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A note on transaction costs and the existence of derivatives markets

Frank Fehle [a,b,*]

[a] *Moore School of Business, University of South Carolina, 1705 College Street, Columbia, SC 29208, USA*
[b] *Barclays Global Investors, 1 Royal Mint Court, London EC3N 4HH, UK*

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Abstract

This note connects the idea of arbitrage pricing under transaction costs to the existence and structure of derivatives markets. It illustrates the "paradox" of pure arbitrage pricing via replicating portfolios and the existence of markets for redundant securities in a general multi-period model as in Duffie (1996). A general result under homogeneous transaction costs regarding the choice between derivatives markets and replicating portfolios is derived. For the case of differential transaction costs of replication, it is argued that dealer markets for derivative securities dominate both "home-made" replication and order-driven markets; furthermore, the setting of derivatives prices within arbitrage bounds may be driven by the level of competition among dealers.

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Few concepts in finance have enjoyed the success of the use of replicating portfolios for the pricing of derivative securities. At the core of many pricing mechanisms for derivative securities lies a no-arbitrage condition prescribing the equality of the derivative security's price and the price of the corresponding replicating portfolio.

Most of the fundamental pricing results in this area such as the option pricing equation by Black and Scholes (1973) or the two-factor term structure model by Heath,

* Tel.: +1-803-777-6980; fax: +1-803-777-6876.
E-mail address: ffehle@moore.sc.edu (F. Fehle).

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Jarrow, and Morton (1990) ignore the existence of imperfections such as portfolio constraints and transactions costs. This omission is interesting in light of the fact that without imperfections markets for derivative securities should not exist or at least their existence should be a matter of indifference: investors wishing to obtain the payoff pattern from the derivative security could simply buy the corresponding replicating portfolio at the same cost. Yet derivative markets do exist and their explosive growth over the past 20 years is testament to the success of derivative securities as financial innovations.

This “paradox” can be interpreted as a violation of the second fundamental theorem of asset pricing, as introduced by Harrison and Kreps (1979), and Harrison and Pliska (1981), which shows that the uniqueness of the equivalent martingale measure is a necessary and sufficient condition for market completeness. In incomplete markets, the existing securities do not span a complete payoff space. Derivative securities are not truly “derivative” or redundant in the sense that they provide previously unobtainable payoffs. Examples of the incomplete markets literature are Adler and Detemple (1988), Cvitanić and Karatzas (1992), Duffie and Zariphopoulou (1993), He and Pages (1993), Jouini and Kallal (1995a), Scheinkman and Weiss (1986), and Svensson and Werner (1993). With incomplete markets replicating portfolios may not exist even though approximating portfolios may exist and the “paradox” vanishes.

One potential source of market incompleteness are frictions such as transactions costs and portfolio constraints. Beginning with Leland (1985) this literature extends the theory of derivatives pricing to consider the effects of such frictions. In an economy with transactions costs, the payoffs of derivative securities can (potentially) be replicated but the securities may still exist, if replication is costlier than the derivative security itself. In this case arbitrage bounds can be derived which depend on the structure of the transactions costs among other things. Allowing for heterogeneous consumers and heterogeneous transaction costs the equilibrium price of a derivative security must lie inside the arbitrage bounds but may not lie at the no-arbitrage price without transactions costs. Other work in this area is Boyle and Vorst (1992), Bergman (1995), Cvitanić and Karatzas (1993), Davis, Panas, and Zariphopoulou (1993), Edirisinghe, Naik, and Uppal (1993), Grannan and Swindle (1996), Korn (1995), and Whalley and Wilmott (1997).

The focus of this note is on the interaction of transaction costs and derivatives market existence, and its implications for the market design and for the provision of liquidity services of derivative securities. To simplify the analysis transaction costs are explicitly modeled for redundant securities or their replicating portfolios only, thereby avoiding problems of existence and equilibrium pricing. The thrust of the analysis is directed at agents’ choices between holding derivatives and their replicating portfolios, an area mostly neglected by previous research. Using the concept of arbitrage bounds, it is then argued that market structure, such as the level of competition in a derivatives markets, may have implications for observed derivatives prices.

The remainder of the note is structured as follows: Section 1 introduces the model of the economy without transaction costs, while Section 2 provides the main results regarding the existence of derivatives markets with transaction costs. Section 3 concludes. Proofs and derivations are contained in Appendix A.

