Expert Systems With Applications 46 (2016) 426–438

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

On the effect of subjective, objective and combinative weighting in multiple criteria decision making: A case study on impact optimization of composites

Mohammad Alemi-Ardakani[a], Abbas S. Milani[a,*], Spiro Yannacopoulos[a], Golnaz Shokouhi[b]

[a] School of Engineering, University of British Columbia, 3333 University Way, Kelowna, BC, Canada V1V 1V7
[b] AS Composite Inc., 835 Rue Bancroft, Pointe-Claire, QC, Canada H9R 4L6

ARTICLE INFO

Keywords:
Fiber reinforced polymer composites
Impact design
Fiber architecture selection
Multiple criteria decision making
Criteria weighting

ABSTRACT

To date, no specific framework has been developed to guide composite structure designers to select the optimum fiber types and fabric weave patterns for a given application. This article aims to, first, investigate the effect of weighting methods in multiple criteria decision making (MCDM) and then arrive at a systematic framework for optimum weave pattern selection in fiber reinforced polymer (FRP) composites. Namely, via measured data from an industrial case study, the TOPSIS MCDM technique has been applied to choose the best candidate among different polypropylene/glass laminates. As an input to TOPSIS, different types of subjective and objective weighting methods were initially compared to assess the role of relative importance values (weights) of design criteria. These included the Entropy method, the modified digital logic (MDL) method, and the criteria importance through inter-criteria correlation (CRITIC) method. Next, two new subjective weighting methods, named 'Numeric Logic (NL)' and 'Adjustable Mean Bars (AMB)' methods, were introduced to give more practical and effective means to the decision makers during the weighting of criteria. In particular, compared to the MDL, the NL method increased the accuracy of assigned weights for an expert DM. On the other hand, the AMB provided a more interactive, visual approach through MCDM weighting process for less experienced DMs. Finally, a generalized combinative weighting framework is presented to show how different types of weightings may be combined to find more reliable rankings of alternatives. The combinative weighting could specifically accommodate different scenarios where a group of designers are involved and have different levels of experience, while given a large number of alternatives/criteria in highly nonlinear applications such as impact design of composite materials.

Abbreviations

AE	Absorbed Energy
AMB	Adjustable Mean Bars weights
CRITIC	Criteria Importance through Inter-criteria Correlation
CM	Combinative Weights
DM	Decision Maker
EVD	Exterior Visible Damage
EW	Equal Weights
FEA	Finite element analysis
ID	Interior Damage
RLFT	Relative Loss of Flexural Toughness due to impact
RLUFS	Relative Loss of Ultimate Flexural Strength due to impact
MCD	Maximum Central Deflection
MCDM	Multi-Criteria Decision Making
MDL	Modified Digital Logic method
NL	Numeric Logic weights
FT	Flexural Toughness (of healthy sample)
UFS	Ultimate Flexural Strength (of healthy sample)
PW	Plain Woven
PM	Project Manager
RF	Reaction Force
ROC	Rank Order Centroid weights
RR	Rank Reciprocal weights
RS	Rank Sum weights
TW	Twill Woven
UD	Unidirectional
UW	Unbalanced Woven
WPM	Weighted Product Model
XMT	X-ray Microtomography Technique

* Corresponding author. Tel.: +250 807 9652; fax: +250 807 9850.
E-mail addresses: mohammad.alemi@alumni.ubc.ca (M. Alemi-Ardakani), abbas.milani@ubc.ca (A.S. Milani), spiro.yannacopoulos@ubc.ca (S. Yannacopoulos), golnaz.shokouhi@ascomposite.com (G. Shokouhi).

http://dx.doi.org/10.1016/j.eswa.2015.11.003
0957-4174/

Variables

A_i	Alternatives or materials ($i = 1, \ldots, m$)
$A*$	Positive-Ideal Solution
A^-	Negative-Ideal Solution
C_j	Criterion j or material properties ($j = 1, \ldots, n$)
C_i^*	Similarities to Positive-Ideal Solution
C_{jk}	Comparative Weight
r_{ij}	Normalized element of decision matrix
S_i^*	Separation from Positive-Ideal Solution
S_i^-	Separation from Negative-Ideal Solution
v_{ij}	Weighted normalized element of decision matrix
w_j	Weight or importance of criteria j
x_{ij}	Elements of decision matrix, ith alternative or material, jth criterion

Equations

1	Equal weights
2	Rank sum weights
3	Rank reciprocal weights
4	Rank order centroid weights
5	Adjustable mean bars weights
6	Modified digital and numeric logic weights
7–10	Entropy weighting method
11–12	CRITIC weighting method
13	Combinative weights
14	Modified combinative weights
15–19	Modified combinative weights for 3 scenarios
A1–A7	TOPSIS formulation

1. Introduction

Despite several advantages offered by fiber reinforced polymer (FRP) composites, such as low weight and yet high mechanical performance, their optimum use in specific applications including high-speed impact still requires development of new design methodologies, along with accurate numerical modeling and prediction tools (Alemi-Ardakani, Milani, Yannacopoulos, & Borazghi, 2015a). Currently, in order to avoid severe impact failures in composite structures, trial and error methods are most often employed by manufacturers through varying design parameters such as the composite lay-up and the shape of structures. In addition, most design decisions are made under *multiple*, often *conflicting* and *inter-dependent,* criteria. To exemplify this state of complexity in impact design of FRPs, the recent experimental case study (Alemi-Ardakani, Milani, Yannacopoulos, & Shokouhi, 2015b) suggested that the application of 'Multiple Criteria Decision Making' (MCDM) methods is paramount to select structures that can satisfy a multitude of design criteria at the same time.

Industrial practitioners and researchers frequently employ different MCDM and criteria weighting techniques through their design of expert systems. For example, Monghasemi, Nikoo, Khaksar Fasaee, and Adamowski (2015) used a multi-objective algorithm incorporating the NSGA-II (non-dominated sorting generic algorithm) during a highway construction project to find optimum design alternatives. In the weighting process, they used Shannon's entropy technique to weigh the conflicting criteria of time, cost and quality. In another research, Cobuloglu and Büyüktahtakın (2015) proposed a stochastic analytical hierarchy process (AHP) weighting method for MCDM in design of an expert system for sustainable biomass crop selection. Their selection matrix included 16 sub-criteria from three main sustainability criteria categories; namely economic, environmental and social categories. Yavuz, Oztaysi, Cevik, and Kahraman (2015) used a hierarchical hesitant fuzzy linguistic model to utilize the linguistic evaluation of multiple experts in selection of alternative-fuel vehicles. Different MCDM approaches such as ELECTRE, TOPSIS and the Grey Theory were studied and compared in the work by Özcan, Elebi, and Esnaf (2011), specifically for expert selection of warehouse locations. The results of TOPSIS and ELECTRE appeared to be similar, despite their very different calculation algorithms. However, given the high sensitivity of criteria weights in most design case studies similar to those above, it has not been shown how different 'subjective', 'objective' and 'combinative' criteria weighting methods in MCDM would differently capture the expertise of the same (given) designer, along with statistical characteristics of the measured data.

To address and exemplify the above effect, in the present work a set of common weighting methods from the literature (Modified Digital Logic, CRITIC, and Entropy methods) have been selected and tested against a same composite designer and the same data matrix from a FRP impact design case study. In addition, two new weighting methods, namely an adjustable mean bars (AMB) method and a modified digital logic (NL) method, have been introduced for the first time, along with a new generalized framework for combining different types of weighting methods, to accommodate different experience levels of decision makers. It is argued that the NL can increase the accuracy of weighting for an expert designer, while the AMB can provide a more intuitive direction through weighting process for a decision maker with potentially less experience/information, especially for complex design problems such as FRP impact where the underlying mechanical theories are still under development. For the selection/ranking stage of MCDM problem, among various techniques, the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) (Hwang & Yoon, 1981) has been employed owing to its popularity and efficiency. Examples of other common selection methods in the reported literature include Lexicographic (Paul Yoon & Hwang, 1995), Elimination by Aspect (Tversky, 1972), Simple Additive Weighting (SAW) (Fishburn, 1967), Weighted Product Method (Bridgman, 1922), ELECTRE (Roy, 1991), Median Ranking (Cook & Seiford, 1978), PROMETHEE (Vincke & Brans, 1985), and Analytic Hierarchy Process (AHP) (Saaty, 1980).

1.1. Case study description

The MCDM problem herein is based on the experimental data obtained by Alemi-Ardakani et al., 2015b via an industrial case study where the ultimate goal is to choose the most promising fiber reinforcement architecture for impact applications. Four thermoplastic composite candidates have been presented: plain woven (PW), twill woven (TW), unbalanced woven (UW), and unidirectional fiber tape (UD). Nine attributes (design criteria) were recommended including the reaction force during impact (RF), absorbed impact energy (AE), the maximum central deflection of the laminate (MCD), areal fraction of induced interior damage (ID), the exterior visible damage area (EVD), ultimate flexural strength of the healthy (non-impcated) sample (UFS), the relative loss of ultimate flexural strength due to impact (RLUFS), flexural toughness of healthy sample (FT), and the relative loss of flexural toughness due to impact (RLFT). Table 1 summarizes the matrix of experimental data obtained from drop tower impact testing, four-point flexural testing, and non-destructive damage evaluation (visual inspections and x-ray microtomography). For a general impact-resistant structure such as a roadside barrier (Fig. 1), the criteria AE, UFS and FT would be benefit-like (i.e., the higher the better), while RF, MCD, ID, EVD, RLUFS and RLFT would be cost-like (i.e., the lower the better). Table 2 shows the order of preference of candidate materials within each column (design attribute) of Table 1; i.e., one-factor-at-a-time (OFAT) optimization based on the objective related to each specific criterion. Notably, the order of preference of candidate materials is not identical between any two columns in Table 2, showing *the highest level of criteria conflicts,* hence the critical need for a robust weighted MCDM approach in composite impact optimization problems. Methodological considerations of the proposed methods are presented in Section 2, followed by the case study results and discussions in Section 3. Section 4 includes concluding remarks and potential future work.

