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Performance analysis of OFDM system with transmit antenna selection using delayed feedback

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ABSTRACT

We consider orthogonal frequency division multiplexing (OFDM) in a multiple input single output (MISO) system. In the presence of spatially uncorrelated time-varying frequency selective channel, we use subcarrier by subcarrier antenna selection using delayed feedback. We derive closed-form expressions for the pdf of the received SNR and BER for MQAM constellation. The expressions have been obtained as a function of the correlation (ρ) between perfect channel state information (CSI) and delayed CSI, where $0 \le \rho \le 1$. We have verified our analytical expressions by comparing them with simulation results. We have also reduced the BER expression for some special cases and compared them with the results available in the literature. We conclude that the diversity gain of the considered system is reduced to one for $\rho < 1$, i.e. not having perfect antenna selection for each subcarrier. However, we get some coding gain compared to single input single output system, the coding gain reduces with decreasing the correlation.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is used in wireless standards for high speed data transmission in frequency selective channels. Using OFDM, multiple data streams can be transmitted in parallel without inter symbol interference. Moreover, if each subchannel is narrow enough then the multipath fading can be characterized as flat fading. Thus, OFDM enhances spectral efficiency by increasing data rate, however it results in poor BER performance due to flat fading nature of effective channel. Therefore, spatial diversity with multiple antennas, popularly known as multiple input multiple output (MIMO) systems, is used to mitigate the effect of fading. Hence, the use of OFDM in MIMO systems has been proposed as an efficient solution to meet the current demand of high data rate with reliable communication in wireless standards such as LTE [1], IEEE 802.11 (WLAN) and IEEE 802.16 (WiMAX) [2].

However, combination of MIMO and OFDM has some inherent bottlenecks also. One of them is that MIMO systems require channel state information (CSI) at the transmitter for precoding or transmit beamforming [3]. In case of frequency division duplex (FDD), this required CSI can be conveyed to the transmitter by employing dedicated feedback channel. Unfortunately, in MIMO—OFDM systems, the amount of CSI data grows linearly with number of subcarriers and number of antennas, which makes this combination difficult in a practical wireless system. Therefore, various techniques have been proposed to reduce the feedback data in MIMO–OFDM systems. For example, [4] has exploited time and frequency correlations between subcarriers to reduce feedback for precoding and beamforming. In [5], opportunistic scheduling and beamforming schemes have been proposed in multi-user environment. In [6–9], finite rate transmit beamforming has been considered with limited feedback.

The other popular technique to reduce feedback data is transmit antenna selection. Performance analysis of MIMO systems with antenna selection is well documented in literature, a few of them are [10,11]. However, most of them have considered flat fading channels. The benefit of antenna selection in MIMO systems is that the diversity gain of reduced MIMO systems (with antenna selection) is the same as the diversity gain of full MIMO systems (without antenna selection). Moreover, in MIMO-OFDM systems, antenna selection reduces inter carrier interference also. In [12–16], subcarrier by subcarrier antenna selection using perfect CSI at the transmitter in MIMO–OFDM systems has been considered. However, since wireless channels are time varying and the feedback link introduces non zero delay, it is difficult to provide perfect CSI at the transmitter, even if we assume perfectly estimated CSI at the receiver and a noiseless feedback link. Therefore, in [17–19], performance analysis of different MIMO systems with antenna selection using delayed CSI at the transmitter has been done in flat fading channels.

In this paper, we consider subcarrier by subcarrier transmit antenna selection using delayed CSI in MISO–OFDM systems for frequency selective channel. We assume perfect CSI for all the subcarriers at the receiver and delayed feedback link. For MQAM constellation, we derive closed-form expressions for the pdf of

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received SNR and for the BER as a function of correlation (ρ) between perfect CSI and delayed CSI. We reduce the expression of BER for some special cases and compare them with the prevailing results in literature. We also present simulation results of the considered system and compare the analytical results with them. Moreover, we have also presented simulation results for spectral efficiency (R) of the considered system and shown the degradation in R due to decreasing the value of ρ.

