



The effects of online social networks on tacit knowledge transmission

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HIGHLIGHTS

- We propose a tacit knowledge transmission model on networks with even mixing.
- Two routes of tacit knowledge transmission are considered.
- We derive the threshold that governs whether or not a kind of tacit knowledge can be shared.
- The degree distribution of the users' contact network has an important impact.

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ABSTRACT

Due to the popular use of online social networks in today's world, how to propagate employees' tacit knowledge via online social networks has attracted managers' attention, which is critical to enhance the competitiveness of firms. In this paper, we propose a tacit knowledge transmission model on networks with even mixing based on the propagation property of tacit knowledge and the application of online social networks. We consider two routes of transmission, which are contact through online social networks and face-to-face physical contact, and derive the threshold that governs whether or not a kind of tacit knowledge can be shared in an organization with few initial employees who have acquired it. The impact of the degree distribution of the users' contact network on the transmission is investigated analytically. Some numerical simulations are presented to support the theoretical results. We perform the sensitivity analysis of the threshold in terms of the propagation parameters and confirm that online social networks contribute significantly to enhancing the transmission of tacit knowledge among employees.

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1. Introduction

Knowledge transmission is critical to firms' success in today's highly competitive environment [1]. Based on Polanyi's conceptualization, Nonaka suggested that knowledge can be classified as explicit and tacit [2,3]. Tacit knowledge, as opposed to formal or explicit knowledge, refers to a category of knowledge that is difficult to transfer to another person by means of writing it down or verbalizing it, however, it is significant in humans' knowledge system and the core resource in humans' brains that dominates the behavior [4]. A new transmission route of tacit knowledge applying online social networks (OSNs) to communicate with one another emerged and has gradually become dominant in these decades, different from the traditional one, through face-to-face physical contact and communication. Online Social Networks have incrementally

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become the most popular applications such as Facebook, Twitter, Google+ and MySpace, which enable employees to share information, opinions, and knowledge simultaneously with a large number of peers. It allows rapid communication and interaction among employees as the distance barriers have been minimized [5]. The development of online social networks plays an essential role in the emergence of knowledge transmission activities in today's Internet world, and the knowledge is comprised of explicit and some tacit knowledge, as it provides a platform for people to do activities such as posting questions and answers, discussions, messaging, story-telling as well as sharing experiences [6].

Complex networks are widely applied to describe the features of complex systems in the real world, including social, biological, and communication systems [7,8]. And numerous studies focused on social network and its application have been published over the last years, due to the increasing development of human society. This interdisciplinary research field has attracted much attention of scientific communities from diverse disciplines, such as statistical physics, information science, sociology, and complexity science [9]. The new insights are contributed to the understanding the dynamics of information propagation such as knowledge transmission, which provides valuable support for leaders' management. Accordingly, this paper intends to study the effects of online social networks on employees' tacit knowledge transmission. The previous studies aimed at the propagation of tacit knowledge seem to be limited in analytic and empirical research, which analyzes and summarizes the existing phenomena, and predicts the future one [10,11]. Some researchers discuss this matter from the perspective of theoretical research, while this study is done mainly under the guidance of reductionism [12,13]. As a matter of fact, the spread of tacit knowledge between employees in an organization is a typical and complicated system. Therefore, this study does some analysis applying the method of transmission dynamics from the perspective of theoretical and quantitative research.