Table 1
Decision matrix of the MCDM case study based on the drop weight tower, visual damage inspection, x-ray microtomography and four-point flexural bending experiments conducted by Alemi-Ardakani, Milani, Yannacopoulos, and Shokouhi, 2015; The criteria AE, UFS and FT are assumed to be benefit-like (i.e., the higher the better), while RF, MCD, ID, EVD, RLUFS and RLFT are cost-like (i.e., the lower the better).

Materials	Impact test (dynamic properties)			Nondestructive evaluation		Pre- and post-impact flexural test (quasi-static properties)			
	RF (N)	**AE** (J)	**MCD** (mm)	**ID** (%)	**EVD** (mm^2)	**UFS** (Mpa)	**RLUFS** (%)	**FT** (kN/m^2)	**RLFT** (%)
PW	32,018	36.15	11.87	0.413	3.50	262.22	18.60	358.12	17.83
TW	34,121	37.21	12.43	0.322	3.03	222.67	7.34	294.41	11.08
UW	31,286	42.76	13.22	0.504	3.07	225.80	20.17	226.04	14.70
UD	29,514	69.14	12.37	0.173	0.44	206.59	31.95	148.25	29.79

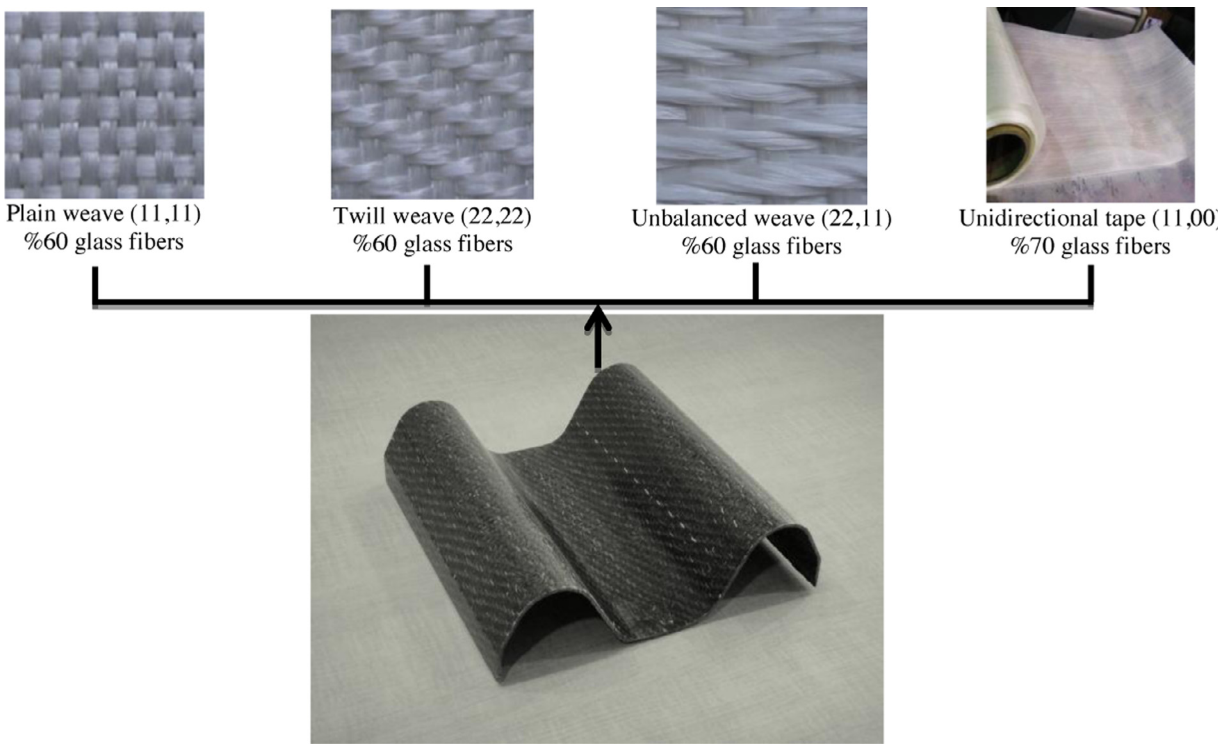

Fig. 1. Prototype of a semi-rigid PP/glass guardrail with four given material options for the composite plies.

Table 2
Ranking of candidate materials based on individual criteria (i.e., single objective optimizations).

Materials	Impact test (dynamic properties)			Nondestructive evaluation		Pre- and post-impact flexural test (quasi-static properties)			
	RF	AE	MC	ID	EVD	UFS	RLUFS	FT	RLFT
PW	3	4	1	3	4	1	2	1	3
TW	4	3	3	2	2	3	1	2	1
UW	2	2	4	4	3	2	3	3	2
UD	1	1	2	1	1	4	4	4	4

2. Methods

2.1. TOPSIS multiple criteria decision making

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) was originally proposed by Hwang and Yoon, 1981 and later modified by Yoon, 1987 and Hwang, Lai, & Liu, 1993. The concept of this method is to find the optimum alternative in a given decision space such that it has simultaneously the shortest distance from the so called 'positive-ideal solution' and the farthest distance from the 'negative-ideal solution'. The positive- and negative-ideal solutions are often artificial (infeasible practically) and are merely hypothesized in TOPSIS for ensuring the best performance of the chosen alternative. It should also be noted that TOPSIS is among the compensatory type of MCDM methods where trade-offs between decision attributes (criteria) are allowed. More specifically, a good performance of a material candidate under one design attribute can compensate the poor performance of the material under some other attributes. This feature of the method can be suitable in near-the-end stages of a design process where short-listed material candidates have met the minimum design requirements and the question remained is which material can maximize the overall performance of the structure under 'all' given criteria. Due to its simplicity and efficiency, TOPSIS has been widely used in the past in a diverse range of areas. As a few examples, this technique was used by Davoodi, Sapuan, Ahmad, Aidy, and Khalina, (2011), for the design of a car bumper beam made of hybrid bio-composites, by Pakpour, Olishevska, Prasher, Milani, and Chénier, 2013, for optimum DNA extraction from agricultural soil samples, and by Jee and Kang, 2000 for the material selection of a flywheel. Behzadian, Khanmohammadi Otaghsara, Yazdani, and Ignatius, 2012 reviewed 266 scholarly articles on the use of TOPSIS in other applications such as (a) supply chain management and logistics, (b) design, engineering and manufacturing systems, (c) business and marketing management, (d) health, safety and environment management, (e) human resources management, (f) energy management, (g) chemical engineering, (h) and water resources management. The review referred to 103 different journal publications between 2000 and 2012. Implementation steps of the TOPSIS method can be found in Appendix A.

2.2. Criteria weighting

In order to successfully apply any standard compensatory MCDM solution method such as TOPSIS, some critical assumptions should be taken into account: (1) the decision matrix must satisfy design requirements, (2) the desired attribute values are normally in a monotonically increasing or decreasing form (i.e., the higher the better, or the lower the better), (3) measured units should be commensurable (Behzadian et al., 2012) otherwise test data should be normalized, (4) criteria weights should be assigned by the designer as related to each given application, and (5) in the case of changes in the decision matrix (e.g., adding/removing one material), the entire calculation process must be repeated as it can affect the final ranking scheme. Perhaps the most critical input in TOPSIS, similar to most other MCDM methods, is the assignment of criteria weights which can be based on subjective, objective, or combinative techniques.

Subjective weighting methods rely on the expert-opinion while the emphasis of the objective methods is on the statistical evaluation of data given in a decision matrix. Each of these techniques has its own advantages and disadvantages. Potential uncertainty in expert judgment is the main disadvantage of the subjective methods, while the objective methods do not benefit from the expertise and experience of designers. Examples of the well developed subjective weighing techniques include the Digital Logic and Modified Digital Logic methods (Dehghan-Manshadi, Mahmudi, Abedian, & Mahmudi, 2007); Weighted Least-Square Method (Chu, Kalaba, & Spingarn, 1979); Delphi method (Hwang & Lin, 1987); Simple Multiattribute Rating Technique (SMART) (Edwards, 1977) and its modified versions including SMARTS (von Winterfeldt & Edwards, 1986) and SMARTER (Barron & Barrett, 1996a, 1996b; Edwards & Barron, 1994); Simos' procedure (Simos, 1990); Revised Simos' procedure (Figueira & Roy, 2002) for single decision making and the extended version for group decision making (Shanian, Milani, Carson, & Abeyaratne, 2008). These techniques specify the weights solely based on the preferential judgments of the decision maker (DM) and as the number of attributes increases, they can become intricate. As stated earlier, due to several conditions such as the lack of experience, imprecise information, limited capability of the DM for analyzing and correlating attributes and intangible nature of criteria, sometimes the DM may not be able to assign precise weights to criteria (Kahneman, Slovic, & Tversky, 1982; Weber, 1987). In order to solve this problem, well-stablished objective weighting techniques such as Entropy (Hwang & Yoon, 1981) and Criteria Importance through Intercriteria Correlations (CRITIC) (Diakoulaki, Mavrotas, & Papayannakis, 1995) are employed to extract statistical (unbiased) weights through dispersion analyses of a given decision matrix. Such objective methods, however, ignore the valuable input that can be gained from the DM's expertise via subjective weighting. Accordingly, some researchers such as (Jahan, Mustapha, Sapuan, Ismail, & Bahraminasab, 2011, 2011b) highly recommend 'combinative weighting' methods to account for both types of subjective and objective weighting and arrive at a single, aggregated set of criteria weights.

