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Modeling a flexible manufacturing cell using stochastic Petri nets with fuzzy parameters

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ABSTRACT

In this paper, an approach for modeling and analysis of time critical, dynamic and complex systems using stochastic Petri nets together with fuzzy sets is presented. The presented method consists of two stages. The first stage is same as the conventional stochastic Petri nets with the difference that the steady-state probabilities are obtained parametrically in terms of transition firing rates. In the second stage, the transition firing rates are described by triangular fuzzy numbers and then by applying fuzzy mathematics, the fuzzy steady-state probabilities are calculated. A numerical example for modeling and analysis of a flexible manufacturing cell is given to show the applicability of proposed method. The importance of the proposed approach is that it can take into consideration both dimensions of uncertainty in system modeling, stochastic variability and imprecision.

1. Introduction

A flexible manufacturing system (FMS) is a discrete-event system and contains a set of versatile machines, an automatic transportation system, a decision-making system, multiple concurrent flows of job processes that make different products, and often exploits shared resources to reduce the production cost (Jeng, 1997a; Zuberek & Kubiak, 1994). The layout of a complex FMS is given in Fig. 1 (http://www.denford.co.uk/). These systems require both qualitative and quantitative aspects to be considered in modeling and analysis. Qualitative analysis searches for structural properties like the absence of deadlocks, the absence of overflows or the presence of certain mutual exclusions in case of resource sharing. Quantitative analysis looks for performance properties (e.g. throughput), responsiveness properties (e.g. average completion times) or utilization properties (e.g. average queue lengths or utilization rates). Quantitative analysis concerns the evaluation of the efficiency of the modeled system whereas qualitative analysis concerns the effectiveness of the modeled system.

There are many methods and tools used for modeling and analysis of FMSs such as queueing networks, Markov chains, simulation, and Petri nets. Petri nets (PN) introduced by Petri (1962), as a graphical and mathematical tool, can be used for modeling and analyzing complex systems which can be characterized as synchronous, parallel, simultaneous, distributed, resource sharing, nondeterministic and/or stochastic (Bobbio, 1990; Marsan, Balbo, Conte, Donatelli, & Franceschinis, 1995; Murata, 1989; Zhou & Venkatesh, 1999). The complex systems of these types exhibit characteristics which are difficult to describe mathematically using conventional tools like differential equations and difference equations (Jeng, 1997b; Murata, 1989). On the other hand, Petri nets as a mathematical tool provide obtaining state equations describing system behavior, finding algebraic results and developing other mathematical models. With respect to other techniques of graphical system representation like block diagrams or logical trees, Petri nets are particularly more suited to represent in a natural way logical interactions among parts or activities in a system (Bobbio, 1990). In modeling point of view, Petri net theory allows the construction of the models amenable both for the effectiveness and efficiency analysis (DiCesare, Harhalakis, Proth, Silva, & Vernadat, 1993).

Due to the graphical nature, ability to describe static and dynamic system characteristics and system uncertainty, and the presence of mathematical analysis techniques, Petri nets form an appropriate conceptual infrastructure for modeling and analysis of FMSs.

Although the concept of time was not included in the original work by Petri (1962), for many practical applications, the addition of time is a necessity. Without an explicit notion of time, it is not possible to conduct temporal performance analysis, i.e., to determine production rate, resource utilization. In modeling a FMS with PNs, timing and activity durations for analyzing temporal performance and dynamics of the system should be taken into consideration.

In PNs, time is often associated to transitions. The reason for this is that transitions represent events in a model and it is more natural

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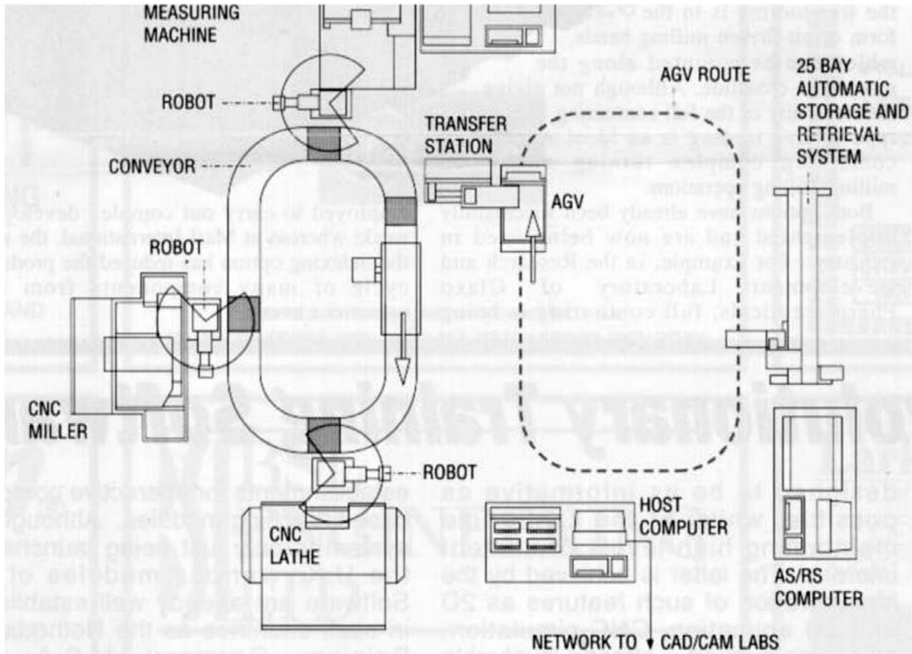

Fig. 1. The layout of a complex FMS (Denford Co., UK).

to consider events to take time rather than time to be related to conditions, that is, places (Bowden, 2000; Gharbi & Ioualalen, 2002; Murata, 1989; Zhou & Venkatesh, 1999). The time delays in a PN model can be specified either deterministically or probabilistically. If the time delays are deterministically given, such a PN model is called as deterministic timed net and if the delays are probabilistically specified, the PN model is called stochastic net. Timed PNs and stochastic PNs are two popular extensions of PNs which are widely used in the application field of manufacturing systems.

A stochastic PN (SPN) is a Petri net where each transition is associated with an exponentially distributed random variable that expresses the delay from the enabling to the firing of the transition. Due to the memoryless property of the exponential distribution of firing delays, Molloy (1982) showed that the reachability graph of a bounded SPN is isomorphic to a finite Markov chain. Queueing networks and Markov chains provide flexible, powerful and easy to use tools for modeling and analysis of complex manufacturing systems and are widely used (Al-Jaar & Desrochers, 1990). However, it is difficult to describe the causal relation of uncertain events explicitly in the complex models using Markov chain and queueing network models because of their unrealistic mathematical assumptions (Hatono, Yamagata, & Tamura, 1991). In SPN models, we can explicitly describe the causal relation of uncertain events by using places, transitions, and arcs. Therefore, using SPNs, we can construct the model of a FMS more easily than using the other models. SPNs combine the modeling power of PNs and the analytical tractability of Markov processes for the purpose of performance analysis (Molloy, 1982).

The limitation of the SPN is that the number of states of the associated Markov chain grows very fast as the complexity of the SPN model increases (Marsan, Bobbio, Conte, & Cumani, 1984, 1995). Marsan et al. (1984) introduced the generalized SPNs to reduce the complexity of solving a SPN model in which the number of reachable markings is smaller than that in a topologically identical SPN. A generalized SPN is basically a SPN with transitions that are either timed (to describe the execution of time consuming activities) or immediate (to describe some logical behavior of the model). Timed transitions behave as in SPNs, whereas the immediate transitions have an infinite firing rate and fire in zero time.

