature review of resilience study, we developed a framework of five steps: (i) online database searching and information clustering, (ii) citation and sample refinement, (iii) abstract review refinement, (iv) full-text review refinement, and (v) final sort. The Web of Science database, one of the most comprehensive multidisciplinary content search platforms for academic researchers [58], was searched to identify the papers.

Using keywords to conduct the search, we selected those papers only relevant to modeling and measuring resilience in engineering fields, including engineering design, supply chain, infrastructure systems, and physical networks, and non-engineering fields, including enterprises/organizations, social networks, and economics. Journal papers were filtered with such keywords as resilience modeling, resilience quantification, resilience metrics, design resilience, disaster resilience, and engineering resilience. This approach was applied to the papers published from 2000 to April 2015, though we focus primarily on recent papers.

2.1. Distribution by Domain

CiteSpace [59], a well-known visualization tool, was used to visualize and analyze trends in the resilience literature. As shown in Fig. 2, the application of resilience in each discipline is represented by a cluster. The largest cluster is dedicated to the Psychology domain, followed by the Environmental, Social, & Ecology domain. The size of cluster of a discipline is relates to the number of papers published in that discipline. Meanwhile, a lesser proportion of resilience-related research exists in the engineering domain, suggesting that greater strides in defining and quantifying resilience have historically been made in non-engineering contexts. As such, opportunities exist in impacting resilience in the engineering domain (e.g., engineering design).
2.2. Distribution by Journal

Several different journals from different disciplines that published work related to resilience quantification approaches were included in this literature review. Table 1 lists 14 journals that contributed more than one article examined in this literature review. Among these, *Reliability Engineering and Systems Safety* is the most significant source of articles related to the resilience research, with *Risk Analysis*, *International Journal of Production Research*, and *Procedia Computer Science* following. The application of resilience in organizations, enterprises, business management, and logistics engineering are mostly published in *International Journal of Production Research*. Mathematical modeling perspectives on resilience have been mostly published in *Computers and Operations Research*, *Transportation Research-Part B*, and *Transportation Research-Part E*.

Table 1. Top journal sources of resilience research, as appropriate for this review.

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<thead>
<tr>
<th>No.</th>
<th>Journal title</th>
<th>No. of papers</th>
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<tbody>
<tr>
<td>1</td>
<td>Reliability Engineering and Systems Safety</td>
<td>10</td>
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<tr>
<td>2</td>
<td>Risk Analysis</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>International Journal of Production Research</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>Procedia Computer Science</td>
<td>3</td>
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2.3. Distribution by Year of Publication

The distribution of resilience-related archival journal articles by year from 2000 to April 2015 is represented in Fig. 3, using Web of Science [60]. The recent government and policy emphasis on resilience is also seen in academic research, according to the increasing appearance of resilience-related research.

![Graph showing distribution of papers by year of publication, as of April 2015.](image-url)

**Fig. 3.** Distribution of papers by year of publication, as of April 2015.
2.4. Classification of Resilience Assessment Approaches

In general, the resilience evaluation procedure can be separated into two major categories: qualitative and quantitative. The qualitative category which includes methods that tend to assess the resilience of system without numerical descriptors, contains two sub-categories: (i) conceptual frameworks that offer best practices, and (ii) semi-quantitative indices that offer expert assessments of different qualitative aspects of resilience. The quantitative methods include two sub-categories: (i) general resilience approaches that offer domain-agnostic measures to quantify resilience across applications, and (ii) structural-based modeling approaches that model domain-specific representations of the components of resilience. Note that the focus of this paper is on quantitative approaches, given our interest in engineering systems. The classification scheme of resilience assessment approaches is visually represented in Fig. 4. Readers interested in qualitative contributions to resilience research can refer to references [61] and [62]. We summarize the classification of reviewed papers, along with their corresponding methods, in Table 2.

![Fig. 4. Classification scheme of resilience assessment methodologies.](image1)

### Table 2: Classification of Resilience Assessment Approaches

<table>
<thead>
<tr>
<th>Qualitative assessment</th>
<th>Quantitative assessment</th>
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<tr>
<td>Conceptual frameworks</td>
<td>General measures</td>
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<td>Semi-quantitative indices</td>
<td>Structural-based models</td>
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<td>Probabilistic approaches</td>
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<td>Optimization models</td>
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<td>Fuzzy logic models</td>
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3. Qualitative Assessment Approaches

This section highlights the qualitative resilience assessment approaches categorized as conceptual frameworks and semi-quantitative indices.