In the present work, next to employing the MDL, CRITIC, and Entropy methods, two new subjective techniques (a Numeric Logic method and an Adjustable Mean Bars method) are proposed with the goal of simplifying the judgment process for DMs with different levels of experience, especially when the number of criteria in the decision space becomes large. In addition, the combinative weighting method proposed by Jahan et al., 2011 has been slightly modified to adjust to different levels of experience of DMs, or for group decision making environments. Finally the discussed weighting methods and the TOPSIS technique are integrated to develop the strategy for impact optimization of FRP composite structures.

2.2.1. Adjustable Mean Bars (AMB) direct weighting method

The simplest method of objective weighting is the direct complete weight elicitation (Hwang & Yoon, 1981) where a highly experienced DM is able to assign the relative importance values for all criteria at once. This method may not be advised for all the DMs because of its highly intuitive nature and potential inaccuracies in final ranking. Accordingly, less experienced DMs often opt to give an equal weighting (EW) to criteria; however this approach can then become too conservative in some sensitive applications. Ordinal weighting techniques have been proposed in the literature to address this problem and assist the DMs; such as the rank sum (RS) weighting and rank reciprocal (RR) weighting (Stillwell, Seaver, & Edwards, 1981), as well as the rank order centroid weighting (ROC) (Edwards & Barron, 1994). In these techniques, the DM first ranks the attributes based on their priorities and then assigns weights in a descending order ($w_1 > w_2 > \cdots > w_n$), starting from the most important to the least important attribute. (Barron & Barrett, 1996b) compared the efficiency of the EW, RS, RR and ROC techniques based on Eqs. (1)–(4) using more than 10,000 test cases. They reported that ROC outperforms the other techniques with the order of ROC > RR > RS > EW.

$$w_j(EW) = \frac{1}{n} \tag{1}$$

$$w_j(RS) = \frac{2(n+1-R_j)}{n(n+1)} \tag{2}$$

$$w_j(RR) = \frac{1/R_j}{\sum_{j=1}^{n} 1/R_j} \tag{3}$$

$$w_j(ROC) = \frac{1}{n}\sum_{k=j}^{n}\frac{1}{k} \tag{4}$$

n represents the number of attributes ($j = 1, 2, \ldots, n$) and R_j is the preferred rank of attribute *j;* $R_j = 1$ represents the most important attribute. In Eq. (4), k indicates the number of alternatives to be found as optimum (usually $k = 1$). In order to satisfy $\sum_{j=1}^{n} w_j = 1$, the RS and RR weights should be normalized, e.g., with respect to the sum of weights.A limitation within the above objective weighting methods may be that they are merely based on criteria ranks and, thus, the distance between each consecutive criteria weights remains constant (i.e., the DM cannot give an extra emphasis to some specific criteria). For example, Table 3 shows the ROC weights for several cases with different number of attributes ranging from 2 to 9. As it can be seen these set of weights are constant and independent of the opinion of DM and data values.

An 'Adjustable Mean Bars' (AMB) weighting method is proposed in this section with the goal of keeping the simple nature of direct weighting method, while allowing the DM to interactively assign more emphasis on specific criteria of interest. Depending on the total number of attributes (n), this method can take up to $n-1$ sub-weighting steps. In each step, the DM picks the most important attribute(s) and assigns a numeric weight between 0 and 1 according to Eq. (5):

$$w_{j,\ AMB} = \frac{1-\sum_{i=1}^{m} w_i}{n-m} + k_j\frac{1}{n^2} \tag{5}$$

w_i represents the weight of attribute i calculated in the previous steps and m is the total number of attributes weighted in the previous

Table 3
ROC weights for different number of attributes.

Rank j	Number of attributes							
	2	3	4	5	6	7	8	9
1	0.7500	0.6111	0.5208	0.4567	0.4083	0.3704	0.3397	0.3143
2	0.2500	0.2778	0.2708	0.2567	0.2417	0.2276	0.2147	0.2032
3		0.1111	0.1458	0.1567	0.1583	0.1561	0.1522	0.1477
4			0.0625	0.0900	0.1028	0.1085	0.1106	0.1106
5				0.0400	0.0611	0.0728	0.0793	0.0828
6					0.0278	0.0442	0.0543	0.0606
7						0.0204	0.0335	0.0421
8							0.0156	0.0262
9								0.0123
$\sum w_j$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

steps. $\frac{1}{n^2}$ is the minimum step size (unit vector) allowed for adjusting the weights, and k_j is the emphasis factor of criteria j defined by the DM. This method is named adjustable mean bars (AMB) because, in each step, the DM sets the height of $(m-n)$ number of mean bars equal to the average of the remaining weights $(\frac{1-\sum_{i=1}^{m} w_i}{n-m})$ and then adjusts the weight of the most important attribute(s) by increasing their height(s) by $(k_j \frac{1}{n^2})$. It is also to note that the DM gives a positive integer value to k_j according to the required emphasis of criteria j, with a constraint that the calculated weight in each step must be smaller than the assigned weights in the previous steps. For instance, $k_j = 1$ would mean that the DM would like to raise the relative importance of criteria j only one unit step (quantified as $\frac{1}{n^2}$) compared to the previous step. The second constraint is that the summation of all final weights should be equal to one ($\sum_{j=1}^{n} w_{j,AMB} = 1$).

Fig. 2 visually illustrates how this method works, as an example where the total number of attributes is $n = 7$. The solid and hollow bars in each step in Fig. 2 demonstrate the weighted and yet-to-be-weighted attributes, respectively. The heights of the solid bars show the AMB weights. For better clarity, the procedure of weighting in this example is outlined below:

Step 0 (Default condition): all attributes are equally important. Thus, the default height of mean bars is $w = \frac{1}{7} \cong 0.14$. The step size is calculated to be $\frac{1}{7^2} \cong 0.02$.

Step 1: the DM specifies that attributes 3 and 7 are the most important attributes with the relative emphasis factor of 3 ($k_3 = k_7 = 3$). This means that the DM considers three unit step rise for attributes 3 and 7 compared to the other attributes. As a result, the new weights for attributes 3 and 7 became: $w_3 = w_7 = 0.14 + (3)(0.02) = 0.20$.

Step 2: the remaining weights with a total value of 0.6 (i.e., $1 - 2(0.2) = 0.6$) should be distributed between the unweighted attributes (1, 2, 4, 5, 6). So, the height of current hollow bars is set to $0.6/5 = 0.12$. Then, according to DM's opinion, the second most important group of factors are attributes 1, 2 and 5 with the importance factor of one unit step more than attributes 4 and 6. Accordingly, the corresponding weights are adjusted based on Eq. (5) as: $w_1 = w_2 = w_5 = \frac{1-2(0.2)}{5} + 1(0.02) = 0.14$

Step 3: the mean height of the two remaining bars (i.e., for attributes 4, 6) becomes $\frac{1-[2(0.2)+3(0.14)]}{2} = 0.09$. Between these attributes, the DM specifies one unit higher importance for attribute 6; i.e., $k_6 = 1$. Subsequently, the AMB weight for attribute 6 becomes:

$$w_6 = \frac{1 - [2(0.2) + 3(0.14)]}{2} + 1(0.02) = 0.11$$

Step 4: the weight of the remaining attribute (w4) is calculated as:

$$\sum_{j=1}^{7} w_j = 1 \ \rightarrow w_4 = 1 - [2(0.2) + 3(0.14) + 0.11] = 0.07$$

It should be noted that after completing one round of weighting, the DM can compare the weights distribution to verify whether it is satisfactory based on his/her initial perception. In the case of dissatisfaction, he/she can go back to any of the above steps and change the k values and repeat the procedure until a suitable distribution is found.

2.2.2. Modified Digital Logic (MDL) method

For applications in which the number of design attributes is fairly large (similar to the current impact optimization case with 9 criteria), assigning the importance weights among multiple criteria simultaneously may be difficult for the DM. The Digital Logic (DL) method has been developed to address this problem by suggesting pair-wise comparisons of criteria (which is essentially similar to the approach behind the Analytic Hierarchy Process/AHP (Saaty, 1980)). The DL method has proven to be successful in increasing the reliability of decision results to a large extent, while providing a simple intuitive procedure for implementation purposes. In this method, two criteria are compared at a time and receive binary scores of 0 or 1 depending on their level of priority to the decision maker (1 for more important criterion, 0 for the less important one). Dehghan-Manshadi et al., 2007 developed a Modified Digital Logic (MDL) method and others researchers including Diakoulaki et al., 1995 and Chakraborty and Chatterjee, 2013 employed it to (a) give the possibility to the decision maker to assign equal weights to two attributes, and (b) not to eliminate the least important criterion from the decision matrix. This enhancements were achieved by changing the aforementioned binary scoring scheme from {0 and 1}, to a digital scoring scheme of {1, 2 and 3} to represent the less (1), equal (2), or more important (3) criteria. After all pair-wise comparisons are made, the MDL weights are calculated as:

$$w_{j,\ MDL} = \frac{\sum_{k=1}^{n} C_{jk}}{\sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk}}, \ \ j \ and \ k = \{1, \ldots, n\} \ and \ j \neq k \tag{6}$$

If two criteria j and k are equally important, then $C_{jk} = C_{kj} = 2$, otherwise $C_{jk} = 3$ and $C_{kj} = 1$ if the attribute k is more important than the attribute j. If the attribute k is less important than the attribute j, then $C_{jk} = 1$ and $C_{kj} = 3$.

