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Fuzzy model of vehicle delay to determine the level of service of two-lane roads

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ABSTRACT

The level of service (LOS) on two-lane highways and, therefore, the quality of traffic flow, is currently estimated based on the delay of the vehicles and, in certain types of roads, the average travel speed. Speed is relatively easy to measure. However, it is important, and not so simple, to determine whether a vehicle is delayed. Traditional methods, generally based on quantitative measurements of average time between vehicles and thresholds, fail to take into account the inherent vagueness of the driving process. In this paper, we have developed a fuzzy model that gives a new and reliable method for determining such vehicle state on two-way two-lane roads, based on drivers' perceptions. The proposed system is composed of seven fuzzy subsystems that take into account imprecise knowledge, human factors, and subjective perceptions regarding the road, the car, the driver, environmental conditions, etc. Simulation results of the system have been successfully compared with the behavior of two-lane road drivers who were interviewed. The level of service of these facilities is obtained using the estimated vehicle delay state and the overtaking maneuver. Therefore, this proposal makes it possible to introduce these existing driving experiences into LOS assessment and accordingly, it is potentially a step forward since LOS must be related, by definition, to user experience. These results could be used in future frameworks. In addition, an extension of the possible states of a vehicle is defined. This approach takes into account the drivers point of view regarding overtaking desire and in this sense, it is closer to reality.

1. Introduction

Both traffic engineers and road users are concerned with the quality of traffic flow. The concept of level-of-service (LOS) is commonly accepted in traffic engineering as a way to assess the quality and flow characteristics on various facilities (Cohen & Polus, 2011). The level-of-service gives an idea of the comfort of drivers while driving and reflects the traffic flow and traffic congestion. But only a quantitative description of traffic quality is not enough. In fact, transportation has increased the need for reliable descriptions of traffic quality flow using measures and concepts that are easily understood by road users. After a century, this search is not over.

The level of service on two-lane roads and, therefore, the quality of traffic flow, is currently estimated based on the delay of the vehicles and, in certain types of roads, the average travel speed. Speed is relatively easy to measure (Corcoba & Muñoz, 2014). On the contrary, being vehicle delay a key factor to calculate the level of service on roads, its estimation depends largely on technical and non-technical parameters. Therefore, it is a very complex task because it is influenced by many different factors and most of them present uncertainties and inaccuracies.

Traditional methods use a few parameters (hardly two) to calculate the delay, and do not deal with uncertainty. Vehicle delay typically consists of two parts, uniform and non-uniform (Dion, Rakha, & Kang, 2003). Most studies focused on the uniform delay, estimated by signal timings and traffic volumes. They are based on historical data, use the statistical approach, and work on urban environments where there are traffic lights and other type of signals. However, the determination of non-uniform delay has been a problem for researchers, as it involves random and uncertain factors. Conventional approaches do not handle many variables and interactions that cannot be defined properly by mathematical models. Indeed, following headway, driver conditions, perception times, weather conditions, among others, are not being used in the existing delay formulas due to the imprecise nature of these variables, while it is obvious that vehicle delay can increase on rainy days or when sight distance is very low. These conditions are not considered in those estimations (Murat, 2006). Furthermore, these traditional approaches do not provide a method for estimating vehicle

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delay for two-lane roads or for multilane highways. They usually calculate the delay only for signalized intersections. Therefore, a model of the vehicle delay that takes into account not only quantitative but also qualitative factors and that can be applied to two-way roads is required.

In this paper, we have used approximate reasoning to develop a fuzzy model that gives a new and reliable method to estimate if a vehicle is delayed, based on imprecise variables. This knowledge based model aims at better understanding drivers' perception, including subjective factors. The motivation of using approximate reasoning is because one of the main features of human decision-making and response processes is their inherent approximate nature. Deterministic models of driver behavior fail to take into account the inherent vagueness of the driving process (Chakroborty & Kikuchi, 1999). Besides, we are interested in two-way two-lane roads (T-W T-L), with typically very few road signals.

The proposed fuzzy model is made up of several fuzzy subsystems. It is structured in three levels. In the first one, three fuzzy systems estimate the quality of the environment as perceived by the driver, the car performance, and the driver conditions. The outcomes of these systems allow the determination of the level of safety while driving and the estimation of the available gap in order to overtake. At the third level, two fuzzy systems estimate the driver willingness to overtake and if the driver perceives he can overtake. Finally, based on this information, the fuzzy system estimates if the individual car is delayed.

We have used this vehicle state to estimate the level of service of the two-lane roads. Furthermore, as the overtaking maneuver has been considered as the combination of these two subjective perceptions of the driver, the intelligent system provides guidance on whether to overtake, thus reducing delays and consequently, improving driving comfort and the level of service.

Additionally, this approach has led to an extension of the possible states of a delayed vehicle considering passing desire as a new factor.

Surveys were conducted to T-W T-L road drivers to test the system. Their answers were compared with those of the system itself, being the expected on more than 90% of the cases.

To summarize, the main contributions of this paper are the following. We have design a fuzzy model of the vehicle delay in two-lane roads completely different from how the delay is defined in most of the previous papers, including many of the subjective variables that are involved in the driving. As far as we know, the estimation of vehicle delay in two-lane two-way roads using fuzzy logic has not been proposed before and, therefore, the fuzzy perception of the level of service of these types of highways is novel. We have also proposed a new categorization of the delayed vehicles based on the subjective consideration of overtaking.

The rest of the paper is organized as follows. Section 2 summarizes how the vehicle delay is estimated by traditional methods, particularly in two-lane roads, and the literature regarding the application of fuzzy logic to traffic flow. Section 3 describes in detail the design and implementation of the fuzzy model of the vehicle delay, showing most of the fuzzy subsystems it includes. Section 4 presents the determination of the level of service of two-lane roads based on drivers' subjective perceptions such as the overtaking desire. In Section 5, simulation results are compared to real data obtained from surveys conducted to two-way road drivers, and an extension of the possible states of vehicles is proposed. The paper ends with the conclusions.

2. Background

The Transportation Research Board (1985), in the *Highway Capacity Manual* (HCM), first defined the level-of-service (LOS) as a mere reflection of the comfort of drivers and, later, in a more specific way including conditions such as speed, timings, safety, signaling, etc. The level-of-service depends largely on the type of road. In fact, it can be calculated as the ratio between traffic flow and road capacity. This may be true for high capacity roads, where the chance of overtaking at any time is supposed to be uniform for all drivers. However, this is not the case for two-lane roads, where some drivers may be delayed despite their desired speed because it is not always possible to overtake another vehicle. Therefore, vehicle delay must be included as a crucial factor to estimate the level of service of multilane roads.

In 2015, the first large research project funded by NCHRP (National Cooperative Highway Research Program) on operational considerations in two-lane roads, project 17-65, has gone under way. The results of the research presented here may be used by the project team in revising the existing framework for measuring LOS in these facilities.

2.1. Vehicle delay calculation by traditional methods

Delay calculations have not been prevalent in uninterrupted flow facilities for individual vehicles. The trend has been to measure delay comparing actual trip times with free flow travel times. This way, an aggregate delay can be measured, and is routinely reported as hours of congestion spent by users in a facility. Therefore, in order to consider the delay of a single vehicle in a facility, the best resource is to compare it with data from interrupted flow facilities.

In conventional models for estimating vehicle delay, only statistical data are usually taken into account, with little consideration of nontechnical factors, as the latter cannot be directly represented in analytical models. The Webster (1958), Highway Capacity Manual (Transportation Research Board, 1985) and its subsequent updates, or the Akcelik's (1981) delay calculation methods have been preferred by traffic engineers for many years. In these studies, the average delay of vehicles is calculated based on deceleration, stopping, acceleration times and queues, and they are mainly focused on signalized intersections or urban traffic flow.