2. Model

Numerous studies on knowledge management (KM) have proven that the transmission of useful knowledge between employees enhances the performance of firms such as their absorptive capacity and innovation capability. Tacit knowledge carries a higher value since it is concerned with direct contact and the observation of employees' behaviors and related to more complex ways of acquiring knowledge from others. Thus, tacit knowledge is more difficult than explicit knowledge to spread among employees [14]. In recent years, studying the propagation of tacit knowledge has become much more feasible due to the wider availability of a variety of databases, fast and cheap computing power and efficient search and model-fitting algorithms. There are a number of ways in which the spread of tacit knowledge can be tracked. In terms of transmission dynamics, the similarities between the transmission of tacit knowledge and that of infectious diseases cannot be ignored. In the study of the characteristics of tacit knowledge and its propagation, we find that the spread of tacit knowledge has something in common with that of infectious diseases because of their similar transmission property, which is achieved through the direct contact between individuals. In the spread of a disease through a population, contact between an infectious and a susceptible individual can lead to the transmission of infection. Similarly, individuals or groups who have acquired a kind of tacit knowledge can motivate other individuals or groups to learn it through contact and communication.

Nowadays, the employees using online social networks to communicate share tacit knowledge with other users via online social networks and through face-to-face physical contact, and they also share tacit knowledge with the ones who do not use online social networks to communicate through face-to-face physical contact. While the employees who do not use online social networks to communicate share tacit knowledge with others only through face-to-face physical contact. We consider the employees of an organization using online social networks to communicate with one another and their contacts as an undirected network, each user in the organization can be regarded as a node in the network, and each contact through online social networks between two users is represented as an edge connecting their nodes. The number of edges emanating from a node, that is, the number of contacts a user has, is called the degree of the node. With the further integration of the global economy, enterprises have changed so fundamentally that managers pay more attention to the sharing of some key knowledge and its maximized use [15]. Therefore, we consider that the tacit knowledge being discussed requires to be mastered by all the employees in the organization. We classify the employees of an organization as the employees without a kind of tacit knowledge (S) and the ones who have acquired it (T). We all know that the employees who have acquired a kind of tacit knowledge may forget it after a period of time and become the ones without it. Furthermore, the employees who have forgotten it may acquire it once again through communication with others. Similarly, the employees without it can acquire it through communication with others and become the ones with it. It is approximately the same with the interconversion between the infectious and susceptible individuals in the spread of most infectious diseases. The classic SIS models described by dynamical systems have been extensively studied to understand the mechanisms of the transmission of these infectious diseases. Kiss et al. presented an SIR model of some infectious diseases transmission on networks with even mixing and considered multiple routes of transmission. They analytically found that these combined transmission mechanisms have a major impact on the final epidemic size [16]. Enlightened by his work, we propose an STS model of tacit knowledge propagation on networks with even mixing and consider two routes of transmission, which are through online social networks and face-to-face physical contact. Suppose $S_0(t)$ and $T_0(t)$ are the numbers of the employees who do not use online social networks to communicate without this tacit knowledge and with it at time t . Suppose $S_k(t)$ and $T_k(t)$ ($k = 1, 2, \dots, n$) are the number of the nodes of degree k without this tacit knowledge and with it at time t . Let N be the total number of employees in the organization and $N = \sum_{k=0}^n N_k(t)$, where $N_k(t) = S_k(t) + T_k(t)$. Nowadays, most

enterprises have a stable scale. Thus, we consider that the total number remains constant. The degree distribution of the network is given by $p(k)$, which is time-independent. The value $\langle k \rangle = \sum_k kp(k)$ is the average number of contacts per node.

In the transmission of a kind of tacit knowledge among employees, face-to-face physical contact between an employee who has acquired it and an employee without it can lead to its spread. And this contact is considered to be random. Meanwhile, network contact between a user who has acquired it and a user without it can also lead to its spread. We define a transmission rate β_0 to be the number of adequate contacts through face-to-face physical contact per individual per unit time. It indicates the ability of an employee who has acquired a kind of tacit knowledge to spread it to others through face-to-face physical contact and reflects such factors as the capacity for action of the employees with it in reality, their tacit knowledge sharing intentions and the externalization level of it. Thus, $\beta_0 S_0(t) \sum_{k=0}^n T_k(t)/N$ is the number of newly-born employees with this tacit knowledge through face-to-face physical contact at time t in unit time. We consider that β_1 is the transmission rate across a network contact between a user who has acquired it and a user without it. $p(l|k)$ is the probability that a node of degree k is contacted to a node of degree l , and $\sum_l p(l|k) = 1$. Therefore, $k \sum_{l=1}^n p(l|k) T_l(t)/N_l(t)$ is the number of the nodes with this tacit knowledge who are contacted by a node of degree k without it at time t . Then, $\beta_1 k S_k(t) \sum_{l=1}^n p(l|k) T_l(t)/N_l(t)$ is the number of the nodes of degree k without this knowledge, to whom it is spread by all the nodes with it through online social networks at time t in unit time. The employees who have acquired it forget it and become the ones without it at rate γ . Accordingly, the dynamic model can be written as follows, where $k = 1, 2, \dots, n$.

