



## Generation expansion planning in electricity markets: A novel framework based on dynamic stochastic MPEC



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### ABSTRACT

This paper presents a novel framework for generation expansion planning (GEP) of restructured power systems under uncertainty in a multi-period horizon, which includes generation investment from a price maker perspective. The investment problem is modeled as a bi-level optimization problem. The first level problem includes decisions related to investment in order to maximize total profit in the planning horizon. The second level problem consists of maximizing social welfare where the power market is cleared. Rival uncertainties on offering and investment are modeled using sets of scenarios. The bi-level optimization problem is then converted to a dynamic stochastic MPEC and represented as a mixed integer linear program (MILP) after linearization. The proposed framework is examined on a typical six-bus power network, MAZANDARAN regional electric company (MREC) transmission network as an area of IRAN interconnected power system and IEEE RTS 24-bus network. Simulation results confirm that the proposed framework can be a useful tool for analyzing the behavior of investments in electricity markets.

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### Introduction

During past decades, power industry has experienced major changes in structure and regulations of the markets in order to improve economic efficiency and promote sustainable development [1]. Regarding the expansion planning and the operation decisions by companies, the issue of profitability has attracted more attentions in the short-term and long-term planning [2]. Since one of the main drivers of generation capacity expansion is the expected electricity price in the future, appropriate market-clearing models should be used to determine the price of electricity in the short and medium term. On the other hand, one of the major challenges of market operators in electricity markets is maintaining the adequacy of generation capacity [3]. To this end, regulators should make appropriate policies to encourage producers to invest new generation capacities [4].

It is very important to apply appropriate model for the generation expansion and study the impact of different markets on the investment. Accordingly, researchers have been providing models and programming methods to treat the investment problem. A bi-level model for generation expansion (Cournot modeling) is presented in [5]. The impact of transmission congestion and competition in generation investment is reviewed by the Cournot

model in [6]. The presented GEP models in [5,6] are static without considering uncertainties. In [7], a probabilistic dynamic programming model has been proposed to solve the investment problem in the presence of demand uncertainty. Strategic producer behavior has been investigated in [8] using a bi-level model considering uncertainties related to demand and rival offers. A static model is proposed in [9] to solve the generation investment problem from a strategic producer point of view. The static model of [9] has been extended in [10] to consider demands uncertainties, behavior of rival producers and their offers at a specified time horizon. Also, Bender's decomposition has been used to solve the investment problem. A hybrid DP/GAME framework is proposed in [11] to deal with GEP problem in which DP was applied to solve the investment problem and Cournot game was used to model strategic behavior of the producers in the spot markets. In [12], the expansion planning has been solved for a set of non-strategic producers in the liberalized electricity market. The market clearing problem is modeled using conjecture price approach in the lower level problem. An open-loop model is used in [13] as an approximation of closed-loop model for reducing computational time where the problem is modeled as an EPEC without considering uncertainties. In [14,15], a bi-level model was proposed to characterize generation investment equilibria in a single horizon pool-based electricity market neglecting uncertainties where the producers behave strategically. Also, strategic offers of producers were considered through stepwise supply function.

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This paper provides a multi period framework to study the generation expansion of a strategic producer under uncertainties in electricity markets. The investment problem is modeled as a bi-level optimization problem. The upper level includes decisions taken by a strategic producer who investigates installments of new generating units in a given time period and the future possible productions, to maximize the total profit in the planning horizon. The lower level problem models the responses provided by a competitive fringe in terms of production bids, which are sorted by a market operator, who clears the market obtaining Locational Marginal Prices (LMPs) as dual variables of the nodal balancing constraints and the objective of maximizing social welfare. Rival uncertainties on offering and investment are modeled using sets of scenarios. The considered bi-level optimization problem is then converted into a single level optimization problem. The single level optimization problem is considered as a Mathematical Program with Equilibrium Constraints (MPEC) [21]. Then, the single level problem is linearized and converted to an MILP. The proposed framework uses dynamic stochastic MPEC. To the best of our knowledge, dynamic nature of investment decisions have not been considered in the MPEC models presented in the literatures to solve GEP problems in a dominant producer point of view. Thus, the multi period stochastic MPEC model is the main contribution of this paper, which also considers transmission network constraints. The presented model can also be treated as a mixture of operation and expansion problems of power systems. The supply function model is also used as offer strategies of the producers in the spot market, which is a more realistic model of spot market than the other models. It should be noted that the supply function model is a more detailed description of the actual electricity market compared to Cournot, Bertrand and conjecture variations models. Moreover, the considered multi period bi-level problem is formulated as an MPEC problem.

The paper is organized as follows. In Section ‘The proposed framework’, the proposed framework and the bi-level model is introduced. The mathematical formulation of the problem is presented in Section ‘Mathematical formulation’. Section ‘Case studies’ provides simulation results and analysis for three case studies. Finally, Section ‘Conclusion’ provides some relevant conclusions.

### The proposed framework

The proposed framework is generally depicted in Fig. 1. The main block of the proposed framework represents the bi-level

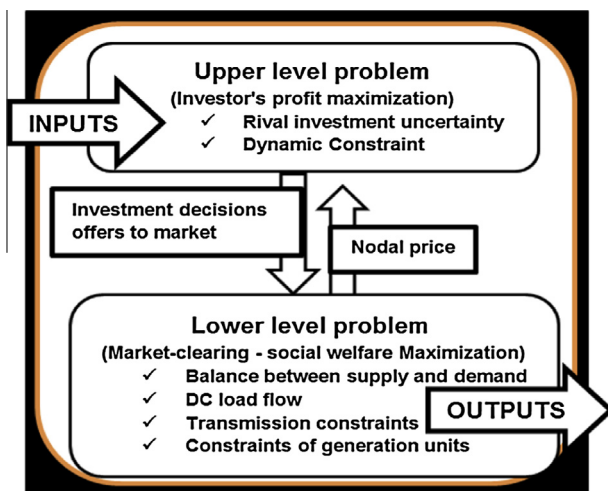


Fig. 1. Schematic of the proposed framework.

model. The upper level represents the investment problem of a dominant producer who is seeking to maximize the present value of the total profit of investment. In the upper level, investment decisions of rival producers are modeled using set of scenarios. Investment decisions of producers and their offers to the spot market are outputs of the upper level problem. Due to dynamic nature of the planning problem, dynamic constraints exist in the upper-level. The lower-level problem represents the market-clearing. The clearing of the market for any given operating condition is represented as an optimization problem that identifies the operating decisions through maximizing social welfare. The market clearing problem is constrained by DC power flow equations and limitations of transmission network and units' capacity. Output of the lower level problem is nodal prices (dual variables associated to the power balance constraints), which are fed back to the upper level.

### Mathematical formulation

Mathematical formulation of the proposed framework is presented in the following sub-sections. In order to introduce the model, we define the following sets, parameters and decision variables.

#### Indices

- $d$ : index for demand,
- $h$ : index for size of investment option,
- $i/k$ : index for new/existing generation unit of strategic producer,
- $j$ : index for generation unit belonging to other producers,
- $n/m$ : index for bus,
- $t$ : index for demand blocks,
- $y$ : index for year,
- $w$ : index for scenario.