The first and most general model for estimating the delay is given by:

$$d = d_u + d_o \tag{1}$$

In Eq. (1), the first term represents the average delay assuming uniform vehicles arrivals, that is, the uniform part. The second term represents the additional delay due to the randomness of vehicle arrivals and over-saturation queues. This non-uniform delay, also called the overflow term, is attributed to the probability of sudden surges arrivals and cycle failures. A third term can be considered, meaning a semi-empirical adjustment term that is introduced into the model to account for specific field conditions.

The Webster well-known delay formula to estimate the delay for isolated intersections is:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0.65\left(\frac{c}{q^2}\right)^{1/3} x^{2+5\lambda} \tag{2}$$

where:

- d = Average delay per vehicle on the particular approach of the intersection (seconds)
- c = Cycle length in seconds
- q = Flow
- λ = Proportion of the cycle which is effectively green for the phase under consideration, that is, the ratio of effective green to cycle time
- x = Degree of saturation (volume to capacity ratio)

The values of the parameters give particular forms to the delay expressions depending on the specific traffic situation (country, urban flow, etc.).

Following this classical work, numerous studies were conducted in the field of estimating delay at signalized intersections. As a result of these studies, a number of delay models based on deterministic queuing theory were proposed to suite different field conditions. Among these, the most notable are the models developed by Akçelik (1981), Akçelik and Rouphail (1993), Miller (1961) in Australia, and the models proposed in the continuous updates of the HCM in the United States. These and other subsequent definitions are analytically superior to Webster's classical model, in the sense that they can successfully deal with oversaturated conditions and the effect of progression and platooning (Hoque & Imran, 2007). Murat (2006) summarizes several studies related to vehicle delay modeling applicable for signalized junctions, as in a more recent paper Cheng, Du, Sun, and Ji (2015) also do, who provide a comprehensive review of the theoretical delay estimation model over the last ca. 90 years.

2.2. Vehicle delay and level of service in two-lane roads

Very few are the studies on estimating delay for two-lane facilities. The latest editions of the Highway Capacity Manual (Transportation Research Board, 2011) suggested two factors to determine the level-of-service of two-lane rural roads: speed and delay. To calculate this, it is necessary to measure two variables: Average Travel Speed, ATS, and Percent Time-Spent Following, PTSF. The latter, representing delay and platooning structure, has been included since 2000. However, the calculation of the delay percentage based only on these parameters is not an easy task. Indeed, it is rather difficult to measure PTSF due to the complexity of collecting data from fast-traveling vehicles that follow slower cars. In fact, the applicability of PTSF as a service measure is being questioned (Cohen & Polus, 2011; Luttinen, 2001).

Percent time-spent following can be defined as the proportion of time that the vehicle cannot travel at the desired speed, i.e., the proportion of time that fast vehicles travel in platoons behind slower vehicles. The calculation of PTSF for two-way highways is as follows:

$$PTSF = BPTSF + f_{d/np}, \tag{3}$$

$$BPTSF = 100(q - exp(-.000879Vp)) \tag{4}$$

where:

- $f_{d/np}$ = adjustment for the combined effect of the directional distribution of traffic and of the percentage of no-passing zones on PTSF
- $BPTSF$ = base percent time spent following for both directions, given by (4)
- V_p = the percent no passing zones, that is, two-way passenger-car equivalent flow rate for peak 15-minutes period (pc/h).
- $q = flow$

There are three road types defined for these facilities, which are the following. Type I, facilities used for mobility, and therefore where speed is relevant; type II, facilities used for accessibility, where only delay and platooning structure matter, and therefore only PTSF is relevant; and type III, facilities in suburban areas where there are many roadside accesses and uses, and where the criterion is not the delay, but percentage of the free-flow speed. The HCM gives the following classification of level-of-service regarding the PTSF (Table 1).

On the other hand, the average travel speed is defined as:

$$ATS = (M \times 3600\,\text{seconds perhour})/ATT \tag{5}$$

where ATT (Average Travel Time) is the sum of TT (Travel Time)/Total number of runs. Travel Time (TT) is time in seconds from one control point (CP) to the next.

Table 1
Level of service for US two-lane highways, type II.

LOS	HCM (PTSF)
A	$<$ 40
B	$>$ 40–55
C	$>$ 55–70
D	$>$ 70–85
E	$>$ 85
F	Flow rate exceeds segment capacity

The determination of what proportion of traffic flow in two-lane roads is delayed is complex. According to the above approach, a vehicle is considered to be delayed if the preceding headway is under a critical headway. This factor is a measure of the time headway between two consecutive vehicles that allows the vehicle to be considered isolated, that is, if the speed is not influenced by any other vehicle. If the time headway is lower than this value, the vehicle is delayed. Many attempts have been made since 1965, proposing critical values of between 3 and 9 s. In the HCM, this value has gone down from 5 s in 1985 to 3 s in 2000, in an attempt to replicate the values of the variable that better approaches the delay, the Percent Time Spent Following, or PTSF, obtained through simulations (NCHRP Project 3-55). PTSF cannot be established in the field currently, being only observable in simulation.

Several approaches have been proposed in order to estimate this critical interval. Headways are seldom lower than 0.5 s or over 11 s at different traffic volumes but, within that interval, very different values are obtained depending on the main criterion considered (reaction time, speed of the vehicle from behind, if it is legal to overtake another vehicle on that stretch of the road, and other statistical measures). Athol (1965) investigated the effects of critical intervals of 1.2, 1.5, 2.1 and 2.7 s on platoon behavior. He selected a critical headway of 2.1 s corresponding to a traffic volume of 1500 vehicles per hour per lane (vphpl), according to two parameters: reaction time and response time. Other authors adopted criteria based on either the speed or the difference of speed between two consecutive vehicles. Hoban (1984) found intervals of up to 6 s, and Pahl (1971) between 4 and 5 s. The literature on this topic shows that critical gaps can be as low as 1.60 s, and there is a significant variation (12–38%) on the estimated values by different methods.

Statistical models have also been applied to obtain the delay. According to Blank (1980), a free vehicle (not delayed) is easy to characterize. Assuming the arrivals of the vehicles to be independent, the probability model is a Poisson distribution, and the time between arrivals follows an exponential distribution. Using this method, Branston (1979) proposed intervals between 3.75 and 4 s; Miller (1961) tried an exponential distribution that gives an interval of 8 s, and Buckley's (1968) about 4 s. Highway Capacity Manual used 3 s (average percent of time that vehicles spend in platoons behind slow vehicles due to inability to overtake, that is, percentage of vehicles traveling at headways of 3 s or less). The issue is far from resolved, probably due to the use of a single value as threshold.

However, papers that take into account the driver's feelings regarding the traffic flow are scarce. A notable exception is the work by Greenshields, Channing, and Miller (1935), who developed a quality index for traffic flow based on "frustration factors". In their view, the frequency and amount of speed changes are undesirable factors that irritated drivers and increased the cost of operation. The quality index (Q) was defined as:

$$Q = 1000S/\left(\Delta_S\sqrt{f}\right) \tag{6}$$

where:

- S = average speed (km/h (mph))
- Δ_S = absolute sum of speed changes per kilometer (mi)
- f = number of speed changes per kilometer (mi)

Other documents have also addressed the delay calculation with respect to the chance of overtaking including human behavior, being more general. Romana and López (1996) reported that for rural two-lane roads, due to limited opportunities to overtake other vehicles, some drivers are moving slower than desired until they can overtake. The extra time wasted can be stressful for the driver and passengers. More recently, Llorca, Moreno, Lenorzer, Casas, and Garcia (2015) included the passing desire in a microscopic driving model.