1. Derivatives markets without transaction costs

This section uses the same setting of a discrete-time multi-period model of the economy and discusses the standard arbitrage pricing results for derivatives in the absence of transaction costs as in Duffie (1996). It is shown that in this setting agents are indifferent between transacting in derivatives markets and holding replicating portfolios. For simplicity, all the notation from Duffie (1996) is used: probability space (Ω, F, P), T dates, N basic securities with adapted dividend process δ, and price process S, trading strategy θ; the standard notion of arbitrage is a trading strategy with $\delta^{\theta} > 0$; agents have strictly increasing utility functions U, and endowment process e in L_{+}; let π denote the state-price deflator.

Now consider the introduction of a derivative or redundant security with dividend process $\hat{\delta}$ and price process $\hat{S}$. The security is redundant, if there exists a trading strategy θ based solely on the N basic securities such that $\delta_t^{\theta} = \hat{\delta}_t$ for $t \geq 1$. Call this trading strategy the replicating portfolio θ^{RP}. Note that market completeness is not necessary for this definition. Duffie (1996) shows that in the absence of arbitrage the price process of the redundant security follows:

$$\hat{S}_t = \frac{1}{\pi_t} E_t \left(\sum_{j=t+1}^{T} \pi_j \hat{\delta}_j \right), \quad t < T. \tag{1}$$

Lemma 1. *The price process $\hat{S}$ and the value of the replicating portfolio defined as θ^{RP}. S are equal for all t. Note that at T the processes are trivially equal to zero for all $t < T$.*

Proof. Given in Appendix A. □

Proposition 1. *Suppose (δ, S) is arbitrage-free with state-price deflator π. Let $\hat{\delta}$ be a redundant dividend process with price process $\hat{S}$. Then any agent is indifferent between a trading strategy θ_1 that includes θ^{R} holdings of the redundant security and a trading strategy θ_2 that includes θ^{R} holdings of the replicating portfolio θ^{RP}.*

Proof. Given in Appendix A. □

The result can easily be extended to economies with more than one redundant security. Note that this result has several interesting implications. In the absence of transaction costs occurrence and trading volume of derivative securities should be random as agents are perfectly indifferent between buying and selling derivative securities and holding the appropriate replicating portfolios. However, if we make the assumption that agents incur some type of search cost to find a counterparty for a transaction in a derivative security, it appears that absent any other transaction costs agents prefer to hold replicating portfolios and thus derivative securities should not exist. In this case, arbitrage pricing without transaction costs and the existence of derivative markets should not hold simultaneously.

2. Derivatives markets with transaction costs

In the following, a general formulation of transaction costs for the buying and selling of securities in the economy is introduced. Under mild technical conditions it is shown that the transaction costs can be considered in the individual agent's optimization problem as an additional one-time cost. With transaction costs being the same for all agents, trading in derivative securities versus replicating portfolios depends on the magnitude of transaction costs for both alternatives. Finally, a potentially more realistic assumption of differential transaction costs for classes of agents is introduced and it is shown that for a finite number of agents in the class with the lowest transaction cost profit opportunities may exist.

2.1. "Home-made"' replication versus derivative securities

Derivatives markets do exist and arbitrage pricing methods appear to do a good even though not perfect job of pricing derivative securities. Thus, one choice of relaxing the proposition in the previous section is via the introduction of transaction costs. Let the total transaction costs of a replicating portfolio θ^{RP} be a positive adapted process τ^{RP}. Let the total transaction costs of a redundant security θ^{R} be a positive adapted process τ^{R}. The transaction costs are best understood in terms of the costs of administering a replicating portfolio or the cost of finding a counterparty for a derivatives transaction. The transaction costs of buying and selling the basic securities are not explicitly modeled to simplify the exposition. In reality, the trading of basic securities also gives rise to transaction costs, which in turn affect the transaction costs θ^{RP} of replicating portfolios. As mentioned in the introduction, there is a substantial literature on the topic of replication when there are transaction costs for the basic securities, but recall that the focus of the present analysis is on the choice between "home-made" replication and derivatives markets rather than the interaction of basic security transaction costs and arbitrage pricing bounds.

Lemma 2. *Suppose that $\tau \in \Phi$ and that there are no transaction costs of administering a replicating portfolio for τ. Then the transaction costs of a given strategy θ^{RP} or θ^{R} can be bought or sold at a price Υ:*

$$\Upsilon_t = \frac{1}{\pi_t} E_t \left(\sum_{j=t+1}^{T} \pi_j \tau_j \right). \tag{2}$$

Note that the transaction costs for the current period are excluded assuming that initiation of the strategy is costless.

