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Tax evasion, tax corruption and stochastic growth

Fred Célimène [a], Gilles Dufrénot [b], Gisèle Mophou [c,*], Gaston N'Guérékata [d]

[a] Université des Antilles et de la Guyane, Laboratoire CEREGMIA. Universit é des Antilles et de La Guyane, Campus Schoelcher, B.P. 7209, 97275 Schoelcher Cédex (FWI), France
[b] Aix-Marseille Université (Aix-Marseille School of Economics) & CNRS & EHESS, Banque de France, CEPII, Château Lafarge, route des Milles, 13290 Aix-en-Provence Les Milles, France
[c] Université des Antilles et de la Guyane, Laboratoire CEREGMIA. Universit é des Antilles et de La Guyane, Campus Fouillole, 97159 Pointe-à-Pitre Guadeloupe (FWI), France
[d] Morgan State University, Baltimore, MD, USA

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ABSTRACT

This paper presents a continuous time stochastic growth model to study the effects of tax evasion and tax corruption on the level and volatility of private investment and public spending that are both factors of growth. The model highlights several channels through which the mean and volatility of these variables are affected. We first stress the role of equity markets, showing that the evasion outcome for the private sector is not necessarily viewed as a burden. Equity market performs here have the same role as a policy of tax exemption. In societies in which the share of private investment in percentage of GDP is growing, in which tax cheaters usually choose to shelter the proceeds of their illegal activities from the official financial institutions, and in which the productivity of public spending is often low, tax evasion and tax corruption may contribute to the development of private capital if people find an opportunity to invest the proceeds of their illegal activities in equity markets.

1. Introduction

This paper studies the impact of tax evasion and tax corruption on private investment and government spending, two key determinants of the growth rate and volatility of per-capita GDP. When the public sector is an essential contributor to the economic growth, stagnation and severe swings in economic growth are related to the deficient tax collection systems which do not allow providing the minimum amount of public goods and services necessary for productive activities like infrastructure, education, or investment (see Friedman et al., 2000). Many countries are still stuck in a vicious circle of both tax corruption and tax evasion, a phenomenon to which the theoretical and empirical literature has paid a great attention (see, among others, Mauro, 2004). According to the literature, corruption is an important factor contributing to growth volatility (see Denizer et al., 2010).

This paper suggests that when a government is unable to reduce the level of corruption and tax evasion, an alternative solution could be, either to allow the resources of the evaded tax to be invested in equities (by fostering the development of equity markets) or to raise the efficiency of public spending in order to attenuate the negative externalities of tax evasion on productive public expenditure. To develop these ideas, we use a standard portfolio argument by adopting an open economy stochastic growth model, in line with previous models like Turnovsky (1993), Grinols and Turnovsky (1993), Turnovsky (1999). Public goods and private investment are both productive inputs in the production function.

The uncertainty in our model is endogenous to the functioning of institutions. It comes from the fact that people hide income from the tax administration and offer bribes to inspectors. Cheating is a risky activity because there is a probability of being detected and a probability of being confronted to a corrupted inspector. The model considers tax evasion, private capital and public spending as endogenous variables and creates a loop between them.

We build upon the idea that tax evasion and tax corruption are non-separable when tax collection is performed by corruptible inspectors (see Hindrinks et al., 1999; Sanyal et al., 2000). However, our model differs from previous models on the same topic in several respects. Lin and Yang (2001) also consider a stochastic growth model of tax evasion, but with no specific role for corruption and no role for public spending as an input in the production function. Chen (2003) also considers a model of tax evasion with productive public capital. Unlike the author, we do not consider any optimizing behavior from the government side. Further, in our model tax evasion generates a source of uncertainty on production. Dzhumashev (2007) uses a framework like ours, but his model applies to a closed economy. In our case, opening the economy allows introducing wealth effects in the model. Considering a general CRRA utility function (with Constant Relative Risk Aversion), we show that the impact on capital accumulation of tax corruption and tax evasion depends upon a trade-off between the risk aversion and the saving behavior. Finally, Corquetti and Coppier (2011) address the issue of the effects of tax evasion and tax corruption on economic growth and they

* Corresponding author.
E-mail addresses: fred.celimene@martinique.univ-ag.fr (F. Célimène), gilles.dufrenot@univ-amu.fr (G. Dufrénot), gmophou@univ-ag.fr (G. Mophou).

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apply a game-theoretical approach to a Ramsey model. The authors focus on the strategic behaviors of consumers and bureaucrats and this issue is out of the scope of this paper.

The remainder of the paper is organized as follows. Section 2 sets out the main findings of the paper. In Section 3 we present the model while Section 4 analyzes the optimal choice of the domestic agent. Section 5 presents the steady state distributions, and Section 6 contains the results of a comparative dynamics analysis. Finally Section 7 concludes.

2. How do tax evasion and tax corruption affect the economies? Main findings

In order to clarify the understanding of the model proposed in the next section, we briefly summarized our main findings and explain how our work is related to the existing literature on similar topics. The general message of the paper is that, when private capital and public spending are substitutes in the productive sector, the usual externalities of tax evasion can be internalized by private agents and compensate their negative impact of economic growth. But this can be done only at the cost of a higher volatility in production.

In countries with a minimal level of financial development, the proceeds of tax evasion are not necessarily consumed or thrown abroad in foreign banks, but can be used for rising funds to finance private domestic investment. This argument in contrast with a widespread literature suggesting a negative link between tax evasion and economic growth, especially in the developing countries (see, for instance, Barreto (2000), Brevik and Gartner (2008), Ehrlich and Lui (1999)).