In this paper, as we claimed in the introduction, the approach is completely different from how the delay is defined in most of these papers. We are interested in two-lane roads, with typically very few road signals, but from the driver's subjective point of view.

2.3. Application of fuzzy logic to traffic flow

Fuzzy logic was first introduced by Zadeh in 1965. It was proposed as an extended version of the classic logic by Aristotle. Fuzzy logic is useful when dealing with vague, uncertain, and complex environments. The imprecise information that characterizes the elements of a universe can be interpreted as a linguistic variable and modeled with fuzzy sets.

Given a universe of discourse U, a fuzzy set is a mapping μ: $U\rightarrow [0, 1]$ that gives a membership degree to every element of U in the interval [0, 1]. A semantic label is assigned to this fuzzy set and its membership degree is used to measure a characteristic of the elements of the universe U.

Fuzzy systems are typically used to formulate human knowledge, which is represented by a fuzzy rule base with the canonical form:

$$Ru^{(1)}: \text{ IF } (x_1 \text{ is } A_1^l) \text{ and } (x_n \text{ is } A_n^l) \text{ THEN } (y \text{ is } B^l) \quad (7)$$

where $\mathrm{Ru}^{(l)}$ is the lth rule, A_i^l and B^l are fuzzy sets in $U_i \subset R$ and $V \subset R$, respectively, and $\mathrm{x} = (x_1, x_2, \ldots, x_n)^\mathrm{T} \in U$, $y \in V$ are input and output variables of the fuzzy system, respectively.

The fuzzy logic is then worked out by the compositional rule of inference that determines the membership functions of the fuzzy propositions in the conclusions.

A fuzzy system consists of four modules, which can be briefly defined as:

1. Fuzzification: the linguistic variables associated to the application under study are selected based on experience. Several fuzzy sets are assigned to each of these linguistic variables. Each fuzzy set is defined by a semantic label and a membership function. In the fuzzy system we present in this paper, we are working with both, crisp and fuzzy input and output values. Not only some of the external inputs are crisp measurements, but even outputs of the fuzzy subsystems can also be crisp ones if the outcome is the result of a zero-order Sugeno-type fuzzy system (Jang & Sun, 1995). In the same way, other values are fuzzy because they have been defined as such or because they are the fuzzy output of a fuzzy subsystem. For each input, its membership degree to every fuzzy set is calculated. These values will fire the fuzzy rules.
2. Knowledge base: the fuzzy rules are expressed as combinations of antecedents and consequents (Eq. 7). The rule base is defined comprising the knowledge of an expert.
3. Inference: the inference method, such as max–min (Mamdani), max-product (Larsen), or Sugeno type is selected depending on the application and the nature of the raw data available. The fired rules result from using the inference method considered. A combine or singleton fuzzy result is obtained at the end of each inference.
4. Defuzzification: the fuzzy results are converted to crisp values by applying a defuzzification method. Different methods can be applied (center of area, weighted average, mean of maximum, etc). The centroid is the most widely used.

After Zadeh, many researchers have applied this approach to different areas, including transportation and traffic flow (Santos, 2011; Santos & López, 2012; Teodorović, 1999). One of the early works was done by Pappis and Mamdani (1977), who developed a fuzzy controller for a traffic junction. Chang and Shyu (1993) designed a fuzzy expert system to advice the driver the need of traffic light at intersections with respect to the traffic volume, peak hours, number of lanes, etc. The same application was also solved by fuzzy logic by Teodorović, Lucic, Popovic, Kikuchi, and Stanic (2001). Fuzzy traffic control in high capacity urban roads can also be found in the literature (Hegyi et al., 2001). Zaied and Othman (2011) developed a fuzzy logic traffic system that considers two-way intersections and is able to adjust changes in time intervals of a traffic signal based on traffic situation level. Mucsi, Khan, and Ahmadi (2011) applied Adaptive Neuro-Fuzzy Inference System (ANFIS) to estimate the number of vehicles in a detection zone.

Quite interesting is the work by Kaczmarek (2005) that proposes a fuzzy description of traffic flow in street networks. Although it mainly works with vehicle groups (time position and time length variables), the fuzzy approach has allowed him to incorporate more information about traffic at road side. Other related papers by Rossi, Gastaldi, Gecchele, and Meneguzzer (2012), (2014), and Gastaldi, Meneguzzer, Gecchele, and Rossi (2015) present fuzzy logic models for representing gap-acceptance behavior at priority intersections at roundabouts, based on data collected from driving simulator tests.

Nevertheless, regarding the focus of this work, papers are scant. The most related example is the application of fuzzy logic to the analysis of the capacity and level of service of highways by Chakroborty y Kikuchi (1990). They modeled some parameters such as road capacity, traffic volume or vision distance using fuzzy logic, and defined the level-of-service as a fuzzy output. They also showed the variability of some of the parameters as a function of the driver's perception. In a more recent paper, the same authors (Chakroborty & Kikuchi, 1999) compare a fuzzy inference model for car-following with the so-called generalized model (GM), stating that fuzzy models possess many of the features that are desirable in a model of car-following but are not available in the GM models, such as approximate nature, asymmetric response, etc. Khodayari, Ghaffari, Kazemi, Alimardani, and Braunstingl (2014) propose an adaptive neuro fuzzy inference system (ANFIS) to simulate and predict the car-following behaviour based on the reaction delay of the driver-vehicle unit, unlike other previous works where the reaction delay is considered to be fixed. The same technique is applied by Wanga, Zhanga, Lub, and Wangc (2015), where authors develop a car-following model with consideration of driver's behavior based on an adaptive neuro fuzzy system. In the car-following model they propose, relative speed, distance headway between leading and following vehicle, and speed of following vehicle are the three input parameters, while acceleration of the following vehicle is the output. Real data are available to train the system. The paper by Hasiloglu, Gokdag, and Karsli (2014) also presents an adaptive neuro-fuzzy inference system, which has been adapted as an alternative to other classical models for estimating the vehicle delays at signalized junctions. The most interesting part of this recent paper is the comparison with Observation, Webster, HCM, Multiple Regression Analyses, and Signal Simulation (SSM) models of the