#### Parameters

- $B_{nm}$ : Susceptance of line  $n-m$  (p.u.).
- $CO_{tjw}$ : Price offer of units  $j$  of other producers in demand block  $t$  and scenario  $w$  ( $\text{€}/\text{MW h}$ ).
- $C_i^S / C_k^{ES}$ : Marginal cost of new/existing unit of strategic producer ( $\text{€}/\text{MW h}$ ).
- $f$ : Discount rate.
- $F_{nm}^{\max}$ : Transmission capacity of line  $n-m$  (MW).
- $K^{\max}$ : Available investment budget (M€).
- $K_{yt}$ : Annual investment cost of new generating unit ( $\text{€}/\text{MW}$ ).
- $p_j^{0\max}$ : Capacity of generation unit  $j$  of other producer (MW).
- $p_k^{ES\max}$ : Capacity of existing generation unit  $k$  of strategic producer (MW).
- $p_{ytd}^{D\max}$ : Maximum load of demand  $d$  in block  $t$  and year  $y$  (MW).
- $U_{ytd}^D$ : Price bid of demand  $k$  in demand block  $t$  and year  $y$  ( $\text{€}/\text{MW h}$ ).
- $X_{yih}$ : Option  $h$  for investment capacity of new unit  $i$  (MW).
- $\sigma_{yt}$ : Weight of demand block  $t$  in year  $y$ .
- $\varphi_w$ : Weight of scenario  $w$ .

#### Decision variables

- $P_{ytkw}^{ES} / P_{ytiw}^S$ : Power produced by existing/new unit  $k/i$  of strategic producer in year  $y$ , demand block  $t$  and scenario  $w$  (MW).
- $P_{ytw}^O$ : Power produced by unit  $j$  of other producers in year  $y$ , demand block  $t$  and scenario  $w$  (MW).

$P_{ytdw}^D$ : Power consumed by demand  $d$ , in year  $y$ , demand block  $t$  and scenario  $w$  (MW).

$u_{yih}$ : Binary variable that is equal to 1 if the  $h$ th investment option of technology  $i$  is selected in year  $y$ , otherwise it is equal to 0.

$X_{yi}$ : Capacity investment of new unit  $i$  of the strategic producer in year  $y$  (MW).

$\alpha_{ytkw}^{ES}/\alpha_{ytw}^S$ : Price offer by existing/new unit  $k/i$  of the strategic producer in year  $y$ , demand block  $t$  and scenario  $w$  (€/MWh).

$\theta_{ytmw}$ : Voltage angle of bus  $n$ , in year  $y$ , demand block  $t$  and scenario  $w$ .

### The bi-level model

The multi period stochastic investment problem is formulated using the following bi-level model, which comprises an upper level problem (i.e. (1)–(4)) and a collection of lower level problems (i.e. (5)–(13)). The objective function (1) is the present value of the minus expected profit (investment cost minus expected revenue) of strategic producer in the planning horizon, which comprises two terms of investment cost and expected profit obtained by selling energy in the spot market. Note that  $i \in \Psi_n/k \in \Psi_n$  identifies the new/existing generating units  $i/k$  located at bus  $n$ . Constraint (2) states that investment options are only available in discrete blocks. This equation imposes that only one technology is binding and determines the new technology to be installed at each bus of the system. It should be noted that based on constraint (2) the producer can either open exactly one new plant each year, or choose one option for installing. Dynamic constraints on the investment decision variables and investment budget limit are represented in constraints (3) and (4), respectively. The market clearing problems are represented by the minus social welfare (5) and constraints (6)–(13). Constraints (6) represent the energy balance at each bus, being the associated dual variables LMPs or nodal prices. Constraints (7)–(9) impose power bounds for generation constraints blocks and constraints (10) represent demand limits. Constraints (11) define the power flow through transmission lines using a lossless DC model. Constraints (12) and (13) enforce angle bounds and fix the voltage angle at the reference bus, respectively. Note that  $m \in \Phi_n$  identifies the buses  $m$  is connected to bus  $n$ . Dual variables are indicated at the relevant constraints following a colon.

$$\text{Minimize } \sum_y \left( \frac{1}{1+f} \right)^y \sum_i K_{yi} X_{yi} - \sum_w \varphi_w \sum_y \left( \frac{1}{1+f} \right)^y \sum_t \sigma_{yt} \left\{ \begin{array}{l} \left( \sum_{i, [n: i \in \psi_n]} P_{ytw}^S \lambda_{ytmw} - \sum_i P_{ytw}^S C_i^S \right) + \\ \left( \sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw} - \sum_k P_{ytkw}^{ES} C_k^{ES} \right) \end{array} \right\} \quad (1)$$

Subject to:

$$X_{yi} = \sum_n u_{yih} X_{yih}, \sum_n u_{yih} = 1, u_{yih} \in \{0, 1\}, \forall i, \forall y \quad (2)$$

$$0 \leq X_{yi} \leq X_{(y+1)i} : \forall y, \forall i \quad (3)$$

$$\sum_y \left( \frac{1}{1+f} \right)^y \sum_{i \in \psi_s} K_i X_{yi} \leq K^{\max} \quad (4)$$

$$\lambda_{ytmw}, P_{ytw}^S, P_{ytkw}^E \in \arg \min \left\{ \sum \alpha_{ytw}^S P_{ytw}^S + \sum \alpha_{ytkw}^{ES} P_{ytkw}^{ES} + \sum CO_{tkw} P_{ytw}^O - \sum U_{td}^D k P_{ytdw}^D \right\} \quad (5)$$

Subject to:

$$\sum_{d \in \psi_n} P_{ytdw}^D + \sum_{m \in \Phi_n} B_{nm}^{P,U} S_b (\theta_{ytmw} - \theta_{ytmw}) - \sum_{i \in \psi_n} P_{ytw}^S - \sum_{k \in \psi_n} P_{ytkw}^{ES} - \sum_{j \in \psi_n} P_{ytw}^O = 0 : \lambda_{ytmw}, \forall n \quad (6)$$

$$0 \leq P_{ytw}^S \leq X_{yi} : \mu_{ytw}^S \min, \mu_{ytw}^S \max, \forall i \quad (7)$$

$$0 \leq P_{ytkw}^{ES} \leq P_k^{ES \max} : \mu_{ytkw}^{ES \min}, \mu_{ytkw}^{ES \max}, \forall k \quad (8)$$

$$0 \leq P_{ytw}^O \leq P_j^O \max : \mu_{ytw}^O \min, \mu_{ytw}^O \max, \forall j \quad (9)$$

$$0 \leq P_{ytdw}^D \leq P_{ytd}^{D \max} : \mu_{ytdw}^{D \min}, \mu_{ytdw}^{D \max}, \forall d \quad (10)$$

$$-F_{nm}^{\max} \leq B_{nm}^{P,U} S_b (\theta_{ytmw} - \theta_{ytmw}) \leq F_{nm}^{\max} : V_{ytmw}^{\min}, V_{ytmw}^{\max}, \forall n, \forall m \in \Phi_n \quad (11)$$

$$-\pi \leq \theta_{ytmw} \leq \pi : \zeta_{ytmw}^{\min}, \zeta_{ytmw}^{\max}, \forall n \quad (12)$$

$$\theta_{ytmw} = 0 : \zeta_{ytmw}^1, n = 1 \} \forall t, \forall y, \forall w. \quad (13)$$

### MPEC

The bi-level problem (1)–(13) can be converted to a single level problem (MPEC) by enforcing KKT conditions to the lower level problems [16,17]. These are represented by (14)–(36).