$$\begin{cases} \frac{dS_0(t)}{dt} = -\beta_0 \frac{S_0(t)}{N} \sum_{l=0}^n T_l(t) + \gamma T_0(t), \\ \frac{dT_0(t)}{dt} = \beta_0 \frac{S_0(t)}{N} \sum_{l=0}^n T_l(t) - \gamma T_0(t), \\ \frac{dS_k(t)}{dt} = -\beta_0 \frac{S_k(t)}{N} \sum_{l=0}^n T_l(t) - \beta_1 k S_k(t) \sum_{l=1}^n p(l|k) \frac{T_l(t)}{N_l(t)} + \gamma T_k(t), \\ \frac{dT_k(t)}{dt} = \beta_0 \frac{S_k(t)}{N} \sum_{l=0}^n T_l(t) + \beta_1 k S_k(t) \sum_{l=1}^n p(l|k) \frac{T_l(t)}{N_l(t)} - \gamma T_k(t). \end{cases} \tag{1}$$

Assume the degree of the nodes in the network is uncorrelated, then $p(l|k) = lp(l)/\langle k \rangle$. Let $x_k(t) = S_k(t)/N, y_k(t) = T_k(t)/N$, where $k = 1, 2, \dots, n$. Then (1) is rephrased as

$$\begin{cases} \frac{dy_0(t)}{dt} = \beta_0 [p_0 - y_0(t)] \sum_{k=0}^n y_k(t) - \gamma y_0(t), \\ \frac{dy_k(t)}{dt} = \beta_0 [p_k - y_k(t)] \sum_{l=0}^n y_l(t) + \beta_1 \frac{k}{\langle k \rangle} [p_k - y_k(t)] \sum_{l=1}^n l y_l(t) - \gamma y_k(t). \end{cases} \tag{2}$$

We calculated the expression of the threshold R_0 applying the method of the next generation matrix [17]. Moreover, the equilibrium without this knowledge of the system (2) is $E_0 = (0, 0, \dots, 0, x_0^0, x_1^0, \dots, x_k^0, \dots, x_n^0)^t$, where $x_k^0 = p_k, k = 0, 1, \dots, n$. By calculating, we know $R_0 = \rho(FV^{-1})$, where $\rho(A)$ denotes the spectral radius of a matrix A . We can obtain,

$$F = \begin{bmatrix} \beta_0 p_0 & \beta_0 p_0 & \beta_0 p_0 & \cdots & \beta_0 p_0 & \cdots & \beta_0 p_0 \\ \beta_0 p_1 & \beta_0 p_1 + \frac{\beta_1 p_1}{\langle k \rangle} & \beta_0 p_1 + \frac{2\beta_1 p_1}{\langle k \rangle} & \cdots & \beta_0 p_1 + \frac{k\beta_1 p_1}{\langle k \rangle} & \cdots & \beta_0 p_1 + \frac{n\beta_1 p_1}{\langle k \rangle} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_0 p_k & \beta_0 p_k + \frac{k\beta_1 p_k}{\langle k \rangle} & \beta_0 p_k + \frac{2k\beta_1 p_k}{\langle k \rangle} & \cdots & \beta_0 p_k + \frac{k^2\beta_1 p_k}{\langle k \rangle} & \cdots & \beta_0 p_k + \frac{kn\beta_1 p_k}{\langle k \rangle} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \beta_0 p_n & \beta_0 p_n + \frac{n\beta_1 p_n}{\langle k \rangle} & \beta_0 p_n + \frac{2n\beta_1 p_n}{\langle k \rangle} & \cdots & \beta_0 p_n + \frac{kn\beta_1 p_n}{\langle k \rangle} & \cdots & \beta_0 p_n + \frac{n^2\beta_1 p_n}{\langle k \rangle} \end{bmatrix},$$