2.2.3. Numeric Logic (NL) method

A slight modification is applied to the MDL method to allow the more experienced DMs apply more precise weighting among criteria. In the proposed Numeric Logic (NL) method, during each pair-wise comparison, the decision maker can assign any numeric weight (w_1) between 0 and 1 to the first criterion, and $w_2 = 1 - w_1$ becomes the weight of the other criterion. In other words, in this case, weights during pair-wise comparisons are not limited to 0.25, 0.50 and 0.75 as in the MDL method. The final NL weights can be calculated again via the same Eq. (6) with the only difference being that C_{jk} this time represents the arbitrary numeric weights between criteria j and k (rather than digital scores).

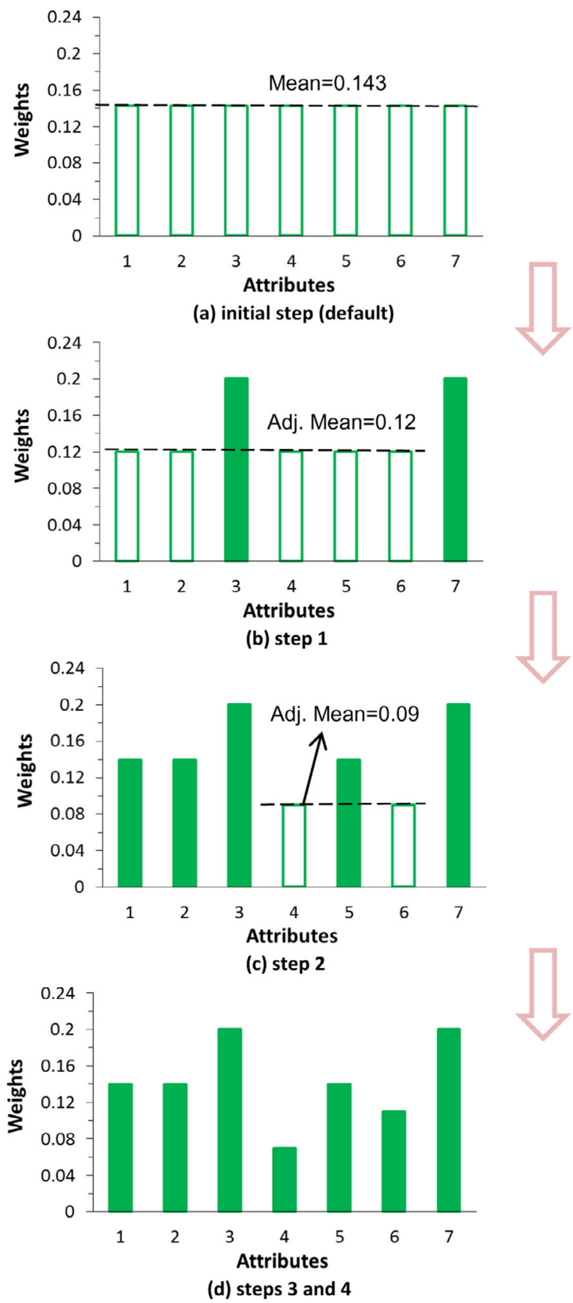

Fig. 2. Adjustable Mean Bars (AMB) weighing steps for a hypothetical example with 7 attributes: (a) initial state with equal weighing, (b) attributes 3 and 7 are weighted, (c) next attributes 1, 2 and 5 are weighted, and (d) finally attributes 4 and 6 are weighted; solid and hollow bars demonstrate the weighted and yet-to-be weighted attributes in each step, respectively.

2.2.4. Entropy method

Depending on a given application, the term "entropy" has different meanings. For example, in physics it implies the level of disorder in a system, and in transportation models it shows the dispersal of trips between two locations (Islam & Roy, 2006). For some events or applications it quantifies the degree of randomness or fuzziness (Güneralp, Gertner, Mendoza, & Anderson, 2007). In MCDM, entropy relates to the degree of diversity within an attribute dataset. The greater the degree of the diversity, the higher the weight of that attribute. In another words, the smaller the entropy within the data associated to the attribute, the greater the discrimination power of the attribute in changing the ranks of alternatives. The steps for calculating Entropy weights are summarized below (Hwang & Yoon, 1981; Lotfi & Fallahnejad, 2010).

Step 1. Normalization

Since measured data under different criteria can be of different units or scales, a given decision matrix (e.g., Table 1) should be first transformed into a dimensionless space via:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}; \quad i = 1, \ldots, m \;\&\; j = 1, \ldots, n \tag{7}$$

where x_{ij} is an element of the decision matrix corresponding to the ith alternative and the jth criterion. m is the total number of alternatives (here $m = 4$; i.e., materials PW, TW, UW and UD). n is the number of criteria (here $n = 9$; i.e., the nine impact design criteria).

Step 2. Calculation of the entropy (E_j) and the degree of diversity (d_j)

Entropy within the datasets of the normalized decision matrix for the jth criterion can be calculated via:

$$E_j = -\frac{1}{\ln(m)} \sum_{i=1}^{m} p_{ij} \ln p_{ij} \tag{8}$$

The degree of diversity (d_j) is calculated as:

$$d_j = 1 - E_j \tag{9}$$

Step 3. Calculation of objective weights (w_j)

The last step is the linear normalization of d_j to find the relative weight of each criterion:

$$w_{j,entropy} = \frac{d_j}{\sum_{k=1}^{n} d_k} \tag{10}$$

2.2.5. Criteria Importance through Inter-criteria Correlation (CRITIC) method

In addition to the contrast intensity of attribute datasets in the decision matrix (the notion that was quantified by the Entropy method), there is another concept that is more recently taken into consideration by MCDM researchers. Diakoulaki et al., 1995 noticed that the higher the level of interdependency between attributes, the larger error in the ranking outcome. Criteria importance through inter-criteria correlation (CRITIC) was proposed by Diakoulaki et al., 1995, as a new objective weighting method that can consider correlations between all given criteria. The method by Diakoulaki et al., 1995 also included the contrast intensities (by means of standard deviations of criteria) and combined them with the weights from correlation analysis. Alternatively, the model by Jahan et al., 2011 employed the Pearson product-moment correlations only and excluded the standard deviations of criteria in weighting formulations. Since in the present work the contrast intensities will be taken into account by means of the Entropy method, we follow the same formulation as by Jahan et al., 2011 to calculate the correlation weights as follows .

Step 1. Finding the correlation coefficients

R_{jk} values via the Pearson product-moments represent the correlation between the criteria j and k:

$$R_{jk} = \frac{\sum_{i=1}^{m} (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)}{\sqrt{\sum_{i=1}^{m} (x_{ij} - \bar{x}_j)^2 \sum_{i=1}^{m} (x_{ik} - \bar{x}_k)^2}}; \quad (\mathrm{j\,\&\,k} = 1, \ldots, \mathrm{n}) \tag{11}$$

Where m, $\bar{x}_j$ and $\bar{x}_k$ are the number of materials and the average values of criteria j and k, respectively. R_{jk} close to +1 or −1 indicates highly correlated criteria, while R_{jk} close to 0 indicates no correlation.

Step 2. Calculating the CRITIC weights

The next step is to calculate the weight of each criterion using its correlation to all other criteria:

$$W_j = \frac{\sum_{k=1}^{n} (1 - |R_{jk}|)}{\sum_{j=1}^{n} (\sum_{k=1}^{n} (1 - |R_{jk}|))}; \; j \text{ and } k = 1, 2, \ldots, n; \; k \neq j \tag{12}$$

Note that the above correlation weighting formulation is regardless of the type of criteria (i.e., they can be of the higher the better, or the lower the better type).

2.2.6. A Modified Combinative Weighting (MCW) framework

Based on the above presented weighting methods, since there are more than one set of weights (some subjective and some objective), for a final decision making it is needed to assemble these weights into one single set. Jahan et al., 2011 proposed a combinative weighting (CW) method to combine three sets of weights corresponding to subjective, objective, and correlation weights ($w_{j,1}$, $w_{j,2}$ *and* $w_{j,3}$) via the following formulation:

$$w_{j,CW} = \frac{[(w_{j,1}).(w_{j,2}).(w_{j,3})]^{1/3}}{\sum_{j=1}^{n}[(w_{j,1}).(w_{j,2}).(w_{j,3})]^{1/3}},\ j = \{1, 2, \ldots, n\} \quad (13)$$

The assumption of Eq. (13) is that the authority/power of all three sets of weights is equal. A Modified Combinative Weighting (MCW) method is proposed in this work to enable the analyst to assign different powers, α_p, to different weighting systems. The MCW suggests:

$$w_{j,MCW} = \frac{[(w_{j,1})^{\alpha_1}.(w_{j,2})^{\alpha_2}\cdots.(w_{j,m})^{\alpha_p}]^{1/(\alpha_1+\alpha_2+\cdots+\alpha_p)}}{\sum_{j=1}^{n}[(w_{j,1})^{\alpha_1}.(w_{j,2})^{\alpha_2}\cdots.(w_{j,m})^{\alpha_p}]^{1/(\alpha_1+\alpha_2+\cdots+\alpha_p)}},$$
$$j = \{1, 2, \ldots, n\} \quad (14)$$

where p and n are the number of weighting methods and attributes, respectively. This method may also be used during group decision making if, for example, the project manager is interested to give more power to the weights from a more experienced designer/DM in the group.

3. Case study results and discussions

The above described multi-criteria material selection approach and weighting methods are now applied on the decision making case study of the laminate impact optimization problem (Table 1), with a potential application of the selected material, e.g., in a composite highway guardrail (Fig. 1). In doing this, the five fundamental methods of AMB, MDL, NL, Entropy, and CRITIC are considered. For different objective methods (AMB, MDL, and NL), a DM (designer) with the same level of experience was employed. In addition, to mimic more real life decision-making circumstances, four scenarios are presumed based on how the project manager (let's say the final decision maker) would combine the set of weights under the MCW framework depending on his/her experience in systems engineering, as well as the level of confidence he/she has on the expertise of the company's designer in the composites impact field. Each ensuing set of combinative weights are then used as an input in the TOPSIS solution method to find the best laminate among the given four candidates (UD, UW, PW, TW).