$$(1) - (4) \quad (14)$$

$$-U_{ytdw}^D + \lambda_{yt(n: d \in \psi_n)w} + \mu_{ytdw}^{D \max} - \mu_{ytdw}^{D \min} = 0 \quad \forall y, \forall t, \forall d, \forall w \quad (15)$$

$$\alpha_{ytw}^S - \lambda_{yt(n: i \in \psi_n)w} + \mu_{ytw}^{S \max} - \mu_{ytw}^{S \min} = 0 \quad \forall y, \forall t, \forall i, \forall w \quad (16)$$

$$\alpha_{ytkw}^{ES} - \lambda_{yt(n: k \in \psi_n)w} + \mu_{ytkw}^{ES \max} - \mu_{ytkw}^{ES \min} = 0 \quad \forall y, \forall t, \forall k, \forall w \quad (17)$$

$$CO_{ytw} - \lambda_{yt(n: j \in \psi_n)w} + \mu_{ytw}^{O \max} - \mu_{ytw}^{O \min} = 0 \quad \forall y, \forall t, \forall j, \forall w \quad (18)$$

$$\sum_{m \in \Phi_n} B_{nm}^{P,U} S_b (\lambda_{ytmw} - \lambda_{ytmw}) + \sum_{m \in \Phi_n} B_{nm}^{P,U} S_b (V_{ytmw}^{\max} - V_{ytmw}^{\max}) + \sum_{m \in \Phi_n} B_{nm}^{P,U} S_b (V_{ytmw}^{\min} - V_{ytmw}^{\min}) + \zeta_{ytmw}^{\max} - \zeta_{ytmw}^{\min} + (\zeta_{ytmw}^1)_{n=1} = 0 \quad \forall y, \forall t, \forall n, \forall w \quad (19)$$

$$(6) - (13) \quad (20)$$

$$0 \leq P_{ytw}^S \perp \mu_{ytw}^{S \min} \geq 0 \quad \forall y, \forall t, \forall i, \forall w \quad (21)$$

$$0 \leq P_{ytkw}^{ES} \perp \mu_{ytkw}^{ES \min} \geq 0 \quad \forall y, \forall t, \forall k, \forall w \quad (22)$$

$$0 \leq P_{ytw}^O \perp \mu_{ytw}^{O \min} \geq 0 \quad \forall y, \forall t, \forall j, \forall w \quad (23)$$

$$0 \leq P_{ytdw}^D \perp \mu_{ytdw}^{D \min} \geq 0 \quad \forall y, \forall t, \forall d, \forall w \quad (24)$$

$$0 \leq (X_{yi} - P_{ytw}^S) \perp \mu_{ytw}^{S \max} \geq 0 \quad \forall y, \forall t, \forall i, \forall w \quad (25)$$

$$0 \leq (P_k^{ES \max} - P_{ytkw}^{ES}) \perp \mu_{ytkw}^{ES \max} \geq 0 \quad \forall y, \forall t, \forall k, \forall w \quad (26)$$

$$0 \leq (P_j^O \max - P_{ytw}^O) \perp \mu_{ytw}^{O \max} \geq 0 \quad \forall y, \forall t, \forall j, \forall w \quad (27)$$

$$0 \leq (P_{ytd}^{D \max} - P_{ytdw}^D) \perp \mu_{ytdw}^{D \max} \geq 0 \quad \forall y, \forall t, \forall d, \forall w \quad (28)$$

$$0 \leq [F_{nm}^{\max} + B_{nm}^{P,U} S_b (\theta_{ytmw} - \theta_{ytmw})] \perp V_{ytmw}^{\min} \geq 0 \quad \forall y, \forall t, \forall n, \forall m \in \Phi_n, \forall w \quad (29)$$

$$0 \leq [F_{nm}^{\max} - B_{nm}^{P,U} S_b (\theta_{ytmw} - \theta_{ytmw})] \perp V_{ytmw}^{\max} \geq 0 \quad \forall y, \forall t, \forall n, \forall m \in \Phi_n, \forall w \quad (30)$$

$$0 \leq (\pi - \theta_{ytmw}) \perp \zeta_{ytmw}^{\max} \geq 0 \quad \forall y, \forall t, \forall n, \forall w \quad (31)$$

$$0 \leq (\pi + \theta_{ytmw}) \perp \zeta_{ytmw}^{\min} \geq 0 \quad \forall y, \forall t, \forall n, \forall w \quad (32)$$

### Linearization

The MPEC problem of (14)–(36) is a nonlinear problem because of existing  $\sum_{i, [n: i \in \psi_n]} P_{ytw}^S \lambda_{ytmw} + \sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw}$  in the objective function and (21)–(32). Each of the non-linear relationship is linearized according to their nature as follows:

1. Eqs. (21)–(32) are linearized by (33), (34) [18], where  $M$  is a large enough constant.

$$0 \leq a \perp b \geq 0 \quad (33)$$

$$a \geq 0, b \geq 0, a \leq \tau M, b \leq (1 - \tau)M, \tau \in \{0, 1\} \quad (34)$$

For instance, (25) is linearized based on duality gap and usage of the complementarity conditions. Each of the non-linear relationship is linearized by replacing  $(X_{yi} - P_{ytw}^S)$  and  $\mu_{ytw}^{S \max}$  instead of  $a$  and  $b$  in (34).

$$(X_{yi} - P_{ytiw}^S) \geq 0, \mu_{ytiw}^{S \max} \geq 0, (X_{yi} - P_{ytiw}^S) \leq \tau M, \\ \mu_{ytiw}^{S \max} \leq (1 - \tau)M, \quad \tau \in \{0, 1\}$$

2. To find a linear expression for  $\sum_{i, [n: i \in \psi_n]} P_{ytiw}^S \lambda_{ytmw} + \sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw}$ , the strong duality theorem and some of the KKT equalities are used. According to the strong duality theorem, if a problem is convex, the objective functions of the primal and dual problems have the same value at the optimum. Thus after applying the strong duality theorem to each lower-level problem (5)–(13):

$$\sum \alpha_{ytiw}^S P_{ytiw}^S + \sum \alpha_{ytkw}^{ES} P_{ytkw}^{ES} + \sum C_{otkw} P_{ytjw}^0 - \sum U_{td}^D k P_{ytdw}^D \\ = - \sum_i \mu_{ytiw}^{S \max} X_i + \sum_k \mu_{ytkw}^{ES \max} P_k^{ES \max} - Z \quad \forall t, \forall y, \forall w \quad (35)$$

where

$$Z = \sum_j \mu_{ytjw}^0 P_j^{0 \max} + \sum_d \mu_{ytdw}^D P_{ytd}^{D \max} + \sum_{n(m \in \Omega_n)} V_{ytmw}^{\min} F_{nm}^{\max} \\ + \sum_{n(m \in \Omega_n)} V_{ytmw}^{\max} F_{nm}^{\max} + \sum_n (\zeta_{ytmw}^{\min} + \zeta_{ytmw}^{\max}) \pi. \quad (36)$$

From (25) and (26):

$$\sum_i \mu_{ytiw}^{S \max} X_i = \sum_i \mu_{ytiw}^{S \max} P_{ytiw}^S \quad (37)$$

$$\sum_k \mu_{ytkw}^{ES \max} P_k^{ES \max} = \sum_k \mu_{ytkw}^{ES \max} P_{ytkw}^{ES} \quad (38)$$

Substituting (37) and (38) in (35) renders

$$\sum P_{ytiw}^S (\alpha_{ytiw}^S + \mu_{ytiw}^{S \max}) + \sum P_{ytkw}^{ES} (\alpha_{ytkw}^{ES} + \mu_{ytkw}^{ES \max}) \\ = - \sum C_{otkw} P_{ytjw}^0 + \sum U_{td}^D k P_{ytdw}^D - Z \quad (39)$$