The rest of the paper is organized as follows. Section 2 describes the channel and system models. In Section 3 we present the detailed performance analysis and some special cases have been discussed in Section 4. In Section 5, we present the results and the paper is concluded in Section 6.

Notations: Bold upper (lower) letters denote matrices (column vectors). The transpose, hermitian, absolute value, norm and expectation are denoted by $(\cdot)^T$, $(\cdot)^*$, $|\cdot|$, $\|\cdot\|$ and $E[\cdot]$ respectively. We use $\mathcal{Q}(\cdot)$ and $\mathcal{J}_0(\cdot)$ to denote the Gaussian Q-function and the zeroth order Bessel's function of the first kind respectively.

2. Channel and system models

We consider the wide sense stationary uncorrelated scattering model for the frequency selective mobile radio channel [21]. Mathematically, the baseband impulse response of the channel can be described by

$$h(t,\tau)=\sum_{l=1}^{L}\alpha_l(t)\delta(\tau-\tau_l), \tag{1}$$

where L, τ_l and $\alpha_l(t)$ denote total number of paths, delay of the lth path and the complex amplitude of the lth path respectively. Moreover, the $\alpha_l(t)$'s are wide sense stationary complex Gaussian processes and independent for different paths with $E[|\alpha_l(t)|^2]=q_l^2$. The channel is normalized such that

$$\sum_{l=1}^{L}q_l^2=1.$$

It is further assumed that all the paths have the same normalized correlation function $R(\tau_d)$ in time domain. Therefore,

$$E[\alpha_l^*(t)\alpha_l(t+\tau_d)]=q_l^2R(\tau_d). \tag{2}$$

Let B and K be the total bandwidth of the system and total number of subchannels respectively. It means subchannel spacing is $\Delta f = B/K$ and an OFDM block length is $T = 1/\Delta f = K/B$.

We assume that each subchannel is narrow enough so that each experiences flat fading. Let T_s be the sampling interval, then the frequency response $h[n, k]$ of the kth subcarrier in the nth OFDM block can be expressed as

$$h[n,k]=\sum_{l=1}^{L}\alpha_l(nT)e^{-(2\pi jk\tau_l)/(KT_s)}. \tag{3}$$

Then, $h[n, k]$ have the following two properties:

1 $h[n, k]$ is a complex Gaussian random variable with zero mean and unit variance for any n and k.
2 The correlation function between $h[n, k]$ and $h[n + \Delta n, k]$ is given by

$$R(\Delta nT)=E[h^*[n,k]h[n+\Delta n,k]]. \tag{4}$$

The input data stream is modulated by a MQAM modulator, resulting in a complex symbol stream. This symbol stream is passed through a serial to parallel (S/P) converter, which generates an OFDM block of K parallel MQAM symbols. The nth OFDM block is denoted as $\mathbf{x}[n]$, where $\mathbf{x}[n] = [x[n, 1]\,x[n, 2]\ldots x[n, K]]^T$. Each OFDM block is passed through IFFT and then the cyclic prefix of length L_c data symbols, where $L_c \geq L$, is added. Finally, it is passed through parallel to serial (P/S) converter. We consider a MISO system with N_t transmit antennas and frequency selective channel. We assume perfect CSI at the receiver for all the subcarriers corresponding to each of N_t antennas. Now, we select one antenna for one subcarrier based on maximization of received SNR for the subcarrier. Thus, at each instant one subcarrier is transmitted through only one antenna. This is known as subcarrier by subcarrier antenna selection scheme [12]. The indices of the selected antennas are sent back to the transmitter via a dedicated feedback link. However, in a scenario of time varying channel and nonzero delay in the feedback link, antenna selection can be carried out based on delayed CSI only. At the receiver, we remove cyclic prefix. The remaining stream is serial to parallel converted and passed through a K point FFT. The output of the FFT represents the nth received OFDM block as