Unlike many previous papers, we connect tax evasion and tax corruption. We refer to the empirical observation according to which, when corruption is widespread, a connection is established between corruption activities by bureaucrats and the countries' fiscal policy. Both tax evasion and tax corruption reduce the ability of the administration to raise funds to finance the economic growth, since both are diverted for private use. But, we distinguish between the diversion to bureaucrats' and households' private use. While bribes are very often consumed (rent seeking activities), the proceeds of tax evasion can be re-invested in private ownership of firms.

In our model, the decision to cheat and corrupt a bureaucrat is the result of a rational choice. This decision generates negative externalities in the production activity, because the amount of evaded income yields lower tax revenues that are used to finance public goods and services. Tax evasion and tax corruption are also a source of volatility of per-capita GDP, capital, spending and consumption. In our model the agent internalizes the potential spending externalities on production caused by her behavior. Though she does not obtain utility from public expenditure, the consumer–producer knows that tax evasion and tax corruption impact the amount of per-capita spending in the economy and thus the amount of income she will receive from production. This knowledge could encourage evasion if the return on the equities generated by tax evasion is higher enough so that the positive impact on production of a higher share of private capital exceeds the negative impact of public spending externality. This is likely to happen if the agent faces a favorable gamble, for instance with a low probability of being caught and convicted and if the likelihood of paying a bribe when detected is high. A key parameter is also the degree of risk aversion because the agent may rather decide to consume the extra-income from cheating. In this case, she would reduce her share of domestic and foreign capital out of wealth because, according to her preferences, consuming an unexpected income (random income) is better then taking part in a gamble.

Tables 1 and 2 display our main findings.

Assume that we are in a “poor” country in which consumers have preferences characterized by a strong risk aversion and thus by a high curvature of the utility function (high γ). Further assume that the country also lacks developed equity markets and that the productivity of public goods and services is low, that the tax administration faces difficulties in collecting taxes and that consumers escape tax payments by paying bribes to the bureaucrats. According to the tables, not only will tax evasion and tax corruption reduce the mean growth, but per-capita output will also be highly volatile. This implies situations in which tax evasion deepens recessions. There are several ways in which a government could smooth the cyclical fluctuations of the economy. It could raise the efficiency of public spending in order to reduce the degree of the public spending externality in the presence of tax evasion. Another possibility would be to reduce the incentive for cheating by employing an efficient technology to detect tax evasion or to fight corruption. The government may also want to limit the negative effects of tax evasion on the mean growth, by allowing people to invest their ill-gotten benefits in equity markets. However, if agents have a high risk aversion, the wealth effects on consumption will be important, thereby implying a decrease in their holding of private capital.

Now imagine a country in which a government faces tax noncompliance, but in which taxpayers want to buy domestic and foreign equities (we assume that they have a low risk aversion). Assume that, in this country, the productivity of public spending is low, that people have incentives to pay bribes to government tax collectors, that income tax evasion is widespread. Finally, let us assume that the government is unable to implement an effective fight against corruption and tax evasion.

Table 1
Impact of tax evasion and corruption on private capital and public spending. p: probability of being caught, p_1: probability of facing a corrupted bureaucrat, b: amount of bribe, and $\overline{\theta}$:expected returns of a unit of evaded tax.

Low incentive for cheating		
Impact on private capital	Magnitude increases with degree of financial openness (n_d^*) and risk aversion (γ)	p, p_1, b, and s are high (or increase) (+) Wealth effects on consumption ratio (−) Positive externality of public spending on consumption
	$p, \overline{\theta}, \tau$	(−) Higher risk of investing in private capital: ω_1
Public spending	τ which influences the tax income yield $\alpha^s/(\beta^s)^2$	(+) Output-enhancing public spending
Private capital	Equity market depth (n_d)	(−) Internalization: higher public spending reduces the agent's incentive to accumulate private capital
High incentive for cheating		
Impact on private capital	Magnitude increases with degree of financial openness (n_d^*) and risk aversion (γ)	p, p_1, b, and s are low (or decrease) (−) Wealth effects on consumption ratio (+) Negative externality of public spending on consumption
	$p, \overline{\theta}, \tau$	(+) Lower risk of investing in private capital: ω_1
Public spending	τ which influences the tax income yield $\alpha^s/(\beta^s)^2$	(−) Diversion of productive public spending
Private capital	Equity market depth (n_d)	(+) Internalization: lower public spending increases the agent's incentive to accumulate private capital

Table 2
Impact of tax evasion and corruption on the volatility of growth components.

		Low incentive for cheating	High incentive for cheating
		$1 < \psi^s/(\Omega_1^s)^2 < 5/2$	$\psi^s/(\Omega_1^s)^2 > 5/2$
Low efficiency of public spending	$\xi < 2$	High volatility in public spending Normal volatility in private capital	High volatility in public spending High volatility in private capital
High efficiency of public spending	$\xi > 2$	Normal volatility in public spending Normal volatility in private capital	Normal volatility in public spending High volatility in private capital

Again, this country will experiment volatile fluctuations of the output, in addition to possible negative effects on the mean growth rate due to the diversion of public resources. To reduce the size of the output fluctuations, the government could increase the productivity of public spending. In this case, since bureaucrats cannot manage to fight tax evasion, such a policy will only reduce the volatility of public spending; but private capital will still be volatile. However, the situation would be better than the initial situation in which both components of the growth rate of per-capita output are volatile. To dampen the negative effects associated with the diversion of public spending resources, the government can make the investment in equity markets an attractive activity to taxpayers by, firstly reducing the tax rate, and, secondly, by improving the productivity of public spending (these measures increase n_d). In this case, private equity markets act a substitute for anti-corruption policies and policies to fight against tax evasion.