It is known that roadside barriers can be categorized into three different groups according to their performances: (1) rigid, (2) semi-rigid, and (3) flexible barriers (Bank & Gentry, 2001). A flexible barrier is allowed to deform up to 4 m off of the roadway to stop the vehicle. The maximum acceptable deflection for a semi-rigid guardrail system is 1 m, whereas for a rigid system, no deflection is allowed and the errant vehicle should be redirected into traffic. The American Association of State Highway and Transportation Officials (AASHTO)—which is the institute in charge of developing standards on specifications, test protocols and guidelines for highway design and construction in the United States (Wikipedia, 2015)- in the report AASHTO M-180 (entitled *"Corrugated Sheet Steel Beams for Highway Guardrails"*) recommends semi-rigid steel guardrail systems for highways (AASHTO, 2008). Accordingly, criteria weightings by the DM in our case study are selected based on a semi-rigid design of composite guardrail (Fig. 2). AASHTO in collaboration with Federal Highway Administration (FHWA), has published a *NCHRP Report 350* (Ross, Sicking, Zimmer, & Michie, 1993) entitled *"Recommended Procedures for the Safety Performance Evaluation of Highway Features"*. In that report the post-impact vehicular trajectory, the maximum velocity and ride down acceleration that occupants experience during the crash has been mentioned among the main safety factors. Clearly these requirements can have roots in the impact reaction force, energy absorption, mechanical properties, and damage characteristics of the material used in the guardrail structure.

3.1. Adjustable Mean Bars (AMB) weights

Considering the above guardrail design guides, the DM for AMB weighting ranked the importance of attributes in Table 1 as: $w_{PIUFS} = w_{LUFS} = w_{PIFT} = w_{LFT} > w_{AE} > w_{RF} > w_{MCD} > w_{EVD} > w_{ID}$ with $k_{PIUFS} = k_{LUFS} = k_{PIFT} = k_{LFT} = k_{AE} = k_{RF} = 3\&k_{MCD} = 2\&k_{EVD} = k_{ID} = 1$. The subsequent AMB weighting procedure is summarized in Table 4 and shown graphically in Fig. 3.

3.2. Modified Digital Logic (MDL) weights

For the MDL method, the DM compared each two attributes at a time and assigned digital scores of 1, 2, or 3 to them, based on his/her perception of their relative importance. The results, using Eq. (6), are shown in Tables 5 & 6.

3.3. Numeric Logic (NL) weights

In contrast to the MDL approach where the DM assigned digital scores to the attributes, in the numeric logic (NL) method, he/she could assign any relative weights in a continuous scale during pairwise comparisons. The results of this method are shown in Tables 7 and 8.

3.4. Entropy weights

In order to find the entropy weights, first it was needed to normalize the decision matrix data as they were measured in different units/scales. Linearly normalized values using Eq. (7) are given in Table 9. Next, the entropy, the degree of diversity and the final objective weights (Table 10) were calculated according to Eqs. (8)–(10).

3.5. Criteria Importance through Inter-criteria Correlation (CRITIC) weights

Table 11 shows the inter-criteria Pearson correlations between the nice design criteria (R_{jk}) calculated via Eq. (11). The CRITIC weights were calculated using Eq. (12) and results are presented in Table 12. For illustrative purposes, Table 13 shows the results of TOPSIS and ranks of alternative materials based on different individual sets of weights presented in Table 12. It is clear that individual objective and subjective methods do not always agree.

3.6. Modified Combinative Weighting (MCW) method: different practical scenarios

Individual set of weights (w_{AMB}, w_{MDL}, w_{NL}, $w_{Entropy}$ and w_{CRITIC}) calculated in the previous sub-sections, resonate differences between different weighting methods, given that the DM has been the same for all of them. The last step of weighting procedure is to aggregate these set of weights and arrive at one resultant set of weights to implement in the TOPSIS model and rank the alternative materials.

Table 4
Steps of the AMB weighting by the DM in the impact optimization case study.

Step #	Attribute(s) to be weighted	Weighted attributed in previous steps (m)	Emphasis factor (k_j)	AMB weights (w_j)	Height of remained mean bars (unweighted attributes)
initial	–	–	–	–	0.111
1	UFS, RLUFS, FT, RLFT	0	3	0.147	0.082
2	AE	4	3	0.118	0.074
3	RF	5	3	0.110	0.061
4	MCD	6	2	0.085	0.050
5	EVD	7	1	0.062	0.037
6	ID	8	1	0.037	–

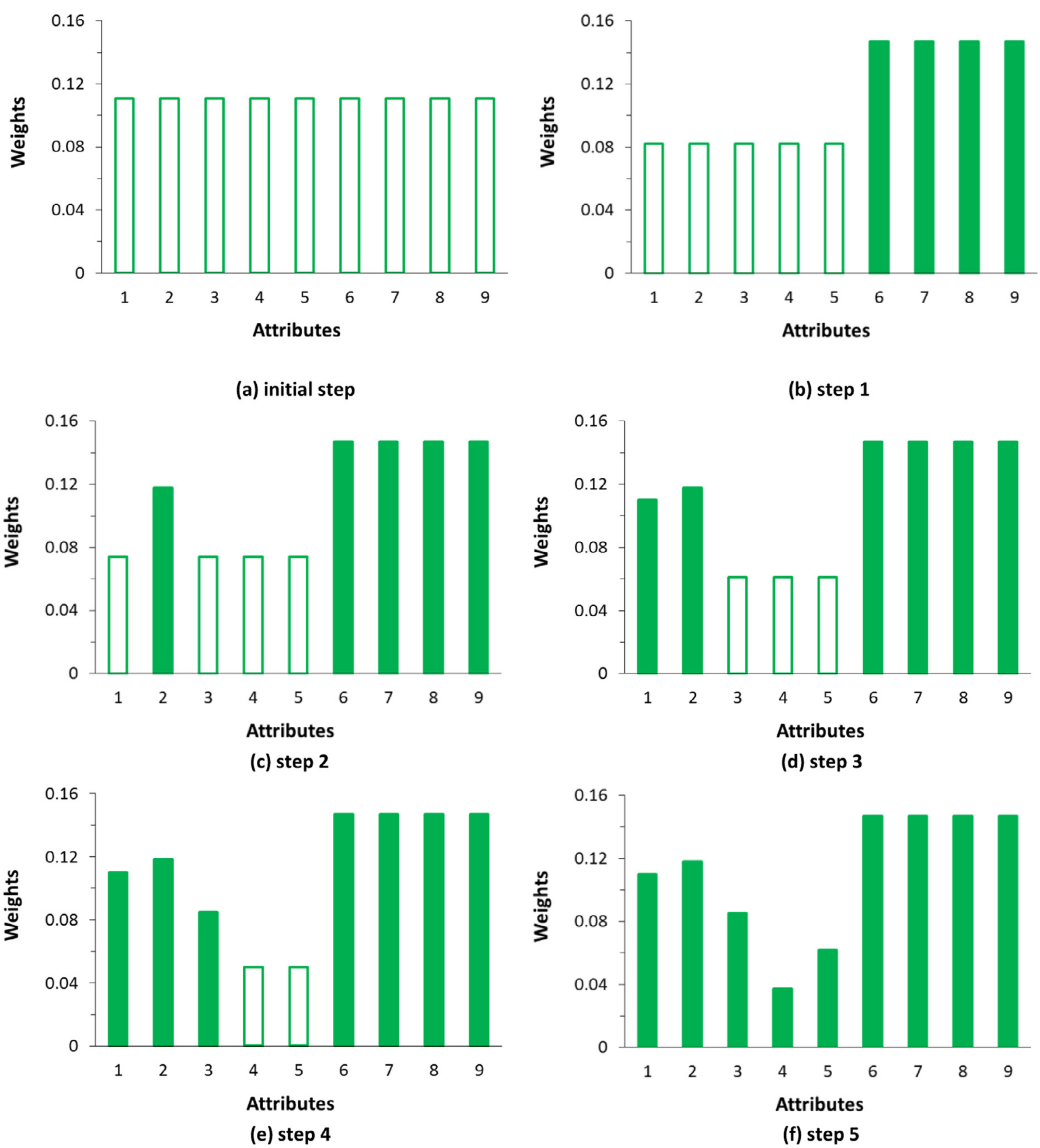

Fig. 3. The AMB weighing procedure for the impact optimization case study with 9 attributes; solid and hollow bars demonstrate the weighted and yet-to-be weighted attributes, respectively.

Table 5
MDL weighting by the DM in the present case study.

Attributes	Relative Digital Weights (C_{jk})																				
RF	1	3	3	3	1	1	1	1													
AE	3								3	3	3	2	2	2	1						
MCD		1							1							3	3	1	1	1	1
ID			1							1						1					
EVD				1							1						1				
UFS					3							2						3			
RLUFS						3							2						3		
FT							3							2						3	
RLFT								3							3						3

Table 6
Continuation of Table 5.

Attributes	Relative digital weights (C_{jk})															#Positive decisions	Weighting factors
RF																14	0.097
AE																19	0.132
MCD																12	0.083
ID	1	1	1	1	1											8	0.056
EVD	3					1	1	1	1							10	0.069
UFS		3				3				2	1	1				18	0.125
RLUFS			3				3			2			1	1		18	0.125
FT				3				3			3		3		2	22	0.153
RLFT					3				3			3		3	2	23	0.160

Table 7
NL weighting by the DM in the present case study.