Petri (1987) presented some criticism related to timed and stochastic PNs about the conceptualization of time and chance. In his latter study Petri (1996) presented many axioms, among which the axioms of measurement and control related to time and nets, and emphasized mainly on uncertainty. These studies turned the attention on fuzzy set theory and fuzzy logic (Zadeh, 1965, 1973) which have been applied successfully in modeling and designing many real world systems in environments of uncertainty and imprecision.

There are several approaches that combine fuzzy sets and Petri nets theories, differing not only in the fuzzy tools used but also in the elements of the nets that are fuzzified. A PN structure is a four tuple consisting of places, transitions, tokens and arcs, and theoretically each of these can be fuzzified (Srinivasan & Gracanin, 1993).

Analysis and design of complex systems often involve two kinds of uncertainty: randomness and fuzziness (Hu, Wu, & Shao, 2002). Randomness refers to describing the behavior of the parameters by using probability distribution functions. In other words, the randomness models stochastic variability. Fuzziness models measurement imprecision due to linguistic structure or incomplete information. In modeling a FMS, input and model parameters are usually in the form of uncertain parameters. The possible sources of imprecision causing uncertainty in system modeling are system inputs, system outputs, and imprecise inner operations (Virtanen, 1995). In some cases, the uncertainty arises from both randomness (stochastic variability) and imprecision (fuzziness) simultaneously. SPNs in which time is the only random variable and time delay is described by probability functions well characterize the uncertainty in the system with the measures of variance and probability distributions. During the analysis, the uncertainty in parameter values can be hidden in the results. The use of fuzzy sets theory to be able to compensate this can be considered as an important alternative.

Although the dominating concept to describe uncertainty in modeling is stochastic models which are based on probability, probabilistic models are not suitable to describe all kinds of uncertainty, but only randomness. Especially the imprecision of data which is for example as a result of the limited precision of measuring is not statistical in nature and cannot be described by using probability (Viertl & Hareter, 2004). The quantification of a one-dimensional quantity is possible by using fuzzy numbers, which are a generalization of real numbers.

In this study, we propose an approach for modeling a FMS by using stochastic PNs together with fuzzy set theory to represent both stochastic variability and imprecision. In this approach, the exponential distribution's parameter is represented by a triangular fuzzy number. By using the fuzzified parameter and fuzzy mathematics, the fuzzy steady-state distribution is obtained. Although fuzzy PNs and stochastic PNs have been separately used in modeling and analysis of FMS, the significant contribution of this paper, as the first, is the suggestion of the use of fuzzy set theory together with stochastic PNs in manufacturing system modeling.

The organization of the paper is as follows. In Section 2, a literature review on PNs is given. In Section 3, the formal definition of stochastic PNs is presented. In Section 4, the proposed approach is explained in detail. In Section 5, a numerical illustration of the proposed approach for a FM cell is given. Finally the conclusions are presented.

2. Literature review

In this section a literature review on PNs and their applications in manufacturing systems are given.

2.1. Classical PNs

Valette, Courvoisier, and Mayeux (1982) were the first who presented the applicability of PNs for flexible production systems by specifying and validating interconnected controllers for a transportation system in a car production system using PNs. They showed that PNs were applicable to this system and indicated that such a PN approach could be based on decomposition and structuring. Valette (1987) indicated that PNs were more convenient than other models for concurrency. Alla, Ladet, Martinez, and Silva (1985) employed coloured PNs to model the same car productions system in Valette et al. (1982). Every token in a colored PN has its own attributes and thus allows different parts and machines to be distinguished among each other. They revealed the possible benefits of using colored PNs over ordinary PNs, i.e., their conciseness made it possible to describe a complex FMS. Narahari and Viswanadham (1985) used PNs to model two manufacturing systems: a transfer line with three machines and two buffers, and a FMS with three machines and two part types. The significance of boundedness, liveness and reversibility in manufacturing systems was presented. They also presented a systematic bottom-up modeling approach. Barad and Sipper (1988) introduced an approach for describing and measuring the flexibility in manufacturing systems by indicating the multi-dimensional characteristics of flexibility. They used PNs for modeling a FMS and comparing different system on the basis of flexibility. Valavanis (1990) introduced extended PNs in which each of tokens, places, transitions and arcs differ from the other so that more information can be carried in a net model. In this study, a FMS containing two workstations, a robot and input and output stations are modeled and simulated. Zhou and DiCesare (1991, 1992, 1993) proposed parallel and sequential mutual exclusions to model resource sharing problems in FMS. The proposed hybrid synthesis approach enables one to develop a Petri net with good behavioral properties for sophisticated manufacturing systems and makes it possible to analyze by avoiding the enumeration of all possible markings.

Zhou, McDermott, and Patel (1993) presented how to model a FM cell having the specified properties and also explained a detailed synthesis process. They also computed the cycle time of the cell by using deterministic timed PNs. Shiizuka and Suzuki (1994) showed the modeling power of PNs in modeling the automated guided vehicle (AGV) networks in FMSs. Using colored Petri nets, they modeled behavior of AGVs such as, go straight ahead, junction of two ways, branch off three ways, junction of three ways, and etc. They also modeled an AGV network using all these behaviors. Kiritsis and Porchet (1996) dealt with process planning using Petri net as a computer-aided design (CAD) or computer-aided process planning (CAPP). The proposed approach consists of three parts. First and second parts are to build a fixed format machining table and a PN model of the system based on the machining table data, respectively. Third is to find a reachability graph of the PN model. Finally reachability graph is reduced, and optimum solution is found by applying simple heuristic. Jeng (1997b) improved a Petri net synthesis theory, which has been proposed for modeling shared-resource automated manufacturing systems. This theory aims at constructing net models by using bottom-up and modular composition approaches. Since only net structure and initial marking are used for analysis, the proposed approach is claimed to be more efficient than state enumeration techniques such as reachability tree. Wang and Wu (1998) introduced a method called colored time object oriented PNs for modeling automated manufacturing systems. The proposed method enables better modeling and analysis of dynamic behavior of automated manufacturing systems. Zimmermann, Rodriguez, and Silva (2001) proposed a two phase optimization method for the PN models of manufacturing systems. In the first phase, by using meta heuristic simulated annealing a near optimal result is obtained. The optimal result is obtained in the second phase. Lefebvre (2001) proposed a method for the estimation of the firing frequencies in discrete and continuous PN models. In case of timed PNs, the production frequencies estimation is obtained from the approximation of the firing sequences and in case of continuous PNs it results directly from the variants of the marking vector. In the proposed method, when several solutions exist, the PNs were extended with additional relations to provide a unique solution.