On the other hand, from (16) and (17)

$$-\lambda_{yt(n: i \in \psi_n)w} = \alpha_{ytiw}^S + \mu_{ytiw}^{S \max} - \mu_{ytiw}^{S \min} \quad (40)$$

$$-\lambda_{yt(n: k \in \psi_n)w} = \alpha_{ytkw}^{ES} + \mu_{ytkw}^{ES \max} - \mu_{ytkw}^{ES \min} \quad (41)$$

Thus:

$$\sum_{i, [n: i \in \psi_n]} P_{ytiw}^S \lambda_{ytmw} = \sum_i \alpha_{ytiw}^S P_{ytiw}^S + \sum_i \mu_{ytiw}^{S \max} P_{ytiw}^S - \sum_i \mu_{ytiw}^{S \min} P_{ytiw}^S \quad (42)$$

$$\sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw} = \sum_k \alpha_{ytkw}^{ES} P_{ytkw}^{ES} + \sum_k \mu_{ytkw}^{ES \max} P_{ytkw}^{ES} - \mu_{ytkw}^{ES \min} P_{ytkw}^{ES} \quad (43)$$

Additionally, from (21) and (22)

$$\sum_i \mu_{ytiw}^{S \min} P_{ytiw}^S = 0, \sum_k \mu_{ytkw}^{ES \min} P_{ytkw}^{ES} = 0 \quad (44)$$

Using (44) to simplify (42) and (43) renders

$$\sum_{i, [n: i \in \psi_n]} P_{ytiw}^S \lambda_{ytmw} + \sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw} \\ = \sum P_{ytiw}^S (\alpha_{ytiw}^S + \mu_{ytiw}^{S \max}) + \sum P_{ytkw}^{ES} (\alpha_{ytkw}^{ES} + \mu_{ytkw}^{ES \max}) \quad (45)$$

Finally, considering (39) and (45)

$$\sum_{i, [n: i \in \psi_n]} P_{ytiw}^S \lambda_{ytmw} + \sum_{k, [n: k \in \psi_n]} P_{ytkw}^{ES} \lambda_{ytmw} \\ = - \sum_j C_{otj}^0 P_{ytjw}^0 + \sum_d U_{ytd}^D P_{ytdw}^D - Z \quad (46)$$

### Case studies

In this section, proposed framework is examined through three case studies. The first case study is a two area power system comprising six buses. The second case is the MAZANDARAN regional electric company (MREC) transmission network as a part IRAN

interconnected power system. Results obtained from simulations on the IEEE 24-bus Reliability Test System [21] are discussed in the third case study.

#### Case study1: six-buses system

The considered network of the first case study is depicted in Fig. 2, which is composed of two areas (north and south) interconnected by two tie-lines. Required data associated to unit characteristics and demands of this network can be found in [9]. It is assumed that the capacities of the tie-lines are limited to 450 MVA. The planning horizon is assumed to be five years. Annual demand growth and annual discount rate are assumed to be 10% and 8.7%, respectively. Available investment budget is assumed 150 million euros over the planning period. Regarding to the investment by the rival producers, it is assumed that they choose peak technology and also construct their new units at bus 4. For the sake of simplicity, four scenarios are considered for investing the other producers over the planning period. These scenarios are indicated as follows.

Scenario 1: No investment over the planning period with the probability of 10%.

Scenario 2: Investing on 400 MW on the bus 4, only in the second year of the planning period, with the probability of 50%.

Scenario 3: Investing on 400 MW on the bus 4, only in the third year of the planning period, with the probability of 25%.

Scenario 4: Investing on 400 MW on the bus 4, only in the fourth year of the planning period, with the probability of 15%.

The proposed model is solved using Solver CPELX [19] software GAMS [20]. In order to validate the simulation results of the model, the static model presented in [9] was implemented at first. According to the results of simulations of [9], the total profit and the total constructed capacity over the planning period have been obtained 32.2 M€ and 700 MW, respectively. After validation of simulation results in the static approach, the model was extended based on the proposed dynamic approach. Results of the dynamic MPEC are given in Table 1. The first row of Table 1 is related to the planning year. The second row illustrates the total installed capacity and the base technology in the parenthesis. For instance, the total and the base installed technologies in the first year are 250 MW

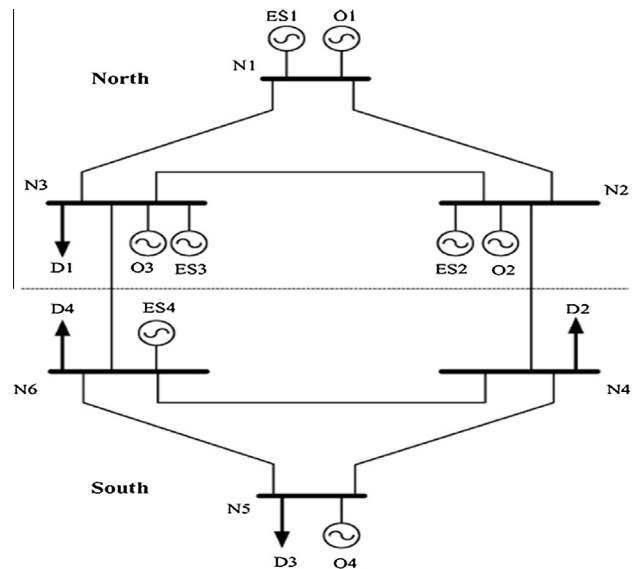


Fig. 2. Six-bus test network [9].

**Table 1**  
Generation capacity expansion results.

Year	1	2	3	4	5
Invested capacity MW (peak, base)	(250,0)	(500, 500)	(0,0)	(0,0)	(200,0)
Demand peak (MW)	1598	1997	2210	2423	2663
Average price (€/MWh)	23.16	19.51	21.36	21.93	22.30
Energy consumption (MMWh)	7.3	8.2	9.14	10.04	11.03
Net surplus of consumers (M€)	11.21	27.86	20.9	17.48	14.78
Net profit of investor (M€)	22.84	10.39	17.75	20.69	24.95
Social welfare (M€)	82.75	111.8	110.6	107.9	107.9
Net surplus of strategic producer (M€)	26.29	45.31	49.87	50.24	54.12
Net surplus of other producers (M€)	45.24	38.64	39.83	40.16	38.98

and 0 MW, respectively. That is, investment in the peak technology is 250 MW. The third and fourth rows show the energy consumptions and peak demand during the planning period, respectively. The average market prices, the net profit of the strategic producer, the net surplus of the rival producers and of consumers, are shown in rows 2 to 7 of Table 1.