$$V = \begin{bmatrix} \gamma & 0 & \cdots & 0 \\ 0 & \gamma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma \end{bmatrix}_{(n+1) \times (n+1)},$$

where $k = 1, 2, \dots, n$. Hence R_0 is given by

$$R_0 = \frac{1}{2\gamma} \left(\beta_0 + \beta_1 \frac{\langle k^2 \rangle}{\langle k \rangle} \right) + \frac{1}{2\gamma} \left[\left(\beta_0 - \beta_1 \frac{\langle k^2 \rangle}{\langle k \rangle} \right)^2 + 4\beta_0\beta_1\langle k \rangle \right]^{1/2}. \quad (3)$$

If $R_0 < 1$, then equilibrium E_0 of the system (2.2) is asymptotically stable. Otherwise the equilibrium E_0 of the system (2.2) is unstable. In summary, the value of $R_0 = 1$ is the threshold that governs whether or not a kind of tacit knowledge can be shared. If $R_0 > 1$, then this tacit knowledge can be shared by employees. If $R_0 < 1$, then it will gradually disappear in the organization.

If $\beta_0 = 0$, then $R_0 = \beta_1 \langle k^2 \rangle / \gamma \langle k \rangle$. If $\beta_1 = 0$, then $R_0 = \beta_0 / \gamma$. If the users' contact network has a scale-free degree distribution, $p(k) \propto k^{-\mu}$. If $\mu \leq 3$, then the second moment of the nodes' degree distribution diverges ($\langle k^2 \rangle \rightarrow \infty$) and according to Eq. (3) R_0 diverges too ($R_0 \rightarrow \infty$). Therefore, for sufficiently high heterogeneity, even infinitesimally small transmission rates can make a kind of tacit knowledge exist and spread among employees in an organization for ever. If $\mu > 3$, then the second moment of the degree distribution ($\langle k^2 \rangle$) is finite. Accordingly, the degree distribution of the users' contact network has an important impact on the transmission of tacit knowledge.

3. Numerical simulations and results

In this section, we first present some numerical simulations to support the above obtained theoretical results and choose as far as possible the parameter values in line with the actual situation in numerical simulations.

According to the rule of 150, managers must make sure the number of employees in an organization is less than 150 if the organization means to run efficiently and becomes the incubator of the propagation of knowledge and thoughts. 150 is a tipping point. If the number of employees is more than 150, many organizations will break up [18,19]. Accordingly, here we select 150 as the total number of employees in the organization, i.e., $N = 150$. In reality, some firms have a relatively small number of employees about 20 persons, yet, the results of this study can still apply to them. Suppose the number of nodes in the users' interactive network is 120, and the number of employees who do not use online social networks to communicate is 30. According to statistics, most knowledge is in the grasp of fewer people, and knowledge transmission activities are very uncommon. As a result, more than half of the knowledge assets, such as patents, are wasted because of not being shared fully, which is very common in small and medium sized firms [15]. Therefore, here we select 5 as the initial number of employees who have acquired this tacit knowledge. Assume four of the five persons are users and the others are not. Therefore, $S_0 = 29$, $T_0 = 1$, $\sum_{k=1}^n S_k = 116$, $\sum_{k=1}^n T_k = 4$. The number of employees selected above with this tacit knowledge and that of ones without it at the initial time vary in a reasonable range, which is independent of the results.