$$\mathbf{y}[n]=\mathbf{h}_{\widehat{\mathrm{m}}}[n]\mathbf{x}[n]+\mathbf{w}[n], \tag{5}$$

where $\mathbf{w}[n] = [w[n, 1]\,w[n, 2]\,\ldots\,w[n, K]]$ and each entry denotes independent and identically distributed complex Gaussian additive random variable with mean zero and variance N_0. Further in (5),

$$\mathbf{h}_{\widehat{\mathbf{m}}}[n]=[h_{\widehat{m}_1}[n,1]\,h_{\widehat{m}_2}[n,2]\,\ldots\,h_{\widehat{m}_K}[n,K]], \tag{6}$$

where

$$\widehat{m}_k=\arg\max_{1\le u\le N_t}\{|\widehat{h}_u[n,k]|^2\},\quad 1\le k\le K. \tag{7}$$

In (7), $\widehat{h}_u[n, k]$ denotes delayed version of $h_u[n, k]$. Then, we convert symbols from parallel to serial form and pass through MQAM demodulator. Finally, the detection variable $\tilde{x}[n, k]$ corresponding to the kth subcarrier (or modulated symbol $x[n, k]$), using well known zero forcing principle, can be expressed as

$$\tilde{x}[n,k]=\frac{y[n,k]}{h_{\widehat{m}_k}[n,k]}.$$

In the next section, we will derive the expression of BER for MQAM.

3. Performance analysis

In this section, we derive the closed form expression of received SNR and then derive BER in the case of MQAM. For brevity, we avoid the indices n and k. Then, the received instantaneous SNR γ per symbol can be expressed as

$$\gamma=|h_{\widehat{m}}|^2\gamma_s, \tag{8}$$

where $\gamma_s = E_s/N_0$ and E_s denotes average symbol power for MQAM constellation. Let us denote the correlation between $h_{\widehat{m}}$ and $\widehat{h}_{\widehat{m}}$ as ρ, where $0 \leq \rho \leq 1$ or in general

$$E[\widehat{\boldsymbol{h}}^*_{\widehat{\boldsymbol{m}}}\boldsymbol{h}_{\widehat{\boldsymbol{m}}}]=\rho\mathbf{I}_K,$$

where $\mathbf{I}_K$ denotes a $K \times K$ identity matrix. Now, using order statistics [20], we can easily determine the pdf of $|\widehat{h}_{\widehat{m}}|^2$. However, it is difficult to determine the pdf that we need i.e. the pdf of $|h_{\widehat{m}}|^2$ in (8).

To solve this problem, we first determined the pdf of $|h_{\widehat{m}}|^2$ conditioned on $|\widehat{h}_{\widehat{m}}|^2$. Then using the pdf of $|\widehat{h}_{\widehat{m}}|^2$, we determine the pdf of $|h_{\widehat{m}}|^2$. The details of the approach used follow.

To express $|h_{\widehat{m}}|^2$ in terms of $|\widehat{h}_{\widehat{m}}|^2$, let us represent $h_{\widehat{m}}$ as a function of $\widehat{h}_{\widehat{m}}$ and an independent error term by using a Gauss–Markov process model as done in [11]

$$h_{\widehat{m}}=\rho\widehat{h}_{\widehat{m}}+\sqrt{1-\rho^2}\delta. \tag{9}$$

Since $h_{\widehat{m}} \sim \mathcal{CN}(0,1)$ and $\widehat{h}_{\widehat{m}} \sim \mathcal{CN}(0,1)$, we can easily show that $\delta \sim \mathcal{CN}(0,1)$ and δ is independent of $\widehat{h}_{\widehat{m}}$. Therefore, for a given $\widehat{h}_{\widehat{m}}$, we can write [21]

$$h_{\widehat{m}} \sim \mathcal{CN}(\rho \widehat{h}_{\widehat{m}}, 1-\rho^2). \tag{10}$$