3. The model

This model applies to a developing country with a minimum level of financial market development, as observed in the so-called "frontier" or "emerging" economies. This section presents a continuous time stochastic growth model. We describe a representative agent's choice and present the dynamics of saving and public spending. People who cheat can be caught, but they may face corruptible bureaucrats to whom they propose bribes. Tax corruption thus occurs through bribery to avoid paying the penalty for tax evasion. Bribes and corruption can be viewed as a cost to the detection technology, a negative externality generated by anti-fraud policy.

3.1. Tax fraud and tax corruption as a source of random income

3.1.1. Production

We consider an open economy, with infinitely lived representative agents, called the domestic country and the rest of the world referred as the foreign country. In each country we consider a society populated with a continuum of individuals with measure 1. A consumer supplies her labor force inelastically to the productive sector (we normalize the labor supply to 1). In addition to consumers and firms, politicians live in both the domestic and foreign economies. They provide a productive input in public spending financed out of tax revenues.

Private firms in the domestic economy produce a consumption good with the following production technology:

$$c(t) = y(t) = A(t)k(t), \quad A(t) = \xi[g(t)]^{1/\xi}, \quad (k(t), g(t)) \in [0, +\infty) \times [0, +\infty), \tag{1}$$

where $c(t)$, $y(t)$ are per-capita consumption and output, $k(t)$ and $A(t)$ are the (private) capital–labor ratio and productivity. The latter is assumed to depend on public goods and services (roads, public health, education, etc) provided by the bureaucrats or politicians and we assume decreasing returns of the technology for public goods ($\xi > 1$). The price of the consumption good is normalized to 1. $g(t)$ is per-capita public spending. Similarly, the production technology in the foreign country is given by

$$c^*(t) = y^*(t) = A^*(t)k^*(t), \quad A^*(t) = \xi^*[g^*(t)]^{1/\xi^*}, \quad (k^*(t), g^*(t)) \in [0, +\infty) \times [0, +\infty). \tag{2}$$

For simplicity, we assume that private capital does not depreciate. g is a pure public good (government goods and services are neither rival nor excludable).

3.1.2. Tax evasion and tax corruption

Our modeling of tax evasion relies upon Allingham and Sandmo (1972) and Yitzhaki (1974). Taxes are used to finance public goods and services.

An agent chooses to hide a fraction $e(t)$ of her income from the government and we assume that $0 < e(t) < 1$. Yet, politicians try to detect tax evasion. The probability of being detected is p ($0 < p < 1$). A consumer who is detected is asked to pay the legal tax $\tau e(t)y(t)$ plus a penalty defined as a fraction s of the undeclared income, $s\tau e(t)y(t)$. τ is the legal tax rate ($0 < \tau < 1$) and we have a similar definition for the legal tax rate in the foreign country ($0 < \tau^* < 1$). To avoid paying the penalty, the detected evader can pay a bribe to inspectors. The latter are corruptible with a probability p_1 ($0 < p_1 < 1$). Denoting θ the penalty rate when there are no bribes, we assume that the detected evader can pay back less than θ and we denote b the penalty rate when politicians are corrupted ($b < \theta$). This assumption means that the bureaucrat prefers to receive b rather than nothing, especially when corruption is widespread. b is lower than θ and strictly positive meaning that, in any case, the consumer pays more than the initial due tax.

The penalty rate is thus a random variable

$$\theta_1 = \left\{ \begin{array}{lll} \theta, & w.p. & 1-p_1 \\ b, & w.p. & p_1 \end{array} \right| \tag{3}$$

and the expected value of the penalty rate is $E[\theta_1] = \overline{\theta} = p_1 b + (1-p_1)\theta$. Therefore, the random return of a unit of evaded tax is

$$x_1 = \left\{ \begin{array}{lll} 1-\overline{\theta}, & w.p. & p(1-p_1) \\ 1-b, & w.p. & pp_1 \\ 1, & w.p. & 1-p \end{array} \right| . \tag{4}$$

The expected return on a unit of evaded tax is thus $E[x_1] = \overline{x}_1 = 1-(\overline{\theta}-b)pp_1-\overline{\theta}p$. Assuming that the domestic economy is composed of an infinite number of consumers who behave in a similar way, both processes tends to a normal law. Therefore x_1 converges to a normal law with mean $\overline{x}_1$ and a variance $\sigma_1^2 = V(\overline{x}_1)$ defined as a function of b, p and p_1.

The dynamics of the random gain induced by tax evasion is described by the following stochastic differential equation (SDE):

$$dx_1(t) = x_1\tau e(t)y(t)dt + \sigma_1\tau e(t)y(t)dz_1(t), \tag{5}$$

where $z_1(t)$ is a Brownian motion process.

We have a similar dynamics in the foreign country.

3.1.3. Portfolio diversification and the dynamics of wealth

A household spends a fraction of his income in consumption and uses the remaining income to buy equities whose values represent a share of the physical capital of the domestic country and of the foreign country. We assume that the population size is the same in both countries. We define

$$k(t) = k_d(t) + k_f(t) \text{ and } k^*(t) = k_d^*(t) + k_f^*(t), \tag{6}$$

where

$k_d(t)$ is the domestic per-capita capital owned by the domestic agent,
$k_f(t)$ is the domestic per-capita capital owned by the foreign agent,
$k_d^*(t)$ is the foreign per-capita capital owned by the domestic agent,
$k_f^*(t)$ is the foreign per-capita capital owned by the foreign agent.