In some degree, the structure determines the characteristics and function of networks [20]. If each user has the basically equal number of contacts in unit time, we suppose the users' interactive network follows Poisson degree distribution. The concept of channel capacity in cognitive psychology refers to the memory space of our brain when it accepts some kind of information. For instance, a user may communicate with many good friends in a day, but most of us pay attention to messages from only about seven of them. Our nervous system and acquired habits decide that we are subject to some limitations by birth, which keeps our channel capacity in a rough range, seven. And that is why telephone numbers have seven digits [21]. Accordingly, here we select 7 as the average number of contacts a user has, i.e. $\langle k \rangle = 7$. We fix $n = 50$. First, we choose $\beta_0 = 0.002$, $\beta_1 = 0.001$, $\gamma = 0.01$. Then $R_0 = 1.4658 > 1$. Fig. 1(a) shows that the total number of the employees who have acquired it changes with time. The number of employees with it increases gradually with time, and tends to be stable at about 115, which shows that this tacit knowledge exists and spreads in the organization for ever. Next, we take $\beta_0 = 0.0005$, $\beta_1 = 0.0004$, $\gamma = 0.01$. Then $R_0 = 0.5583 < 1$. Fig. 1(b) shows that the number of the employees with it decreases with time gradually to 0, which means it will disappear in the organization. Numerical simulations confirm the validity of the theoretical results. All the values of the parameters selected above vary in a reasonable range, which are independent of the results.

Usually social networks as well as the social and activity graphs of online social networks are scale-free, i.e., obey power-law degree distribution [22]. If many users have a very small number of contacts and fewest users have exceedingly large number of contacts (i.e., hubs). Now, we can assume that the network follows power-law degree distribution, and the probability takes the form $p(k) \sim k^{-\mu} / \sum_{k=1}^n k^{-\mu}$, where $\mu = 3.1$. We fix $n = 80$. First, we choose $\beta_0 = 0.005$, $\beta_1 = 0.003$, $\gamma = 0.01$. Then $R_0 = 1.2201 > 1$. Fig. 1(c) shows that the number of the employees who have acquired it changes with time. The number of employees with it increases gradually with time, and tends to be stable at about 98, which shows that this tacit knowledge exists and spreads in the organization for ever. Next, we take $\beta_0 = 0.002$, $\beta_1 = 0.001$, $\gamma = 0.01$. Then $R_0 = 0.4303 < 1$. Fig. 1(d) shows that the number of the employees with it decreases gradually to 0. It shows this tacit knowledge will disappear in the organization. Obviously, the numerical simulations confirm the validity of the theoretical results.

In summary, first, it is obvious that for values of $R_0 > 1$ this tacit knowledge exists and spreads in the organization for ever. Whereas, it will disappear in the organization if $R_0 < 1$. We can visually see that R_0 increases as transmission parameters (β_0 and β_1) increase. It indicates that the managers should enhance employees' ability of spreading tacit knowledge via online social networks and through face-to-face physical contact. Second, the findings show that the degree