Let us denote $|\widehat{h}_{\widehat{m}}|^2$ and $|h_{\widehat{m}}|^2$ as $\widehat{Z}$ and Z respectively. Now, take $\sqrt{\rho^2 \widehat{Z}}$ as A. Then, for a given A, the pdf of Z is noncentral chi square with two degrees of freedom which can be represented as

$$p_{Z|A}(z|a) = \frac{e^{-(z+a^2)/(1-\rho^2)}}{(1-\rho^2)} \mathcal{J}_0 \left(\frac{2i\sqrt{z}a}{1-\rho^2} \right). \tag{11}$$

Now, we require the pdf of A, which can easily be derived from the pdf of $\widehat{Z}$ using order statistics [20] as

$$\begin{aligned} p_{\widehat{Z}}(\widehat{z}) &= N_t(1-e^{-\widehat{z}})^{N_t-1} e^{-\widehat{z}}, \quad \widehat{z} \geq 0. \\ &= N_t \sum_{m=0}^{N_{t-1}} \binom{N_t-1}{m} (-1)^m e^{-(m+1)\widehat{z}}. \end{aligned}$$

Now, as $A = \sqrt{\rho^2 \widehat{Z}}$, the pdf of A can be expressed as

$$p_{A(a)} = \frac{2aN_t}{\rho^2} \sum_{m=0}^{N_{t-1}} \binom{N_t-1}{m} (-1)^m e^{-((m+1)a^2/\rho^2)}. \tag{12}$$

Now, $p_Z(z)$ can be determined by

$$p_Z(z) = \int_0^\infty p_{Z|A}(z|a) p_A(a) da. \tag{13}$$

Plugging (11) and (12) in (13) and solving the integral using equation (6.631.1) of [23], we get

$$p_Z(z) = N_t \sum_{m=0}^{N_{t-1}} \binom{N_t-1}{m} \frac{(-1)^m e^{-(m+1)z/(m+1-m\rho^2)}}{m+1-m\rho^2}. \tag{14}$$

For MQAM constellation with β bits/symbol, where $\beta = \log_2(M)$ and for a given channel h or z (where $z = |h|^2$), the instantaneous BER can be approximated by [22] as

$$P(\epsilon|z) = 0.2e^{-1.6z\gamma_s/(2^\beta-1)}. \tag{15}$$

Then, the average BER can be expressed as

$$P_e = \int_{z=0}^\infty P(\epsilon|z) p_Z(z) dz. \tag{16}$$

Now, substituting (14) and (15) in (16) and solving the integral

$$P_e = 0.2N_t \sum_{m=0}^{N_t-1} \binom{N_t-1}{m} \frac{(-1)^m (2^\beta-1)}{1.6\gamma_s(m+1-m\rho^2)+(m+1)(2^\beta-1)}. \tag{17}$$

Now, we determine spectral efficiency using adaptive modulation, where different modulation schemes are used for different subcarriers and the assignments vary over time. One way to choose the modulation schemes to achieve the target BER is to set the instantaneous BER $P_e(\epsilon|z)$ equal to P_{tar} [21]. Then, using (15), we get bits per symbol corresponding to channel power gain z as

$$\beta(z) = \log_2 \left[\frac{1.6\gamma_s z}{\ln(0.2/P_{tar})} + 1 \right] \tag{18}$$

Now, neglecting the guard interval (cyclic prefix), the average spectral efficiency (R) over all the subcarriers can be determined as [21]

$$\begin{aligned} R &= E_z[\beta(z)] \\ &= E_z \left[\log_2 \left(\frac{1.6\gamma_s z}{\ln(0.2/P_{tar})} \right) + 1 \right] \end{aligned} \tag{19}$$

4. Special cases

In this section, we reduce the expression of the BER, i.e. (17), for some special cases.

4.1. Case 1: $N_t = 1$

In this case, (17) can be reduced as

$$P_e = \frac{0.2(2^\beta-1)}{1.6\gamma_s + 2^\beta - 1} \tag{20}$$

This case is same as single input single output (SISO) [21], where all the subcarriers will be transmitted by one transmit antenna. Hence, ρ has no role to play.