Abdallah, Elmaraghy, and Elmekkawy (2002) proposed a scheduling algorithm based on PNs by considering the deadlock-free scheduling problem of manufacturing systems. The objective of the model is to minimize the average flow time and the proposed algorithm gives optimal or near optimal deadlock-free schedule by considering the sequence of the transition firings. Based on the application results, they claim that PN based scheduling is more appropriate in comparison to mathematical programming approaches. Chen and Chen (2003) proposed a object oriented method for performance modeling and evaluation of dynamic tool allocation in FMS by using colored PNs. The FMS is divided into subclasses so that the complexity of the system is decreased. By using PN based simulation the system performance is evaluated. Uzam (2002, 2004) introduced the use of PN reduction approach for obtaining an optimal deadlock prevention policy for FMSs. In the first study, the proposed procedure uses the reachability graph of PN model of a FMS. Due to the state explosion of the reachability graph, it is difficult to obtain the optimal solution. In his latter study, by using the PN reduction approach the procedure to obtain an optimal result is simplified and explained on an example. Wang, Zhang, and Chan (2005) proposed a hybrid PN approach for modeling of nerworked manufacturing systems and control system architecture. Both discrete and continuous system variables are used in the PN model and the control mechanism for the system is developed based on the net model.

2.2. Stochastic PNs

Balbo, Bruell, and Ghanta (1988) presented a hierarchical modeling approach that combines queueing network models and generalized SPNs for the solution of complex models of system behavior. They applied the proposed approach on two examples and demonstrated that how the combination of the two techniques preserves the inherent accuracy of each individual technique in the evaluation of the overall model. Zhou, DiCesare, and Guo (1990) modeled and analyzed the performance of two resource sharing and deadlock-free manufacturing systems. The stochastic PN modeling process is given based on top-down and bottom-up approaches and the system performance indices such as throughput are derived and the comparison results are presented. Al-Jaar and Desrochers (1990) presented the use of generalized SPNs for performance evaluation of automated manufacturing systems for several manufacturing case studies. They showed the use of generalized SPNs, as a flexible and powerful tool for performance evaluation, on a machining workstation controller, transfer lines and production networks. They also presented the advantages of using generalized SPNs for modeling and analysis of complex manufacturing systems. Choi, Kulkarni, and Trivedi (1993) showed that the marking process underlying a deterministic and stochastic PN, which is an extension of SPNs including both exponentially distributed and deterministic delays, is a Markov regenerative stochastic process. They introduced a numerical method for transient analysis based on numerical inversion of Laplace–Stieltjes transforms.

German (1995) developed a numerical method for transient and stationary analysis of deterministic and stochastic PNs based on the method of supplementary variables. Based on the method of supplementary variables general states equations are derived. He also presented an approximation technique for more general cases. Koriem and Patnaik (1997) proposed a method called as generalized stochastic high-level PNs for the performance evaluation of parallel and distributed systems. The presented method is a hybrid of the predicate/transition nets and the generalized SPNs. The main advantage of this method is the reduction in the state space size of the generalized stochastic high-level PN model. Yan, Wang, Zang, and Cui (1998) introduced a method for manufacturing system modeling and analysis which is named as extended stochastic high-level evaluation Petri nets. Based on the simulation results, they showed that by using this method the dynamic rescheduling of the system is possible. In this method, in addition to immediate and exponentially distributed transitions, the firing times that are not exponentially distributed can be used. Lindermann and Thümmler (1999) introduced a numerical algorithm for transient analysis of deterministic and stochastic PNS and other discrete-event stochastic systems with exponential and deterministic events. The proposed approach is based on the analysis of a general state space Markov chain whose state equations constitute a system of multi-dimensional Fredholm integral equations. This method requires three orders of magnitude less computational effort than the approach based on the method of supplementary variables.

Zimmermann and Hommel (1999) used a new method based on colored PNs for manufacturing system modeling. By using the method named as colored stochastic PNs, the structure of a manufacturing system and the work plans can be modeled separately. Bucholz (2004) presented a new approximate solution technique for the numerical analysis of SPNs and related models. His approach combines numerical solution techniques and fixed point computations. In contrast to other approximation methods, the proposed method is adaptive by considering states with high probability in detail and aggregating states with small probabilities. Chen, Amodeo, Chu, and Labadi (2005) introduced a new model called batch deterministic and stochastic PNs by enhancing the deterministic and stochastic PNs with batch places and batch tokens. They presented methods for structural and performance analysis of the model developed. This class of stochastic PNs is suitable for modeling and analysis of inventory systems and real-life supply chains.

2.3. Fuzziness in PNs

Valette, Cardoso, and Dubois (1989) introduced fuzzy-time PNs which is based on the association of fuzzy enabling duration with the transition which results in attaching a fuzzy firing date to the transition. They used the definition of time PNs (Merlin, 1974) with the only modification that the enabling durations associated to transitions are defined by fuzzy intervals. Murata (1996) and Murata et al. (1999) proposed fuzzy-timing high-level PNs and applied to the simulation of communication protocols. Based on the possibility theory, this approach introduced four fuzzy theoretic functions of time called fuzzy timestamp, fuzzy enabling time, fuzzy occurrence time and fuzzy delay. For each of the fuzzy functions of time, trapezoidal or triangular possibility distributions were used. Yeung, Liu, Shiu, and Fung (1996) proposed a net based structure called fuzzy coloured PNs to model both the dynamic behavior and inexact production inference of FMSs. They applied this approach to a printed circuit board production system and demonstrated the model's capability in simulating system behavior. Pedrycz and Camargo (2003) introduced a new version of fuzzy PNs called fuzzy timed PNs by incorporating a concept of time with the interval and fuzzy set-based models of temporal relationship. The factor of time is incorporated into the structure of the net at the level of transitions and places. In this approach the impact of the time factor on the performance of the net is expressed in terms of firing of the transitions and the distribution of the level of marking of the input and output places. Ding, Bunke, Schneider, and Kandel (2005) and Ding, Bunke, Kipersztok, Schneider, and Kandel (2006) presented a new fuzzy timed PN model in which each transition firing is associated with a fuzzy number and during transitions firing tokens are removed from input and added to output places. The performance analysis of the model is based on the reachability state graph.

The papers including different applications of PNs mainly in the field of manufacturing are; modeling and analysis of manufacturing systems by using process nets with resources (Jeng, Xie, & Peng, 2002), reliability design of industrial plants using PNs (Bertolini, Bevilacqua, & Mason, 2006), scheduling of FMSs with timed PNs (Kim, Suzuki, & Narikiyo, 2007; Hsu, Korbaa, Dupas, & Goncalves, 2008; Lee & Korbaa, 2006; Zuberek & Kubiak, 1999), modeling and scheduling of FMSs with PNs (Huang, Sun, & Sun, 2008), reduced PN models of discrete manufacturing systems (Rangel, Trevino, & Mellado, 2005), property-preserving subnet reductions for designing manufacturing systems with shared resources (Huang, Jiao, & Cheung, 2005), PN approach for error recovery in manufacturing systems control (Odrey & Mejia, 2005), PN modeling of discrete-event dynamic systems (Koriem, Dabbous, & El-Kilani, 2004), Modeling and simulation of a bottling plant using hybrid Petri nets (Giua, Pilloni, & Seatzu, 2005), PN based object-oriented modeling of hybrid and complex productive systems (Liu, Jiang, & Fung, 2005; Villani, Pascal, Miyagi, & Valette, 2005), structuring and composition in PN models (Gomes, 2005), hybrid PN and digraph approach for deadlock prevention in automated manufacturing systems (Maione & DiCesare, 2005), fuzzy PN modeling of FMSs (Venkateswaran & Bhat, 2006), modeling, analysis and control of hybrid dynamic systems (Lefebvre, Delherm, Leclercq, & Druaux, 2007), fuzzy PN modeling of intelligent databases (Korpeoglu & Yazici, 2007), dynamic fuzzy PNs for course generation (Huang, Chen, Huang, Jeng, & Kuo, 2008), scheduling by using timed PNs (Ghaeli, Bahri, & Lee, 2008; Kim et al., 2007), cyclic scheduling of FMSs by using PNs and genetic algorithm (Hsu et al., 2008), resource-oriented PN for deadlock avoidance in flexible assembly systems (Wu, Zhou, & Li, 2008), multiparadigm modeling of hybrid dynamic systems using PNs (Lee, Zhou, & Hsu, 2008), reachability and state space analysis of PNs (Fronk & Kehden, 2009; Praveen

& Lodaya, 2008; Reinhardt, 2008), modeling and scheduling of job shops by timed PNs (Zhang & Gu, 2009).