3.1. Conceptual Frameworks

Work that we classified as conceptual frameworks constitute the majority of the qualitative approaches for assessing the system resilience. Alliance [63] proposed a generic framework for evaluating the resilience of social-ecological systems, composed of seven steps: (i) defining and understanding the system under study, (ii) identifying appropriate scale to evaluate resilience, (iii) identifying the system drivers and external and internal disturbance, (iv) identifying the key players in the system, including people and governance, (v) developing conceptual models for identifying necessary recovery activities, (vi) implementing the results of step 5 to inform policymaker, and (vii) incorporating the findings of the previous step. Speranza et al. [64] developed a notional framework for analyzing resilience of livelihoods, or the “resources that people have and the strategies they adopt to make a living.” The framework provides a few attributes of three dimension of resilience: buffer capacity (the amount of change that a system can undergo), self-organization (the emergence of society through inherent social structure), and
capacity for learning (an ability to adapt). In a homeland security context, Kahan et al. [65] proposed a broad conceptual framework for system resilience using eight guiding principles: (i) threat and hazard assessment, (ii) robustness, (iii) consequence mitigation, (iv) adaptability, (v) risk-informed planning, (vi) risk-informed investment, (vii) harmonization of purposes, and (viii) comprehensive of scope. Labaka et al. [66] proposed a holistic resilience framework that has been defined in close collaboration with general management. Their proposed framework consists of two resilience types: internal resilience and external resilience with resilience policies and resilience sub-policies. Several qualitative resilience studies have addressed critical infrastructure applications. Sterbenz et al. [67] presented a framework for resilience and survivability of communication networks and also a survey that includes the resilience disciplines. The results of their study indicate that six factors of defend, detect, diagnose, remediate, refine, and recover contribute to designing resilient networks that can be extended to other domains. This framework provides only a conceptual insight and does not quantify system resilience. In similar work, Vlacheas et al. [68] identified properties of resilience in the scope of telecommunication networks. They found that reliability, safety, availability, confidentiality, integrity, maintainability, and performance, along with their interactions, are most influential properties of networks resilience. Bruyelle et al. [69] suggested some technological solutions and behavior management to improve the resilience of mass transportation systems in the case of bomb attacks. Patterson et al. [70] proposed three key factors for achieving resilience in medication delivery: (i) advanced information visualization techniques, (ii) scenario-based design and evaluation of treatments, and (iii) teamwork during the elicitation of requirements. Vugrin et al. [71] introduced resilience as function of absorptive capacity, adaptive capacity, and restorative capacity. Absorptive capacity is the degree to which a system is able to absorb shocks posed from disruption, implying to preparedness activities, adaptive capacity is the degree to which a system is able to adapt itself temporarily to new disrupted conditions, and restorative capacities is the degree to which a system is able to restore itself if adaptive capacity is not effective. Note that adaptive and restorative capacities refer to recovery activities. This definition of resilience capacity accounts for both preparedness and recovery. A feature of their research that is distinct from others is about introducing resilience cost index (RCI), which is composed of two elements: loss costs posed by disruptive events, and recovery costs. Shirali et al. [72] distilled the main barriers for achieving resilience in a chemical plant, indicating that the main barriers to achieving resilience are: (i) a shortage of experience about resilience engineering, (ii) intangibility of resilience engineering level, (iii) choosing production over safety, (iv) lack of reporting system, (v) religious beliefs, and (vi) out-of-date procedures and manual, poor feedback loop, and economic problems. Shirali et al. [73] found seven indicators of resilience from safety culture perspective which includes: schedule delays, safety committees, meeting effectiveness, safety education, worker’s involvement, competence, safety training. The authors also found that resilience measurements is dependent on four managerial factors including centralization or decentralization control systems, management of change, risk management and accident analysis, management commitment to safety and resilience.

Ainuddin and Routary [74] proposed a community resilience framework for an earthquake prone area in Baluchistan of Pakistan, constructed from a household survey that was conducted among 200 residences. The proposed framework comprises the following: (i) identifying hazard/disaster characteristics, (ii) determining individual/community vulnerability, (iii) risk reception and
awareness preparedness, and (iv) finally improving social (educational, health coverage), economic (housing capital, employment), and physical (shelter, housing age) resources. Other conceptual framework for analyzing resilience, as well as some guiding principles and characteristics of resilient systems, can be found in [75-80].

3.2 Semi-quantitative Indices
The semi-quantitative index approach is usually constructed with a set of questions designed to assess different resilience-based system characteristics (e.g., redundancy, resourcefulness) on a Likert (0 to 10) or percentage scale (0 to 100). Assessments of the characteristics from expert opinion are aggregated in some way to produce an index of resilience. For example, Cutter et al. [81] fist identified 36 resilience variables of communities to natural disasters, including redundancy, resourcefulness, and robustness. Each variable was then scored between 0 and 100 according to the data observation from a government source. These 36 variables were grouped into five sub-indices, including economic, infrastructure, social, community capital, and institutional. The score for each sub-index was calculated using an unweighted average of each variable, and the total score was calculated by taking unweighted average of all sub-index scores. Pettit et al. [82] distilled the two key drivers of resilience in an industrial supply chain: (i) level of the supply chain’s vulnerability, and (ii) capability of the supply chain to withstand and recover from disruption. The authors measured vulnerability and capability of supply chains by providing a set of 152 questions divided into six sections of vulnerability and 15 sections of capability. The importance of each factor was weighted by policymakers, and finally the responses to the questions were calculated using the weighted sum approach. Shirali et al. [83] used semi-quantitative approach to assess resilience engineering in a process industry, introducing six critical process industry resilience indicators: (i) top management, (ii) commitment, (iii) learning culture, (iv) awareness, (v) preparedness, and (vi) flexibility. Data related to these six indicators were collected from 11 units of a process industry using a survey, and the data were analyzed and scored using principal component analysis approach.

4. QUANTITATIVE ASSESSMENT APPROACHES
This section describes several quantitative resilience assessment approaches that serve as the focus of this review.

4.1. General Measures
General resilience measures provide a quantitative means to assess resilience by measuring performance of system, regardless of the structure of system. These measures are comparable across different system contexts with similar underlying logic. As we have defined them, generic resilience metrics determine resilience by comparing the performance of system before and after disruption without concentrating on system-specific characteristics (though modeling performance may require understanding underlying system behavior). We broadly characterize these general measures as deterministic and stochastic, each of which have been used to describe static and dynamic system behavior.

- **Deterministic vs. probabilistic:** A deterministic performance-approach does not incorporate uncertainty (e.g., probability of disruption) into the metric, while probabilistic performance-based approach captures the stochasticity associated with system behavior.
• Dynamic vs. static: A dynamic performance-based approach accounts for time-dependent behavior, while a static performance-based approach is free of time dependent measures of resilience.

4.1.1. Deterministic Approaches

Bruneau et al. [84] defined four dimensions for resilience in the well-known resilience triangle model in civil infrastructure: (i) robustness, the strength of system, or its ability to prevent damage propagation through the system in the presence of disruptive event, (ii) rapidity, the speed or rate at which a system could return to its original state or at least an acceptable level of functionality after the occurrence of disruption, (iii) resourcefulness, the level of capability in applying material (i.e., information, technological, physical) and human resources (i.e., labor) to respond to a disruptive event, and (iv) redundancy, the extent to which carries by a system to minimize the likelihood and impact of disruption.

