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Int. J. Production Economics 93–94 (2005) 225–229
international journal of production economics
www.elsevier.com/locate/dsw

Production planning: An improved hybrid approach

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Abstract

This paper proposes an extended linear programming model for the hybrid approach proposed by Byrne and Bakir (International Journal of Production Economics 59 (1999) 305) and Kim and Kim (International Journal of Production Economics 73 (2001) 165). In this new model the workload of jobs is sub-divided to introduce the unit load concept of JIT. While an optimum plan is sought, due to this unit load concept, the model takes account of the requirement of small lot sizes which is one factor of the JIT approach. The effective loading ratio (ratio of the output quantity to the input quantity) is modified by omitting the slack time for each job. This helps to ensure that correct quantity of product is produced in each period, thus minimising any excess inventory or backlogging. Omission of slack time will also improve equipment utilisation and throughput. A flexible capacity constraint is also introduced which takes into account the availability of resources based on their previous histories.

The incorporation of the unit load concept and modification of resource requirements and constraints in the proposed LP formulation are expected to help to improve the planning model by reducing the level of WIP and total flow time.
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Keywords: Production planning; Hybrid approach; Simulation; Optimization

1. Introduction

Linear programming (LP) models for production planning have been well known for many years. A typical LP planning model has the objective of minimising the total cost (generally covering the production cost, inventory cost, shortages cost, etc.) over a fixed planning horizon. The usual constraints are inventory balances, production quantity, demand quantity and capacity constraints in each period of the planning horizon.

Material requirement planning (MRP) systems are widely used in production planning. Billington et al. (1983) presented mathematical programming formulations for the general MRP planning problem, together with a method to reduce the

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doi:10.1016/j.ijpe.2004.06.021

problem size. However, the complexity of real production systems makes it difficult for MRP systems to deal with the real characteristics of system demand. Segerstedt (1996) further developed the models of Billington et al. (1983) and identified the issue of scheduling constraints, which can lead to infeasibility of mathematical solutions. To overcome some of these difficulties the hybrid solution approach, which gives the advantages of both analytical and simulation solution procedures has been proposed and investigated by Byrne and Bakir (1999) and Kim and Kim (2001).

This paper describes an extended LP model for the hybrid approach proposed by Byrne and Bakir (1999) incorporating JIT concepts.

2. Background review

The different types of production planning models are discussed in Bakir (1996) and Byrne and Bakir (1999). It appears however that LP models and simulation models are the most widely used. For simple production scenarios LP models are effective and more nearly optimal than simulation models. However, for complex situations simulation models can be more effective. It is possible to take advantage of both approaches by using the hybrid solution approach. These approaches are discussed by Nolan and Sovereign (1972) and Hoover and Perry (1989). To take advantages of both, an integrated has been discussed by Shanthikumar and Sargent (1983).

Byrne and Bakir (1999) showed that the solution from the classical LP planning model may be infeasible for real production system due (*inter alia*) to non-linear behaviour of the workloads at the machines. The two major parts of the LP model, the workloads and capacity constraints, are the issues of concern. They proposed the adjustment of capacity constraints based on the results of the simulation runs to obtain more realistic capacity constraints.

Hung and Leachman (1996) proposed a similar approach modifying the workloads on the left-hand side of the LP model. Kim and Kim (2001) combined and extended the ideas proposed by the previous researchers by applying the loading ratio for workloads and the effective utilization at machines for capacity adjustment.

Estrada et al. (1997) explored the number of production kanbans (NPK) and the unit load size (ULS) for the introduction of Just-in-Time (JIT) techniques. Their model they used determines the combination of NPK and the ULS that results in the lowest probability of stockout (PS). Bard and Golany (1991) also presented a model for determining the optimal number of kanbans at each workstation in a manufacturing system.

Yellig and Mackulak (1997) incorporated knowledge of past machine performance into the scheduling logic by capacity hedging. The optimal capacity hedge is based on a machine's history of interrupted production or unplanned downtime.

In this paper, we apply and extend all these ideas to develop a capacity feasible production plan using the hybrid (LP/simulation) approach.

3. Proposed approach

We propose the following model by applying ideas from the previous section to develop a new formulation of the LP model.

The objective function is:

$$\text{Min} \sum_{i=1}^{N}\sum_{t=1}^{T}(c_{it}Y_{it} + h_{it}I_{it} + \pi_{it}B_{it}),$$

subject to

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i} e_{ijk}a_{ijk}\alpha_i G_{it} \leqslant P_a C_{kt} c,$$

$$I_{it} - B_{it} = I_{it-1} - B_{it-1} + Y_{it} - d_{it},$$

$$X_{it} = \alpha_i G_{it},$$

$$G_{it} \leqslant \text{ULS} \leqslant \text{PC}_i,$$

where T is the time period, 1, 2, 3, ..., t, ... T; N the number of products, 1, 2, 3, ..., i, ... N; c_{it} the production cost of one unit for product i in time period t; Y_{it} the output quantity for product i in time period t (relationship with input quantity is: $Y_{it} = e_{ijk}X_{it}$); h_{it} the inventory carrying cost of one unit for product i in time period t; I_{it} the inventory of product i after time period t; π_{it} the backlogging cost of one unit for product i in time period t; B_{it} the backlogged quantity for product i in time period t; M_i the number of operations j for product i where $j = 1, 2, 3, \ldots, M_i$; e_{ijk} the effective loading ratio of product i for operation j at machine k; a_{ijk} the processing time of product i for operation j in machine k; G_{it} the unit input quantity of product i in time t; α_i the number of unit loads of product i; P_a the probability of availability of machine hours based on the previous machine history; C_{kt} the capacity of machine k in time t; d_{it} the demand quantity of product i in time period t; X_{it} input quantity of product i in time period t; ULS the unit load size, PC_i the container capacity of product i; all variables $\geqslant 0$.

