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ORIGINAL ARTICLE

A proposed mathematical model for closed-loop network configuration based on product life cycle

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Abstract Products may be returned over their life cycle. Industrial experiences show that there are three main return–recovery pairs. Commercial returns are repaired. End-of-use returns often are remanufactured. In addition, end-of-life returns are recycled. However, up to now, no optimization model is proposed for closed-loop configuration based on three return–recovery pairs. The repaired and remanufactured products can be sold in the same or secondary market. In this paper, we design and configure a general closed-loop supply chain network based on product life cycle. The network includes a manufacturer, collection, repair, disassembly, recycling, and disposal sites. The returned products are collected in a collection site. Commercial returns go to a repair site. End-of-use and end-of-life returns are disassembled. Then, end-of-life returns are recycled. The manufacturer uses recycled and end-of-use parts and new parts to manufacture new products. The new parts are purchased from external suppliers. A mixed-integer linear programming model is proposed to configure the network. The objective is to maximize profit by determining quantity of parts and products in the network. We also extend the model for the condition that the remanufactured products are sent to the secondary market. The mathematical models are validated through computational testing and sensitivity analysis.

Keywords Reverse logistics (RL) · Closed-loop supply chain (CLSC) · Mixed-integer linear programming (MILP) · Product life cycle

1 Introduction

Nowadays, the majority of companies try to reuse and remanufacture products because of economic incentives and a growing environmental concern [7]. There are three main requirements for sustainable development: resource conservation, environmental protection, and social development. Reverse logistics is an important concept that emphasizes on decreasing and reusing disposal [25]. Reverse logistics is defined as the movement of product or material in the opposite direction for the purpose of creating or recapturing value, or for proper disposal [30]. Implementation of reverse logistics would allow not only for savings in inventory carrying cost, transportation cost, and waste disposal cost, but also for the improvement of customer loyalty and futures sales [16, 18].

Recovery options for returned products consist of reuse, resale, repair, refurbishing, remanufacturing, cannibalization, and recycling [38]. In the remanufacturing process, used products are disassembled in disassembly sites. Usable parts are cleaned, refurbished, and they are transmitted into part inventory. Then the new products are manufactured from the old and new parts [13, 22]. By adding issues such as remanufacturing, recycling, and refurbishing to the supply chain, two problems will be come to exist: firstly, collection and transportation of these products, containers, and packaging. Secondly, the uncertainty associated with the recovery process [5, 22, 38].

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In reality, three main return–recovery pairs exist. Commercial returns are repaired. End-of-use returns often are remanufactured. In addition, end-of-life returns are recycled [8, 39]. However, to the best of our knowledge no quantitative model is proposed based on three return–recovery pairs. It is noticeable that not only the quantity of manufactured products depends on the market demand, but also it is related to commercial returns because they can be used as new products after light repairs. Another challenge appears when some external suppliers and recycling sites exist. In this condition, the manufacturer prefers to minimize the costs. Although the majority of remanufactured products can compete with newly manufacturing products, markets tend to be separated for new and remanufactured products [1]. In other words, the new products may be sold in the same market and the remanufacturing products may be sent to the secondary market.

In this paper, we propose a general network based on product life cycle and return–recovery pairs. The closed-loop supply chain network consists of manufacturer, collection, repair, disassembly, recycling, and disposal sites. Demand can be either satisfied by commercial returns (after light repair) or new products. The manufacturer uses recycled parts, end-of-use returns, and new parts to produce new products. New parts are purchased from external suppliers. To our knowledge, no investigation has examined a general network for return–recovery pairs including commercial, end-of-life, and end-of-use returns. We propose a mixed-integer linear programming model to maximize the profit and determine the number of products and parts in each part of the network. The model is designed for multiproducts, parts, suppliers, and recycling sites. Not only manufacturing, purchasing, collecting, disposing, disassembly, and repairing costs are taken into account, but also set up costs of disassembly, and repair sites are considered. Besides, the model determines the number of recycling sites. We also extend the model for a secondary market. In this condition, demands of same and secondary markets should be satisfied separately. The mixed-integer linear programming (MILP) models are solved, and they are validated through computational testing and sensitivity analysis.

The remainder of the paper is organized as follows. Section 2 presents the literature review. In Section 3, the problem is defined. Section 4 is devoted to the proposed mathematical model. In Section 5, we present computational testing to validate the model. In Section 6, a sensitivity analysis is examined. Section 7 consists of the extended model. Finally, in Section 8 we present conclusions.