4.2. Case 2: $\rho = 0$

In this case, (17) can be reduced as

$$\begin{aligned} P_e &= N_t \left\{ \sum_{m=0}^{N_t-1} \binom{N_t-1}{m} \frac{(-1)^m}{m+1} \right\} \left\{ \frac{0.2(2^\beta-1)}{1.6\gamma_s+2^\beta-1} \right\} \\ &= N_t \left\{ \int_{-1}^{0} (1+x)^{N_t-1} dx \right\} \left\{ \frac{0.2(2^\beta-1)}{1.6\gamma_s+2^\beta-1} \right\} \\ &= N_t \left\{ \frac{1}{N_t} \right\} \left\{ \frac{0.2(2^\beta-1)}{1.6\gamma_s+2^\beta-1} \right\} \\ &= \frac{0.2(2^\beta-1)}{1.6\gamma_s+2^\beta-1} \end{aligned} \tag{21}$$

This expression is same as (20), because this case is a case of random antenna selection. Therefore, no antenna selection gain can be achieved and this case is equivalent to SISO.

5. Results

In this section, the BER performance of the considered system has been evaluated using simulations over a frequency selective fading channel. We generate channels for $L = 10$ multipaths, where all the paths are independent and identically distributed as complex Gaussian with mean zero and variance of $1/L$. The OFDM system includes $K = 64$ subcarriers (K-point FFT) and cyclic prefix L_c of 16 subcarriers. As $L_c > L$, the channel will be ISI free. Moreover, we generate delayed CSI corresponding to each path using (2), where correlation $R(\tau_d) = \rho$, where $0 \leq \rho \leq 1$. Antenna selection for the kth subcarrier has been done using delayed CSI $\widehat{h}[n,k]$ using (7). Average SNR per bit is determined by $\gamma_s/\log_2(M)$, where $M = 2^\beta$. In the figures, Avg. SNR is per bit and in dB. Fig. 1 shows BER for different values of ρ such as 0.8, 0.95, 0.99 and 1, $N_t = 2$ transmit antennas and $M = 32$ QAM constellation. It can be seen that the simulation results are closely matching with their analytical counterparts. As ρ decreases, the degradation in performance can be observed due to non-selection of the best antenna for each subcarrier.

In Fig. 2, we have shown BER performance of $M = 32$ QAM system for different values of ρ such as 0.95, 0.999 and 1 for two transmit antennas. This is denoted by (2, 1 ; 1). We have also shown BER performance of 1×1 (diversity gain one) and 1×2 MRC (diversity gain

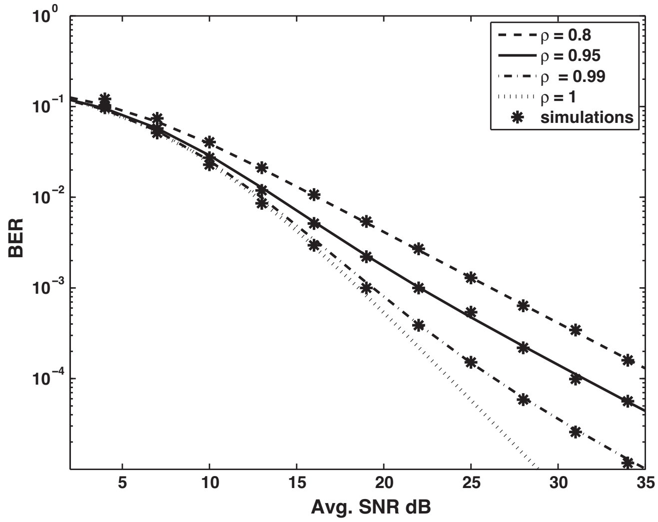

Fig. 1. BER vs. Avg. SNR for $N_t = 2$ and 32 QAM.