Bruneau et al. [84] then proposed a deterministic static metric for measuring the resilience loss of a community to an earthquake with Eq. (1). The time at which the disruption occurs is \( t_0 \), and the time at which the community returns to its normal pre-disruption state is \( t_1 \). The quality of the community infrastructure at time \( t \), which could represent several different kinds of performance measures, is denoted with \( Q(t) \).

\[
RL = \int_{t_0}^{t_1} [100 - Q(t)] dt
\]  

In this approach, the quality of degraded infrastructure is compared to the as-planned infrastructure quality (100) during the recovery period. \( RL \) can be illustrated as the shaded area in Fig. 5. Larger \( RL \) values indicate lower resilience while smaller \( RL \) imply higher resilience. The privilege of this method is its general applicability. Although this approach is utilized for the context of earthquake; however it can be extended to many systems as quality is a general concept. As such, its general applicability is an important advantage of the resilience triangle metric. The proposed metric by Bruneau et al. [84] assumes that the quality of community infrastructure is at 100% before earthquake, perhaps an unrealistic assumption. Some issues with the resilience triangle [85] include that the area associated with \( RL \) could be a difficult measure for decision makers to comprehend even when given as a percentage and that the disruptive event has an assumed instantaneous impact and the recovery efforts begin immediately.
The resilience triangle paradigm has been applied in several contexts [86,87], as well as by Zobel [88], whose proposed metric is specified by “calculating the percentage of the total possible loss over some suitably long time interval $T^*$” as shown in Eq. (2). Parameters include, $X \in [0,1]$ as the percentage of functionality lost after a disruption, $T \in [0, T^*]$ as the time required for full recovery, and $T^*$ as a suitably long time interval over which lost functionality is determined. Zobel, who recognized that the same level of resilience found from the resilience triangle could be found from different combinations of $X$ and $T$, provided a visualization of the tradeoffs between lost functionality and recovery time for the same level of “resilience.”

$R(X, T) = \frac{T^* - XT/2}{T^*} = 1 - \frac{XT}{2T^*}$ \hspace{0.5cm} (2)

From Fig. 6, it can be seen that the total possible loss can be calculated as triangular area ($XT/2$) for a single disruptive event. Zobel and Khansa [89] then extended the metric in Eq. (2) to measure the onset of and partial recovery from multiple, sequential disruptive events. An advantage of this proposed metric is its simplicity, though its linear recovery may not be realistic for some systems and events. Further, the conceptual illustration of resilience represented in Fig. 6 suggests that the degradation of performance after a disruptive event is immediate, which may be true for some systems, though some systems may see a more gradual decrease over time (e.g., a manufacturing plant that maintains inventory as preparedness mechanism). This is also true for the conceptual illustration of resilience presented in Fig. 5 by Bruneau et al. [84].
Rose [44] defined economic resilience as “the ability of an entity or system to maintain system functionally when a disruption occurs.” This metric measures the ratio of the avoided drop in system output and the maximum possible drop in system output, as shown in Fig. 7. The proposed metric, provided in Eq. (3), is classified as a deterministic static model where $\% \Delta Y$ is the difference in non-disrupted and expected disrupted system performance and $\% \Delta Y^{\max}$ is the difference in non-disrupted and worst case disrupted system performance. It may be difficult to estimate the expected degraded performance level, especially for unknown disruptions, because depth, width, and intensity of unknown disruptions might not be precisely estimable. Cox et al. [90] used a similar metric to calculate the resilience of London’s transportation system, where the worst case disruption referred to the maximum reduction in passenger journeys for the attacked transportation modes.

$$R = \frac{\% \Delta Y^{\max} - \% \Delta Y}{\% \Delta Y^{\max}}$$  

(3)

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**Fig. 6.** A reinterpretation of the resilience triangle (adapted from [88]).

**Fig. 7.** Static economic resilience quantification ([44]).
Rose [44] also considered the time-dependent aspects of recovery in the definition of dynamic resilience. Dynamic resilience can be obtained by hastening repair and reconstructing capital stock, such that investment becomes an important input in the dynamic resilience formula in Eq. (4). The measure, $DR$, is a function of $SO_{HR}$, the output of the system under hastened recovery, and $SO_{WR}$, the system’s output without hastened recovery, where $t_i$ is the $i$th time step during recovery and $N$ is the number of time steps considered. $DR$ is depicted graphically in Fig. 8.

$$DR = \sum_{i=1}^{N} SO_{HR}(t_i) - SO_{WR}(t_i)$$  \hspace{1cm} (4)

Although the computation of dynamic resilience proposed by Rose [44] is relatively simple, however it does not quantify the resilience value within range of 0 and 1. In contrast to the dynamic resilience proposed by Rose [44], economic resilience proposed by the same author is bounded between 0 and 1, which provides a convenient understanding of system resiliency, especially for intermediate resilience values (0.2 to 0.8).

Henry and Ramirez-Marquez [91] developed a time-dependent resilience metric that quantifies resilience as ratio of recovery to loss. Given that the performance of the system at a point in time is measured with performance function $q(t)$, three system states that are important in quantifying resilience are represented in Fig. 9: (i) the stable original state which represents normal functionally of a system before disruption occurs, starts from time $t_0$ and ends by time $t_e$, (ii) the disrupted state, which is brought about by a disruptive event $(e^j)$ at time $t_e$ whose effects set in until time $t_d$, describes the performance of the system from time $t_d$ to $t_s$, (iii) the stable recovered state which refers to the new steady state performance level once the recovery action initiated at time $t_s$ is over. Important dimensions of resilience that are depicted in Fig.9. include
reliability, or the ability of the system to maintain typical operation prior to a disruption, vulnerability, or the ability of the system to stave off initial impacts after event $e^j$, and recoverability, or the ability of the system to recover in a timely manner from $e^j$. The time-dependent measure of resilience is defined in Eq. (5), noting that resilient behavior is a function of $e^j$. Notation $R(t|e^j)$ was adopted by Whitson and Ramirez-Marquez [81], as $R$ is commonly reserved for reliability.

$$R(t|e^j) = \frac{\varphi(t|e^j) - \varphi(t_d|e^j)}{\varphi(t_0) - \varphi(t_d|e^j)} \quad (5)$$

As it explained above, the numerator of this metric implies recovery up to time $t$, while the denominator refers to the total loss due to disruption $e^j$. The authors also calculated the total cost of recovered system followed by disruption as sum of implementing cost for resilience action and loss cost incurred due to system’s non-operability due to disruption. Several subsequent developments in the context of resilience measurement and planning [92-95] are based on the system state transition represented in Fig. 9 and the metric in Eq. (5) by Henry and Ramirez-Marquez [91].