Proposition 2. *Suppose (δ, S) is arbitrage-free with state-price deflator π. Let $\hat{\delta}$ be a redundant dividend process with price process $\hat{S}$. Suppose agents choose between a trading strategy θ_1 that includes θ^{R} holdings of the redundant security and a trading strategy θ_2 that includes θ^{R} holdings of the replicating portfolio θ^{RP}.*

(a) *For $\Upsilon_t^{\mathrm{R}} > \Upsilon_t^{\mathrm{RP}}$ all agents choose to hold replicating portfolios and derivative markets do not exist.*

(b) *For $\Upsilon_t^{\mathrm{R}} > \Upsilon_t^{\mathrm{RP}}$ all agents choose to hold derivative securities.*

Proof. Given in Appendix A. □

The trivial case of the costs being equal is ignored. The above result has an interesting implication for the success of financial engineering. Financial innovations in the form of derivative securities have a higher probability of success the higher the transaction costs of "home-made" replicating portfolios and the lower the transaction costs of derivative securities markets.[1]

2.2. Derivatives markets with differential transaction costs

The above model can be made more realistic and can be used to motivate empirical features of existing derivatives markets by analyzing the effects of differential transaction costs of replication. For $\Upsilon_t^{\mathrm{R}} > \Upsilon_t^{\mathrm{RP}}$, Proposition 2 suggests that agents are willing to incur transaction costs in the form of search costs to find counterparties for their desired derivatives position, since these search costs are lower than the transaction costs of administering a replicating portfolio. In market microstructure terms this mechanism corresponds to a purely order-driven market without dealer intermediation where the search costs Υ_t^{R} can, for example, be interpreted as the costs of participating in an electronic trading system.

However, many existing derivatives markets use dealers as liquidity providers: some well-known examples are the Chicago Board Options Exchange (CBOE), the Eurex derivatives exchange in Europe, and the over-the-counter market for interest rate swaps with an active dealer market centered in London.[2] Thus, introduce to the present analysis a subset of agents called dealers (denoted by subscript D) such that $\Upsilon_{\mathrm{D}}^{\mathrm{RP}} < \mathrm{Min}[\Upsilon^{\mathrm{R}}, \Upsilon^{\mathrm{RP}}]$. The assumption of differential transaction costs for administering replicating portfolios can be motivated by a simple economies of scale effect given that there is a fixed component to the transaction costs of replication; in this case, a dealer, who does not wish to take a derivatives position, can nonetheless sell derivatives contracts to many agents, and simultaneously create an off-setting replicating (hedging) portfolio. If the per contract transaction costs of the dealer's replicating portfolio are lower than the search costs of an order-driven market, the dealer market will be preferable.[3] Note that in this interpretation, derivatives dealers do not simply serve as intermediaries between agents with off-setting demands for derivative securities, such as, for example, an option buyer and an option writer. Rather, the above analysis allows for a derivatives market in which economic agents all have the same demand, for example, desiring to buy options.

Derivatives markets with dealers give rise to a set of arbitrage bounds for observed derivatives prices: we have already established in the previous section that all other agents choose to hold derivative securities and initially are willing to pay $\hat{S} + \Upsilon^{\mathrm{R}}$ for the right to

[1] There have been several notable failures of derivatives markets: Horrigan (1987) discusses CPI inflation futures markets, which briefly existed on the New York Coffee, Sugar, & Cocoa Exchange (CSCE) in the mid 1980s. Johnston and McConnell (1989) describe the decline of the futures market for GNMA bonds, which they attribute to a design flaw in the contract's specifications.

[2] Tsetsekos and Varangis (1998) provide an international comparison of the structure of derivatives exchanges.

[3] The proof (not shown) that for $\Upsilon_{\mathrm{D}}^{\mathrm{RP}} < \Upsilon^{\mathrm{R}}$ agents choose dealer markets over order-driven markets proceeds exactly like the proof of Proposition 2.

hold securities with pure payoffs. There exists an arbitrage opportunity for dealers as they can replicate the pure payoff at a total cost of hat $\hat{S} + \Upsilon_{\mathrm{D}}^{\mathrm{RP}}$. Market prices of the derivative security must lie between these arbitrage bounds, but their exact location depends on the structure of the market: for example, markets with intense competition among dealers should exhibit lower prices closer to $\hat{S} + \Upsilon_{\mathrm{D}}^{\mathrm{RP}}$ than market with less intense competition. This notion is consistent with empirical findings by Mayhew (2002) showing that US equity options listed on multiple exchanges (proxying for more intense competition) have lower bid-ask spreads than otherwise similar single-listed options listed.