Denoting $w(t)$ the average wealth of the domestic agent (per-capita wealth or saving), $n_d(t)$ and $n_d^*(t)$ the shares of domestic and foreign capital in the domestic agent' total wealth, we have

$$n_d(t) = \frac{k_d(t)}{w(t)}, \quad n_d^*(t) = \frac{k_d^*(t)}{w(t)}, \quad w(t) = k_d(t) + k_d^*(t). \tag{7}$$

We have similar relationships for the foreign consumer:

$$n_f^*(t) = \frac{k_f^*(t)}{w^*(t)}, \quad n_f(t) = \frac{k_f(t)}{w^*(t)}, \quad w^*(t) = k_f(t) + k_f^*(t). \tag{8}$$

where $w^*(t)$ is per-capita wealth in the foreign country. We assume perfect capital mobility without restrictions on asset trade. We further assume that there is a demand for portfolio diversification. This implies that $n_d(t)$, $n_d^*(t)$, $n_f^*(t)$ and $n_f(t)$ are strictly positive and less than 1.

Wealth (or saving) is a random variable because the expected return on tax evasion is a random variable. Each unit of hidden income yields $\bar{x}_1$ on average with more or less σ_1. Assuming that per-capita consumption evolves at a deterministic rate $c(t)dt$, we have

$$\begin{aligned} dw(t) = \{&[1-\tau+\bar{x}_1\tau e(t)]A(t)k_d(t) + \left[1-\tau^*+\bar{x}_1^*\tau^*e^*(t)\right]A^*(t)k_d^*(t) \\ &-c(t)\}dt + \sigma_1\tau e(t)A(t)k_d(t)dz_1(t) + \sigma_1^*\tau^*e^*(t)k_d^*(t)dz_1^*(t), \end{aligned} \tag{9}$$

from which we deduce the rate of accumulation of assets by the domestic agent:

$$\frac{dw(t)}{w(t)} = \psi(t)dt + \omega_1(t)dz_1(t) + \omega_1^*(t)dz_1^*(t), \tag{10}$$

where

$$\begin{aligned} &\psi(t) = R(t)n_d(t) + R^*(t)(1-n_d(t)) - \frac{c(t)}{w(t)}, \\ &R(t) = (1-\tau+\bar{x}_1\tau e(t))A(t), \\ &R^*(t) = \left(1-\tau^*+\bar{x}_1^*\tau^*e^*(t)\right)A^*(t). \end{aligned} \tag{11}$$

and

$$\begin{aligned} \omega_1(t) &= \sigma_1\tau e(t)A(t), \\ \omega_1^*(t) &= \sigma_1^*\tau^*e^*(t)A^*(t). \end{aligned}$$

$R(t)$ and $R^*(t)$ are the gross rates of returns of one unit of capital invested respectively in the domestic and in the foreign countries. They depend upon the tax rates, the expected returns of a unit of evaded tax and the proportions of hidden revenues. $\omega_1(t)$ and $\omega_1^*(t)$ are the risk of one unit of capital invested in the home and foreign countries. Therefore $R(t)n_d(t) + R^*(t)\ (1 - n_d(t))$ is the gross rate of return of the domestic agent's portfolio. For the foreign agent, we have similar relationships.

We assume that the following inequalities hold simultaneously $R(t) > R^*(t)$ and $\omega_1(t) > \omega_1^*(t)$ or $R(t) < R^*(t)$ and $\omega_1(t) < \omega_1^*(t)$.

3.2. The utility function

The consumer's preferences are represented by an isoelastic utility function. We assume that she obtains utility from private consumption. The objective function is

$$U = E_0\int_0^\infty (1/\gamma)(c(t))^\gamma e^{-\beta t}dt. \tag{12}$$

We assume that $-\infty < \gamma < 0, \beta > 0$. β is the time preference rate. $1-\gamma$ is the Arrow–Pratt coefficient of relative risk aversion. E_0 is the expectation at time $t = 0$. Unlike other models developed in the stochastic growth literature, we assume that public spending do not enhance the marginal utility of consumption, but only the productivity of private capital. This is a major difference with, for example, Turnovsky (1999).

3.3. The dynamics of public spending

Public goods and services are financed out of tax income. The random return to income taxation is

$$\mu(t) = \begin{cases} \mu_1(t) = \tau\left(1+\bar{\theta}e(t)\right)A(t)k(t), & w.p. \quad p \\ \mu_2(t) = \tau(1-e(t))A(t)k(t), & w.p. \quad 1-p \end{cases}. \tag{13}$$

Tax revenue is a random variable and so is per-capita public spending. Assuming a zero fiscal balance, the stochastic process describing the dynamics of public spending is therefore

$$dg(t) = \lambda_1(t)g(t)^{1/\xi}dt + \lambda_2(t)g(t)^{2/\xi}dZ_g(t), \tag{14}$$

where $Z_g(t)$ is a Brownian motion process and

$$\begin{aligned} \lambda_1(t) &= p\mu_1(t) + (1-p)\mu_2(t) \\ &= \xi k(t)\left\{p\tau\left(1+\bar{\theta}e(t)\right) + (1-p)\tau(1-e(t))\right\}, \end{aligned} \tag{15}$$

and

$$\lambda_2(t) = p(1-p)\xi^2k(t)^2\left\{\tau^2\left(1+\bar{\theta}e(t)\right)^2 + \tau^2(1-e(t))^2\right\} \tag{16}$$

$$-2\tau^2\left(1+\bar{\theta}e(t)\right)(1-e(t)). \tag{17}$$

Eq. (14) is a nonlinear SDE with drift and diffusion components which both depend on tax evasion behavior and tax corruption.