Attributes	Relative numeric weights (C_{jk})																				
RF	0.3	0.8	0.9	0.9	0.2	0.2	0.1	0.1													
AE	0.7								0.8	0.9	0.9	0.5	0.5	0.5	0.4						
MCD		0.2							0.2							0.8	0.6	0.2	0.2	0.1	0.1
ID			0.1							0.1						0.2					
EVD				0.1							0.1						0.4				
UFS					0.8							0.5						0.8			
RLUFS						0.8							0.5						0.8		
FT							0.9							0.5						0.9	
RLFT								0.9							0.6						0.9

Table 8
Continuation of Table 7.

Attributes	Relative numeric weights (C_{jk})															Positive decisions	Weighting factors
RF																3.5	0.097
AE																5.2	0.144
MCD																2.4	0.067
ID	0.4	0.1	0.1	0.1	0.1											1.2	0.033
EVD	0.6					0.2	0.2	0.1	0.1							1.8	0.050
UFS		0.9				0.8				0.5	0.4	0.4				5.1	0.142
RLUFS			0.9				0.8			0.5			0.4	0.4		5.1	0.142
FT				0.9				0.9			0.6		0.6		0.5	5.8	0.161
RLFT					0.9				0.9			0.6		0.6	0.5	5.9	0.164

Table 9
Normalized decision making matrix ($\boldsymbol{p_{ij}}$); note that the criteria become dimensionless.

Materials	Impact testing (dynamic properties)			Nondestructive evaluation		Pre- and post-impact flexural testing (quasi-static properties)			
	RF	AE	MCD	ID	EVD	UFS	RLUFS	FT	RLFT
PW	0.25	0.20	0.24	0.29	0.35	0.29	0.24	0.35	0.24
TW	0.27	0.20	0.25	0.23	0.30	0.24	0.09	0.29	0.15
UW	0.25	0.23	0.26	0.36	0.31	0.25	0.26	0.22	0.20
UD	0.23	0.37	0.25	0.12	0.04	0.23	0.41	0.14	0.41

Table 10
Calculated entropy (E), degrees of diversity (d) and weights of importance (w) for different criteria according to the Entropy method.

Measures	RF	AE	MCD	ID	EVD	UFS	RLUFS	FT	RLFT
$\boldsymbol{E_j}$	0.999	0.972	0.999	0.953	0.886	0.997	0.922	0.965	0.950
$\boldsymbol{d_j}$	0.001	0.028	0.001	0.047	0.114	0.003	0.078	0.035	0.050
$\boldsymbol{w_{Entropy}}$	0.004	0.079	0.003	0.131	0.319	0.009	0.216	0.098	0.140

Table 11
Inter-criteria correlation factors ($|R_{jk}|$) according to the CRITIC method.

Attributes	RF	AE	MCD	ID	EVD	UFS	RLUFS	FT	RLFT
RF	1	0.83	0.12	0.34	0.73	0.34	0.99	0.72	0.89
AE		1	0.06	0.76	0.99	0.71	0.86	0.89	0.92
MCD			1	0.41	0.01	0.50	0.02	0.49	0.24
ID				1	0.85	0.58	0.43	0.52	0.70
EVD					1	0.75	0.77	0.87	0.88
UFS			Sym.			1	0.33	0.89	0.38
RLUFS							1	0.70	0.93
FT								1	0.66
RLFT									1

Table 12
Summary of the four subjective (EW, AMB, MDL and NL) and the two objective (Entropy & CRITIC) weighting methods.

Weights	RF (N)	AE (J)	MCD (mm)	ID (%)	EVD (mm2)	UFS (Mpa)	RLUFS (Mpa)	FT (kN/m2)	RLFT (kN/m2)
w_{EW}	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
w_{AMB}	0.110	0.118	0.085	0.037	0.062	0.147	0.147	0.147	0.147
w_{MDL}	0.097	0.132	0.083	0.056	0.069	0.125	0.125	0.153	0.160
w_{NL}	0.097	0.144	0.067	0.033	0.050	0.142	0.142	0.161	0.164
$w_{Entropy}$	0.004	0.079	0.003	0.131	0.319	0.009	0.216	0.098	0.140
w_{CRITIC}	0.109	0.071	0.221	0.123	0.077	0.126	0.106	0.081	0.086

Table 13
TOPSIS results based on the individual subjective and objective weighting sets presented in Table 12.

	TOPSIS scores (C^*)						Ranks					
	EW	MDL	NL	AMB	Entropy	CRITIC	EW	MDL	NL	AMB	Entropy	CRITIC
PW	0.44	0.55	0.54	0.56	0.319	0.45	3	2	2	2	4	3
TW	0.57	0.68	0.66	0.67	0.494	0.61	1	1	1	1	2	1
UW	0.38	0.49	0.49	0.50	0.325	0.37	4	3	3	3	3	4
UD	0.48	0.31	0.35	0.31	0.560	0.48	2	4	4	4	1	2

Based on Eq. (14), the following general formula may be written for the present case study:

$$w_{j,MCW} = \frac{\left[(w_{j,subjective})^{\alpha}.(w_{j,Entropy})^{\beta}.(w_{j,CRITIC})^{\gamma}\right]^{1/(\alpha+\beta+\gamma)}}{\sum_{j=1}^{n}\left[(w_{j,subjective})^{\alpha}.(w_{j,Entropy})^{\beta}.(w_{j,CRITIC})^{\gamma}\right]^{1/(\alpha+\beta+\gamma)}},\quad j=\{1,2,\ldots,n\} \tag{15}$$

Four different scenarios were proposed to use Eq. (15) by the project manager (PM) based on the background and the level of expertise of the original DM (designer), as well as the level of complexity that the PM would like to include in a sensitive application such as roadside barrier. These scenarios are as follows.

Scenario I: the DM has a relatively low level of experience and as a result he/she is not fully confident in assigning subjective weights. Hence, the PM opts to select the most conservative approach and assign equal weights to all criteria (i.e., the EW method in Eq. (1)). In this case, in Eq. (15) we have $\alpha = 1$ and $\beta = \gamma = 0$, and the MCW weights are equal to the EW weights:

$$w_{j,MCW-I} = w_{j,EW},\ j=\{1,2,\ldots,n\} \tag{16}$$

Scenario II: the DM has a high level of experience and, therefore, is capable of assigning subjective/application-based weights. Depending on his/her level of confidence, from low to high, he/she might use the MDL, NL or AMB techniques. Still, the PM does not take into account the statistical/objective weights (Entropy and CRITIC) and merely relies on the designer's input. Parameters in Eq. (15) for this case again are $\alpha = 1$ and $\beta = \gamma = 0$, and the MCW weights are equal to one of subjective weights (MDL, NL or AMB):

$$w_{j,MCW-II} = \left\{\begin{array}{c} w_{j,MDL} \\ or \\ w_{j,NL} \\ or \\ w_{j,AMB} \end{array}\right\},\quad j=\{1,2,\ldots,n\} \tag{17}$$

Scenario III: the PM is interested to include equally powered sets of objective weights into the final weighting scheme, with the aim of arriving at more accurate results. Hence, $\alpha = \beta = \gamma = 1$, and the final weights are calculated as:

$$w_{j,MCW-III} = \frac{\left[(w_{j,NL}).(w_{j,Entropy}).(w_{j,CRITIC})\right]^{1/3}}{\sum_{j=1}^{n}\left[(w_{j,NL}).(w_{j,Entropy}).(w_{j,CRITIC})\right]^{1/3}},\quad j=\{1,2,\ldots,n\} \tag{18}$$

Note that instead of NL, the results of MDL or AMB could be used in Eq (18).

Scenario IV: the DM possesses a very high level of expertise in impact design of composite structures. Accordingly, because of his/her high confidence in all assigned subjective weights, the PM opts to give more power to the designer's experience by increasing α from one to two; i.e. $\alpha = 2$ and $\beta = \gamma = 1$. Alternatively, he/she can make a group decision making and employ two different methods of subjective weights by two different DMs (so that potential methodological inconsistencies during weighting are taken into account). In this case study, the well-experienced DM has enforced the effect of the NL weights by power of two via the following equation:

$$w_{j,MCW-IV} = \frac{\left[(w_{j,NL})^{2}.(w_{j,Entropy}).(w_{j,CRITIC})\right]^{1/4}}{\sum_{j=1}^{n}\left[(w_{j,NL})^{2}.(w_{j,Entropy}).(w_{j,CRITIC})\right]^{1/4}},\quad j=\{1,2,\ldots,n\} \tag{19}$$

The final combinative weighting results of these four scenarios are presented in Table 14.

Table 14
Results of the four combinative weights based on four proposed scenarios to mimic the DM's level of experience.