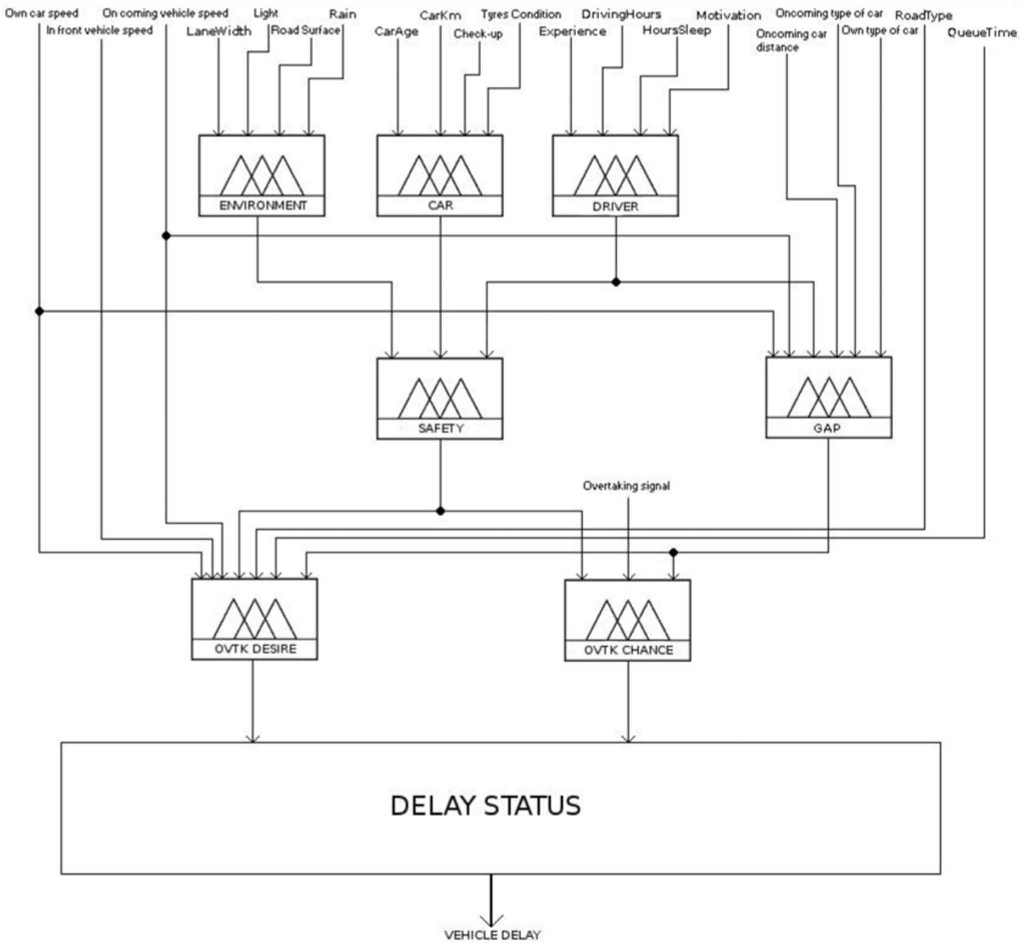


Fig. 1. Fuzzy system to estimate vehicle delayed.

delay, but again at signalized intersections. Another work on modeling the vehicle delay at junctions using fuzzy logic can be found in Murat (2006), who compared fuzzy logic and neural networks taking into account some values as average queue, traffic volume, environment conditions and period time of the traffic lights. Qiao, Yi, Yang, and Devarakonda (2002), and Su et al. (2009) developed a fuzzy logic based system for estimating vehicle delay at an intersection, the latter in China.

To our best knowledge, the estimation of vehicle delay on two-way roads using fuzzy logic has not been proposed before and, therefore, the fuzzy perception of the level of service of these roads is novel.

3. Fuzzy system designed to estimate the vehicle delay

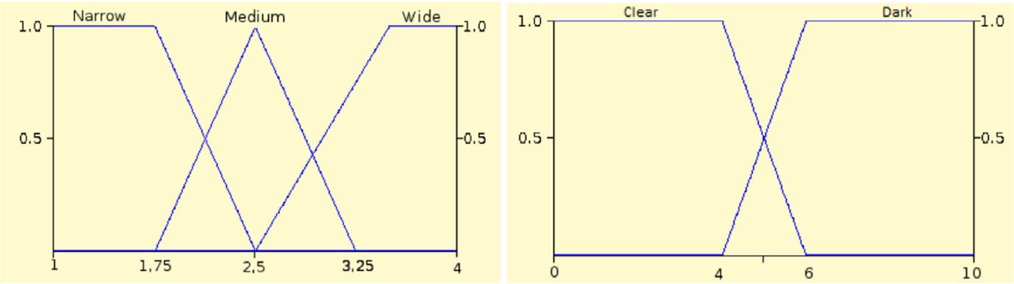
The perception of the delay is different for each driver. Certain headway can be comfortable for a driver but too small for another. There are drivers that travel close to the vehicle in front but do not want to overtake, and others who keep larger distances. Even the same driver does not consider the same factors in different trips (commuting, going to the movies…). Moreover, depending on weather and road conditions, for example, a larger gap may be necessary to be considered comfortable and safe by a driver. These drivers' subjective perceptions have been included in the fuzzy system we have developed, together with other different factors proposed in the literature.

The intelligent system considers a range of different factors as a whole, not just few variables as other methods. Some of these variables (road conditions, weather, driver visibility, etc.), are taken into account without the need of being measured accurately. In addition, some parameters, which had been previously considered in the literature, have also been included (Santos & Romana, 2012; Valverde, Santos, & López, 2009). Nevertheless, most of the variables used by other authors are not applicable in this case as they are defined for signalized intersections, such as red time ratio, traffic volume, etc. (Murat, 2006).

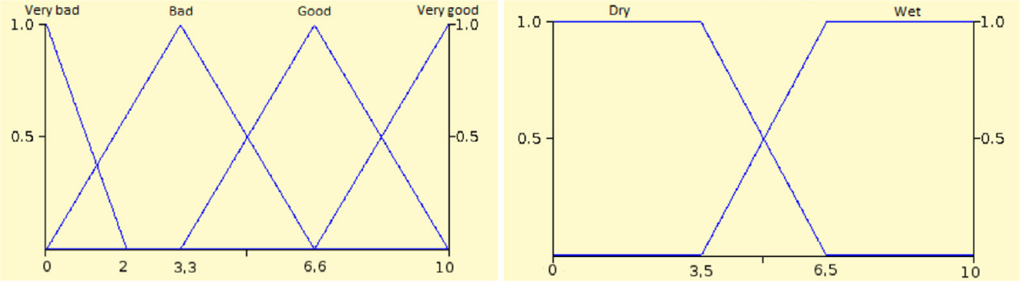
The fuzzy model is a MISO (Multi-Input Single-Output) system, designed and developed in order to determine if a vehicle is delayed. It consists of seven different MISO fuzzy sub-systems that are interrelated. The outputs of some of them are the inputs of others. The structure of the decision fuzzy system is shown in Fig. 1, where OVTK stands for overtaking.

Different input variables have been used (up to 23). Each one is described by a different number of fuzzy sets (3 or more membership functions, mainly trapezoidal or triangular shape). Even some of them are crisp, such as "solid line", which means that overtaking is forbidden or permitted on that stretch of the road, according to the traffic law. The decision-making system has two main outputs: the chance of overtaking and the driver's passing desire. Both variables, along with the speed of the vehicle itself and the speed of the car in front, will be used to calculate if the vehicle is delayed.

In this fuzzy system, the expert knowledge is represented by if-then rules. These fuzzy rules have between 3 and 7 antecedents, depending on the number of linguistic input variables of the corresponding fuzzy subsystem. The combination of all the variables gives up to 1296 rules for the fuzzy subsystem



a) Lane width b) Light



c) Road surface d) Rain

Fig. 2. Fuzzy inputs of the subsystem "Environment".

"Overtaking_Desire" and 972 for the so-called "Gap" (the two largest). The rules were generated following a decision tree, allowing to prune some branches and thus reducing the number of rules needed. For example, the following rule includes the 144 possible combinations of **low** *Safety* and **high** *Infront_vehicle_speed*. The other variables of that system (*Vehicle_type, Actual_speed, Road_type, Time_in_queue*, etc.) can be ignored, as they do not affect the output when these more restricting conditions are met.

> $\text{Ru}^{(l)}$: IF Safety is **low** and Infront_vehicle_speed is **fast** THEN Overtaking_desire is **not**

However, there are rules where many inputs have to be evaluated in order to obtain the conclusion, as in the following:

> $\text{Ru}^{(l)}$: IF Safety is **middle** and Infront_vehicle_speed is **fast** and Infront_vehicle_size is **big** and Vehicle_size is **big** and Road_bendiness is **high** and Time_in_queue is **small** and Vehicle_speed is **fast** THEN Overtaking_desire is **not**

The t-norm product has been chosen to implement the AND operator, as is usual in engineering applications. It has the advantage of providing scaling of the membership functions instead of just clipping as the t-norm minimum does. This allows us to achieve better results. The NOT operator is implemented as the function (1−x).

The fuzzy subsystems of the first level ("Environment", "Car", and "Driver") (see Fig. 1) are going to be described in detail in this Section. The rest of the systems have been implemented in the same way.