4. The optimal choice of the domestic agent

An agent faces the following intertemporal utility maximization problem. She maximizes Eq. (12) subject to the constraint (10) with $w(0) = w_0$.

Proposition 1. *The optimal choice of a consumer in the domestic country is given by the following unique interior solution (see the proof in Célimène et al., 2013):*

$$\begin{cases} (1-\gamma)\dfrac{\tilde{c}(t)}{w(t)} = \beta - \dfrac{\gamma}{2}[1-\gamma]\left[(\omega_1(t))^2 + \left(\omega_1^*(t)\right)^2\right]\tilde{n}_d^2 \\ \qquad\qquad + \dfrac{\gamma}{2}[1-\gamma]\left(\omega_1^*(t)\right)^2 \\ \qquad\qquad -\gamma R^*(t), \\ \tilde{n}_d(t) = \dfrac{(1-\tau)A(t) - R^*(t)}{[1-\gamma]\left(\omega_1^*(t)\right)^2} + 1, \\ \tilde{e}(t) = \dfrac{A(t)\ \bar{x}_1\tau}{[1-\gamma][\sigma_1\tau A(t)]^2\tilde{n}_d(t)}. \end{cases} \tag{18}$$

The first equation is obtained from the equality between the marginal utility of consumption and the marginal utility of wealth, which leads:

$$\widetilde{c}(t) = \{V'(w(t))\}^{\frac{1}{\gamma-1}} \tag{19}$$

where V is the value function.

The second equation is an arbitrage equation obtained from the first-order condition of the objective function obtained using the Jacobi–Hamilton–Bellman equation with respect to $n_d(t)$. This yields

$$R(t) - AP(w)\omega_1(t)^2 n_d(t) = R^*(t) - AP(w)\omega_1^*(t)^2 n_d^*(t), \tag{20}$$

where $AP(w)$ is the absolute value of the Arrow–Pratt relative risk aversion coefficient assumed to be constant:

$$AP(w) = \frac{wV''(w)}{V'(w)}. \tag{21}$$

Eq. (20) says that the risk-adjusted gross returns of one unit of capital invested in the domestic and foreign countries are equalized. The risk can be decomposed into several components. It depends upon the share of capital invested out of total wealth in the domestic and foreign countries, upon the uncertainty from tax evasion and corruption and upon the agent's behavior towards risk. The risk premium is therefore a function of the degree of relative risk aversion and of the difference in the uncertainty of fraud and corrupting bureaucrats in both countries:

$$R(t) - R^*(t) = AP(w)\left[\omega_1(t)^2 n_d(t) - \omega_1^*(t) n_d^*(t)^2\right]. \tag{22}$$

The third equation is obtained by equalizing to zero the derivative of the objective function with respect to $e(t)$. This yields

$$\widetilde{e}(t) = \left(\frac{1}{-AP(w)}\right)\left(\frac{\overline{x}_1}{\sigma_1^2}\right)\left(\frac{1}{\tau}\right)\left(\frac{1}{y^d(t)/w(t)}\right). \tag{23}$$

The optimal decision of tax fraud varies positively with the risk-adjusted return of fraud and with the degree of risk aversion, negatively with the tax rate and the domestic revenue as share of the agent's wealth. A system in which the tax rate is high is an incentive to cheat. Conversely, the motivation for a tax fraud diminishes as domestic production represents a high proportion of an individual's total wealth.

5. Steady state distributions

5.1. Definition of the equilibrium

For a given sequence of $\left\{A^*(t), e^*(t), \widetilde{n}_f^*(t), \frac{\widetilde{c}^*(t)}{w^*(t)}, y^*(t), k^*(t)\right\}_0^\infty$ and initial values (0), $\frac{\widetilde{c}(0)}{w(0)}$, $\tilde{n}_d(0)$, $g(0)$, $y(0)$, $k(0)$, the equilibrium is a sequence

$$\left\{A(t), e(t), \widetilde{n}_d(t), \frac{\widetilde{c}(t)}{w(t)}, y(t), k(t)\right\}_0^\infty,$$

where each variable is defined by a distribution, that satisfies the following conditions:

i) these variables satisfy the agent's optimal choice,
ii) domestic capital growths at the same rate as saving,
iii) the government's budget constraint is described by the SDE (14),
iv) the economy's capital and financial account is balanced.

Condition (i) implies that the equilibrium path must satisfy the system in Eq. (18). As shown in Célimène et al. (2013) in Appendix 1, the convexity of the maximization problem implies the unicity of the optimal solution.