![Fig. 9. System performance and state transition to describe resilience (adapted from Henry and Ramirez-Marquez [91]).](image)

Wang et al. [96] proposed a metric to measure the resilience of enterprise information systems, defined in Eq. (6), where $m$ is the number of operations in the enterprise information system, $d_i$ is the demand time for the recovery of operation $i$ ($i = 1, 2, ..., m$), $c_i$ is the completion time of operation $i$, and $z_i$ is the weight given to the importance of operation $i$.

$$R = \max \sum_{i=1}^{m} z_i \frac{d_i}{c_i} \quad (6)$$
This resilience measure can take on values greater than 1 when all operations can be recovered within demand time, and a larger value of this metric implies a more resilient system. The major limitation of this metric is that the number of recovery actions and number of operations are assumed to be known, while in reality systems are dealing with unknown situations.

Omer et al. [97] proposed a resilience metric for infrastructure networks, calculated as the ratio of the closeness centrality of the network before and after disruption respectively. The closeness centrality is determined based on the accessibility of a node to the rest of the network. This resilience metric gives a value between 0 and 1, where the larger value is more desirable.

Chen and Miller-Hooks [98] introduced an indicator for measuring resilience in transportation networks. The resilience indicator, represented in Eq. (7), quantifies the post-disruption expected fraction of demand that, for a given network, can be satisfied within pre-determined recovery budgets. Parameter $d_w$ quantifies the maximum demand that can be satisfied for origin-destination (O-D) pair $w$ following a disruption, and $D_w$ is demand that can be satisfied for O-D pair $w$ prior to the disruption. A limitation of this formulation includes its lack of specificity of the contribution of pre-disaster and post-disaster recovery activities, specifically in accounting for recovery time.

$$\text{Resilience} = \frac{E \left( \sum_{w \in W} d_w / \sum_{w \in W} D_w \right)}{\sum_{w \in W} D_w} = \frac{1}{\sum_{w \in W} D_w} E \left( \sum_{w \in W} d_w \right)$$

(7)

Janic [99] used the proposed indicator by Chen and Miller-Hooks [88] for assessing airport resilience, defined as a ratio between the on-time flights and the total number of planned flights.

Orwin and Wardle [100] introduced a measurement metric by linking resilience with instantaneous and maximum disturbance as shown in Eq. (8). $E_{\text{max}}$ refers the maximum intensity of absorbable force without perturbing the system’s function, and $E_j$ refers to the magnitude of the disturbance’s effect on safety at time $T_j$. The instantaneous resilience at time $T_j$ can take on values between 0 and 1, where the value 1 indicates the maximum system resilience. The maximum resilience can be obtained when the disturbance’s impact is fully recovered ($E_j = 0$). A disadvantage of this metric is that it does not consider time to recover and, much like the resilience triangle, could return the same resilience value for two systems with different recovery times.

$$\text{Resilience} = \left( \frac{2 \times |E_{\text{max}}|}{|E_{\text{max}}| + |E_j|} \right) - 1$$

(8)

Enjalbert et al. [101] introduced local and global resilience assessment metrics, found in Eqs. (9) and (10), respectively, to model the resilience of public transportation systems from a safety management perspective. Function $S(t)$ is a safety indicator of the system, measured as the “sum of effect of factors which can affect the system safety” [101]. Local resilience measures instantaneous resilience based on the safety indicator, and global resilience is obtained by integrating of local resilience over time, between when the disturbance effect commences
(represented by \( t_b \)) and end time of disturbance effect (represented by \( t_e \)). Ouedraogo et al. [102] expands the application of these local and global resilience metrics to air transportation systems.

**Local resilience**

\[
\text{Local resilience} = \frac{dS(t)}{dt}
\]  

(9)

**Global resilience**

\[
\text{Global resilience} = \int_{t_b}^{t_e} \text{local resilience} = \int_{t_b}^{t_e} \frac{dS(t)}{dt}
\]  

(10)

Francis and Bekera [103] proposed a dynamic resilience metric. \( \rho_t \) for event \( i \), shown in Eq. (11). \( S_p \) refers to the speed of recovery, \( F_o \) is the performance level of the system at its original state, \( F_r \) is the performance level at a new stable level after recovery efforts, and \( F_d \) is the performance level immediately following the disruption. Speed of recovery in Eq. (12) assumes exponential growth, with \( t_e \) representing slack time or the maximum amount of time post-disaster that is acceptable before recovery ensues, \( t_r \) representing the time to final recovery or time to reach a new equilibrium state, \( t_r^* \) representing the time to complete the initial recovery actions, and \( \alpha \) representing the parameter controlling the “decay” in resilience until the new equilibrium is met. This metric describes the absorptive capacity in terms of the proportion of original steady-state functionality maintained after the new steady-state functionality, \( F_r/F_o \). It is notable that this metric is not constrained on \([0, 1]\), thereby making extreme values difficult to comprehend, and further the exponential growth function governing the improvement in resilience may not always represent system behavior. The relationship between absorptive capacity and adaptive capacity is a bit unclear. The authors suggest ratio \( F_d/F_o \) represents the capability of the system to absorb shocks without recovery action, and \( F_r/F_o \) represents adaptive capacity which relates to those post-disaster activities taken after the disruption. Adaptive capacity might be more effectively formulated to reflect the ability of the system to recover the lost performance level not initially absorbed by absorptive capacity. That is, for the adaptive capacity ratio, the recovered performance level \( F_r \) could be compared with the difference between the initial performance level \( F_o \) and the performance level after disruption \( F_d \), 

\[
\frac{F_r}{F_o - F_d}.
\]  

(11)

\[
\rho_t = S_p \frac{F_r}{F_o} \frac{F_d}{F_o}
\]

\[
S_p = \begin{cases} 
(t_s/t_r^*) \exp[-\alpha(t_r - t_r^*)] & \text{for } t_r \geq t_r^* \\
(t_s/t_r^*) & \text{otherwise}
\end{cases}
\]  

(12)