3.1. "Environment" fuzzy subsystem

The fuzzy system we have called "environment" refers to driver perception of the quality of the environment, meaning how some external factors could affect the driver behavior, i.e., the environment conditions from the driver point of view. This fuzzy system has as inputs some variables that reflect the environmental conditions as perceived by the user, regardless of the driving: *lane_width, light, road_surface*, and *rain*. They are external inputs and, therefore, their values are set by the user or taken from sensors. The output is *environment_conditions*, from 1 (bad) to 10 (good). The fuzzy description of these fuzzy variables is the following.

- *Lane_width*
  A narrow road makes the cars of adjacent lanes go too close and, therefore, the driver will have to drive more carefully. The domain has been defined in the interval [1, 4], which represents the lane width (in meters). Three symmetric trapezoidal and triangular fuzzy sets have been defined (narrow, medium, wide) (Fig. 2a).
- *Light*
  The lighting directly affects the driver visibility, and it is one of the main parameters that define the quality of the environment as perceived by the user. The range of possible values is [0, 10], where 0 is clear and 10 dark (Fig. 2b).
- *Road surface*
  If the road surface is under poor conditions, with potholes, gravel, etc, it causes a bad grip of the tyres and can make driving difficult. Four fuzzy sets have been defined from 0 (very bad) to 10 (very good conditions of the road surface) (Fig. 2c).
- *Rain*
  A slippery road is probably the most dangerous situation for driving. This factor has been the most significant according to the drivers who were asked. Two trapezoidal fuzzy sets describe if the road is dry or wet (Fig. 2d).

The output gives the environment quality as perceived by the driver (Fig. 3). The value of the output, between 0 and 10, corresponds to bad, medium or good. The centroid defuzzification method has been used to determine the output.

3.2. "Car" fuzzy subsystem

This system refers to the car state. There are different fuzzy inputs related to the car conditions and different ways to deal with them. Although many factors can affect the quality of the car, not all of them are so obvious for the driver. For this subsystem, only some of the factors that can influence the perception that the driver has of his own car have been selected. These variables are the age of the car, kilometers, time since last checkup, and state of the tyres. The output is the car condition.

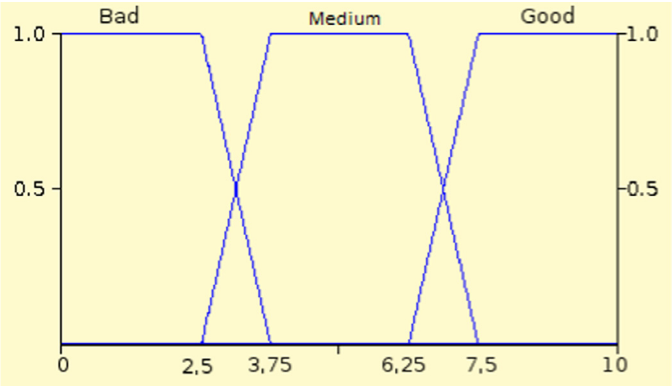


Fig. 3. Fuzzy output of the subsystem “Environment” (*environment_conditions*).

- *Car Age:*
  In addition to the relationship that usually exists between the mileage and the age of the car (older cars usually have run more kilometers), an old car generates the impression that its performance is much worse than newer vehicles that are also circulating. The values that represent the car age are in the range [1, 30], and have been divided as shown in Fig. 4a): two triangular membership functions (new and medium), and a trapezoidal fuzzy set for old.
- *Kilometers:*
  With the increase in the number of kilometers traveled by the car, the vehicle’s reliability decreases, and so does the image the driver has of the safety and performance of the vehicle. The range goes from 0 to 300,000 km. The fuzzy sets assigned are Few, Medium and Many, represented by asymmetric triangular membership functions (Fig. 4b).
- *Checkup:*
  Although the time since the last car checkup is not a factor taken into account by most drivers, knowing that the vehicle has passed the annual inspection recently provides a little extra assurance of safety, especially in older vehicles. This input, between 0 and 24 months, is grouped under two triangular membership functions for recent and earlier (meaning distance in time) (Fig. 4c).
- *Tyres:*
  The poor condition of the wheels is one of the main reasons for low safety in a vehicle. However, according to the polls, many drivers do not consider the state of the tyres. In contrast, those who are aware of the good or bad condition of the wheels considered this factor important when planning to overtake. The state of the tyres is evaluated between 0 and 10, from poor (bad) to good condition (three sets with triangular membership functions, Fig. 4d).

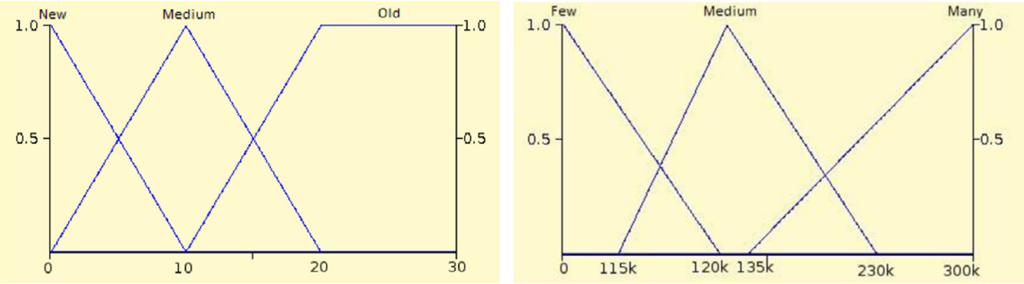
The output gives the condition of the car. As it was the case with the quality of the environment, this output shows the perception of the driver of the quality of the vehicle by a value between 0 and 10. Trapezoidal membership functions have been defined for low, medium or high (Fig. 5). The centroid defuzzification method has been used.

3.3. “Driver” fuzzy subsystem

Driver’s ability is a key factor in determining the safety of a traveling vehicle. The personal features of the driver also influence the perception of the driving. The same driver can interpret the same situation in two completely different ways depending on, for example, how tired he is according to the number of hours he has been driving.

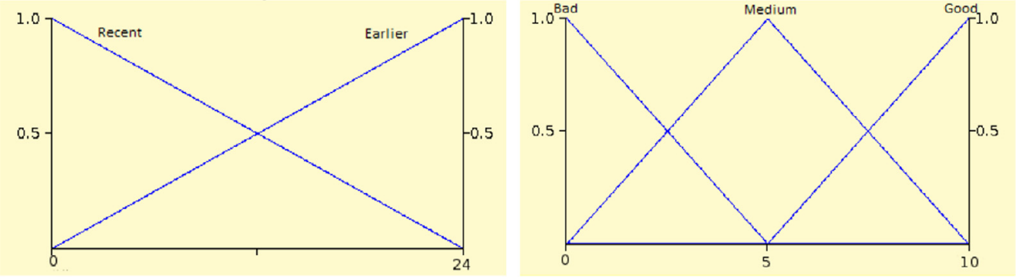
This fuzzy system takes into account variables such as years of experience, along with others that depend on the specific time while driving, as the number of consecutive hours already driven, the resting time of the night before, and the motivation. The inputs are the following.

- *Experience:*
  Inexperienced drivers tend to overreact to unforeseen events. They are not used to traffic problems, driving in a more



a) Car age

b) Kilometers



c) Last checkup

d) Tyres

Fig. 4. Fuzzy inputs of the subsystem “Car”.