Weights	RF (N)	AE (J)	MCD (mm)	ID (%)	EVD (mm2)	UFS (Mpa)	RLUFS (Mpa)	FT (kN/m2)	RLFT (kN/m2)
w_{MCW-I}	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
w_{MCW-II}	0.097	0.144	0.067	0.033	0.050	0.142	0.142	0.161	0.164
$w_{MCW-III}$	0.044	0.119	0.044	0.103	0.136	0.069	0.188	0.138	0.159
w_{MCW-IV}	0.056	0.128	0.050	0.080	0.109	0.085	0.180	0.147	0.165

3.7. Final TOPSIS ranking of the candidate laminates under MCW

The obtained sets of weights in Table 14 under different design scenarios were next applied to the TOPSIS to rank the four alternative laminates in the composite impact optimization problem under question. According to the results in Table 15, the degrees of similarity of each material option to the positive-ideal solution (i.e., Eq. (A7) in Appendix A), also called the TOPSIS scores, indicate that the first rank material is the TWILL woven laminate, under all the four weighting scenarios. The TOPSIS method based on the advanced weighting case (scenario 4) ranked and scored the candidates as:

$$\begin{pmatrix} Twill\ Woven \\ Plain\ Woven \\ Unbalanced\ Woven \\ Unidirectional \end{pmatrix} \rightarrow \begin{pmatrix} Rank\ 1 \\ Rank\ 2 \\ Rank\ 3 \\ Rank\ 4 \end{pmatrix};\ Scores = \begin{pmatrix} 0.65 \\ 0.49 \\ 0.46 \\ 0.38 \end{pmatrix}$$

Interestingly, for this case study the results of Scenarios 2 to 4 are identical. The only observed difference was between Scenario 1 (i.e. equal weights) and other scenarios. According to scenario 1 the UD laminate is ranked second while scenarios 2 to 4 all ranked PW as second. One main reason is that the UD laminate has a superior rank with respect to the individual RF, AE, ID, and EVD criteria as seen in Table 2 (specially looking at the actual measured values in Table 1, this laminate in terms of minimized external visible damage/EVD has been by far the best option). In contrast, UD has shown the poorest results under UFS, RLUFS, FT and RLFT criteria. In scenarios 2 to 4, the subjective weights like NL and AMB have been part of the weighting process. Namely in these subjective techniques, DM has given higher weights of importance to the criteria in which UD were weak (i.e. UFS, RLUFS, FT and RLFT) and lower importance to the criteria in which UD were strong, specially ID and EVD (see Table 14). Under scenario 2, which was purely based on designer's opinion, he/she has perceived that both the external and internal damage areas (EVD and ID) would be somewhat automatically reflected in the post-impact (residual) mechanical properties of the material and, hence, should not receive high weights. This notion on the DM's perceptions can also be clearly seen from the lower number of positive decisions that he/she has given during the MDL and NL method (Tables 6 and 8). Recalling the correlation coefficients in Table 11 (which were used in the CRITIC method), it becomes evident that as the DM had perceived, indeed the EVD has a high correlation (77%) with the relative loss of ultimate flexural strength (RLUFS), and 88% with the relative loss of flexural toughness (RLFT). However, the DM's experience may have been less accurate regarding the correlation of the internal damage (ID) with other properties. From a micromechanics point of view, the mode of induced failure mechanism (e.g., fiber pull-out, fiber breakage, matrix cracking, kinking, delamination, etc.) would play much more important role in the residual mechanical properties than the internal damage area fraction. Table 11 has also revealed that reaction force and absorbed energy have the highest correlation with the loss of mechanical properties. Mechanically, using unbalanced (twill) woven plies, the laminate can still be symmetric by applying [0/90] stacking sequence. However, each ply due to less crimping would be closer to the unweaved (UD) configuration and perhaps that is why the unbalanced twill woven (UW) laminate has absorbed more energy compared to the balanced plain weave in the current case study. Overall, from Table 1 it can be noted that the TW laminate performs comparably well under all criteria while showing the lowest loss of mechanical properties due to impact (i.e., both low values of RLUFS and RLFT), hence it has been chosen as the preferred weave pattern option within all the four MCW weighting scenarios.

Surprisingly, given the same decision maker and decision matrix, the individual sets of subjective weighting methods mostly resulted in the same ranking order of criteria and alternatives. However, different objective methods (here namely, Entropy and CRITIC) ranked some criteria and materials very differently in the performed example, and is suspected to be coincident in many other application areas because each objective weighting method is based on particular aspects/statistical measure of given decision matrix; whereas most subjective methods are merely based on the same expertise of a given DM and hence high-priority criteria should remain the same regardless of the selected method. For most effective decision makings, the latter observations highlight the importance of using a combinative weighting framework. Subsequently, the proposed MCW based on practical scenarios may be implemented to assist the selection of candidate materials based on the level of experience of each designer and/or the project manager.

The above discussions in this example show, on one hand, the importance and usefulness of MCDM models to the decision makers for complex systems such as impact of fiber reinforced composites, and on the other hand, the critical need of trying different weighting and solution methods before making a final decision.

4. Concluding remarks

A multi-criteria impact optimization of FRP laminates was conducted via some popular weighting methods in multiple criteria decision making (MCDM). Along with these weights, TOPSIS was employed as the solution/ranking method to consider severe conflicts among different impact design criteria and to arrive at an aggregated score for each laminate candidate. Weighting of the criteria were performed based on both subjective and objective techniques, as well as their combinations. Among several results of the work, some may be contributed to the design of FRP composite structures, while others relate to some new insight on MCDM weighting techniques in expert and intelligent systems, as follows.

From the viewpoint of composites design, a systematic MCDM approach was developed and exemplified to help designers in selecting optimum fiber reinforcement architectures. The lack of application of MCDM in the state-of-the-art decision making framework in the composites design practice is still noticeable in the literature, while dealing with such highly nonlinear material systems. A strong motivation of using MCDM is that the decision making under multiple composite design criteria can be archived systematically both at the individual and team levels.

In terms of specific MCDM weighting techniques, as discussed in the introduction section, there still exists a lack of understanding how different subjective, objective and combinative weighting techniques would differently capture the expertise of the same DM in practice. The conducted industrial case study on composite materials was designed to preliminarily address this gap and identify the role of different techniques during an actual decision making process. In addition, two new subjective (AMB and NL) and one combinative weighting methods (MCW) were proposed with the goal of enhancing the

Table 15
Final MCDM results for the laminate options; under the four different weighting scenarios of Table 14.

	TOPSIS scores (C^*)				Ranks			
	Scenario I	Scenario II	Scenario III	Scenario IV	Scenario I	Scenario II	Scenario III	Scenario IV
PW	0.44	0.56	0.47	0.50	3	2	2	2
TW	0.57	0.67	0.64	0.66	1	1	1	1
UW	0.38	0.50	0.43	0.46	4	3	3	3
UD	0.48	0.31	0.40	0.37	2	4	4	4

effectiveness of decision making processes in expert systems. Both advantages and limitations associated with the methods are identified as follows.

The AMB in comparison with other subjective methods such as the direct complete weight elicitation enables a less experienced DM to make interactive and intuitive decisions as the DM can adjust the weighting factors in a flexible, step-by-step manner. Compared to the equal weighting (EW) method recommended in some literature for unexperienced DMs, the AMB can assign more accurate and realistic weights. The AMB method can also solve the limitation of some earlier weighting techniques that are merely based on criteria ranks (such as RS, RR and ROC techniques). The distance between each consecutive criteria weights in these techniques are constant while the DM in the AMB can change the distances by giving extra emphasis to some specific criteria. The other advantage of the AMB may be that it relies on a graphical method which is often easier to implement. Thanks to the interactive basis of this approach, the chance of assigning inappropriate weights is expected to be less. On the other hand, for experienced DMs, an advantage of the NL method over the MDL method is that the DM is not limited to three pre-defined digital levels of scoring during pair-wise comparisons. In the NL weighting, the DM can assign arbitrary numeric weights at each pair-wise comparison node, hence increasing the precision of weighting outcomes. Finally, in comparison with the original combinative weighting (CW), the modified combinative weighing (MCW) method can (i) allow for assigning different importance levels to specific objective and/or subjective weightings, and (ii) be adaptable to group decision making environments.

The perceived limitations associated with the AMB method, e.g., compared to EW, RS, RR, ROS, would be that this technique is more time-consuming. Regarding the NL, e.g., in comparison to the DL and MDL, it may be suitable only for more experienced designers who are confident to assign numeric weights (instead of digital weights) and hence assist more precise decisions. Finally, a restriction of the MCW method is that the ultimate DM or the head of decision making team (e.g., project manager) should possess a very high level of experience and acquaintance of the group to allocate correct powers in Eq. (14) to the opinion of more experienced members.

As potential future work, it should be added that the MCDM methodology discussed herein was applied to composites with flat geometries with uniform lay-ups. Further application of the approach and comparison of woven composites impact performance under more complex lay-ups and 3D part geometries is worthwhile. Also, in more realistic design cases, next to an optimum material selection for a given structure, some other aspects of the final product such as cost, performance associated with the manufacturing process itself, assembly conditions, etc., should be taken into account under the MCDM framework, where the appropriateness of discussed weighting techniques could also be assessed more effectively.

Acknowledgments

The authors would like to acknowledge financial support from the Natural Sciences and Engineering Research Council (NSERC) of Canada. Thanks are due to all technical staff at AS Composite Inc. as well as the Industrial Materials Institute-National Research Council Canada (IMI-NRC), for their valuable support and help in different stages of the work. Constructive comments from the anonymous reviewers are highly acknowledged.

Appendix A. TOPSIS Implementation Steps

This section presents a summery of ranking of the alternatives (materials) using the TOPSIS method (Hwang & Yoon, 1981).

Step 1: Normalization

Since each attribute is measured on a different scale, normalization is required. The normalization is achieved by:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \; i = 1, \; \ldots, \; m; \; j = 1, \; \ldots, \; n \tag{A1}$$

Where, i and j represent the corresponding row (material) and column (criterion) in the given decision matrix.

Step 2: Weighting

Based on the importance of each given criterion, the weight (w_j) from one of the subjective/objective/combinative methods should be applied to the corresponding normalized values of the decision making matrix:

$$v_{ij} = w_j r_{ij} \tag{A2}$$

Note that summation of all weights must be equal to one ($\sum_{j=1}^{n} w_j = 1$).