Cimellaro et al. [104] expressed resilience in terms of quality of service, as shown in Eq. (13), where \( \alpha \) is a weighting factor representing the importance of pre- and post-disruption service qualities, \( Q_1(t) \) and \( Q_2(t) \) are the quality service of the system before and after the disruption, respectively, and \( T_{LC} \) is the control time of the system. The authors applied this metric to measure healthcare resilience, using waiting time that a patient spends in the queue for treatment as an index of service quality. The resilience value obtained in Eq. (13) is highly dependent upon the value of the weighting factor. Therefore, different resilience values can be attained due to differences in decision maker preferences. Although the authors introduced four properties of
resilience including rapidity, robustness, redundancy, and resourcefulness, these properties were not explicitly included in the resilience metric.

\[ R = \alpha \int_{T_{LC}}^{T_{LC}} \frac{Q_1(t)}{T_{LC}} dt + (1 - \alpha) \int_{T_{LC}}^{T_{LC}} \frac{Q_2(t)}{T_{LC}} dt \]  

(13)

4.1.2. Probabilistic Approaches

Chang and Shinozuka [105] introduced a probabilistic approach for assessing resilience, measured with two elements: (i) loss of performance and (ii) length of recovery. Resilience is defined as the probability of the initial system performance loss after a disruption being less than the maximum acceptable performance loss and the time to full recovery being less than the maximum acceptable disruption time. This measure is represented in Eq. (14), where \( A \) represents the set of performance standards for maximum acceptable loss of system performance, \( r^* \), and maximum acceptable recovery time, \( t^* \), for a disruption of magnitude \( i \).

\[ R = P(A|i) = P(r_0 < r^* \text{ and } t_1 < t^*) \]  

(14)

A series of disruption simulations were performed, and the probabilities in Eq. (14) were generated from the proportion of simulation runs not meeting the standards defined by \( A \). Although Chang and Shinozuka [105] applied this approach to measure the resilience of infrastructure and communities following an earthquake, it can be generally applied to any other systems and disruptions. The distinguishing feature of the proposed metric is its acknowledgement of uncertainty in quantification of resilience. However, the proposed metric does not consider an extra penalty when both performance loss and length of recovery exceed their maximum acceptable values.

Ouyang et al. [106] developed a stochastic time-dependent metric for measuring “annual resilience” under multi-hazard events, shown in Eq. (15). Their primary metric measures the mean ratio of the area between the actual performance curve, \( P(t) \), and the time axis to the area between the target performance curve, \( TP(t) \), and the time axis over a length of time \( T \) (considered to be a year by the authors). \( AR \) is a stochastic metric as \( P(t) \) is modeled as a stochastic process, and \( TP(t) \) can be represented as a stochastic process or some deterministic function. Multiple hazards can be included with the \( \sum_{n=1}^{N(t)} AIA_n(t_n) \) term, where \( n \) refers to the \( n \)th event, \( N(T) \) is the total number of events that occur during \( T \), \( t_n \) is a random variable describing the time at which the \( n \)th event occurs, and \( AIA_n(t_n) \) is the area between \( P(t) \) and \( TP(t) \) for the \( n \)th event. The authors considered different types of disruptions, making the approach more applicable for real world applications. Further, uncertainty is incorporated by modeling target performance curve as stochastic process.

\[ AR = E \left[ \frac{\int_0^T P(t)dt}{\int_0^T TP(t)dt} \right] = E \left[ \frac{\int_0^T TP(t)dt - \sum_{n=1}^{N(t)} AIA_n(t_n)}{\int_0^T TP(t)dt} \right] \]  

(15)
Youn et al. [14] considered both mitigation and contingency strategies to define their resilience metric. The metric, provided in Eq. (16), is defined as the sum of the passive survival rate (reliability) and proactive survival rate (restoration) following a disruption.

\[ \Psi \text{ (resilience)} = R \text{ (reliability)} + \rho \text{ (restoration)} \]  

(16)

In Eq. (16), restoration is defined to be the degree of reliability recovery and is calculated as the joint probability of a system failure event, \( E_{sf} \), a correct diagnosis event, \( E_{cd} \), a correct prognosis event, \( E_{cp} \), and a successful recovery action event, \( E_{mr} \) [14]. The formulation for restoration is provided in Eq. (17).

\[ \rho = P(E_{mr}|E_{cp}E_{cd}E_{sf}) \times P(E_{cp}|E_{cd}E_{sf}) \times P(E_{cd}|E_{sf}) \times P(E_{sf}) \]  

(17)

In contrast to the most of studies reviewed in this paper (e.g., [91,96]), the metric in Eq. (16) accounts for reliability, or a preventive means to stave off the occurrence of a disruption as a component in quantifying resilience, while most other resilience assessment metrics are primarily a function of the level of initial impact and duration of recovery. It is noteworthy to point out that this metric is bounded on [0,1]. \( \Psi \) takes on the value 0 when the restoration activity does not occur or otherwise fails, and takes on the value 1, its upper bound, when the system is completely restored. The advantage of resilience formula given in Eq. (16) is the consideration of both pre-disaster and post-disaster activities. As shown in Eq. (17), restoration is not time-dependent, therefore does not consider the length of restoration. Due to its inclusion of reliability, such a metric is perhaps more suitable for measuring the resilience of engineering systems because reliability can more effectively be calculated for engineering systems through failure testing studies. The calculation of conditional probability may be difficult, especially when a disruption occurs for the first time. Any errors in estimating the conditional probability by expert knowledge can result in a mischaracterization of restoration, and consequently, resilience.