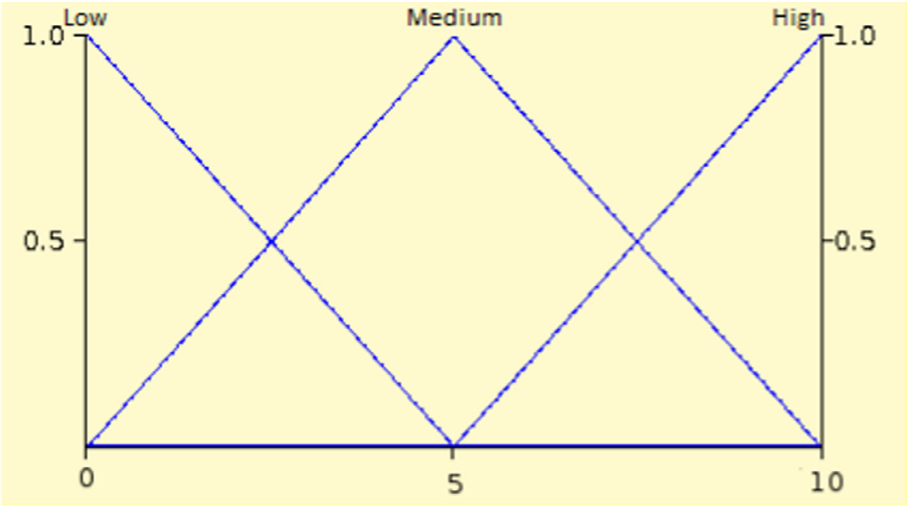
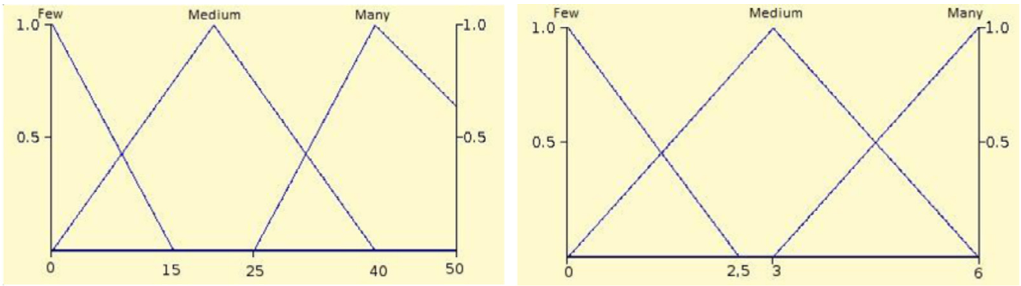
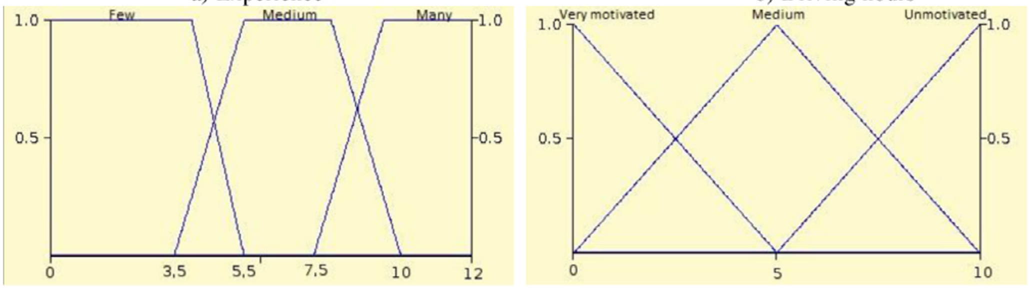


Fig. 5. Fuzzy output of the subsystem "Car" (*car_conditions*).



a) Experience b) Driving hours



c) Hours of sleep d) Motivation

Fig. 6. Fuzzy inputs of the subsystem "Driver".

insecure way than those who have spent many years at the wheel. The range of this variable goes from 0 to 50 years. The membership functions are triangular and correspond to the fuzzy sets few, medium or many (Fig. 6a). Older people sometimes behave as less experienced ones due to they lose reflexes, this explains why the last fuzzy set drops more abruptly.

- *Driving hours:*
  The accumulated fatigue after long hours at the wheel produces a relaxation of the driver's attention and loss of responsiveness among other reactions that affect safety. Its range, [0, 6], shows the number of consecutive hours of driving (Fig. 6b) with symmetrical triangular sets for few, medium and many.
- *Sleep:*
  Just as fatigue after long hours driving, fatigue caused by a short break before starting the trip also affects driving. For this reason, the hours of sleep the night before are considered an important factor. Over the range of [0, 12] hours, three fuzzy sets: few, medium and many, with non-symmetrical trapezoidal functions are defined (Fig. 6c).
- *Motivation:*
  Driving is better when it comes to something pleasant, without tension, and not when it is an obligation or a routine. For example, a driver will be more attentive while traveling to holiday destination than commuting to work, a route he repeats day after day. Motivation has a range [1, 10]. The triangular membership functions correspond to very motivated, medium, unmotivated (Fig. 6d).

The driver subsystem output, described by three triangular membership functions, in an interval [0, 10] represents the conditions of the driver for traveling: poor, normal, and good (Fig. 7). The centroid defuzzification method has been used to obtain the output of this Mamdani system.

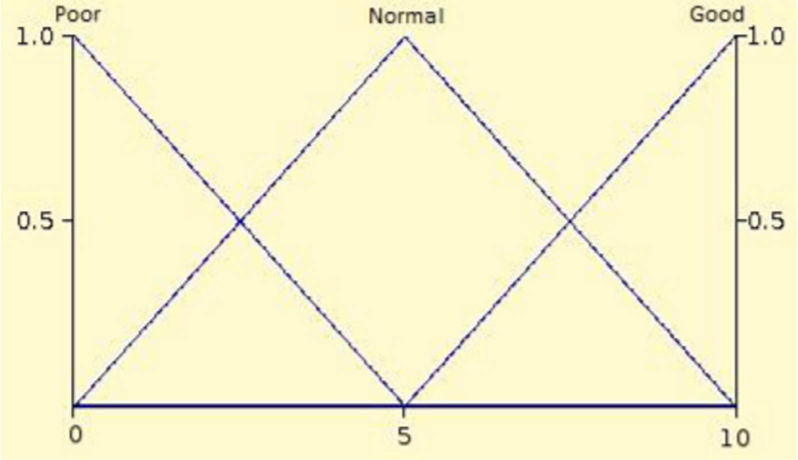


Fig. 7. Fuzzy output of the subsystem “Driver” (*driver's conditions*).

3.4. “Safety” and “Gap” fuzzy subsystems

At another level of the system (see Fig. 1), there are other modules whose inputs come not only from external sources but also from other fuzzy subsystems, such as “Safety” and “Gap”.

(a) *Safety fuzzy subsystem*
The inputs of the Safety fuzzy system are the outputs of the three just mentioned: “Environment”, “Car” and “Driver”. The output is the level of safety while driving with that car (Valverde et al., 2009). This output is very important to estimate the overtaking chance.

(b) *Gap fuzzy subsystem*
It has been introduced in this paper to analyze the chance of overtaking, in order to reduce the vehicle delay. To carry out the overtaking maneuver, a crucial factor is to determine if there is sufficient distance in the opposite direction on two-way two-lane roads.

To calculate this safety margin, several inputs have been taken into account: actual speed of the own vehicle and of the oncoming car, distance between them, type of vehicles involved in the maneuver (car/truck), and driver's condition, which is the output of the Driver fuzzy subsystem. Three symmetrical trapezoidal fuzzy sets have been defined for the linguistic variable speed: slow, medium, and fast, in a universe of discourse between 0 and 140 km/h. The type of vehicle can be a car (which is classified by the engine size (cm^3), between 0 and 3000, corresponding to small, medium or large), or a truck (over 3000 cm^3, only lorry type). As heavy vehicles are not often found on two-lane roads, the case of the own vehicle being a lorry was not considered. This selection was also due to the lack of trucks' drivers during the validation surveys. Nevertheless, oncoming traffic can be a lorry. The output (available gap in the opposite lane) is given by three fuzzy sets: short, enough or large.