2 Literature review

Reverse logistics has absorbed a lot of attention. Some literature reviews and surveys have been published in the reverse logistics (RL) field. The summary of these articles are written in Table 1.

Some researchers have investigated network configuration in reverse logistics. They examined the application of facility location models in RL. Jayaraman et al. [10] presented a MILP model that simultaneously determines the location of remanufacturing/distribution facilities, the transhipment, production, and stocking of the optimal quantities of remanufactured products and cores (used parts). Fleischmann et al. [6] presented a generic model for recovery network configuration. The model is designed based on forward facility location model. They illustrated the model by means of two examples: copier

Table 1 Literature reviews in reverse logistics context

Reference	Subject	Abstract
[5]	Reverse logistics	They categorized the field into three main areas: distribution planning, inventory, and production planning. They reviewed mathematical models
[20]	Sustainable supply chain	They described the relationship between supply chain and sustainability
[31]	Reverse logistics	They provided abstract of the reverse logistics papers that were published between 1995 and 2005
[8]	Closed-loop supply chain	They categorized closed-loop supply chain networks to five phases: 1—the golden age of remanufacturing as a technical problem, 2—from remanufacturing to valuing the reverse logistics process, 3—Coordinating the reverse supply chain, 4—closing the loop, 5—prices and markets
[22]	Supply chain management	They examined facility location models in the context of SCM. The literature dedicated to reverse logistics is divided to closed-loop networks and recovery networks
[28]	Reverse logistics	They focused on all aspects of RL from collection of used products, their processing and finally to the outputs of processing, namely, recycled materials, spare parts, remanufactured products, and waste material disposal
[9]	Reverse logistics	The author identified 45 papers dealing with issues of RL networks. A few of them are case studies

remanufacturing and paper recycling. Krikke et al. [15] proposed a mathematical model to support both product design and logistics network, simultaneously. The model is applied for refrigerators with real data. Kim et al. [13] proposed a network and a mathematical model to maximize the total cost savings by determining the quantity of parts to be processed at each remanufacturing facilities, and the number of purchased parts from external supplier. Although the model is adopted for multiple periods, it is designed for single supplier. Debo et al. [2] studied the effects of new and remanufacturing products in the same market over the life cycle. In addition, they examined the production system when demand for new and remanufactured products is segmented into same and secondary markets. Ko and Evans [14] presented a mixed-integer nonlinear programming (MINLP) model to configure forward and return networks. Moreover, they utilized genetic algorithm to solve the problem. Listes [21] considered a stochastic model for design of networks comprising both supply and return channels in a closed-loop supply chain (CLSC). The model is solved by a decomposition approach. Salema et al. [32] presented a general model for reverse logistics network where capacity limits, multiproduct management, and uncertainty on product demands and returns exist. The model minimizes the cost function. Lieckens and Vandaele [19] developed an MINLP model on the basis of stochastic lead time and queuing theory. The model is solved by genetic algorithm. However, it is designed for a single product. Selim and Ozkarahan [34] proposed a fuzzy goal programming approach for a RL network. The uncertainty in demand and decision makers' aspiration levels for the goals are taken into account. Du and Evans [3] proposed a bi-objective model including minimization of total costs and minimization of the overall tardiness of cycle time. In addition, scatter search is used to solve the model. Srivastava [37] according to the literature and interviews with 84 stakeholders developed a conceptual model for simultaneous location–allocation of facilities for a cost effective and efficient reverse logistics network. Pati et al. [24] formulated a mixed-integer goal programming model to assist in proper management of the paper recycling logistics system. The model takes into account multiple objectives of a recycled paper distribution network. Lee and Dong [17] developed a stochastic programming model that can consider uncertainty. They also proposed a heuristics solution. Kannan et al. [11] utilized genetic algorithm and particle swarm to optimize a CLSC. Francas and Minner [7] studied the network design problem of a firm that manufactures new products and remanufactures returned products in its facilities. They examined the capacity decisions and expected performance of two alternative manufacturing network configurations when demand and return flows are uncertain. They distinguished between the case that products are sold in the same market and the case that they are remanufactured for secondary market. Mutha and Pokharel [23] proposed a mathematical model for design of a RL network with deterministic parameters. Lee et al. [16] proposed a model for minimizing shipment costs of a CLSC and opening costs of disassembly centers and processing centers. In other words, the model can determine the optimal numbers of disassembly and processing centers. But, it does not include inventory costs such as holding costs. In addition, the model is designed for single supplier. Wang and Hsu [40] designed a closed-loop network and utilized a spanning-tree based genetic algorithm. Pishvaee et al. [26] developed a bi-objective MILP model to minimize the total costs and maximize the responsiveness of a logistics network. They applied a memetic algorithm. El-Sayed et al. [4] proposed a multi-period forward and reverse logistics network. They considered both deterministic and stochastic demands. Kannan et al. [12] investigated a closed-loop network by mixed-integer programming and a heuristics algorithm. Qin and Ji [29] utilized fuzzy programming in RL to take into account uncertainty. Pishvaee et al. [27] applied MILP model and simulated annealing to configure a RL network. Sasikumar et al. [33] developed a MINLP model to configure a CLSC network. They applied the model for truck tire manufacturer. Shi et al. [35] proposed a mathematical model to maximize the profit of a remanufacturing system using Lagrangian relaxation approach. Shi et al. [36] studied the production planning problem for a multiproduct closed-loop system with uncertain demand and return.