Ayyub [107] defined a stochastic resilience metric in terms that also considered the effects of aging on the system. The system’s performance is defined as the difference between the system’s strength and system’s load. Robustness and resourcefulness are considered as two dimensions of resilience in this metric. The metric is shown in Eq. (18), where \( T_i \) is the time to incident, \( T_f \) is the time to failure, \( T_r \) is the time to recovery, \( \Delta T_f = T_f - T_i \) is the duration of failure, and \( \Delta T_r = T_r - T_f \) is the duration of recovery.

\[ R_e = \frac{T_i + F \Delta T_f + R \Delta T_r}{T_i + \Delta T_f + \Delta T_r} \]  

(18)

The failure profile, \( F \), in Eq. (18) is a measure of robustness and redundancy, calculated using Eq. (19). Ayyub offers several trajectories of \( f \) for brittle, ductile, and graceful failures. Similarly, the recovery profile, \( R \), measures recoverability with Eq. (20), with several example trajectories of \( r \) depending on the convexity (e.g., “as good as old”) or concavity (e.g., “as good as new”) recovery. The plot of system performance in [107] is similar to that of Fig. 9, but offers
(i) explicit vulnerability and recoverability trajectories with specific meanings and (ii) specifically incorporates the effects of aging in its graphical representation.

\[ F = \frac{\int_{t_i}^{t_f} f \, dt}{\int_{t_i}^{t_f} Q \, dt} \]  

(19)

\[ R = \frac{\int_{t_f}^{t_f} r \, dt}{\int_{t_f}^{t_f} Q \, dt} \]  

(20)

Note that the time to failure \( T_f \) is characterized by its probability density function which is the negative of the derivative of reliability function. This metric by Ayyub [107] is among the most comprehensive resilience measures, prescribing both mitigation (reliability) and contingency (recovery duration) strategies. Ayub [107] modeled the ratio of robustness to redundancy and the ratio of resourcefulness to rapidity by introducing failure profile \( F \) and recovery profile \( R \), respectively.

Hashimoto et al. [108] defined the resilience of a system as conditional probability of a satisfactory (i.e., non-failure) state in time period \( t + 1 \) given an unsatisfactory state in time period \( t \), shown in Eq. (21). \( S(t) \) is the state of the system at time \( t \), and \( NF \) and \( F \) represent non-failure and failure states, respectively.

\[ R = P\{S(t + 1) \in NF|S(t) \in F\} \]  

(21)

Franchin and Cavalieri [109] introduced a probabilistic metric for assessing infrastructure resilience in the presence of earthquake. Their definition of resilience is based on the efficiency of the spatial distribution of an infrastructure network. The efficiency of two nodes in an infrastructure network is defined as being inversely proportional to their shortest distance. The resilience metric is provided in Eq. (22), where \( P_D \) is the fraction of displaced population, \( E_0 \) is the efficiency of the city network before the earthquake, \( P_r \) is the measure of progress of recovery, and \( E(P_r) \) is the recovery curve of the fraction of the displaced population. In their study, the efficiency of a city road network is measured in terms of population density.

\[ R = \frac{1}{P_D E_0} \int_0^{P_D} E(P_r) \, dP_r \]  

(22)

The metric in Eq. (22) is probabilistic due to the stochastic nature of \( P_D \). This resilience metric is restricted between zero and one, since normalization is performed with \( P_D \) and \( E_0 \). Although the authors used this metric for assessing resilience of road city network, it is applicable to the other infrastructures such as electric power and water supply networks, assuming a suitable function for efficiency exists. An time-dependent extension could model \( P_D \) using a dynamic simulation model.
Pant et al. [93] introduced three stochastic resilience metrics to implement the resilience formulation in Eq. (5). *Time to Total System Restoration* measures the total time spent from the point of time when recovery activities commence to the time that all recovery activities are finalized. From recovery point of view, this metric refers to the man-hours require to repair the disrupted component individually. In their work a set of recovery activities are defined based on order of importance, assuming that the recovery order and probability distributions for the components recoveries are known. The second resilience measure, *Time to Full System Service Resilience*, measures the total time spent from the point of time when recovery starts to the time that system service is fully restored. Finally, *Time to α×100%-Resilience*: it measures the total time spent from the point of time when recovery commences until the time that $α×100\%$ of system functionality (e.g., capacity, inventory) is restored.

Attoh-Okine et al. [110] quantified the value of system resilience using belief functions or Dampster-Shafer theory, a generalization of the Bayesian theory of subjective probability that uses imprecise probabilities. The discrete belief functions was used to calculate the resilience of system which is beneficial for the systems with high degree of interdependencies like those connecting infrastructure systems.

Barker et al. [92] proposed two stochastic resilience-based component importance measures (CIMs) for identifying the primary contributors to network resilience, also based on the resilience formulation in Eq. (5). The modeling of these two metrics is devoted to vulnerability and recoverability in a network following a disruption. The first CIM metric, analogous to the risk reduction worth importance measure in the reliability engineering field, quantifies the proportion of restoration time attributed to each network component. The second resilience-based CIM, similar to the reliability achievement worth importance measure, quantifies how network resilience is improved if a specific network component is invulnerable. The authors then concluded that the network resilience can be obtained in the form of two ways: vulnerability reduction strategy or accelerating the speed of recovery activities through evaluating CIM metrics.

4.2. Structural-based Models

The structural-based approaches examine how the structure of a system impacts its resilience. System behavior must be observed and characteristics of a system must be modeled or simulated. We characterize structural-based models into three kinds of approaches: optimization models, simulation models, and fuzzy logic models.

4.2.1. Optimization Models

Faturechi et al. [111] proposed a mathematical model for evaluating and optimizing airport resilience, aiming to maximize the resilience of an airport’s runway and taxiway network. The main strategy used in their mathematical model is the quick restoration of post-event take-off and landing capacities to the level of capacities before disruption by taking into account time, physical, operational, space, resource, and budget restrictions. Two types of decision variables, including pre-event and post-event decisions, were considered. The main feature of their work is that preparedness and recovery activities are taken into account in the stochastic integer model.
Faturechi and Miller-Hooks [112] introduced a multi-objective, three-stage stochastic mathematical model to quantify and optimize travel time resilience in road networks. The three stages of decision process include: (i) pre-event mitigation, (ii) preparedness, and (iii) post-event response. The resilience of the road network is defined as network’s ability to withstand and adapt to a disruption, with travel time used to assess resilience. The objective function of their model seeks to maximize the expectation of road network resilience over all possible disruption scenarios and minimize the total travel time simultaneously.

Azadeh et al. [113] investigated the concept of resilience engineering in a petrochemical plant using data envelopment analysis (DEA), a linear programming method for measuring the efficiency of multiple decision-making units (DMUs) when production process consists of multiple input and outputs. The authors first introduced ten indicators of resilience contributed into petrochemical plant including management commitment, reporting, learning, awareness, preparedness, flexibility, self-organization, teamwork, redundancy, and fault-tolerance. In their petrochemical plant study, eleven departments such as chemical operation, information technology, maintenance, and polymer operation are considered and denoted as DMUs. Finally, DEA is utilized to measure the efficiency of the petrochemical plant’s departments based on ten introduced indicators.