3. Stochastic Petri nets

Before giving the formal definition of SPNs we present the definition of PNs introduced by Petri (1962).

A marked Petri net (PN) $Z = (P, T, I, O, m)$ is a five-tuple where

1. $P = \{p_1, p_2, \ldots, p_n\}, n > 0$ and is a finite set of places pictured by circles
2. $T = \{t_1, t_2, \ldots, t_s\}, s > 0$ and is a finite set of transitions pictured by bars, with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$
3. $I : P \times T \rightarrow N$ and is an input function that defines the set of directed arcs from P to T where $N = \{0, 1, 2, \ldots\}$
4. $O : T \times P \rightarrow N$ and is an output function that defines the set of directed arcs from T to P
5. $m_i : P \rightarrow N$ and is a marking whose ith component represents the number of tokens in the ith place. An initial marking is denoted by m_0. The tokens are pictured by dots.

A marked PN and its elements are shown in Fig. 2. The four-tuple (P, T, I, O) is called a PN structure that defines a directed graph structure. A PN models system dynamics using tokens and their firing rules. Introducing tokens into places and their flow through transitions make it possible to describe and study the discrete-event dynamic behavior of the PN. Additional information about structures and properties of PNs, firing rules and analysis techniques can be found in Murata (1989).

An ordinary continuous-time stochastic PN is a PN with a set of positive, finite and exponentially distributed firing rates $\Lambda = (\lambda_1, \ldots, \lambda_m)$, possibly marking dependent, associated with all its transitions. An enabled transition can fire after an exponentially distributed time delay with parameter $\frac{1}{\lambda}$ elapses.

Live and bounded SPNs are isomorphic to continuous-time Markov chains due to the memoryless property of exponential distribution (Molloy, 1982). This important property allows for the analysis of SPNs and the derivation of the some important performance measures. The states of the Markov chain are the markings in the reachabillity graph, and the state transition rates are the exponential firing rates of the transitions in the SPN. By solving a system of linear equations representing the Markov chain, performance measures can be computed.

Assume that every transition in a PN is associated with an exponentially distributed random delay from the enabling to the firing of the transition. Then the firing time of each transition can be characterized as a firing rate.

A stochastic PN $Z = (P, T, I, O, m_0, \Lambda)$ is a six-tuple where

1. $P = \{p_1, p_2, \ldots, p_n\}, n > 0$ and is a finite set of places
2. $T = \{t_1, t_2, \ldots, t_s\}, s > 0$ and is a finite set of transitions with $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$
3. $I : P \times T \rightarrow N$ and is an input function that defines the set of directed arcs from P to T where $N = \{0, 1, 2, \ldots\}$
4. $O : P \times T \rightarrow N$ and is an output function that defines the set of directed arcs from T to P
5. $m_i : P \rightarrow N$ and is a marking whose ith component represents the number of tokens in the ith place. An initial marking is denoted by m_0.
6. $\Lambda : T \rightarrow R^+$ and is a firing function whose ith component represents the firing rate of the ith transition where λ_i denotes the firing rate of t_i and R^+ is the set of all positive real numbers.

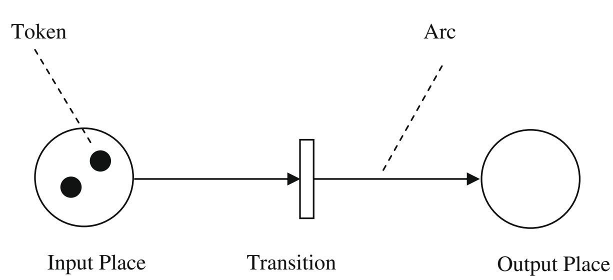

Fig. 2. A marked PN and its elements.

In a SPN, when a transition is enabled at marking m, the tokens remain in its input places during the firing time delay. At the end of the firing time, tokens are removed from its input places and deposited in its output places. The number of tokens in the flow depends on the input and output functions.

After generating the reachability graph $R(m_0)$, the Markov process is obtained by assigning each arc with the rate of the corresponding transition. The steady-state probability distribution $\Pi = (\pi_0, \pi_1, \ldots, \pi_q)$ of a SPN is obtained by solving the linear system

$$\Pi A = 0, \quad \sum_{i=0}^{q-1} \pi_i = 1 \tag{1}$$

where $A = (a_{ij})_{q \times q}$ is the transition rate matrix. For $i = 0, 1, \ldots, q-1$, A's ith row elements, i.e., $a_{ij}, j = 0, 1, \ldots, q-1$ are determined as follows:

1. if $j \neq i, a_{ij}$ is the sum of all outgoing arcs from state m_i to m_j.
2. Since any row elements in A satisfies $\sum_{j=0}^{q-1} a_{ij} = 0$, then $a_{ii} = -\sum_{j \neq i}^{q-1} a_{ij}$, where a_{ii} represents the sum of firing rates of transitions enabled at m_i, i.e., transition rates leaving state m_i.

From the steady-state distribution Π and transition firing rates Λ, the required performance indices of the system modeled by the SPN can be obtained.

4. Stochastic Petri nets with fuzzy parameters

The steady-state probability distribution $\Pi = (\pi_0, \pi_1, \ldots, \pi_q)$ of a SPN is obtained by solving the linear system in Eq. (1). Here the transition rate matrix A is a square matrix whose off-diagonal elements are the rates of the exponential distribution associated with the state to state transitions. The elements on the main diagonal are chosen so that the elements of each row sum to zero.

Every transition in a SPN is associated with an exponentially distributed random delay from the enabling to the firing of the transition. Then λ_i denotes the firing rate of t_i. While modeling manufacturing systems by using SPNs, the λ is used for exponentially representing the activity durations like machining, transferring of materials or parts, machine failure, inspection etc. probabilistically. In conventional Markov method and SPNs, λ values are estimated statistically from crisp data with a confidence level based on measurement, and are accepted as constant values. Such a way of describing the behavior of the parameters used for modeling and analysis by using probability distribution functions takes into consideration the probabilistic or stochastic variability only, in short randomness. As mentioned before the imprecison of data, as a result of the limited precision of measuring, is not statistical in nature and can not be described by using probability theory.