The combination of the outputs of all the fuzzy subsystems, as shown in Fig. 1, gives the information required to determine the chances of overtaking and, therefore, allows the calculation of whether the vehicle is delayed.

4. Overtaking to reduce the vehicle delay and improving the LOS

This fuzzy system can advise the driver on the chance of overtaking. Thus, an action that may help to reduce the delay is suggested. Regarding the passing maneuver, it is important to note two subjective perceptions. On the one hand, the desire of overtaking that arises when the driver feels he is being delayed because he would like to move at a faster speed. On the other hand, the chance of overtaking that depends on certain external conditions and some driver perceptions. Two fuzzy systems have been designed to represent these two perceptions.

4.1. Overtaking desire fuzzy system

Passing maneuver allows faster drivers to pass slower vehicles and drive at their own desired speed (Llorca et al., 2015). Desired speed is defined as the driver selected speed without the effect of neither traffic nor highway alignment. As already mentioned, if a vehicle is slowed due to the vehicle in front, this does not necessarily mean that it is delayed. There may be some other reasons that make him want to keep that lower speed and not to overtake. We will call it an in-line vehicle but not a delayed one.

The factors that influence this passing desire are: the speed and type of the vehicles involved in the maneuver, type of road (bendiness), the time the driver has been queued, and his/her perception of the available gap and safety. We will briefly describe some of them.

- *Vehicle speed*
  If the difference between the speed of the own vehicle and the one in front is small but both speeds are high, the driver might not want to overtake. Conversely, if the car in front of us is running at a very low speed, it is very likely that we want to pass it, especially if the own car speed is high. Besides, we have to consider the speed of the oncoming vehicle. This fuzzy variable, speed, has been described as part of the Gap subsystem (Section 3.4b).
- *Vehicle type*
  As proposed for the Gap subsystem (Section 3.4b), the type of vehicle can be a small, medium or large car, or a truck. We found that the type of vehicle does influence the driver passing desire, especially if the oncoming vehicle is a lorry because of its size.
- *Road bendiness*
  Two fuzzy sets have been assigned to this variable, depending on road bendiness, which is measured as the sum of horizontal deflections from curves in the road, and reported in degrees/km. Thus, roads can be (relatively) windy, with many curves and high bendiness, or (relatively) straight. Driving on a straight two-lane highway, with long straight stretches, makes the overtaking maneuver easier because the visibility is usually

Table 2
Driver contribution to level of service (LOS).

IF	*ovtk_desire*	AND	*ovtk_chance*	THEN	Level of service	Delay	LOS	Vehicle
	Yes		**Not**		**bad**	Yes	D, E	Delayed
	Not		**Not**		**can_be_good**	Not	B, C	In line
	Not		**Yes**		**good**	Not	A	Free/Isolated

much better. On the other hand, while driving on secondary roads, type II, the driver may prefer to slow down. Bendiness is not a variable where a clear threshold exists, and therefore a fuzzy variable is more adequate to deal with it.

- *Time in queue*
  If a driver is kept at a lower speed than desired for a long time, his anxiety may increase and may also change the perception of overtaking desire. Two fuzzy sets, little and much, have been defined in a range of [0 60] minutes.

This fuzzy subsystem, overtaking desire, also includes as input the safety as perceived by the user, which is the output of the safety fuzzy system (Section 3.4a) that takes into account the Environment, Vehicle, and Driver modules (see Fig. 1). Additionally, the output of the gap subsystem is another key input. The output of this system is the final decision made by the driver. When travelling behind a slower vehicle, drivers experience a growing desire to pass, and finally reach a decision on to try it or not, if the perception of safety is high. This final decision (crisp) is obtained as the result of a zero-order Sugeno model, where the output is a constant.

4.2. Overtaking chance fuzzy system

In studies of vehicular gap-acceptance behavior, the choice to accept or reject a gap of a certain size is generally considered the result of a driver decision process, which includes, as inputs, subjective estimates of a set of explanatory variables, given specific objective factors. These subjective evaluations are usually affected by a high degree of uncertainty, which can be properly treated both by classical and probabilistic models (Rossi et al., 2012).

The fuzzy subsystem Overtaking chance requires as inputs both crisp and fuzzy values. Overtaking may be forbidden or allowed (crisp) by traffic law; on the other hand, driver's perception of the safety margin, estimation of the gap, the distance to other vehicles, etc, are considered fuzzy variables. Some of these inputs are the outputs of the corresponding fuzzy subsystems (see Fig. 1). The input variables of this fuzzy subsystem are:

- *Overtaking signal*
  It is important to know if there is a solid line on the road surface (in some countries) or any other vertical signal forbidding passing. This is an external input to the system (solid line).
- *Gap*
  The opposite lane occupation and whether there is enough gap on the opposite lane for overtaking are important factors. Otherwise, the fuzzy system will not advice the driver to overtake. This input is the output of the Gap fuzzy subsystem (short, enough, large).
- *Safety*
  The perception of this value is entirely subjective; it depends on the slow or fast reflexes of the driver, sight distance, visibility, etc. It is calculated by the fuzzy subsystem Safety that has been previously described (Section 3.4b).

The output of this zero-order Sugeno-type fuzzy system is Yes or Not, depending on whether overtaking the other vehicle is possible or not. That is, finally, after considering all the variables, the driver decides that there is a chance of passing. As explained before with the desire of overtaking, the variable focuses on the final judgement: yes, I can try to pass, or no, this is not a good occasion for passing. The zero-order Sugeno model gives this result.

Therefore, we have obtained as a result if it is possible to overtake and if the driver wants to do it, which can be used to assess if a vehicle is delayed.

4.3. Final output of the fuzzy system: vehicle delay and level of service

We present a proposal for the determination of the level of service (LOS) of two-way two-lane roads based on the *overtaking desire* and *overtaking chance*, outputs of the described fuzzy system.

There are some studies on particular situations that define the level of service (LOS) as a function of the road capacity, but most relate LOS with user perception, through a stratifies measure. In (Transportation Research Board, 2011), LOS is based on density, and a categorization of LOS (vehicle/hour/lane) is given, from A (free flow) to F (breakdown), going through stable (B, C), high density (D), and near capacity (E) (Table 1). It was defined under ideal conditions (US freeway with a lane width of 3.5 m, with maximum capacity slightly over 2000 vehicles per hour per lane). In these facilities, passing is unrestricted with lower flows, and friction appears restricting free passing between vehicles when flows grow; slow traffic does not interfere with fast traffic. Therefore, quality perception by the user decreases when delay increases and when other vehicles in the traffic flow restrict the users' speed. We have been inspired by this classification of the LOS to label our results (see Table 2).

However, like some authors comment, other factors (such weather or road surface conditions) also influence the capacity significantly, e.g., the rain (Chung, Ohtani, Warita, Kuwahara, & Morita, 2006). Even more, the capacity of the road can be very influenced by the behavior of the drivers. In countries with disciplined traffic behavior the capacity seems to be considerable higher than in countries where traffic behavior is less disciplined. Less disciplined traffic tends to block traffic and cause considerable delays. Such kind of subjective factors should be included in a vehicle delay model. Therefore, we will relate the level of service to the delay state of a vehicle, where those perceptions have been taken into account. The level of service is then given by the following rules (Table 2):

The proposed fuzzy system also allows us to analyze how changing certain parameters the state of a vehicle can be improved. For example, if the visibility increases then the chance of overtaking is higher and, if the driver does so, he is no longer delayed. Therefore, depending on some variables, the driver is advised on the action to take.

In the existing framework, LOS is an aggregate result for a given demand, taken from an aggregate measure of the traffic flow characteristics. This is done in freeways with density and in two-lane highways with average travel speed, percent time spent following or percentage of the free flow speed, depending on road types. This paper offers an alternative, or, at least, a complementary possibility to estimate LOS considering how each driver assesses their driving experience. These results could be used in future frameworks.