Jin et al. [114] developed a two-stage stochastic programming model for analyzing a metropolitan public transportation network’s resilience. The authors defined the resilience of the network as the fraction of travel demand that can be satisfied by the disrupted network after occurrence of disruptive event. The proposed mathematical model generates alternative paths under disruptive conditions.

Baroud et al. [94] quantified vulnerability and recoverability of waterway network using the two stochastic resilience-based component importance measures (CIM) introduced by Barker et al. [92]. The links of waterway network were ranked with respect to their importance calculated by two CIM indicators. The waterway links were prioritized using a multicriteria comparison technique to generate a stochastic order. The authors did not take into account the impact of cascading of a disruptive event through the waterway network.

Cardoso et al. [115] proposed a mixed integer linear model to design both forward and closed-loop supply chains. The proposed model takes into account two situations: (i) when the disruption occurs with certainty, and (ii) when there is a probability associated with the occurrence of disruption. Six indicators are considered into the model for designing a resilience network including flow and node complexity, node and density criticality, customer service level and investment.

Khaled et al. [116] proposed a mathematical model and solution approach for evaluating critical railroad infrastructures to maximize rail network resilience. Identifying critical components can enable stakeholders to prioritize protection initiatives or add necessary redundancy that will maximize rail network resilience during a disruptive event. In this paper, the criticality of an infrastructure element is evaluated based on the increased delay incurred when that element is disrupted. They developed a system wide optimization model and heuristic solution approach for making-up and routing of trains in a disruptive situation considering the congestion effects and
capacity restrictions to minimize the overall transportation time. The authors conducted a case study for major Class-I railroad network of USA based on publicly available data. The mathematical model considers individual component (links and nodes) disruptions separately to determine the impact, where considering multiple component disruptions simultaneously might be more meaningful as a disruptive event may realistically impact multiple adjacent components.

Vugrin et al. [117] proposed a multi-objective optimization model for transportation network recovery, where resilience is defined by the optimal recovery of disrupted links. The model consists of two levels: (i) a lower-level problem that involves solving for network flows, and (ii) an upper-level problem that identifies the optimal recovery sequences and modes. The proposed model embedded only recovery actions, including the level of recovery for a disrupted link, but not preparedness actions.

Ash and Newth [118] attempted to optimize complex large-scale networks for resilience against cascading failures. Cascading failures are very common in power transmission, communication, and transportation networks. Cascading failures usually triggers by failing a node of network due to overloading and its effect nonlinearly propagate through network that eventually may results in network shutdown. Many efforts have been put to study the behavior of cascading failure in complex interdependent networks like [119-124]. Ash and Newth [118] first modeled cascading failures and then developed failure resilient networks based the notion of network topology indices including common neighbors, modularity, and assortativeness.

Alderson et al. [125] proposed a mixed integer non-linear programming (MINLP) to quantify the operational resilience of critical infrastructures. Resilience is defined in terms of defense strategies with little attention given to the important recovery dimension of resilience found in most works. Their proposed model aims to find out the best defense strategy in the case of attacks such that the total cost of the defense strategy is minimized. The concentration of MINLP model is on preparedness actions, but not on recovery.

Sahebjamnia et al. [87] proposed a multi-objective mixed integer linear programming (MOMILP) to find efficient resource allocation patterns among candidate business continuity and disaster recovery plans while considering features of organizational resilience. The objective of proposed model is to minimize the total loss of operating level of key products, as well as to minimize the total recovery time of key products.

With respect to supply chains. Li and Zhao [126] developed a model for assessing supply chain resilience with a series relationship among the supply chain components along with self-adaptive and self-recovery abilities. Mari et al. [116] proposed a resilient supply chain by minimizing the expected disruption costs through considering disruption probability of suppliers, manufacturers, and warehouses.

4.2.2. Simulation Models

Albores and Shaw [127] proposed a discrete event simulation model to evaluate the preparedness of a fire and rescue service department in the presence of terrorist attacks. The authors considered preparedness as key driving factor of pre-event disruption resilience. Two simulation models were: (i) the first model mimics the mass decontamination of a population following a
terrorist attack, while (ii) the second model deals with the harmonization of resource allocation across regions.

Carvalho et al. [128] applied discrete event simulation to assess the resilience of a supply chain. Two strategies of flexibility and redundancy are taken into account as elements of resilience in their simulation model. Redundancy is modeled by keeping additional inventory to successfully withstand disruptions, and flexibility is modeled by restricting the extent of the disrupted transportation system. Six different scenarios are investigated with the simulation model. There are several limitations for this research study. First, the results found in this research may not be universally applicable across different sectors, as a redundancy strategy may not be a cost-efficient solution in comparison to a flexibility strategy due to high inventory holding costs, or vice versa.

Virginia et al. [129] proposed a dynamic simulation approach for simulating supply chain resilience. The authors considered readiness (preparedness), responsiveness, and recovery as key elements of resilience. The Integral of Time Absolute Error (ITAE), used commonly in the control engineering field, is employed as measure of resilience. The simulation model attempts to capture the minimum value of ITAE which is corresponding to the best response and recovery with lowest deviation from the target level following by disruption.

Jain and Bhunya [130] used Monte Carlo Simulation to study the resilience of a water storage reservoir, calculated using conditional probability introduced by Hashimoto et al. [108]. The behavior of a reservoir’s resilience is investigated under diverse adversary scenarios.

With respect to critical infrastructure networks, Adjetey-Bahun et al. [131] used a time-dependent simulation model to measure the resilience indicators of a railway transportation system. A set of disruptive events are modeled through simulation model with consequences of increase of travel time and reduction of train capacity. Sterbenz et al. [132] proposed an approach based on integrating analytical simulation, topology generation, and experimental emulation to improve the resilience and survivability of Internet networks. The resilience of the Internet network is defined as the ability of the network to provide a desired service level when it is challenged by large-scale disasters or intense failures.