Condition (ii) implies that the dynamics of capital obeys the following SDE:

$$dk(t) = \frac{dw(t)}{w(t)}k(t) = \psi(t)k(t)dt + \Omega_1(t)k(t)dZ_k(t), \tag{24}$$

where we have substituted a new diffusion component $\Omega_1(t)dZ_k(t)$ for the two local martingale terms $\omega_1(t)dz_1(t)$ and $\omega_1^*(t)dz_1^*(t)$ in Eq. (10). The solution of this SDE can be written as

$$k(t) = k(0)exp\left\{\int_0^t\left(\psi(s) - \frac{1}{2}(\Omega_1(s))^2\right)ds\right\}. \tag{25}$$

Condition (iii) implies that the dynamics of $A(t)$ can be found by applying the Ito lemma. We have

$$A(t) = \xi[g(t)]^{1/\xi} \text{ and } dg(t) = \widetilde{\lambda}_1(t)dt + \widetilde{\lambda}_2(t)dz_g(t), \tag{26}$$

where $\widetilde{\lambda}_1(t) = \lambda_1(t)[g(t)]^{1/\xi}$ and $\widetilde{\lambda}_2(t) = \lambda_2(t)[g(t)]^{2/\xi}$ with $\lambda_1(t)$ and $\lambda_2(t)$ defined by Eqs. (15) and (16). Applying the Ito lemma, we have

$$dA(t) = \alpha(t)A(t)dt + \beta(t)A(t)dz_g(t), \tag{27}$$

where

$$\alpha(t) = \widetilde{\lambda}_1(t)(\xi/g(t)) + \left(\frac{1-\xi}{\xi}\right)\left(\xi/g(t)^2\right)\widetilde{\lambda}_2(t)^2, \tag{28}$$

$$\beta(t) = (\xi/g(t))\widetilde{\lambda}_2(t). \tag{29}$$

Eq. (27) implies

$$A(t) = A(0)exp\left\{\int_0^t\left(\alpha(s) - \frac{1}{2}(\beta(s))^2\right)ds\right\}. \tag{30}$$

Eqs. (25) and (30) are not closed-form solutions because $k(t)$ and $A(t)$ also appear in $\psi(t)$, $\Omega_1(t)$, $\alpha(t)$ and $\beta(t)$ and in Eq. (18). $k(t)$ and $A(t)$ are the two important state variables in the model, since they determine the dynamics of all the other variables. The equilibrium is described by a random sequence of the variables or by a distribution. Indeed, as is seen from our equations, the dynamics is the results of a deterministic drift component and of a diffusion component where the variance of the variables is used to define their distribution. The stochastic nature of the model entirely comes from the uncertain income caused by tax evasion and tax corruption.

5.2. Steady state distributions for $g(t)$ and $k(t)$

We focus on the dynamics of the variables of our model in the neighborhood of the long-run stochastic steady state. Such a state is characterized, in systems of SDE, by a steady stable distribution. We study the conditions for the existence of such a distribution for per-capita GDP. Since the latter depends upon $k(t)$ and $A(t)$, we search for their limit stable distribution. For the existence conditions we refer the reader to Célimène et al. (2013)

Proposition 2. *A closed-form expression of the invariant steady-state distribution for public spending is given by the following upper incomplete Gamma distribution*:

$$P(g) = K_1^s K_3^s \frac{\Gamma\left(\alpha, K_2^s g^{\frac{\xi-3}{\xi}}\right)}{\Gamma(\alpha)}, \quad \xi \in (1,3), \quad \alpha = \frac{4-\xi}{3-\xi}, \tag{31}$$

where K_1^s, K_2^s *and* K_3^s *are constants*:

$$K_1^s = \frac{1}{(\lambda_2^s)^2} exp\left\{K_2^s g(0)^{(\xi-3)/\xi}\right\}, K_2^s = \frac{2\lambda_1^s}{(\lambda_2^s)^2} \times \frac{\xi}{3-\xi}, \tag{32}$$

$$K_3^s = \left(\frac{1}{K_2^s}\right)^{(-4+\xi)/(\xi-3)} \frac{\xi}{\xi-3}. \quad (33)$$

λ_1^s and λ_2^s are Eqs. (15) and (16) defined in the neighborhood of the random steady state and

$$\Gamma(\alpha) = \int_0^\infty g^{\alpha-1} exp(-g)dg \quad \text{and} \quad \Gamma(\alpha, y) = \int_y^\infty g^{\alpha-1} exp(-g)dg, \quad \alpha>0, x>0.$$

The proof of this proposition is in Célimène et al. (2013). The main characteristics of the invariant distribution of public spending g depends upon the properties of the distribution of an "äuxiliary" variable $z = g^{\frac{\xi-3}{\xi}}$. The distribution of g is an upper incomplete Gamma distribution defined by using both the upper incomplete Gamma function and the Gamma function.

Notice that the upper incomplete Gamma function (the numerator of Eq. (31)) can be rewritten as follows:

$$\Gamma\left(\alpha, K_2^s g^{\frac{\xi-3}{\xi}}\right) = \Gamma(\alpha) - \gamma(\alpha, K_2^s z), \quad z = g^{\frac{\xi-3}{\xi}}, \quad (34)$$

where $\gamma(\alpha, K_2^s z)$ is the lower incomplete gamma function defined by

$$\gamma(\alpha, K_2^s z) = \int_0^{K_2^s z} g^{\alpha-1} exp(-g)dg. \quad (35)$$

Therefore, we have

$$P(g) = K_1^s K_3^s \left(1 - \widetilde{P}(\alpha, K_2^s z)\right), \quad \widetilde{P}(\alpha, K_2^s z) = \frac{\gamma(\alpha, K_2^s z)}{\Gamma(\alpha)}. \quad (36)$$