To be able to better represent uncertainty, both stochastic or probabilistic variability and imprecision, we present an approach for modeling FMSs by using stochastic PNs which is based on the fuzzification of transition firing rates. In this way, we will obtain the fuzzy steady-state probabilities by using fuzzified parameters and applying fuzzy mathematics. Although the proposed approach is applied to a FM cell, its usage is not restricted to modeling and analysis of FMSs. It can be used for modeling and analysis of any time critical, dynamic and complex system modeled by SPNs.

Before we present our approach, we will explain some important concepts about fuzzy sets that are used in our method. For detailed information about fuzzy set theory, see Zadeh (1965), Zimmerman (1994) and Ross (1995).

Definition 4.1. Fuzzy numbers are the fuzzy sets that are normalized and convex. However our fuzzy numbers will always be triangular (shaped) fuzzy numbers. A triangular fuzzy number $\tilde{N}$ is defined by three numbers $a < b < c$ where the base of the triangle is the interval $[a, c]$ and its vertex is at $x = b$ which is shown in Fig. 3.

To be triangular shaped fuzzy number the graph is required to be continuous and: (1) monotonically increasing on $[a, b]$, and (2) monotonically decreasing on $[b, c]$.

Definition 4.2. α-cuts are slices through a fuzzy set producing regular (non-fuzzy) sets. If $\tilde{A}$ is a fuzzy subset of some set Ω, then an α-cut of $\tilde{A}$, written $\tilde{A}(\alpha)$ is defined as

$$\tilde{A}(\alpha) = \{x \in \Omega | \tilde{A}(\alpha) \geqslant \alpha\}, \tag{2}$$

for all $\alpha, 0 < \alpha \leqslant 1$. The $\alpha = 0$ cut, or $\tilde{A}(0)$, must be defined separately.

For any fuzzy number $\tilde{Q}$, $\tilde{Q}(\alpha)$ is a closed, bounded, and interval for $0 \leqslant \alpha \leqslant 1$ and can be written as

$$\tilde{Q}(\alpha) = [q_1(\alpha), q_2(\alpha)] \tag{3}$$

where $q_1(\alpha)(q_2(\alpha))$ will be an increasing (decreasing) function of α with $q_1(1) \leqslant q_2(1)$. If $\tilde{Q}$ is a triangular shaped fuzzy number then: (1) $q_1(\alpha)$ will be a continuous, monotonically increasing function of α in $[0, 1]$; (2) $q_2(\alpha)$ will be a continuous, monotonically decreasing function of α in $[0, 1]$; and (3) $q_1(1) = q_2(1)$. It is possible to check monotone increasing (decreasing) by showing that $dq_1(\alpha)/d\alpha > 0$ $(dq_2(\alpha)/d\alpha < 0)$ holds.

Fuzzy arithmetic operations for fuzzy numbers can be performed by using two basic methods of computing: the extension principle, and interval arithmetic with α-cuts. In our study, we will use the interval arithmetic with α-cuts which is equivalent to the extension principle, but it is more user and computer friendly.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers. Since α-cuts are closed, bounded, intervals so let $\tilde{A}(\alpha) = [a_1(\alpha), a_2(\alpha)]$ and $\tilde{B}(\alpha) = [b_1(\alpha), b_2(\alpha)]$. Then basic fuzzy operations can be done by using the following equations:

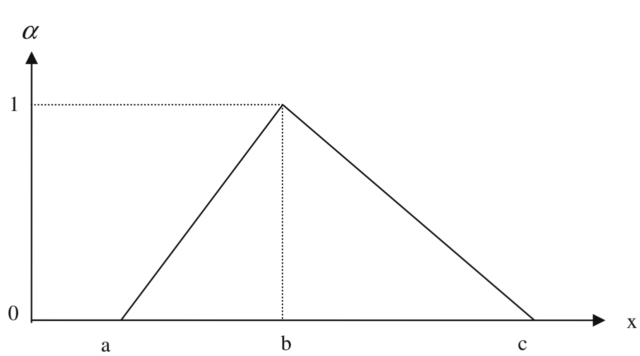

Fig. 3. A triangular fuzzy number, $\tilde{N}$.

$$\tilde{A}(\alpha) + \tilde{B}(\alpha) = [a_1(\alpha) + b_1(\alpha), a_2(\alpha) + b_2(\alpha)] \tag{4}$$

$$\tilde{A}(\alpha) - \tilde{B}(\alpha) = [a_1(\alpha) - b_2(\alpha), a_2(\alpha) - b_1(\alpha)] \tag{5}$$

$$\tilde{A}(\alpha) \cdot \tilde{B}(\alpha) = [c(\alpha), d(\alpha)] \tag{6}$$

where

$$c(\alpha) = \min\{a_1(\alpha)b_1(\alpha), a_1(\alpha)b_2(\alpha), a_2(\alpha)b_1(\alpha), a_2(\alpha)b_2(\alpha)\} \tag{7}$$

$$d(\alpha) = \max\{a_1(\alpha)b_1(\alpha), a_1(\alpha)b_2(\alpha), a_2(\alpha)b_1(\alpha), a_2(\alpha)b_2(\alpha)\} \tag{8}$$

$$\tilde{A}(\alpha)/\tilde{B}(\alpha) = [a_1(\alpha), a_2(\alpha)] \cdot \left[\frac{1}{b_2(\alpha)}, \frac{1}{b_1(\alpha)}\right] \tag{9}$$

when $\tilde{A}(\alpha)/\tilde{B}(\alpha)$, provided that zero does not belong to $\tilde{B}(\alpha)$ for all α.

The memoryless property of exponential distribution of firing delays is very important since live and bounded SPNs are isomorphic to continuous-time Markov chains due to the memoryless property of exponential distribution (Molloy, 1982). In our approach, we describe the exponentially distributed transition firing rates as triangular fuzzy numbers. Since our aim is to take into consideration both randomness and fuzziness, the memoryless property for fuzzy exponential function must be satisfied in order to perform the stochastic analysis of fuzzy parameters.

The exponential $E(\lambda)$ has density

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geqslant 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

The mean and variance of $E(\lambda)$ is $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$, respectively.

The probability statement of the memoryless property of crisp exponential is

$$P[X \geqslant t + \tau | X \geqslant t] = P[X \geqslant \tau] \tag{11}$$

If we substitute $\tilde{\lambda}$ for λ in Eq. (10) we obtain the fuzzy exponential, $E(\tilde{\lambda})$. If $\tilde{\mu}$ denotes the mean, we find its α-cuts as

$$\tilde{\mu} = \left\{ \int_0^\infty x\lambda e^{-\lambda x} dx \middle| \lambda \in \tilde{\lambda}(\alpha) \right\} \tag{12}$$

for all α. However, each integral in the above equation equals $\frac{1}{\lambda}$. Hence $\tilde{\mu} = \frac{1}{\tilde{\lambda}}$. If $\tilde{\sigma}^2$ is the fuzzy variance, then we write down an equation to find its α-cuts and we obtain $\tilde{\sigma}^2 = \frac{1}{\tilde{\lambda}^2}$. It can be seen that the fuzzy mean (variance) is the fuzzification of the crisp mean (variance).