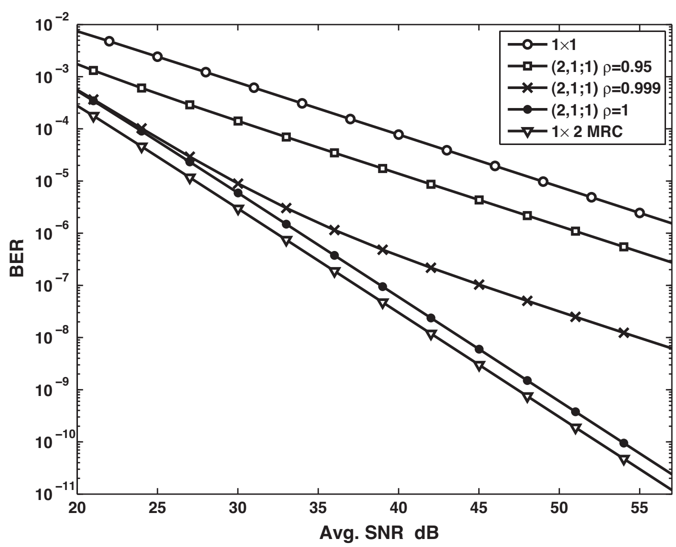

Fig. 2. BER vs. Avg. SNR.

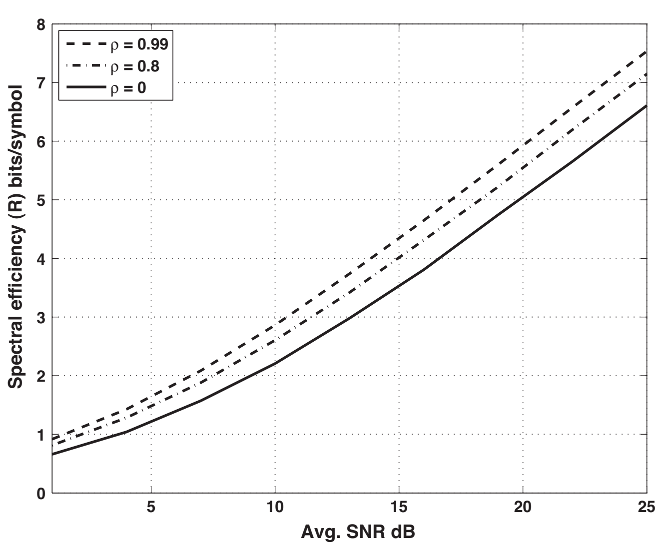

Fig. 3. Spectral efficiency R vs. Avg. SNR for $N_t = 2$.

two) systems. It can be seen that the diversity gain of the considered system is two for $\rho = 1$ but it is reduced to one for $\rho < 1$. However, we can get some coding gain with respect to 1×1 system, the coding gain reduces with decreasing ρ.

Fig. 3 shows spectral efficiency R vs. average SNR per symbol, i.e. γ_s, for $N_t = 2$ and target BER $P_t = 0.01$ using simulations according to (19) for different values of ρ like 0, 0.8 and 0.99. For an SNR of 15 dB, spectral efficiencies are 3.5, 4 and 4.3 bits/symbol corresponding to ρ of 0, 0.8 and 0.99 respectively. This observation is also in line with BER performance. As ρ decreases, R also decreases.

6. Conclusions

We considered subcarrier by subcarrier transmit antenna selection with delayed CSI in MISO–OFDM systems in frequency selective channel. We derived closed-form expressions for the pdf of received SNR and the BER, assuming MQAM constellation, as a function of correlation (ρ) between CSI at the receiver and (delayed) CSI at the transmitter. We have presented simulation results and found a close match with their analytical counterparts. We conclude that the diversity gain of the considered system is reduced to one, irrespective of multiple transmit antennas, while not selecting perfect antenna (i.e. $\rho < 1$) for each subcarrier. However, we have some coding gain with respect to SISO system for $\rho < 1$, the coding gain reduces with decreasing ρ. We have also discussed some special cases of the considered system and compared them with the prevailing results in the literature. Moreover, we have also presented simulation results for spectral efficiency (R) and we observed reduction in R with decreasing ρ.

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