3. Conclusion

This note connects the idea of arbitrage pricing under transaction costs to the existence and structure of derivatives markets. While the effect of transaction costs on derivatives *pricing* has been studied previously in papers such as Delbaen and Schachermayer (1994), Cvitanić, Pham, and Touzi (1999), and Constantinides and Zariphopoulou (1999), the connection among transaction costs and derivatives *market structure* is generally not addressed in this line of research. The note first illustrates the "paradox" of pure arbitrage pricing via replicating portfolios and the existence of markets for redundant securities in a general multi-period model. It then derives a general result under homogeneous transaction costs regarding the choice between derivatives markets and replicating portfolios. Finally, for the case of differential transaction costs of replication, it is argued that dealer markets for derivative securities dominate both "home-made" replication and order-driven markets as dealers with transaction cost advantages can provide other agents desiring a derivative position. In the case of differential transaction costs the setting of derivatives prices within arbitrage bounds may be driven by the level of competition among dealers.

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Appendix A

Proof of Lemma 1.

$$\theta_t^{\mathrm{RP}} S_t = \sum_{k=1}^{N} \theta_t^k \frac{1}{\pi_t} E_t \left(\sum_{j=t+1}^{T} \pi_j \delta_j^k \right) = \frac{1}{\pi_t} E_t \left(\sum_{j=t+1}^{T} \pi_j \sum_{k=1}^{N} \theta_j^k \delta_j^k \right)$$

$$= \frac{1}{\pi_t} E_t \left(\sum_{j=t+1}^{T} \pi_j \hat{\delta}_j \right) = \hat{S}_t. \qquad \square$$

Proof of Proposition 1. The first strategy provides the agent with

$$c_1 = e + \theta_{t-1}^{\mathrm{b}}(S_t^{\mathrm{b}} + \delta_t^{\mathrm{b}}) - \theta_t^{\mathrm{b}} S_t^{\mathrm{b}} + \theta_{t-1}^{\mathrm{R}}(\hat{S}_t + \hat{\delta}_t) - \theta_t^{\mathrm{R}} \hat{S}_t, \tag{A.1}$$

where the superscript b denotes the basic securities in the portfolio. The second strategy provides the agent with

$$c_2 = e + \theta_{t-1}^{\mathrm{b}}(S_t^{\mathrm{b}} + \delta_t^{\mathrm{b}}) - \theta_t^{\mathrm{b}} S_t^{\mathrm{b}} + \theta_{t-1}^{\mathrm{R}}(\theta_t^{\mathrm{RP}} S_t + \theta_t^{\mathrm{RP}} \delta_t) - \theta_t^{\mathrm{R}} \theta_t^{\mathrm{RP}} S_t, \tag{A.2}$$

where the superscript b denotes the basic securities in the portfolio with the exception of the basic securities used to form the replicating portfolio. By the lemma and the definition of the redundant security we can rewrite

$$c_2 = e + \theta_{t-1}^{\mathrm{b}}(S_t^{\mathrm{b}} + \delta_t^{\mathrm{b}}) - \theta_t^{\mathrm{b}} S_t^{\mathrm{b}} + \theta_{t-1}^{\mathrm{R}}(\hat{S}_t + \hat{\delta}_t) - \theta_t^{\mathrm{R}} \hat{S}_t. \tag{A.3}$$

It follows trivially that $U(c_1) \geq U(c_2)$ and $U(c_1) \leq U(c_2)$ which is the standard definition of indifference as for example in Varian (1992). □

Proof of Proposition 2(a). Suppose some agents hold derivative securities and that there is no arbitrage (Proof by contradiction). Agents holding derivative securities are indifferent between either paying $\hat{S} + \Upsilon_t^{\mathrm{R}}$ and receiving only the payoffs of the redundant security or paying only $\hat{S}$ and incurring future transaction costs. Now consider the following strategy: buy θ^{RP} and sell $\theta^{\mathrm{RP}}\tau^{\mathrm{RP}}$. By definition the payoff of this strategy is equal to the pure payoff of the replicating portfolio at a cost of $\theta^{\mathrm{RP}} S + \Upsilon_t^{\mathrm{RP}}$. By definition $\hat{S} = \theta^{\mathrm{RP}} S$ and therefore $\Upsilon_t^{\mathrm{R}} > \Upsilon_t^{\mathrm{RP}}$ implies an arbitrage opportunity if agents hold derivative securities. Thus, all agents must hold replicating portfolios which as is easy to see does not allow for arbitrage. □

Proof of Proposition 2(b). This proof uses the same arbitrage argument as above. □

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