The objective function of the proposed LP formulation is to minimise the sum of production cost, inventory carrying cost and backlogging cost. Obviously it should include the direct production cost. Inventory carrying cost is included to aim to reduce the WIP levels for hybrid MRP II-JIT system. All plants, whatever systems they are using, should aim to deliver their products on time, and this is particularly important in the JIT environment. Hence the incorporation of the backlogging cost.

The first constraint is concerned with the capacity requirement. The left-hand side describes the requirement of machine hours for input product quantity and the right-hand side indicates the availability of machine hours. The loading ratio is defined as the ratio of the output quantity to the input quantity. Zero defect production can rarely be achieved in practice, so the use of the loading ratio may avoid production shortages.

The second constraint describes the relationship between the production quantity, inventory and demand for different time periods (the inventory balance equation).

The third constraint relates production input quantity to the number of unit loads required. G_{it} is the lot size that will move together and the total product input quantity will be an integer multiple of G_{it}. The final inequality shows the relation between the lot size, unit load size and the container capacity.

In this LP formulation, the unit load idea is incorporated to get the advantages of both MRP II and JIT systems to reduce the Work in Process (WIP). Lot sizing is traditionally carried out following completion of the production plan and before production scheduling. The proposed LP formulation includes the factor of unit load. This means that while the optimum plan is sought, the model can take account of the requirement of small lot sizes, which is one factor of the JIT system.

This formulation takes no account of manufacturing lead time. While this is something of a limitation for the LP Model taken in isolation, it is considered that in the context of the hybrid solution process lead times could be taken into account in the associated simulation model, where they would not significantly add to the model's complexity.

Steps of the hybrid solution process are proposed as follows:

Step 1. Generate optimum production plan by the LP model
Step 2. Assign optimum production plan from LP model as input to the simulation model
Step 3. Run simulation model subject to operational criteria
Step 4. Check capacity constraints: if capacity permits go to step 7
if capacity does not permit go to step 5
Step 5. Calculate adjusted capacity
Step 6. Go to step 1
Step 7. Generate production schedule for shop floor based on generated unit load size
Step 8. Stop

4. Case study

We have used the same sample case tested in Byrne and Bakir (1999) and Kim and Kim (2001) and compared the results from the proposed model. The system comprises of 4 machine centres, each having 1 machine and 1 input buffer. Their capacities are constant at 2400 minutes/week.

The cost coefficients, demand matrix processing times and routings are given in Tables 1–4. All process parameters are deterministic and known with certainty.

Table 1
Cost components

Cost coefficient	Unit production cost			Inventory holding cost			Backlogging cost		
Period	1	2	3	1	2	3	1	2	3
Product									
1	100	100	100	25	25	100	400	400	400
2	150	150	150	30	30	150	450	450	450
3	125	125	125	35	35	200	500	500	500

Table 2
Demand

Product	Period		
	1	2	3
1	150	125	160
2	100	150	150
3	125	165	125

Table 3
Processing time (minutes)

Product	Machine centre			
	MC1	MC2	MC3	MC4
1	5		4	10
2	7	7	5	
3	7	6	10	

Table 4
Process routings

Product	Machine visit order		
1	MC1	MC4	MC3
2	MC1	MC2	MC3
3	MC1	MC2	MC3

The analytic part of this proposed procedure is modelled as a linear programme (LP), using the formulation described above. For this first step of the solution procedure we have used C++ programming to supplement the AutoMod software package. The simulation part of the procedure was not incorporated at this stage.

Table 5
Results of proposed approach

Period	Product	Product quantity
1	1	126
	2	125
	3	126
2	1	127
	2	126
	3	126
3	1	126
	2	126
	3	127
Total cost without backlogging	193820	
Total cost with backlogging	207370	

The results shown here (in Table 5) are based on 100% availability of machine hours as well as an effective loading ratio 1. Once the simulation model stage is included variation of these parameters will be explored.

Table 6 shows a comparison between these results and those obtained by the procedures of Byrne and Bakir (1999) and Kim and Kim (2001) (taking the results of the first iteration in the latter two cases).

The results shown above indicate that all three approaches have similar total output levels, and the detailed results indicate that the numbers of each product in each period are also similar. This is to be expected, as each procedure uses a first in first out protocol. It is evident that in the case of no backlogging the total costs of each of the solutions are comparable, while in the backlogging

Table 6
Comparison with results from Byrne and Bakir (1999) and Kim and Kim (2001)

	Proposed approach	Byrne and Bakir approach	Kim and Kim approach
Total output	1135	1151	1137
Total cost without backlogging	193,820	187,925	194,138
Total cost with backlogging	207,370	213,068	231,195

case the proposed formulation leads to a lower total cost than the other two.

It is anticipated that once the full hybrid procedure is implemented incorporating this new LP formulation, there will be improvements to the final plan over the solutions provided by the earlier approaches.

5. Conclusion

This paper has presented a new LP formulation for use with the hybrid approach presented by Byrne and Bakir (1999) and Kim and Kim (2001). This new formulation incorporates features allowing JIT-like aspects of manufacturing to be better represented.

The preliminary results presented here indicate that the new formulation may give some benefits over those previously adopted. Further investigation, incorporating the LP model with simulation in the complete hybrid procedure, is expected to identify further benefits which would make the procedure easier to apply in many modern manufacturing systems which incorporate aspects of the JIT approach.

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