The conditional probability of fuzzy event A given a fuzzy event B is defined by Zadeh (1968) as

$$\tilde{P}(A|B) = \frac{\tilde{P}(A \cdot B)}{\tilde{P}(B)}, \quad \tilde{P}(B) > 0 \tag{13}$$

By using the fuzzy exponential α-cuts of the fuzzy conditional probability, relating to the left side of Eq. (11)

$$P[X \geqslant t + \tau | X \geqslant t](\alpha) = \left\{ \frac{\int_{t+\tau}^\infty \lambda e^{-\lambda x} dx}{\int_t^\infty \lambda e^{-\lambda x} dx} \middle| \lambda \in \tilde{\lambda}(\alpha) \right\} \tag{14}$$

for $\alpha \in [0, 1]$. Now the quotient of the integrals in Eq. (14) equals, after evaluation, $e^{-\lambda\tau}$, so

$$P[X \geqslant t + \tau | X \geqslant t](\alpha) = \left\{ \int_\tau^\infty \lambda e^{-\lambda x} dx \middle| \lambda \in \tilde{\lambda}(\alpha) \right\} \tag{15}$$

which equals $\tilde{P}[X \geqslant \tau](\alpha)$. Hence, Eq. (11) holds for the fuzzy exponential which shows the memoryless property.

Our approach is a two stage modeling approach. The first stage is same as the conventional SPN modeling approach. The only difference is that the steady-state distributions are obtained parametrically by using Eq. (1) and no numeric results are calculated. In other words, each steady-state probability, π_i, is described in terms of transition firing rates, as a function of λ_i. Up to the second stage, the system is a crisp one and describing the stochastic nature of the system. At this stage we represent the transition firing rates, λ_i, as triangular fuzzy numbers which may depend on the opinions of experts. After replacing the fuzzy numeric values of transition firing rates, by using the fuzzy calculation theory we obtain the α-cuts of the fuzzy steady-state probabilities of the system. As Buckley (2005) states that whenever we use interval arithmetic with α-cuts is used to compute the functions of fuzzy variables, we may get something larger than that obtained by using the extension principle. To be able to find the α-cuts of the fuzzy-steady probabilities we solve an optimization problem that makes the solution feasible.

The procedure to compute the fuzzy steady-state probabilities is as follows:

Stage 1:

1.1. Model the system using a PN and associate exponential time delays with transitions.
1.2. Generate the reachability graph. Assign each arc with the rate of the corresponding transition. Label all states or markings.
1.3. By using Eq. (1) find the steady-state probabilities parametrically, in terms of transition firing rates.

Stage 2:

2.1. Place the transition firing rates described as triangular fuzzy numbers in parametric steady-state probabilities obtained in Step (1.3).
2.2. Compute the fuzzy steady-state probabilities by using Eqs. (4)–(9) in terms α-cuts.
2.3. For each fuzzy steady-state probability, π_i, find $\alpha = o$ values which gives the largest possible interval. It should be noted that for $\alpha = 1$ the obtained result is the steady-state distribution of the crisp SPN.
2.4. For each π_i, the maximum and minimum value ($\alpha = o$ value) must be in the interval $[0, 1]$. If $\alpha = o$ for each π_i does satisfy this, the result is feasible. If any of the obtained fuzzy probability does not satisfy this, apply the optimization in the next step.
2.5. As mentioned before, interval arithmetic with α-cuts method is mainly based on max and min operators which may produce larger intervals. Theoretically, $\alpha = o$ cut of a fuzzy number gives the largest possible interval of values. Since we want to calculate the fuzzy probabilities the

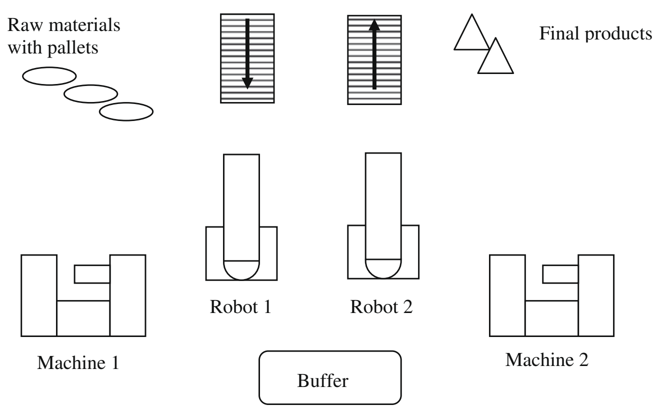

Fig. 4. The illustration of the flexible manufacturing cell.

Table 1
Places, transitions and their firing rates used in the model.

Places	Interpretation	
p1	Pallets with workpieces available	
p2	M1 in process	
p3	Intermediate parts available for processing at M2	
p4	M1 in repair	
p5	M1 available	
p6	Conveyor slots available	
p7	R1 available (redundant from the analysis viewpoint)	
p8	M2 available (redundant from the analysis viewpoint)	
p9	R2 available (redundant from the analysis viewpoint)	
Transitions	Interpretation	Firing rates
t1	R1 loads a part to M1	$\lambda_1 = 40$
t2	M1 machines and R1 unloads a part	$\lambda_2 = 5$
t3	R1 loads/unloads and M2 machines a part	$\lambda_3 = 4$
t4	M1 breaks down	$\lambda_4 = 0.5$
t5	M1 is repaired	$\lambda_5 = 0.5$

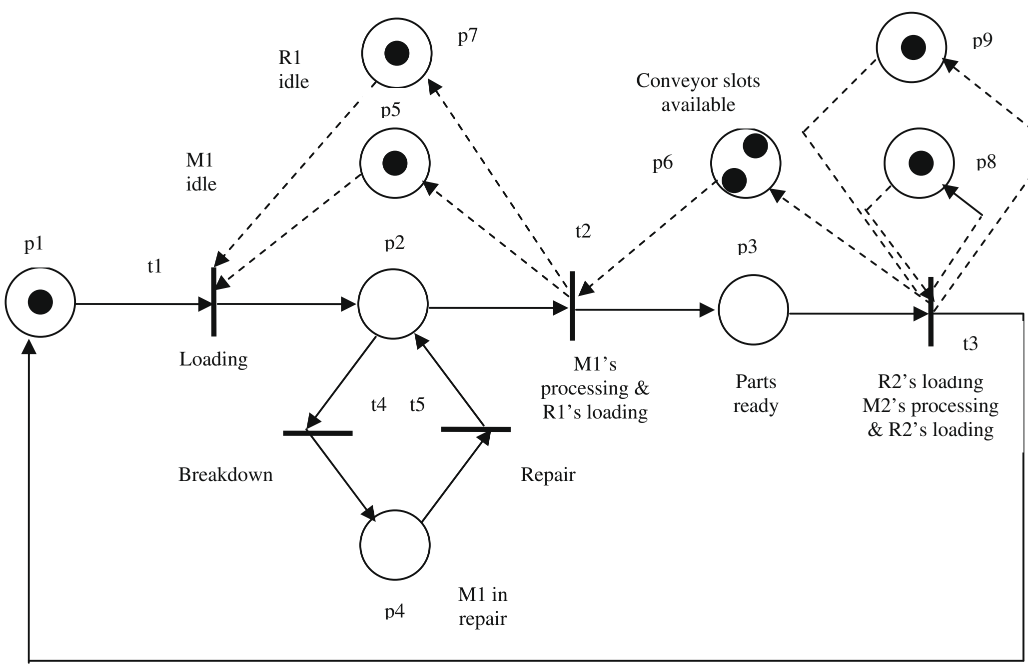

Fig. 5. The stochastic PN model of the system.

largest possible interval is restricted to the interval $[0,1]$. The problem is finding the min α-cut that satisfies this condition which can be found solving the following optimization problem:

Assume that the α-cut representation for the fuzzy steady-state probability is $\pi_i = [\pi_i^-(\alpha), \pi_i^+(\alpha)]$, where $i = 1,2,\ldots,n$ and n is the number of the states. Then the structure of the problem is

$$\begin{aligned} &\text{Min} && (Z) = \alpha \\ &\text{s.t.} && \pi_i^+(\alpha) \leqslant 1 \\ &&& \pi_i^-(\alpha) \geqslant 0 \\ &&& 0 \leqslant \alpha \leqslant 1 \\ &&& \pi_i^-(\alpha) \leqslant \pi_i^+(\alpha) \end{aligned}$$

In the next section a numerical illustration of the proposed approach is given.