Guide and Van Wassenhove [8] categorized product returns according to product life cycle. Besides, they linked product return types to specific recovery activities. However, they did not examine the effects of returns pair on network configuration. Commercial returns are products that are returned by consumers within a certain period of time (for instance, 60 days after buying). These returned products often are repaired. End-of-use returns happen when a functional product is replaced by a technological upgrade. The majority of these products are remanufactured. End-of-life returns are available when the product becomes technically obsolete or no longer contains any utility for the current user. The option of recycling is more suitable for this kind of return. Cell phone industry is a good example of three types of returns

[8]. According to this category, we propose a novel mathematical model.

3 Problem definition

Supply chain networks are divided to open- and closed-loop networks. The degree of complexity in closed-loop networks usually is higher than open ones. There are several types of closed-loop supply chain networks. Unlike the previous investigations that suppose one or two returns, the proposed network is designed based on product life cycle and three types of returns (as a novel innovation). In this study, the reverse logistics consists of a manufacturer, collection, repair, disassembly, recycling, and disposal sites. Figure 1 shows the proposed network. The purchasing decision is a challenge for manufacturer because he must take into account the amount of end-of-use and end-of-life returns. Besides, some of the returned parts are not usable and should be disposed. The number of commercial returns is another challenge for manufacturer. The commercial returns can supply a portion of market demand. The objective of the proposed model is to maximize the profit by simultaneously determining quantity of products and parts in each part of the network. After using the products by customers, some of them are returned. The returned products are taken to the collection site. Then, they are separated to commercial returns, end-of-use returns, and end-of-life returns. Commercial returns are repaired in the repair site. These products can be used as new ones. On the other hand, end-of-use and end-of-life returns are disassembles. In this stage, the wastes are separated. End-of-life returns are recycled in recycling sites. The parts are added to part inventory as new parts. It is noticeable that capacities of manufacturer, repair, disassembly, and recycling sites are limited. According to the number of returned parts, the manufacturer purchases new parts from external suppliers. There are several suppliers who can supply required parts. The capacities of suppliers are known. Besides, it is supposed that suppliers reserve certain key resources for the manufacturer. A cell phone industry is a good example of this general network.

4 Proposed mathematical model

The closed-loop supply chain network can be formulated as a mathematical model. Indices, decision variables, and parameters of the proposed mathematical model are written in Table 2. The following assumptions are made in the designing the model:

- If the quantity of end-of-life and end-of-use returns is not enough for requirement of manufacturer, manufacturer should buy parts from suppliers.
- The demands of products are known.
- Maximum capacity of manufacturer, disassembly, repair, and recycling sites are known.
- The capacity of collection site is unlimited.