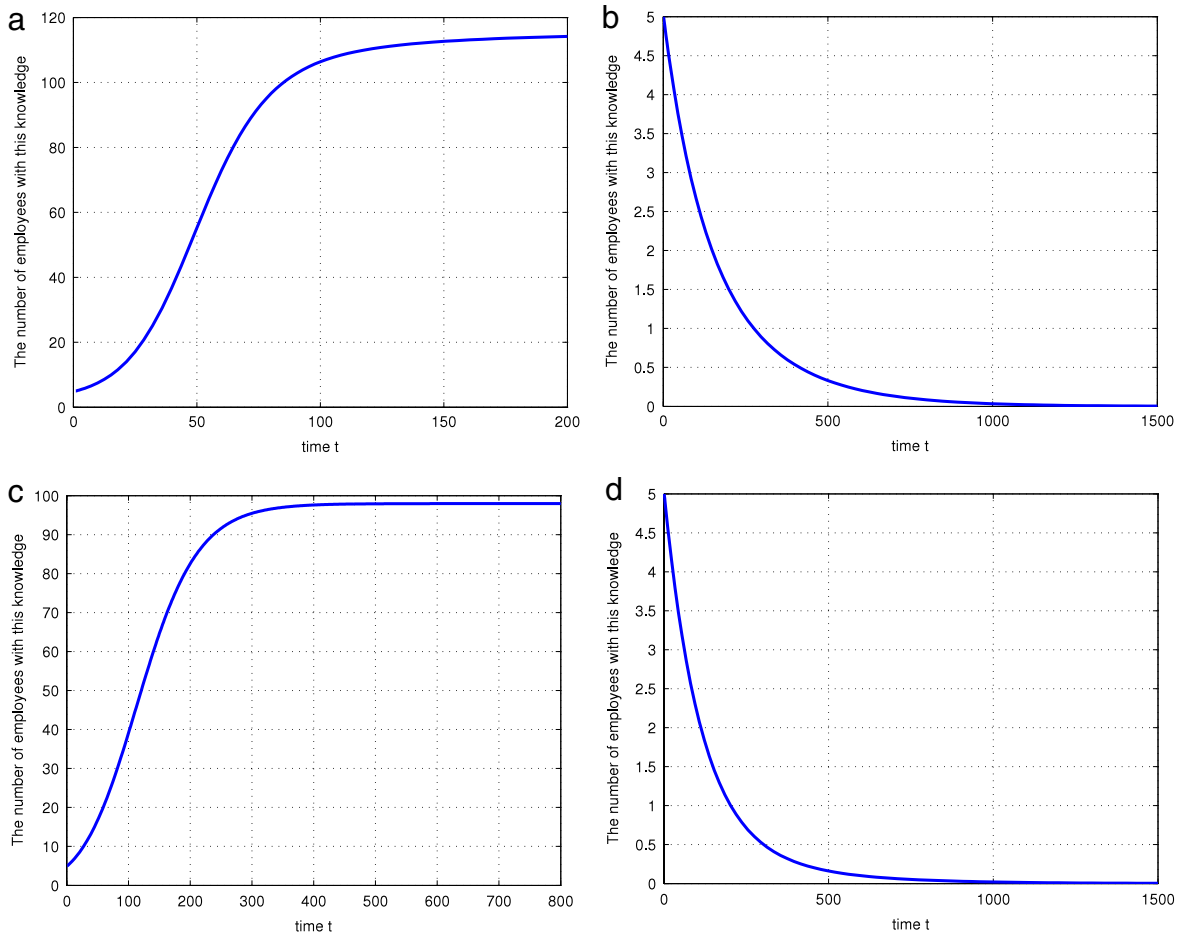


Fig. 1. The total number of employees with this tacit knowledge changes over time t with parameters $\beta_0 = 0.002$, $\beta_1 = 0.001$, $\gamma = 0.01$ ((a) and (d)), $\beta_0 = 0.0005$, $\beta_1 = 0.0004$, $\gamma = 0.01$ (b), and $\beta_0 = 0.005$, $\beta_1 = 0.003$, $\gamma = 0.01$ (c) respectively.

distribution of the users' interactive network has an impact on employees' tacit knowledge transmission and the final size of employees who have acquired it.