5. A numerical example

A new approach for modeling and analysis of complex dynamic systems such as FMSs by using stochastic PNs together with fuzzy set theory to represent both stochastic variability and imprecision has been proposed and the structure of the new algorithm has been given in the above sections. In this section the approach is applied to a FM cell which is selected from Zhou and Venkatesh (1999). This FM cell is illustrated in Fig. 4.

Our FM cell consists of two machines (M1 and M2), each of which is served by a dedicated robot (R1 and R2) for loading and unloading, as shown in Fig. 4. An incoming conveyor carries pallets with raw materials one by one, from which R1 loads M1. An outgoing conveyor takes the finished product, to which R2 unloads M2. There is a buffer with capacity of two intermediate parts between two machines. The system produces a specific type of final parts. Each raw workpiece fixtured with one of three available pallets is processed by M1 and then M2. A pallet with a finished product is automatically defixtured, then fixtured with raw material, and finally returns to the incoming conveyor. Now suppose that

1. M1 performs faster M2 does, however subject to failures when it is processing a part. On the average, M1 takes two time units to break down, and a quarter time unit to be repaired. Thus, its average failure and repair rates (1/time unit) are 0.5 and 4 respectively. M2 and the two robots are failure-free.
2. R1's loading speed is 40 per unit time. The average rate for M1's processing plus R1's loading is 5 per unit time.
3. The average rate for M2's processing plus the related R2's loading and unloading is 4 per unit time.
4. All the time delays associated with the above operations are exponential.

The problem is to find the average utilization of M1, assuming that only one pallet is available.

First, we need to model the system using PNs and associate exponential time delay with transitions. The SPN model of the system is given in Fig. 5. Table 1 gives the explanation and interpretation of the PN elements used in the model.

Note that, the redundant information in each marking on the token values in places p_7, p_8 and p_9 has been eliminated since p_7 holds the same number of tokens as p_5 does and each of the other two always has one token.

The reachability graph and the Markov chain of the modeled system is given in Fig. 6.

By using Eq. (1) we obtain the following system of equations:

$$(\pi_0,\pi_1,\pi_2,\pi_3)\begin{pmatrix} -\lambda_1 & \lambda_1 & 0 & 0 \\ 0 & -\lambda_2-\lambda_4 & \lambda_2 & \lambda_4 \\ \lambda_3 & 0 & -\lambda_3 & 0 \\ 0 & \lambda_5 & 0 & -\lambda_5 \end{pmatrix} = 0 \tag{16}$$
$$\pi_0+\pi_1+\pi_2+\pi_3 = 1$$

The solution of the above system gives the steady-state probabilities parametrically, in terms of transition firing rates, as follows:

$$\Pi^T = \begin{bmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} \lambda_2\lambda_3\lambda_5/\lambda \\ \lambda_1\lambda_3\lambda_5/\lambda \\ \lambda_1\lambda_2\lambda_5/\lambda \\ \lambda_1\lambda_3\lambda_4/\lambda \end{bmatrix} \tag{17}$$

where $\lambda = \lambda_2\lambda_3\lambda_5 + \lambda_1\lambda_3\lambda_5 + \lambda_1\lambda_2\lambda_5 + \lambda_1\lambda_3\lambda_4$.

After obtaining the steady-state probabilities in terms of transition firing rates, in the second stage of our approach we must represent the transition firing rates as triangular fuzzy numbers. The fuzzy number values of each transition firing rate are as in Table 2.

By placing the fuzzy values of Table 2 in the previously obtained parametric steady-state probability representations, Eq. (17), and

Table 2
The fuzzified transition firing rates and their α-cut representations.

Fuzzy λ value	α-cut representation
$\lambda_1 = (30/40/50)$	$\lambda_1 = [30+10\alpha; 50-10\alpha]$
$\lambda_2 = (4/5/6)$	$\lambda_2 = [4+\alpha; 6-\alpha]$
$\lambda_3 = (3/4/5)$	$\lambda_3 = [3+\alpha; 5-\alpha]$
$\lambda_4 = (0.4/0.5/0.6)$	$\lambda_4 = [0.4+0.1\alpha; 0.6-0.1\alpha]$
$\lambda_5 = (0.4/0.5/0.6)$	$\lambda_5 = [0.4+0.1\alpha; 0.6-0.1\alpha]$

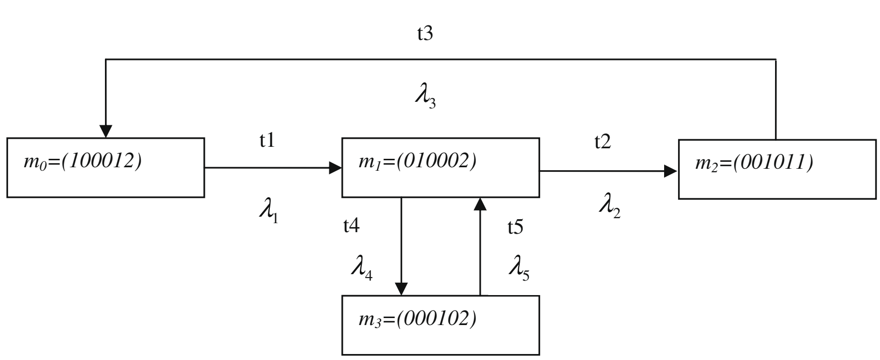

Fig. 6. The reachability graph and Markov chain of the modeled system.

applying fuzzy mathematics given in Eqs. (4)–(9), the following α-cut representations of fuzzy steady-state probabilities are obtained.

$$\pi_0(\alpha)=\left[\frac{0.1\alpha^3+1.1\alpha^2+4\alpha+4.8}{-3.1\alpha^3+50.7\alpha^2-275.6\alpha+498};\frac{-0.1\alpha^3+1.7\alpha^2-9.6\alpha+18}{3.1\alpha^3+32.1\alpha^2+110\alpha+124.8}\right]$$
$$\pi_1(\alpha)=\left[\frac{\alpha^3+10\alpha^2+33\alpha+36}{-3.1\alpha^3+50.7\alpha^2-275.6\alpha+498};\frac{-\alpha^3+16\alpha^2-85\alpha+150}{3.1\alpha^3+32.1\alpha^2+110\alpha+124.8}\right]$$
$$\pi_2(\alpha)=\left[\frac{\alpha^3+11\alpha^2+40\alpha+48}{-3.1\alpha^3+50.7\alpha^2-275.6\alpha+498};\frac{-\alpha^3+17\alpha^2-96\alpha+180}{3.1\alpha^3+32.1\alpha^2+110\alpha+124.8}\right]$$
$$\pi_3(\alpha)=\left[\frac{\alpha^3+10\alpha^2+33\alpha+36}{-3.1\alpha^3+50.7\alpha^2-275.6\alpha+498};\frac{-\alpha^3+16\alpha^2-85\alpha+150}{3.1\alpha^3+32.1\alpha^2+110\alpha+124.8}\right]$$

The graphics of the fuzzy steady-state probabilities are given in Fig. 7a–d.