- (1) The average market price in the second year decreases 15.76% compared to the first year due to construction of 500 MW base technologies by the strategic producer. Thus, profit of the investor in the second year is equal 10.39 M€ which is the lowest level over the planning period. Also, in the second year, the net surplus of consumers and the social welfare have been obtained 27.86 M€ and 111.8 M€, respectively; that are the highest levels over the planning period. In the third year, the average market price increases 9.48% compared to the second year. This is consistent with increasing of the demand equal 10.67% with respect to the second year. In the fourth year, a 400 MW unit is built by the rival producers with related probabilities. Also, the market price has been increased by 2.67% with respect to the third year due to increasing of the demand equal to 9.64% with respect to the third year. Furthermore, the average market price has been increased 1.69% in the fifth year with respect to the fourth year.
- (2) Major of capacity constructions are peak technologies because of their lower investment costs than that of the base technologies. Results regarding to the capacity additions show that during the planning period, investment in peak units leads to increase in the market price due to the high cost of their operation. However, investment in base units reduces the market price. Moreover, the energy consumptions and the net surplus of the consumers have been increased. Also, net surplus of the producers has been increased. As Table 1 shows, in the second year, the consumer surplus has been increased by 148.5% with respect to the first year of the planning period. Therefore, investment in the base unit is more beneficial to consumers in the short term. However, it is profitable for producers in the long term.
- (3) Fig. 3 shows the effect of the rival producer investment on the market prices. It can be seen that although non-strategic producers are considered as price takers in the short term, they can influence market price in the long term through their investment decisions. In this respect, because of investing in the second year by the rival producers, the market price has been decreased in the rest of the planning period.

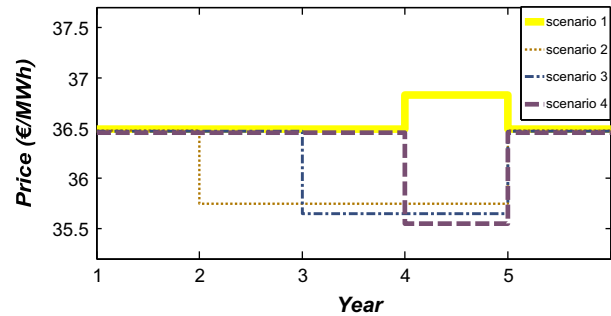


Fig. 3. Impact of investment of other producers on the average price.

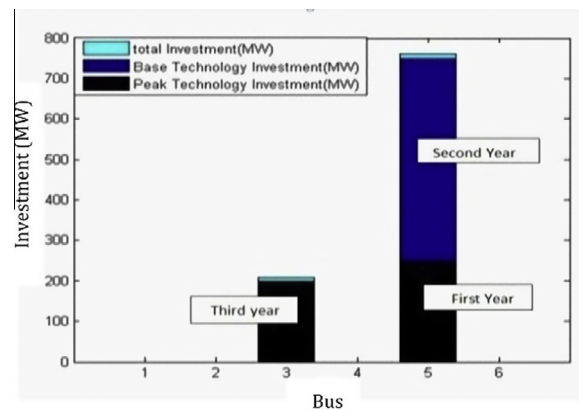


Fig. 4. Strategic producer investments in each bus.

- (4) Fig. 4 shows the capacity additions in the north and southern regions over the planning period. Due to the demand in the first year, only a 250 MW unit is constructed at bus 5. Due to increasing the demand in the first year, construction of a 500 MW (base technology) at bus 5 is profitable. In the third year, due to the investment of 750 MW in the southern region in the first and second years, construction of a new 200 MW of peak technology at bus 3 has been justified. Furthermore, it was observed in the simulations that market price in the southern region was higher than the north region, because of the high demand in this region. Thus, more generation capacities have been added in the southern region. Also, the less the marginal cost of the generating units, the more production of units has been observed. This is the case for the new units, since their marginal costs are less than the existing technologies.
- (5) The total capacity constructed in the dynamic approach and the total profit of the investor over the planning period have been calculated 950 MW and 96.62 M€, respectively. It can be seen that the total investment and the total profit of investor have been increased in the dynamic approach with respect to the static one. Moreover, using of the dynamic versus static approach in the planning of generation capacity leads to accurate and realistic results in the expansion planning. Also, it was observed in the simulation that the budget limitation has postponed the investment.

#### Case study 2: MREC Network

Single-line diagram of MREC transmission network is shown in Fig. 5. Annual growth of demands is assumed 6.2%. Annual discount rate is assumed to be 8.7%. The planning horizon is assumed to be five years, while each year is specified with three different

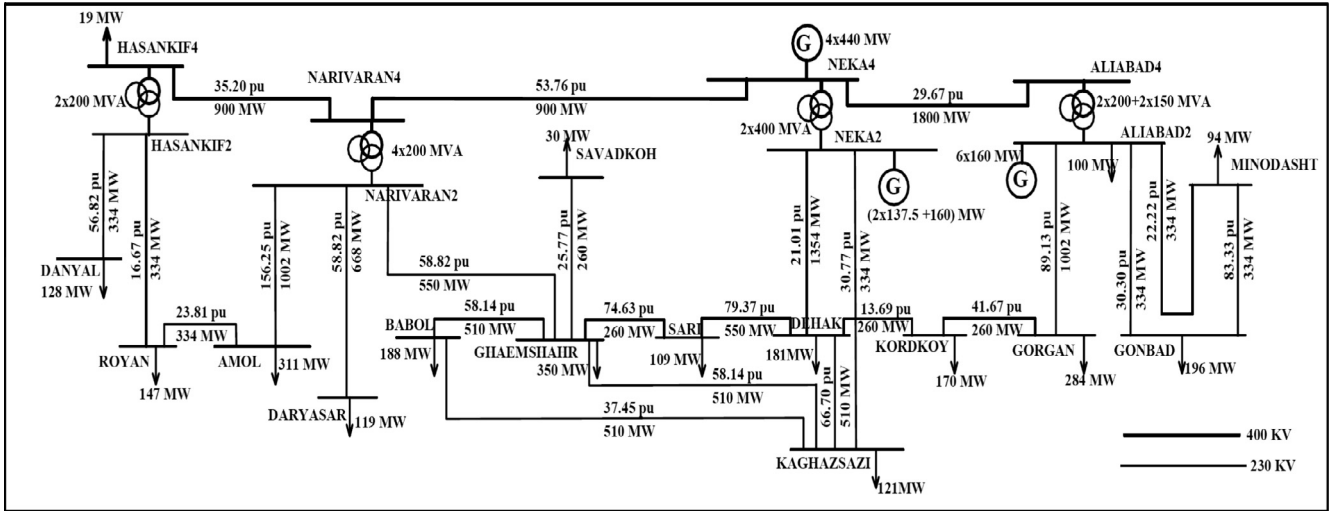


Fig. 5. Single-line diagram of MREC transmission network.

demand blocks; namely peak, shoulder and off-peak. The considered weighing factors associated to each demand blocks (peak, shoulder and off-peak) are assumed to be 20%, 50% and 30%, respectively. The price bids of the demands are 35.75, 28.721, and 27.357 (€/MW h) for peak, shoulder and off-peak blocks, respectively. In each year of the planning period, weighting factor of the off-peak and shoulder blocks are considered to be 25% and 60% of the associated forecasted peak demand. For the sake of simplicity, each demand considers one bid per block. The candidate buses for construction of the new units are assumed to be AMOL, KORDKOY, GORGAN, DARYASAR and MINODASHT, which are 230 kV voltages. It is assumed that the strategic producer has total 2195 MW as existing units which are connected to busses NEKA4 and NEKA2, as indicated in Fig. 5. Moreover, there is one non-strategic producer which has totally 960 MW installed capacity as existing units. The operation costs of the existing units are presented in Table 2. Types and data for investment options are similar to six-bus case study. Here, only one generation block is assumed for new units. The operating costs of the new units are assumed equal to the cost of the first block of units which belong to the six-bus network. Scenarios of investment decisions of the rival producer are also assumed to be the same as six-bus case. Susceptance of the transmission lines in the base of 100 MVA, the available transmission capacity in MW and capacity of existing generation units are shown in Fig. 5. Also, susceptances of existing transformers are given in Table 3. Two cases, namely elastic and inelastic demand are simulated and analyzed. The price cap is assumed to be 25 (€/MW h) in the case of inelastic demand. Results of the simulations are illustrated in Table 4.