$\widetilde{P}(\alpha, K_2^s z)$ is the cumulative distribution function for gamma random variable z with shape parameter α and rate parameter K_2^s (or with a scale parameter $1/K_2^s$ which is the reciprocal of the rate parameter). The distribution of z can be approximated by a Normal distribution, if $\alpha > 10$, which implies the following condition on the efficiency of public spending: $\xi > 0.89$. Since, we have assumed that $\xi > 1$, this condition is always true. Therefore the limit distribution of g can be considered as being the cumulative distribution of a normal law. The limit invariant distribution is thus symmetric. As a consequence, under the assumption of decreasing productivity of public spending the "hocks" affecting public spending and per-capita output in the steady states are Gaussian. Since $\alpha > 1$, g has a unimodal distribution and the maximum is such that

$$z = [(\alpha-1]/K_2^s \quad \text{or} \quad \text{equivalently} \quad g^{max} = \{[(\alpha-1]/K_2^s\}^{\xi/(\xi-3)}. \quad (37)$$

Since we have a Normal distribution, the scale parameter can be interpreted as the variance of the distribution. By definition, the Kurtosis of z equals $(6/\alpha)$. The distribution thus displays heavy tails if $(6/\alpha) > 3$ (or, equivalently, if $\xi < 2$) and "normal" tails if $\xi > 2$.

As we noticed above, since g depends upon tax income, which in turn varies randomly according to the intensity of tax evasion and tax corruption, $A = \xi g^{1/\xi}$ can be interpreted as a public spending externality of tax evasion and corruption. The above results imply that, for small values of public spending productivity ("mall" means lower than 2), public spending externalities can trigger drastic changes in the asymptotic behavior of per-capita public spending and thus on per-capita output. In other words, tax evasion and tax corruption can make the economy become very unstable in terms of the variability of public spending and per-capita output. The occurrence of "extreme events" in spending is linked to the fourth-order central moment of z and depends upon both ξ and the variables of the tax and corruption system. This is easily seen by noting that the fourth-order central moment is $\mu_4 = [3\alpha(2+\alpha)]/(K_2^s)^4$. The likelihood of extreme events increases when μ_4 is big, or, equivalently when K_2^s is small. Given the definition of K_2^s, this implies a low return to income taxation (low ratio $\lambda_1^s/(\lambda_2^s)^2$). This happens when p or s (the probability of being caught and the penalty rate) is small. In other words, tax evasion can make the economy become very unstable in terms of the variability of public spending and thus of per-capita output. Thus the model predicts that, over a long period, we should observe a higher volatility of public spending and of per-capita output growths in those countries in which the tax collection system is highly deficient, tax corruption is widespread and the productivity of public spending is low. However, this instability can be reduced if, public goods and services are highly productive ($\xi > 2$).

Proposition 3. *The density function of $k(t)$ is a power law density function with a scaling parameter $\gamma = -2(1 - \psi^s/(\Omega_1^s)^2)$:*

$$p(k) = \frac{2d_0}{(\Omega_1^s)^2} k^{-\gamma} \quad (38)$$

where d_0 is a normalizing constant.

This density is obtained easily by computing the speed density function as for public spending (see Célimène et al., 2013). We assume that $\gamma > 0$ which implies that $\psi^s/(\Omega_1^s)^2 > 1$. To avoid that $p(k)$ diverges when $k \to 0$, we need to impose a lower bound to k. This bound exists if $k = 0$ is inaccessible (in this case, we need $\psi^s/(\Omega_1^s)^2 > 1/2$). It is straightforward that the normalizing constant is defined by $C = (\gamma-1)k_{min}^{\gamma-1}$ and this yields $d_0 = 0.5(\gamma-1)(\Omega_1^s)^2 k_{min}^{\gamma-1}$. We require at least that the first moment exists, in which case $\gamma > 2$ or $\psi^s/(\Omega_1^s)^2 > 2$. The variance is finite if $2 < \gamma < 3$ or $\psi^s/(\Omega_1^s)^2 < 5/2$ and infinite if $\gamma > 3$ (thus implying heavy tails). Therefore, if p, p_1, b, and s are such that the performance of the agent's portfolio consisting of domestic and foreign equities is high enough ("high" means above 5/2) then changes in per-capita capital can give rise to extreme values (or high volatility in domestic capital accumulation).

6. Impact of tax evasion and tax corruption on private capital and public spending

We first discuss the effects of changes in p_1, p, b, and s on $\frac{dw(t)}{w(t)}$ (or equivalently on $\frac{dk(t)}{k(t)}$ given our definition of the equilibrium) This amounts to examining the impact of changes in $\bar{x}_1$ and σ_1 on the growth rate of saving. For purpose of illustration, we consider a situation in which the domestic agent has an incentive to cheat because she lives in a country where the tax administration is inefficient in collecting taxes and fighting bribery. We discuss the consequences of a lower probability of being caught ($\Delta p_1 < 0$), or a lower expected penalty if caught (that happens if $\Delta\bar{\theta}<0, \Delta p_1<0, \Delta b<0$). These changes imply higher expected return to corruption and tax evasion ($\Delta\bar{x}_1>0$) and a lower uncertainty of fraud activities. An analytical study of a comparative analysis is difficult because we do not have closed-form solutions. We shall instead use heuristic arguments, indicating which equations are affected when changes happen.