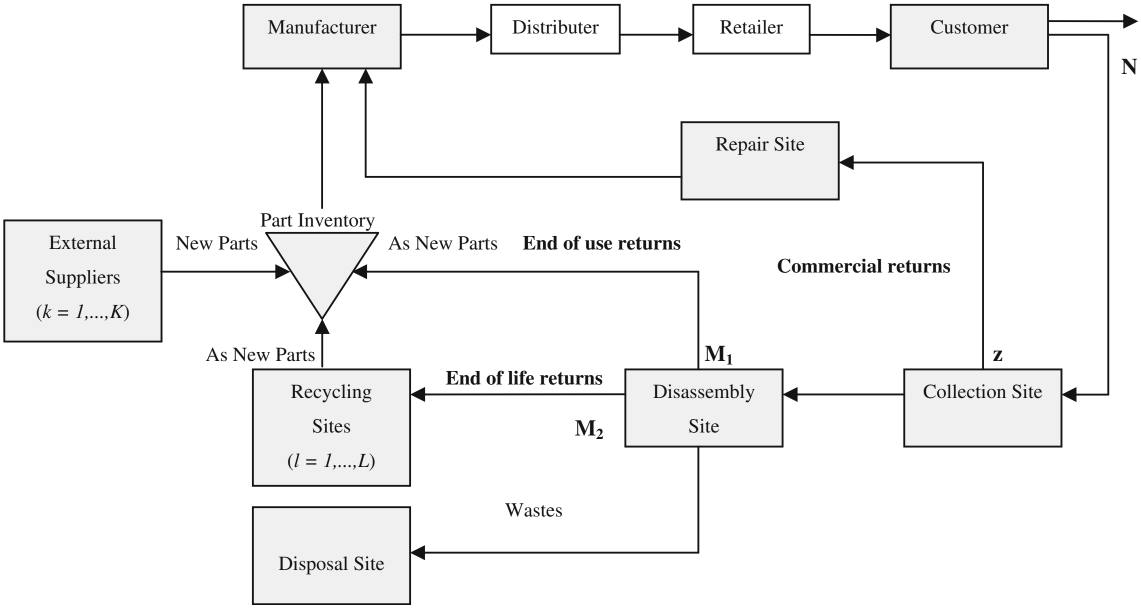

Fig. 1 A closed-loop supply chain network based on product life cycle (highlighted area)

Table 2 Indices, decision variables, and parameters of the proposed mathematical model

Indices		Parameters	
i	Set of parts, $i=1,\ldots,I$	S_j	Unit selling price for the product j
j	Set of products, $j=1,\ldots,J$	a_j	Resource usage to produce one unit of product j
k	Set of suppliers, $k=1,\ldots,K$	H_j	Unit inventory holding cost for collecting product j
l	Set of recycling sites, $l=1,\ldots,L$	y_j	Unit direct manufacturing cost of product j
Decision variables		e_j	Resource usage to repair one unit of product j
X_j	Units of product j to be repaired	C_j	Max capacity of repair site for product j
P_j	Units of product j to be produced	D_j	Demand for product j
Y_j	Units of product j in collection site	c_j	Unit collection cost of product j
Z_j	Units of returned product j to be disassembled	d_j	Unit repair cost of product j
Q_{ik}	Units of part i to be purchased from external supplier k	f_j	Set-up cost of disassembly site for product j
E_i	Units of part i that are obtained in disassembly site	g_j	Set-up cost of repair site for product j
F_{il}	Units of part i to be recycled in recycling site l	B_i	Max capacity of disassembly site to dissemble part i
G_i	Units of part i to be disposed	h_i	Unit disassembly cost for part i
R_i	Units of end-of-use return of part i	m_i	Unit disposing cost for part i
U_{il}	Binary variable for set-up of recycling site l for part i	r_i	Resource usage to disassemble one unit of part i
V_j	Binary variable for set up of disassembly site	n_{il}	Unit recycling cost for part i in recycling site l
W_j	Binary variable for set-up of repair site	o_{il}	Set-up cost of recycling site l for part i
Parameters		s_{il}	Resource usage to recycle one unit of part i in recycling site l
M_1	Max percent of end-of-use returns	O_{il}	Max capacity of recycling site l to recycle part i
M_2	Max percent of end-of-life returns	q_{ij}	Unit requirements for part i to produce one unit of product j
N	Max percent of total returns	p_{ik}	The cost of purchasing part i from external supplier k
z	Max percent of commercial returns	b_{ik}	Internal resource usage of supplier k to produce one unit of part i
A	Max capacity of the manufacturer plant	T_k	Max capacity reserved of external supplier k
t	Max number of recycling sites	M	A big number

- The sum of disassembly and recycling costs of parts is less than purchasing cost of new ones.
- The proposed model is a single-period one. Therefore, the beginning inventory is zero.

$$\begin{aligned} Max\ z\quad & \sum_{j=1}^{J} S_j\left(X_j+P_j\right) - \sum_{i=1}^{I}\sum_{k=1}^{K} p_{ik}Q_{ik} - \sum_{i=1}^{I} h_iE_i - \sum_{l=1}^{L}\sum_{i=1}^{I} n_{il}F_{il} \\ & - \sum_{i=1}^{I} m_iG_i - \sum_{j=1}^{J} y_jP_j - \sum_{j=1}^{J}\left(c_j+H_j\right)Y_j - \sum_{j=1}^{J} d_jX_j \\ & - \sum_{l=1}^{L}\sum_{i=1}^{I} o_{il}U_{il} - \sum_{j=1}^{J} f_jV_j - \sum_{j=1}^{J} g_jW_j \end{aligned} \tag{1}$$