- (1) In the case of inelastic demands, an unexpected increase will be observed in market prices, which may be unrealistic. In such situations, the price is capped in the wholesale market.

Table 2  
Data of existing generating units.

Unit capacity (MW)	Location	$C_i^S/C_k^{ES}$ (€/MW h)
440	NEKA4	11.46
137.5	NEKA2	19.20
160	NEKA2	12.32
960	ALIABAD2	18.60
Units for Investment by other rival investors		
400	DARYASAR	14.72

Table 3  
Susceptance of 400/230 KV transformers.

400/230 KV substation	Susceptance (p.u.)
Narivaran2-narivaran4	20
Neka2-neka4	33
Hasankif2-hasankif4	16.67
Aliabad2-aliabad4	20

Table 4  
Simulation results for MREC network.

Year	1	2	3	4	5
<i>Inelastic demands</i>					
Invested capacity (peak, base) MW	(1550,0)	(0,0)	(0,0)	(0,500)	200,0
Peak demand (MW)	2714	2883	3061	3251	3453
Average price (€/MW h)	22.359	22.353	22.358	22.348	22.486
<i>Elastic demands</i>					
Invested capacity (peak, base) MW	(800,0)	(0,0)	(0,0)	(0,0)	(0,0)
Peak demand (MW)	2714	2793	3061	3171	3282
Average price (€/MW h)	25.49	27.09	23.30	27.098	27.104
Social welfare (M€)	196.5	193.7	204	184.8	179.8
Net surplus of strategic producer (M€)	119.81	119.25	97.1	92.35	93.95
Net surplus of non-strategic producer (M€)	40.24	60.43	45.7	79.1	72.77
Net surplus of consumers (M€)	36.6	14.04	61.26	13.4	13.09

Furthermore, minimization of the total payment to producers is as the objective in the lower level problem. Table 4 shows the results by assuming price cap equal to 25 (€/MW h). It can be seen that the price cap may affect willingness to investment behavior, the net profit and surplus of demands. Moreover, it was observed in the simulation that market prices are not the same in different buses due to transmission constraints.

- (2) In the case of elastic demands and in the first year, the average market price has been obtained equal 25.49 €/MW h because of investing on 800 MW by the strategic producer. In the second year, the average market price has been increased by 6.28%. In the third year, the average price has been decreased by 13.99% with respect to the second year because of investing by the rival producer. In this situation,

it has been observed in the simulations that the total production of the rival units has been increased by 74.78%. Also, the production of the new units of the strategic producer has been increased by 38.0%. The social welfare and the net consumer surplus have been increased by 5.32%, and 336.64%, respectively, with respect to the second year. However, the net surplus of the producers has been decreased by 20%, due to decreasing in the average price. In the fourth and fifth years, a 400 MW unit is constructed at bus DARYASAR by the rival producer. In the fourth year, the market is facing with an increase of 16.27% in the average price compared to the third year. This is due to the demand growth 3.6%, and a decreasing the outputs of strategic producer's existing unit by 27.82%. In the fifth year, the average market price increases by 0.01% with respect to the fourth year. As a result, the net surplus of the consumers has been decreased in the fourth and fifth years, since the biddings of the demands have been assumed constant over the planning period. Furthermore, the average market prices in the fourth and fifth years are 27.098 €/MWh and 27.104 €/MWh, respectively; which are close to the biddings of demands. It can be seen that the market price is influenced by the growth of the demand, investment of the producers. The average price in the absence of elasticity of demands is less than the average price in the presence of elasticity, because of the selected price cap in the simulations. Increasing the price cap leads to spike in the market price.

- (3) Fig. 6a–c show the market clearing for the peak, shoulder and off-peak periods in the case of inelastic demand and in the first year of the planning period. Fig. 6a and b show that the market was cleared in the price cap in the peak and shoulder periods. However, in the off-peak period the market was cleared in the offer of the rival producer (Fig. 6c). On the other hand, Fig. 7a–c shows the market clearing in the peak, shoulder and off-peak periods in the case of elastic demand for the first year of the planning period. The market was cleared based on offers of the strategic producer in the peak and shoulder periods. However, in the off-peak period, the market was cleared in the offer of the rival producer. Therefore, the strategic producer can benefit through clearing the market price in the peak and shoulder periods in the spot market. However, in the off-peak duration the strategic producer does not benefit through the market clearing, since the market is cleared based on offers of the other producer. Table 5 shows variations of reliability indices of the generation capacity over the planning period, and in the case of elastic demand.
- (4) In the off-peak period, the average market prices in all buses are equal with each other due to no deviations in transmission constraints. In the first year and off-peak period, units having less operation costs supply electricity. Also, the offer of the strategic producer is obtained equal to the operating costs of the rival units (i.e. 18.6 €/MWh). In other years,

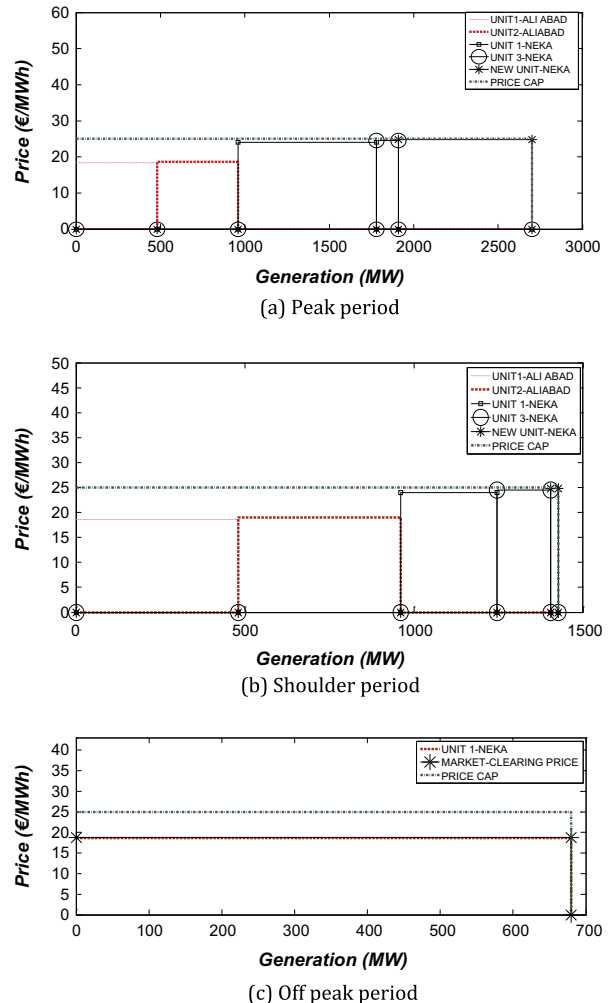
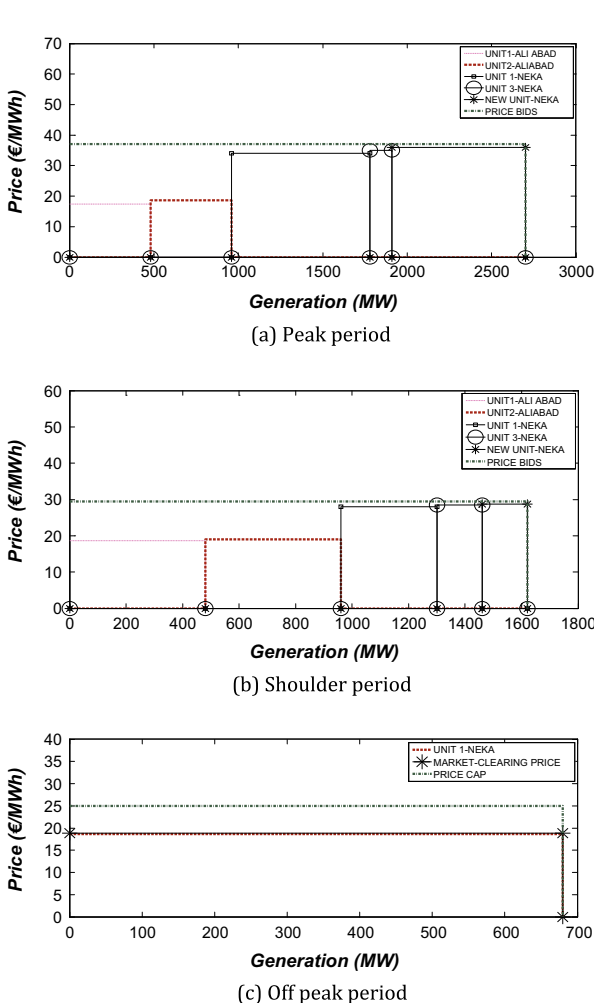