6.1. Consumption

A decrease in the probability of being caught, or lower penalty rate or higher probability of facing a corrupted bureaucrat, has the following consequences on the household's consumption decisions. Firstly, this raises the risk-adjusted return of the unreported income ($\bar{x}_1$ increases and σ_1 decreases). The hidden income is used to buy foreign equities (or equivalently to hold a fraction of the foreign country's physical capital). The gains from this investment are consumed (wealth effects on consumption). The wealth effect is captured by the term $-\gamma R_t^*$ in the consumption equation of Eq. (18) (remember that $\gamma < 0$). This wealth effect reduces saving (and therefore affects the growth rate of private capital negatively) and its magnitude depends upon the curvature of the utility function. The higher the domestic agent's risk aversion, the stronger the negative impact on the growth rate of saving. Further, the financing of public spending declines as tax evasion raises. This in turn reduces the domestic gross return of a unit of concealed income, $R(t)$, and therefore leads to a lower share of the domestic capital held by the household in her total wealth. A decrease in $n_d(t)$ reduces the consumption ratio as shown by the first term in the consumption equation ($\frac{c(t)}{w(t)}$ in Eq. (18) is positively related to $n_d(t)$). This in turn increases the growth rate of saving and therefore has a positive impact on the growth rate of private. Thirdly, a decrease in p_1, p, b, and s reduces the uncertainty of tax evasion (σ_1 decreases) and the risk of domestic equities (ω_1 decreases). For the agent, this is an incentive to reduce the ratio of consumption out of her total wealth. This effect is captured by the term $\frac{\gamma}{2}[1-\gamma](\omega_1(t))^2$ in the consumption equation in Eq. (18). The impact on the growth rate of per-capita private capital is therefore positive.

The total effects are thus ambiguous. It is natural to ask what the net effect will be in general in the developing economies. The important point here is that growth should be affected negatively in case of strong wealth effects. In the poorest countries wealth ownership is low. Therefore an agent has a lot to lose if detected when she hides income. As a consequence, this agent would tend to show a high risk aversion. Conversely, increased wealth levels tend to diminish the marginal utility of income, thereby generating a reduce aversion to cheating. Both these arguments should lead to observe a more negative impact on growth of corruption and tax evasion, through the consumption channel, in the poorest countries.

6.2. Public spending

In our model tax evasion and tax corruption are equivalent to diverting public resources that are productive. A decrease in p_1, p, b, and s results in a higher $\bar{x}_1$ inducing, all things being equal, an increase in $e(t)$. The latter in turn implies a decrease in public spending (provided that the term $\mu_2(t)$ dominates the term $\mu_1(t)$). The magnitude of wasted public resources associated with tax evasion depends upon the taxation rate τ. The negative public spending externalities increase with the amount of lost tax income. The effect on per-capita output is negative (because y is a function of A) with a magnitude which depends on the values of ξ, p, s and τ.

Further, since there is a loop between tax evasion and public spending, a lower $A(t)$ reduces $e(t)$ but increases $n_d(t)$ in Eq. (18) and thereby affects growth positively. Therefore, when the agent internalized the negative externalities of tax evasion and corruption on public spending, this makes per-capita output increase. If this second round effect dominates, we have a situation in which public spending is the main driver of per-capita output and the share of private equity diminishes. Conversely, if the negative externalities dominate, then production will be driven by private capital with a lower share of public spending.

Therefore, tax evasion and tax corruption, in addition to impacting production also influence the composition of the growth rate in terms of private and public investments. On the one side, a higher noncompliance rate and a higher tax corruption do not help the economy to capitalize on the public spending externalities. On the other hand, cheating yields individual benefits to the tax payers if there exists an equity market in which the proceeds of the concealed income can be invested.

7. Conclusion

In this paper, we proposed a theoretical model of the effects of tax evasion and tax corruption on private capital and public spending. These variables are considered as productive inputs in the production function. The model highlights several channels through which the mean and volatility of these variables are affected. We first stress the role of equity markets, showing that the evasion outcome for the private sector is not necessarily viewed as a burden, but as an opportunism and optimal response of individual agents to a governance failure from the tax administration. Tax evasion and tax corruption create a random environment – because illegal activities are risky – and the consumer takes a portfolio decision (by choosing the share of private capital to hold) in conjunction with the evasion rate and her consumption ratio. Equity market performs here have the same role as a policy of tax exemption. In societies in which the share of private investment in percentage of GDP is growing, in which tax cheaters usually choose to shelter the proceeds of their illegal activities from the official financial institutions, and in which the productivity of public spending is often low, tax evasion and tax corruption may contribute to the development of private capital if people find an opportunity to invest the proceeds of their illegal activities in equity markets. We are not claiming that these activities are beneficial in a broader sense for growth, but simply that, conditional on the performance of the taxation system, tax evasion does not necessarily deepen growth or exacerbates growth volatility in an environment in which private investment is the result of a portfolio decision and of a rational choice leading the agents to take their decisions by comparing the returns to cheating and the risk of being caught and/or facing a corrupted inspector.

A second important result is that the returns to tax evasion and tax corruption in private equity markets, the average tax income and the productivity of public spending jointly impact the volatility of the economy, through their influence on the volatility of private capital and public expenditure. We evidence several regimes of volatility for these variables. It is noteworthy that, when there is a high incentive for cheating (because the tax collection system is deficient), the negative externalities on public spending can be attenuated if its productivity is high enough. This implies that there may be a trade-off between tax governance and policies enhancing the efficiency of public goods and services on the economic growth.

Thirdly, we raise the fact that the threshold values of the parameters which determine the different configurations of the mean and volatility of the productive inputs are found endogenously by examining the invariant distributions which prevail in the random steady state. Such distributions depend upon the specification of the production function. In an AK model in which per-capita output is a linear function of per-capita private capital and a power function of per-capita government spending with decreasing returns, we show that the invariant distribution is respectively described by a power law and an upper incomplete gamma function.

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