Step 3: Identifying Positive-Ideal and Negative-Ideal Solutions

The positive-ideal ($A*$) and negative-ideal (A^-) solutions are defined via the weighted normalized criteria values (v_{ij}) as follows:

$$A^* = \left\{ max_i \; v_{ij} | j \in J_1, \; min_i \; v_{ij} \middle| j \in J_2, \; i = 1, \; \ldots, \; m \right\} \tag{A3}$$

And,

$$A^- = \left\{ min_i \; v_{ij} | j \in J_1, \; max_i \; v_{ij} \middle| j \in J_2, \; i = 1, \; \ldots, \; m \right\} \tag{A4}$$

Where J_1 and J_2 are the set of benefit and cost attributes, respectively. Note that the benefit and cost attributes refer to the higher the better and the lower the better type of attributes, respectively.

Step 4: Calculation of Separations from *Positive-Ideal and Negative-Ideal Solutions*

The separation among any two alternatives may be measured by an n-dimensional Euclidean distance. Accordingly, the separation of each alternative from the positive-ideal solution ($A*$) and negative-ideal solution (A^-) in TOPSIS is found as:

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \quad i = 1, \ldots, m \tag{A5}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \quad i = 1, \ldots, m \tag{A6}$$

Step 5: Calculation of Similarities to Positive-Ideal Solution

The next step is to find the closeness of each material candidate to the positive-ideal solution, also called the TOPSIS score:

$$C_i^* = \frac{S_i^-}{(S_i^* + S_i^-)} \tag{A7}$$

C_i^* is between 0 and 1; $C_i^* = 0$ when $\mathrm{A_i} = \mathrm{A}^-$ and $\mathrm{C_i^*} = 1$ when $A_i = A^*$.

Step 6: Rank Preference Orders

The higher the C_i^*, the better the performance of the given material candidate. In another words, the descending order of C_i^* gives the ranking of material candidates.

References

AASHTO (2008). *AASHTO M 180-00: Standard specification for corrugated sheet steel beams for highway guardrail*.

Alemi-Ardakani, M., Milani, A. S., Yannacopoulos, S., & Borazghi, H. (2015). A rapid approach for predication and discrete lay-up optimization of glass fiber/polypropylene composite laminates under impact. *International Journal of Impact Engineering, 84*, 134–144.

Alemi-Ardakani, M., Milani, A. S., Yannacopoulos, S., & Shokouhi, G. (2015). A multicriteria experimental analysis of impact on fiber reinforced polymer composite laminates. *Materials Today Communications, 4*, 6–15.

Bank, L., & Gentry, T. (2001). Development of a pultruded composite material highway guardrail. *Composites Part A: Applied Science and Manufacturing, 32*(9), 1329–1338.

Barron, F. H., & Barrett, B. E. (1996). Decision quality using ranked attribute weights. *Management Science, 42*, 1515–1523.

Barron, F. H., & Barrett, B. E. (1996). The efficacy of SMARTER - simple multi-attribute rating technique extended to ranking. *Acta Psychologica, 93*, 23–36.

Behzadian, M., Khanmohammadi Otaghsara, S., Yazdani, M., & Ignatius, J. (2012). A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications, 39*(17), 13051–13069.

Bridgman, P. W. (1922). *Dimensional analysis*. New Haven, CT: Yale University Press.

Chakraborty, S., & Chatterjee, P. (2013). Selection of materials using multi-criteria decision-making methods with minimum data. *Decision Science Letters, 2*, 135–148.

Chu, A. T. W., Kalaba, R. E., & Spingarn, K. (1979). A comparison of two methods for determining the weights of belonging to fuzzy sets. *Journal of Optimization Theory and Applications, 27*(4), 531–538.

Cobuloglu, H. I., & Büyüktahtakın, İ. E. (2015). A stochastic multi-criteria decision analysis for sustainable biomass crop selection. *Expert Systems with Applications, 42*(15--16), 6065–6074 Retrieved from http://linkinghub.elsevier.com/retrieve/pii/S0957417415002365.

Cook, W. D., & Seiford, L. M. (1978). Priority ranking and consensus formation. *Management Science, 24*(16), 1721–1732.

Dehghan-Manshadi, B., Mahmudi, H., Abedian, A., & Mahmudi, R. (2007). A novel method for materials selection in mechanical design: combination of non-linear normalization and a modified digital logic method. *Materials & Design, 28*(1), 8–15.

Diakoulaki, D., Mavrotas, G., & Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: the critic method. *Computers & Operations Research, 22*(7), 763–770.

Edwards, W. (1977). How to use multiattribute utility measurement for social decision-making. *IEEE Transactions on Systems, Man, and Cybernetics, 7*(5), 326–340.

Edwards, W., & Barron, F. H. (1994). SMARTS and SMARTER- improved simple methods for multiattribute utility measurement. *Organizational Behaviour and Human Decision Processes, 60*, 306–325.

Figueira, J., & Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research, 139*, 317–326.

Fishburn, P. C. (1967). *Additive utilities with incomplete product set: Applications to priorities and assignments*. Baltimore, MD, USA: ORSA Publication.

Güneralp, B., Gertner, G., Mendoza, G., & Anderson, A. (2007). Evaluating probabilistic data with a possibilistic criterion in land-restoration decision-making: Effects on the precision of results. *Fuzzy Sets and Systems, 158*(14), 1546–1560.

Hwang, C. L., Lai, Y. J., & Liu, T. Y. (1993). A new approach for multiple objective decision making. *Computers and Operational Research, 20*, 889–899.

Hwang, C. L., & Lin, M. J. (1987). *Group decision making under multiple criteria: methods and applications*. Berlin: Springer-Verlag.

Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making making: methods and applications*. Berlin: Springer-Verlag.

Islam, S., & Roy, T. K. (2006). A new fuzzy multi-objective programming: entropy based geometric programming and its application of transportation problems. *European Journal of Operational Research, 173*(2), 387–404.

Jahan, A., Mustapha, F., Sapuan, S. M., Ismail, M. Y., & Bahraminasab, M. (2011). A framework for weighting of criteria in ranking stage of material selection process. *The International Journal of Advanced Manufacturing Technology, 58*(1-4), 411–420.

Jee, D.-H., & Kang, K.-J. (2000). A method for optimal material selection aided with decision making theory. *Materials & Design, 21*(3), 199–206.

Kahneman, D., Slovic, P., & Tversky, A. (1982). *Judgment under uncertainty: heuristics and biases*. New York: Cambridge University Press.

Lotfi, F. H., & Fallahnejad, R. (2010). Imprecise shannon's entropy and multi attribute decision making. *Entropy, 12*(1), 53–62.

Davoodi, M. M., Sapuan, S. M., Ahmad, D., Aidy, A., Khalina, A., & Jonoobi, M. (2011). Concept selection of car bumper beam with developed hybrid bio-composite material. *Materials & Design, 32*(10), 4857–4865.

Monghasemi, S., Nikoo, M. R., Khaksar Fasaee, M. A., & Adamowski, J. (2015). A novel multi criteria decision making model for optimizing time–cost–quality trade-off problems in construction projects. *Expert Systems with Applications, 42*(6), 3089–3104 Retrieved from http://dx.doi.org/10.1016/j.eswa.2014.11.032.

Özcan, T., Elebi, N., & Esnaf, A. (2011). Comparative analysis of multi-criteria decision making methodologies and implementation of a warehouse location selection problem. *Expert Systems with Applications, 38*(8), 9773–9779.

Pakpour, S., Olishevska, S. V., Prasher, S. O., Milani, A. S., & Chénier, M. R. (2013). DNA extraction method selection for agricultural soil using TOPSIS multiple criteria decision-making model. *American Journal of Molecular Biology, 3*(4), 215–228.

Paul Yoon, K., & Hwang, C.-L. (1995). *Multiple attribute decision making: An introduction*. Thousand Oaks: Sage Publications, Inc.

Ross, H. E. J., Sicking, D. L., Zimmer, R. A., & Michie, J. D. (1993). NCHRP report 350: recommended procedures for the safety performance evaluation of highway features. Transportation Research. Washington, DC: Transportation Research Board, National Academy Press.

Roy, B. (1991). The outranking approach and the foundations of ELECTRE methods. *Theory and Decision, 31*, 49–73.

Saaty, T. L. (1980). *The analytic hierarchy process*. New York: McGraw Hill.

Shanian, A., Milani, A. S., Carson, C., & Abeyaratne, R. C. (2008). A new application of ELECTRE III and revised Simos' procedure for group material selection under weighting uncertainty. *Knowledge-Based Systems, 21*(7), 709–720.

Simos, J. (1990). *L'évaluation environnementale: Un processus cognitif négocié*. Lausanne: DGF-EPFL These de doctorat.

Stillwell, W. G., Seaver, D. A., & Edwards, W. (1981). A comparison of weight approximation techniques in multiattribute utility decision making. *Organizational Behavior and Human Performance, 28*(1), 62–77.

Tversky, A. (1972). Elimination by aspects: a theory of choice. *Psychological Review, 79*(4), 281–299.

Vincke, J. P., & Brans, P. (1985). A preference ranking organization method. The PROMETHEE method for MCDM. *Management Science, 31*(6), 647–656.

Von Winterfeldt, D., & Edwards, W. (1986). *Decision analysis and behavioral research*: Vol. 604. Cambridge: Cambridge University Press.

Weber, M. (1987). Decision making with incomplete information. *European Journal of Operational Research, 28*(1), 44–57.

Wikipedia (2015). http://en.wikipedia.org/wiki/american_association_of_state_highway_and_transportation_officials.

Yavuz, M., Oztaysi, B., Cevik, S., & Kahraman, C. (2015). Multi-criteria evaluation of alternative-fuel vehicles via a hierarchical hesitant fuzzy linguistic model. *EXPERT SYSTEMS WITH APPLICATIONS, 42*(5), 2835–2848 Retrieved from http://dx.doi.org/10.1016/j.eswa.2014.11.010 .

Yoon, K. (1987). A reconciliation among discrete compromise situations. *Journal of Operational Research Society, 38*, 277–286.