Although for each $\pi_i=\left[\pi_i^-(\alpha),\pi_i^+(\alpha)\right]$, the maximum and minimum value ($\alpha=o$ value) must be in the interval $[0,1]$, it can be seen that π_1^+,π_2^+ and π_3^+ do not satisfy this condition. So we must optimize it to find the min α-cut that satisfies this condition. Since $\pi_1=\pi_3$ and $\pi_i^-(\alpha)\geqslant 0$, the structure of the optimization problem can be reduced to the following:

$$\begin{aligned}
\text{Min}\quad & (Z)=\alpha\\
\text{s.t.}\quad & \pi_0^+(\alpha)\leqslant 1\\
& \pi_1^+(\alpha)\leqslant 1\\
& \pi_2^+(\alpha)\leqslant 1\\
& 0\leqslant\alpha\leqslant 1\\
& \pi_0^+(\alpha),\pi_1^+(\alpha),\pi_1^+(\alpha)\geqslant 0
\end{aligned}$$

By making the necessary simplifications the problem is obtained as follows:

$$\begin{aligned}
\text{Min}\quad & (Z)=\alpha\\
\text{s.t.}\quad & -3.2\alpha^3-30.4\alpha^2-119.6\alpha-106.8\leqslant 0\\
& -4.1\alpha^3-16.1\alpha^2-195\alpha+25.2\leqslant 0\\
& -4.1\alpha^3-15.1\alpha^2-206\alpha+55.2\leqslant 0\\
& 0\leqslant\alpha\leqslant 1
\end{aligned}$$

The solution for the problem can be found by using software packages such as MATLAB or a spread sheet like Excel. The result of the optimization problem is 0.263. This α value is the one that makes the fuzzy steady-state probabilities feasible. The final fuzzy steady-state probabilities are presented in Table 3.

Note that $\alpha=0$ cut value represents the largest interval of probability whereas $\alpha=1$ cut value represents the crisp SPN probability.

M1's utilization is determined by the probability that M1 is machining a raw workpiece. This corresponds to the marking m_1 at which p_2 is marked, or state probability π_1. Therefore, the expected M1's utilization is π_1 which is (0.106/0.296/0.826). This

Table 3
The final fuzzy steady-state probabilities.

	$\alpha=0$ cut	$\alpha=1$ cut
π_0	[0.014; 0.1]	0.037
π_1	[0.106; 0.826]	0.296
π_2	[0.138; 1]	0.37
π_3	[0.106; 0.826]	0.296

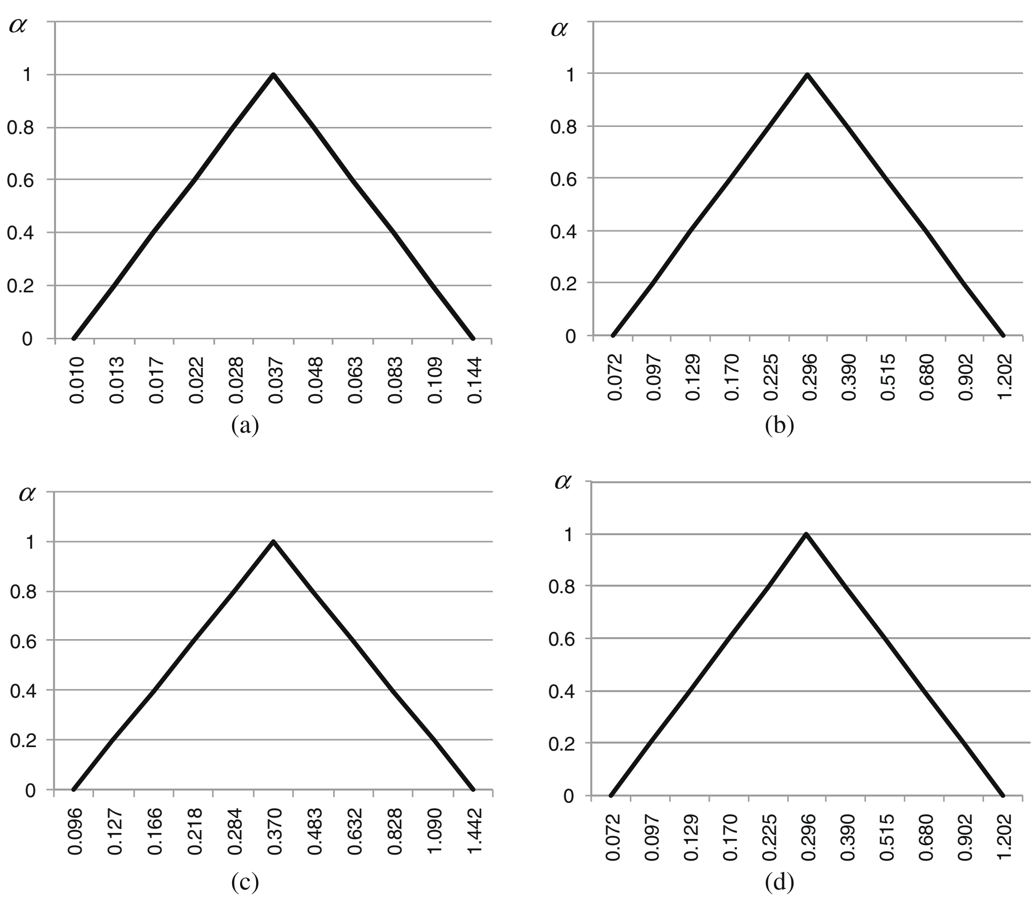

Fig. 7. The graphical representation of the fuzzy steady-state probabilities, (a) π_0, (b) π_1, (c) π_2, (d) π_3.

result informs us that the utilization ratio of M1 can be 0.106 at its lowest level while it can be at most 0.826. The most possible value of M1's utilization is 0.296 with a membership degree of 1.0. The crisp case with a single value does not enable us to have such a result.

6. Conclusion

In this study, we proposed an approach for modeling and analysis of discrete-event dynamic systems which is based on stochastic PNs together with fuzzy set theory to represent both dimensions of uncertainty which are probabilistic (stochastic) variability and imprecision (fuzziness). Our approach is a two stage method which combines two theories, fuzzy sets and PNs, and aims at increasing the power of the modeling and analysis of complex systems. Although the proposed approach was applied to a FM cell, it is not restricted to modeling and analysis of FMSs. It can be used for modeling and analysis of any time critical, dynamic and complex system modeled by SPNs. We believe that the main contribution of our study, in addition to the suggestion of the use of fuzzy set theory together with stochastic PNs in system modeling, is that, a deeper analysis and understanding of the system can be attained. For further research, sensitivity analysis with respect to fuzzy parameters and application of the proposed approach in other fields rather than FMSs are recommended.

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