Fig. 6. Market clearing in peak, shoulder and off-peak periods (inelastic demand).

Fig. 7. Market clearing in peak, shoulder and off-peak periods (elastic demand).

**Table 5**  
Reliability indices of MREC network (elastic demand).

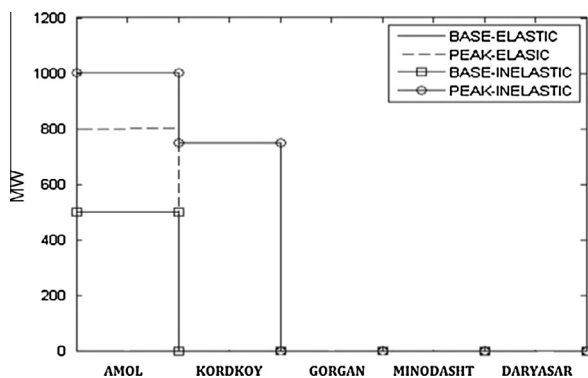
Year	LOLE (d/y)	LOEE (MMW h)
First	3.71	0.0244
Second	3.77	0.0320
Third	4.02	0.0579
Fourth	4.3	0.0697
Fifth	4.44	0.0818

due to investing on the peak unit comprising operating cost equal to 14.72 €/MW h by the rival producer, offers of the strategic producer have been decreased. Therefore, the market price is affected by non-strategic producers in the off-peak periods. Fig. 8 shows capacities constructed by the strategic producer in the cases of elastic and inelastic demands. Also, more capacities have been constructed in bus AMOL due to lack of capacity in this area of the network.

*Case study 3: IEEE 24-bus reliability test system*

This section presents results for a case study based on the IEEE 24-bus reliability test system depicted in Fig. 9 [21]. The data defining the demands and generating units are provided in [22]. Annual demand growth and annual discount rate are assumed to be 10% and 8.7%, respectively. Data regarding to the operation costs of generating units are provided in [23]. The system comprises 17 demands, 32 generating unit, and 38 transmission lines with bus 13 being the reference. Buses 3, 9 and 16 are considered as candidate sites for construction of new units. Also, units 23, 30, 31 and 32 are considered as non-strategic producers. It is assumed that other investors construct their new units at bus 10. Scenarios related to investment strategies of the rival producers, the demand growth, discount rate, budget constraint and other information are assumed the same as the six-bus case study. Table 6 shows the results of simulations.

- (1) The total investment of the strategic producer in the planning period is equal to 1150 MW on the peak technologies. The average market price and its standard deviation in the planning period are 28.89 €/MW h and 0.28 €/MW h, respectively. It can be seen that the market price has experienced small variations over the planning period, because only the same technology (i.e. peak) has been invested over the planning horizon. Fig. 10 shows the operation factor of the units of strategic and non-strategic producers over the planning period. As Fig. 10 shows, the operation factors have slight variations in different years. This is consistent with slight



**Fig. 8.** Results of investment in MREC network.

variations of annual market prices. The total profit of the strategic producer and its standard deviation over the planning period are 1789 M€ and 19.99 M€, respectively. Also, the social wafer and its standard deviation are 2938 M€ and 10.09 M€, respectively. Moreover, variations of the net surplus of strategic producer and social welfare are not much due to small price variations. Simulation results show that in the first year and in the off-peak period, the operation factor of the new units at buses 3, 9 and 16 have been obtained equal to 41.67%, 31.78% and 34.59%, respectively. Furthermore, both the operation factor of plants and their location can affect the investment behavior.

- (2) Fig. 11 shows the sensitivity of the average market price in terms of admissible loading coefficient of transmission (i.e. Fmax). Note that, increasing the Fmax in Fig. 11 corresponds to increasing the transmission capacity or decreasing transmission limitations. The average market price increases by 0.028% by decreasing in the transmission limitations from 0.5 to 0.75. In the simulations, it was observed that outputs of the non-strategic units have been decreased by 6.4% by increasing Fmax from 0.5 to 0.75. Also, output of the strategic producer has been increased by 7.97%. In this situation, market prices have been decreased at buses where there are no non-strategic units. However, the market prices have been increased at buses including non-strategic units, (in the case of Fmax = 0.75) with respect to the case which Fmax equal to 0.5. In this respect, the average market price at bus 10 consisting of a non-strategic new unit (400 MW) has been increased by 1.69%. Also, the average market price at bus 18 consisting of a non-strategic unit (800 MW) has been increased by 0.22%. Again, the average market price at bus 23 consisting of the total capacity equal 1320 MW of non-strategic producers has been increased by 29.66%. At the interval between 0.75 and 1.5 for Fmax, the market price decreases and so the demand increases. As a result, the net surplus of consumers has been increased.

When Fmax is equal 1.5, the following observations should be mentioned: During off-peak period of the third year and in the third scenario, the market price was obtained 14.72 €/MW h which is equal to marginal cost of the new unit of non-strategic producer. It can be seen that the market price has been decreased by 40.86% with respect to the cases that Fmax is assumed 1.25 and 1.75. Note that, in the third scenario it was assumed that a new unit is constructed by non-strategic producers in the third year. In addition, the offer of the strategic producer is equal to 14.72 €/MW h. It should be noted that, output of the strategic producer has been increased by 11.17% and 11.86%, respectively; with respect to the cases which Fmax is assumed 1.25 and 1.75. In this situation, outputs of the non-strategic producers have been decreased by 20%. For Fmax greater than 1.75, both the market prices and the demand are constant. Thus, the net surpluses of consumers do not change for this interval.

Moreover, it was observed in the simulations that the market price decreases when the demand in a few locations are supplied only by non-strategic producers during off-peak periods. In this situation, the net surpluses of these producers were decreased significantly. However, the market price was increased when the strategic producer participates in the off-peak period. Thus, it is beneficial for both the strategic and the non-strategic producers to have productions in the off-peak periods. It can be concluded that the average market price is influenced by the growth of demand, investment behavior of producers, types of producers (strategic or non-strategic) and also their offers, the transmission constraints, network topology, demand utility, and the technology of units.