5. Results and discussion

The computational tool Xfuzzy has been selected for the implementation of this fuzzy system due to its various functionalities. Xfuzzy (IMSE-CNM, 2003) was developed at Seville University, Spain. Xfuzzy is programmed in Java and has GNU license. It consists of several tools covering the different stages of the design process of a fuzzy inference system: description, simulation, debugging, graphical visualization, tuning by the application of learning algorithms, and finally, the synthesis unit that includes tools to generate code in high-level languages (C, C++ and Java), for software and hardware implementations. The simulation environment integrates them under a graphical interface that facilitates the design procedure to users. Xfuzzy uses the specific language XFL (XFuzzy Language) that allows expressing complex relationships between fuzzy variables, hierarchical rule bases, connectives, linguistic modifiers, membership functions, and different methods of defuzzification. It also serves as a link between all the tools.

It is difficult to validate a fuzzy model like this. Although we have obtained reasonable results in the simulations, we decided to collect some real data for comparison, conducting some surveys to drivers of two-way two-lane roads. Most of them were traveling around Madrid and Segovia (Spain). The main goal of the survey was double. On the one hand, we wanted to know the perception of the drivers regarding some variables, the importance they gave to some of them, which the most critical were when overtaking, etc. On the other hand, we proposed some scenarios to learn how the drivers would react in those situations to use them to adjust the system.

The questions were focused on whether they wanted to overtake and if they felt delayed under specific road conditions, vehicle characteristics, etc. Different factors were proposed to the drivers, who had to weigh the importance of each. They were also interviewed on the number of hours they had slept the night before, the motivation of the trip, their reactions depending on the road surface, the weather, etc. Different scenarios were proposed and they answered to these hypothetical situations. This method of knowledge acquisition has been successfully applied by other authors in this field (Pattnaik & Ramesh, 1996).

Some of the external inputs of our model were directly provided by the drivers' answers or even for us, because we were doing the surveys in some establishments at the roadside, where the drivers had stopped to rest or to have something to eat. Therefore, we already knew some data regarding the environment: rain, lane width, road-surface, etc.), even if we asked these questions to the drivers to know their perception. Besides, meteorological stations, historical databases, traffic institutions, traffic regulations, etc. also provided some of these values.

It took us a long time to collect the surveys, and we had to withdraw some of them because they were incomplete or the answers did not give the information we wanted. Eventually, 50 of them were considered appropriate, even if not in all of them all the questions were answered.

Out of these 50 questionnaires, 10 of them were selected to define the fuzzy variables of the system, the terms and labels the drivers used, the range of the possible values, etc. To make the system reliable, we kept in mind that we needed the larger possible set of surveys to validate the system, so we decided to take only ten (20%) that were different enough to cover a wide spectrum of drivers, and to use the other 40 to validate it (80%).

During the adjusting phase, the answers of the 10 selected surveys were used to correct the deviations detected on the results given by the system. This adjustment consisted in narrowing or changing the shapes of the membership functions, establishing the universe of discourse of the different fuzzy sets, etc., in a qualitative way, in order to make the system outputs as close as possible to the survey answers. As it is well known, the design of fuzzy systems is based on expert knowledge and the tuning is usually carried out by trial and error.

The 80% surveys left were used to validate the system, comparing the answers given by the fuzzy system with the answers given by drivers. The results were as follows:

- The answers were the same in more than 90% of the cases.
- 37 out of 40 (92.5%) of drivers who answered that, under certain conditions, they had felt delayed were correctly identified as such by the fuzzy system.
- 90% of the drivers (36 out of 40) who said they were driving at lower speed than desired, but acceptable for them, were also rightly identified as vehicles in line.

The determination of free and isolated vehicles is quite simple according to the fuzzy rules described in Table 2. In fact, every vehicle that is not delayed or in line is classified as free or isolated.

The system is reliable and it closely represents the human way of perceiving some driving conditions. Nevertheless, the number of samples taken was small due to the difficulty of interviewing drivers of two-lane roads. That made the percentages varied greatly. Besides, we have realized that there is not a clear common pattern of the drivers of two-way two-lane roads.

5.1. A new proposal of vehicle states

As another result of this study, this approach has led to an extension of the possible states of a delayed vehicle considering passing desire as a new factor. Most previous approaches found in the literature consider only two possible states of a vehicle, free and delayed, based on measurements such as the critical interval (Rozic, 1992). In this paper, as in our previous work (Santos & Romana, 2012), we propose four states based on subjective factors such as passing desire, road conditions, etc. These states are:

- A vehicle is ISOLATED if the driver is moving at desired speed, without any influence from other vehicles traveling in the same lane.
- A vehicle is IN LINE if its speed is close to but slightly lower than the desired speed. Nevertheless, from his point of view, the driver does not want to overtake. The driver does not think it is worth overtaking and he feels comfortable driving at that speed.
- A vehicle is DELAYED if its speed is significantly lower than desired due to the vehicle in front. The driver would like to overtake it, but he cannot do it because it is not allowed on that stretch of the road, or there is opposing traffic and the gap is not enough.
- A vehicle is FREE when the driver goes at the speed he wants, even though he may be influencing other vehicles. For instance, vehicles behind it can be delayed. This can happen when the vehicle has just been overtaken; then the distance between vehicles is small but both are circulating at desired speeds.

We believe that the proposed classification is closer to reality because it considers two new possible states: IN LINE and ISOLATED. The first one includes a subjective perception of the driver, the desire of overtaking, even if he could be considered delayed.

7. Conclusions and future trends

In this paper, we have designed and implemented a fuzzy system that provides a new and reliable approach to determine whether vehicles on two-way two-lane roads are delayed. Human factors, subjective perceptions, and uncertain variables are taken into account in the fuzzy model. The developed fuzzy system deals with each driver as a unique individual, that is, with a different behavior from the others'. This is one of the main differences with other research papers that deal with the delay in a traditional way, based on measurements of average time between vehicles and thresholds and, therefore, as if all the drivers behave the same. Simulation results of the system have been successfully compared with the behavior of two-lane roads drivers who were interviewed.

This paper also offers an alternative, or, at least, a complementary possibility to estimate level of service (LOS) considering how each driver assesses their driving experience. In our proposal, the level of service of these types of two-lane roads is obtained using the vehicle delay state. These results could be used in future frameworks.

In addition, a new proposal of possible states of a vehicle is defined. This approach takes into account the drivers point of view regarding overtaking.

Currently, the HCM recommends the threshold of 3 s to make field estimations of PTSF. Many HCM users and traffic engineers deduce from this, therefore, that a vehicle travelling at a lower headway than 3 s is delayed. Actually, what the authors of the 3-55 report mean is that 3 s is the value for which a best match between percent delayed vehicles and PTSF has been achieved, when comparing field data and simulation results. We feel that our proposal is more useful to traffic engineers than using a 3-s threshold (indeed, 4 and 5 s have also been proposed in the literature). Really, it is difficult to believe that PTSF, a measure over the whole trip for a single user, or an average of users, can be matched with a measure of the vehicle set in a single point in the road section.

Therefore, this proposal makes it possible to introduce these existing driving modes or experiences into LOS assessment and accordingly, it is potentially a step forward since LOS must be related, by definition in the HCM and elsewhere, to user experience. Nowadays there is a worldwide effort to establish new regulations in two-lane highways, and a large project is under way within NCHRP. The new German Capacity Manual uses density, and follower density and bunching have been proposed, for which a headway threshold must be adopted to see if a vehicle is in queue. Further research must follow up to consider how the individual driver perception can be aggregated into a LOS measure that can be traduced in the current projects and policy measures,

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