4.2.3. Fuzzy Logic Models
Aleksic et al. [133] proposed a fuzzy model for assessing organizational resilience. Fuzzy linguistic variables were used to express the relative importance of the organizational resilience factors.

Azadeh et al. [134] assessed the factors of engineering resilience through a fuzzy cognitive map (FCM). The authors used a FCM to describe the causal reasoning between nine factors of engineering resilience: teamwork, awareness, preparedness, learning culture, reporting, flexibility, redundancy, management commitment, and fault tolerance. A FCM can be represented by a fuzzy graph structure and obtained as a result of neural network and fuzzy logic approaches.
Muller et al. [135] presented a fuzzy architecture for assessing the resilience of critical infrastructure. Redundancy and adaptability were considered to be the primary components of infrastructure resilience. The redundancy and adaptability inputs of the fuzzy architecture, and the resilience output, are expressed using linguistic variables.

Tadic et al. [136] integrated fuzzy forms of the Analytic Hierarchy Process (AHP) and the multicriteria discrete comparison technique TOPSIS for evaluating and ranking organizational resilience based on qualitative assessments. The approach was used to rank several resilience factors including (i) planning strategies, (ii) capability and capacity of internal resources, (iii) internal situation monitoring and reporting, (iv) human factors, (v) quality, (vi) external situation monitoring and reporting, (vii) capability and capacity of external resources, (viii) design factors, (ix) detection potential, and (x) emergency response.

5. RESEARCH DIRECTIONS

Based on the literature review presented in this paper, as well as recent reports and calls for proposals by US funding agencies, we identify a few on-going and upcoming research directions that are of interest to the resilience community.

5.1. Planning for Resilience

We offer a review of a number of measures for quantifying resilience in this paper. But their usefulness is limited unless they can guide planning for resilience.

Highlighted throughout this review, resilience generally focuses on the ability of a process or system to withstand a disruptive event and to recover from it. Both of these components of resilience (e.g., referred to as robustness and rapidity by Bruneau et al. [84], referred to as the complement of vulnerability and as recoverability [91]) are the result of planning and resource allocation. Understanding the tradeoffs among resources available and the resilience achieved through investments in vulnerability reduction and recoverability enhancement enables planning for resilience. Optimization models can be used to model the vulnerability and recoverability of disrupted systems. With respect to network vulnerability, the network interdiction literature provides a formulation for optimal allocation of resources toward network disruption [144]. The recovery problem can be viewed and modeled in similar to project scheduling problem, with the primary difference being that the completion of a subset of repair tasks has value because the performance of infrastructure network can be partially restored [137]. The integration of interdiction approaches and subsequent recovery scheduling approaches may provide a means to optimally allocate resources to building resilience with the tradeoff between vulnerability reduction and recoverability enhancement in mind.

5.2. Resilient Interdependent Processes and Systems

Recognizing the interdependence among infrastructure systems is vital for planning for their operation [138]. There exist highly coupled relationships among transportation, electric power, and telecommunication systems, among other infrastructures. And the resilience of one system can impact the resilience of others. More work is needed from translating a wealth of interdependent infrastructure models [139] to the study of their interdependent resilience [e.g., 140].
5.3. Standards for Resilient Systems
The National Institute of Standards and Technology (NIST) is a federal agency charged with developing and applying measurements and standards for industry practice. NIST has recently promoted the identification of existing standards and guidelines that can be implemented to enhance resilience in the built environment, as well as develop new standards and guidelines to fill remaining gaps. Emphasis is given to performance goals for buildings and infrastructure networks and systems under disruptions, including their recovery [141].

5.4. Community Resilience and the Built Environment
An ultimate measure of the performance of infrastructure networks and other systems is how they enable and enhance daily life. And when such networks and systems are disrupted, how does their resilience impact the resilience of the community that relies on them? The relationship between communities and the built environment is a budding area of research with momentum provided by NIST [142], which is linking the standards for the built environment to their ultimate benefit to the community. Measuring community resilience can come from several perspectives, including human movement, community connectivity, and economic productivity [143].

6. CONCLUDING REMARKS
Over the past decade, the significance of the concept of resilience has been well recognized among researchers and practitioners. Effort has been devoted to measure the resilience of engineering systems, but challenges still exist. The objective of this paper is to provide a taxonomy and review of approaches to quantify system resilience. We first classified four domains for definitions of resilience: organizational, social, economic, and engineering. Across these domains, the traditional definitions of resilience concentrate on the inherent ability of systems to absorb of the effects of a disruption to their performance, referring to preparedness activities, and more recent definitions also account for the recovery of their performance.

We classify the quantification of resilience into two broad classes: qualitative and quantitative approaches. Qualitative assessment approaches include conceptual frameworks and semi-quantitative methods. Conceptual frameworks provide insights about the notion of resilience but do not provide a quantitative value. Semi-quantitative generally involve the aggregation of expert opinion along multiple dimensions into an index. The quantitative assessment category is characterized as either general resilience measures or structural models. General resilience measures generally assess resilience by comparing the performance of a system before and after disruption. Some measures are static in nature [84], while others offer a time-dependent perspective on system performance [91]. A recent trend in resilience measures has been accounting for aleatoric and epistemic uncertainty with stochastic approaches. Structural based approaches emphasize the structure or characteristics of a particular system to derive a measure of its resilience. Work reviewed in this paper is summarized in Table 2.

The term “resilience” is increasingly used in research journals, government documents, and the media, but work still remains on making resilience assessment usable. Methods for resilience planning are still a relatively unexplored area, including tangible resource allocation models,
tradeoffs among the dimensions of resilience, the relationship between community resilience and the resilience of the built environment, and data-driven standards ensuring resilience.

REFERENCES


[60] www.webofscience.com


Table 2. Classification of the literature in modeling and planning for resilience.

<table>
<thead>
<tr>
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A REVIEW OF DEFINITIONS AND MEASURES OF SYSTEM RESILIENCE

Highlights

- A comprehensive review of definitions and measures of system resilience
- Focus given to resilience in engineering systems is provided
- Nearly 150 articles across several domains are reviewed
- Future directions in resilience research are discussed