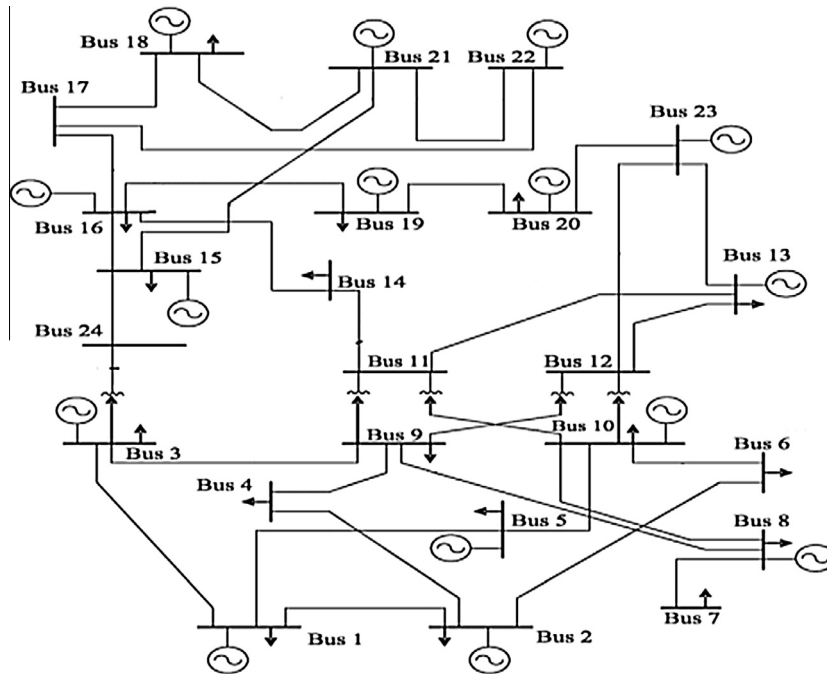


Fig. 9. IEEE RTS system.

Table 6  
Results of RTS network.

Year	Invested capacity (peak, base) MW	Average price (€/MWh)	Net profit of investor (M€)	Social welfare (M€)
1	(450,0)	28.67	340	573
2	(250,0)	28.59	339	589
3	(200,0)	28.84	355	590
4	(0,0)	29.08	369	585
5	(250,0)	29.25	386	601

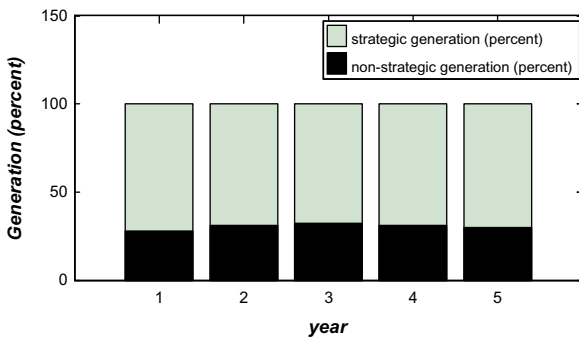


Fig. 10. Share of producers in the market over the planning period.

(3) Fig. 12 shows variations of the market indicators and the strategic producer’s investment in terms of admissible loading coefficient of transmission. Generally, investment in new units increases by increasing  $F_{max}$ . However, in some cases the constructed capacity by the strategic producer has been slightly decreased when  $F_{max}$  is increased. This is the case when  $F_{max}$  increases from 1 to 1.25. Simulation results show that the production of the strategic producer increases by reducing the transmission constraints. However, production of non-strategic producers decreases. Therefore, reduction in the transmission constraints increases the net surplus

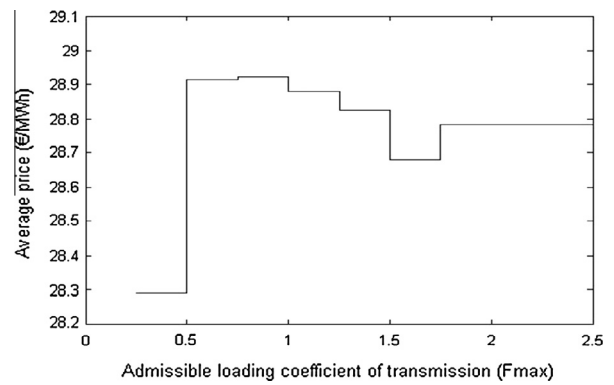


Fig. 11. Average market price in terms of transmission capacity.

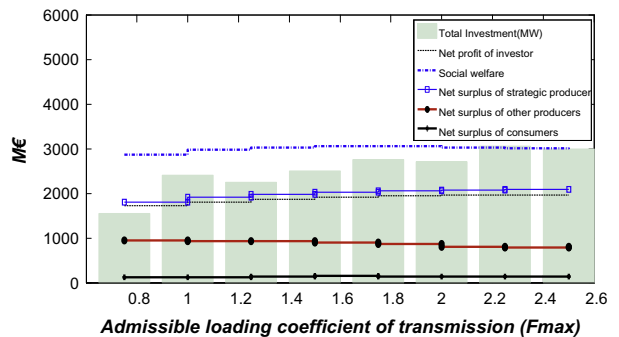


Fig. 12. Market indicators in terms of transmission limitations.

and net profit of the strategic producer, whereas decreases the net surplus of the non-strategic producers. Commonly, social welfare increases when the transmission constraints are decreased. Thus, reducing the transmission constraints improves economic indicators and profit of strategic producer. Table 7 shows computation time for three case studies.

**Table 7**  
Computation time for case studies.

Network	CPU time (S)	GAP%
6 Bus network	71.138	0.00
MREC network (Elastic)	4368.0	16.98
Inelastic MREC network	643.54	1.66
IEEE RTS 24 BUS network	84.012	0.00

## Conclusion

A novel multi-period framework has been presented to study GEP in restructured power systems under uncertainty. The framework includes generation investment from the perspective of the price maker company in a multi-period horizon. Three case studies have been considered and analyzed. The features of the proposed model and simulations results lead to following conclusions:

- (1) Total constructed capacity in the dynamic approach and the total profit of the investor over the planning period have been calculated 950 MW and 96.62 M€, respectively. Total investment and total profit of the investor have been increased in the dynamic approach with respect to the static one. Therefore, using dynamic instead of static approach in GEP leads to accurate and realistic results.
- (2) More peak technologies in energy only markets have been invested in comparison with base technologies. Small variations have been experienced in the market prices over the planning period when only peak technologies have been constructed. The price cap has affected investment willingness, the net profit and surplus of demands in case of inelastic demand. Also, the average market price has been influenced by the type of generation technologies, network topologies and producers.
- (3) Investment on the peak technologies increases the market price because of their operation cost. On the other hand, investment on the base technologies reduces the market price, and increases social welfare. It is also more beneficial to consumers in the short term and makes producers better off in the long term.
- (4) Non-strategic producers do not manipulate market prices in the short term. In the long term, however, these producers may affect the market-clearing price because of investment decisions. Also, share of units' production affects selecting investment options. Therefore, operation factor of plants and their location can affect investment behavior.
- (5) Admissible loading of transmission networks can affect market indicators. In this respect, market price often decreases when admissible loading increases. In addition, the total productions and profit of the strategic producer, the total investment and social welfare have been increased. However, productions and the net surpluses of the price-taker companies have been decreased.

Impact of investment incentives can be included into the proposed model as future work. The proposed model can be expanded to consider impacts of transmission expansion plans, availability of fuel transmission network, regulatory impacts, DSM plans and demand uncertainty.

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