Finally, we perform some sensitivity analysis of the threshold R_0 in terms of the transmission parameters in order to further illustrate the effects of online social networks on the transmission of tacit knowledge among employees. Fig. 2 gives R_0 on parameters β_0 and β_1 , respectively. We can visually see that R_0 increases as each transmission parameter (β_0 or β_1) increases, while the influence of β_1 is greater on R_0 than β_0 . In Fig. 2, the other parameter values are the same as those in Fig. 1(c) except β_0 and β_1 . We consider that the users' interactive network obeys power-law degree distribution, and the probability takes the form $p(k) \sim k^{-\mu} / \sum_{k=1}^n k^{-\mu}$, where $\mu = 3.1$. If we change the parameter values or the initial conditional values in a reasonable range, the conclusions of the sensitivity analysis are roughly the same. In view of this, the strategy of properly improving the employees' capacity of spreading tacit knowledge via online social networks is more effective than the strategy of that of spreading tacit knowledge through face-to-face physical contact, which is of great significance in management science.

4. Conclusions

As suggested by prior studies, the transmission of tacit knowledge among employees is subject to social interaction. This study explores the effects of online social networks on employees' tacit knowledge transmission from the perspective of theoretical and quantitative research. Based on the conclusions from the analytic and empirical studies, we propose a tacit knowledge transmission model on networks with even mixing, and consider two routes of transmission, which are contact through online social networks and face-to-face physical contact. We derive the threshold that governs whether or not a tacit knowledge can be shared by employees in an organization. This method describes the property of employees' tacit knowledge transmission more clearly to make managers acquire some crucial global information. In this paper, the propagation of tacit knowledge through face-to-face physical contact among employees is considered to be a simple

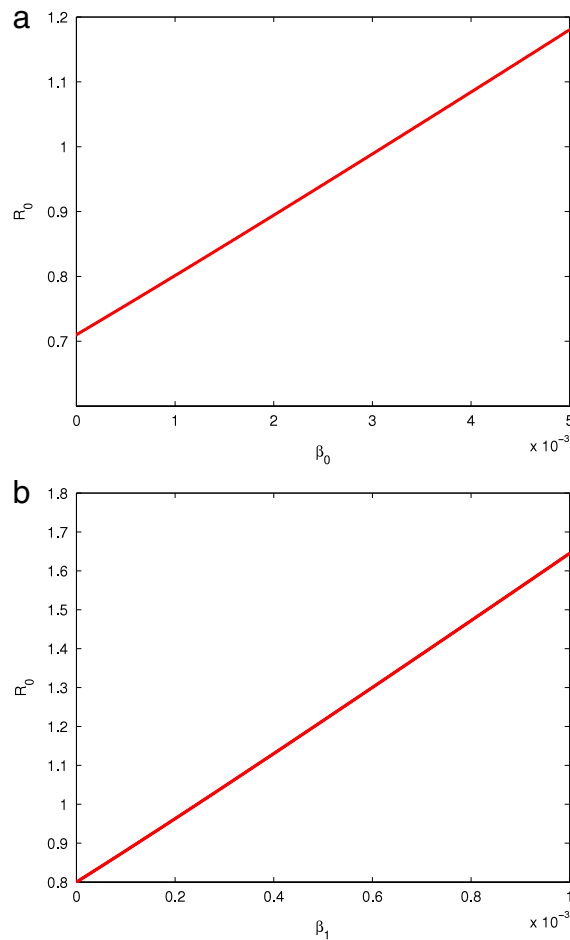


Fig. 2. (Color online) The threshold R_0 in terms of parameters β_0 and β_1 . Here $n = 80$, $\gamma = 0.01$, (a) $\beta_1 = 0.0008$; (b) $\beta_0 = 0.008$.

mean-field transmission, independent of the heterogeneity of individuals. We just considered the case of even mixing in this paper and the case of uneven mixing is more complicated, which we will do some further studies.

The findings show that the degree distribution of the users' interactive network has an important impact on employees' tacit knowledge transmission and the final size of employees who have acquired it. Furthermore, it is more efficient to promote employees' tacit knowledge transmission by means of improving the employees' capacity of spreading tacit knowledge through online social networks than that of spreading tacit knowledge through face-to-face physical contact. This conclusion is of great significance in knowledge management.

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