Subject to

$$\sum_{j=1}^{J} q_{ij}P_j = \sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} Q_{ik} + R_i \quad \forall i \tag{2}$$

$$R_i + \sum_{l=1}^{L} F_{il} + G_i = E_i \quad \forall i \tag{3}$$

$$E_i = \sum_{j=1}^{J} q_{ij}Z_j \quad \forall i \tag{4}$$

$$X_j + Z_j = Y_j \quad \forall j \tag{5}$$

$$\sum_{j=1}^{J} a_jP_j \le A \tag{6}$$

$$\sum_{i=1}^{I} b_{ik}Q_{ik} \le T_k \quad \forall k \tag{7}$$

$$r_i E_i \leq B_i \quad \forall i \tag{8}$$

$$s_{il} F_{il} \leq O_{il} U_{il} \quad \forall i, l \tag{9}$$

$$e_j X_j \leq C_j \quad \forall j \tag{10}$$

$$P_j + X_j = D_j \quad \forall j \tag{11}$$

$$X_j \leq z Y_j \quad \forall j \tag{12}$$

$$Z_j \leq (1 - z) Y_j \quad \forall j \tag{13}$$

$$R_i \leq M_1 E_i \quad \forall i \tag{14}$$

$$\sum_{l=1}^{L} F_{il} \leq M_2 E_i \quad \forall i \tag{15}$$

$$G_i \leq (1 - M_1 - M_2) E_i \quad \forall i \tag{16}$$

$$Y_j \leq N\left(X_j + P_j\right) \quad \forall j \tag{17}$$

$$\sum_{l=1}^{L} \sum_{i=1}^{I} U_{il} \leq t \tag{18}$$

$$Z_j \leq M V_j \quad \forall j \tag{19}$$

$$X_j \leq M W_j \quad \forall j \tag{20}$$

$$U_{il}, V_j, W_j \in \{0, 1\} \quad \forall i, j, l \tag{21}$$

$$P_j, Z_j, Q_{ik}, E_i, F_{il}, G_i, R_i, Y_j, X_j \geq 0 \quad \forall i, j, k, l \tag{22}$$

The objective function (1) maximizes the total profit. The first term of the objective function represents the selling profits of new and repaired products. The second part represents total cost of purchasing parts from external suppliers. Total cost of disassembly site is calculated by the third part, consists of unit disassembly cost multiplied by the amount of disassembled parts. Besides, the forth part represents total recycling costs. The fifth part represents total disposing costs. The sixth part represents total cost of manufacturer happens from the internal production cost, consists of unit manufacturing cost multiplied by the amount of finished product produced by him. Total cost of operation and holding costs of collection site is calculated in the seventh part. The eight parts represents total cost of repair site. In addition, the ninth, tenth and 11th parts include set up costs for recycling, disassembly, and repair sites, respectively.

Constraints (2) ensure that the number of manufactured parts is equal to the number of recycled parts and the number of purchased parts from external suppliers, and the number of end-of-use parts. Constraints (3) show that the number of disassembly parts is equal to the summation of end-of-use and recycled and disposed parts. Constraints (4) ensure the relationship between parts and products in disassembly site. Besides, the constraints (5) represent that collected products are sent to repair or disassembly sites. Constraints (6)–(10) represent maximum capacity of manufacturer, external suppliers, disassembly, and recycling and repair sites. Constraints (11) show that the demand should be satisfied by manufactured products and repaired returns. Constraints (12) and (13) reflect the maximum percent of commercial returns. Furthermore, constraints (14)–(16) show the limitation of end-of-use and end-of-life returns. The maximum percent of total returned products is considered in constraints (17). In addition, constraint (18) represents the limitation of the number of recycling sites. Constraints (19) and (20) are related to the units of returned products to be disassembled and repaired. Finally, decision variables are defined in constraints (21) and (22).

5 Computational testing

In this section, a numerical example is presented. Suppose that a computer manufacturer assembles and sells five models of computer. Each product is produced by five parts. The manufacturer is interested to know how much should be manufactured according to demand. In addition, it is important to know how much should be purchased from each supplier. The required parameters are written in Appendix. In this paper, Generalized Algebraic Modeling System (GAMS) is used to obtain optimal solutions. The GAMS is specifically designed for modeling linear, nonlinear, and mixed-integer optimization problems. The system is especially useful for large and complex problems.

The results are written in Table 3. According to the results of MILP, the manufacturer should produce 1,050 units of product 1. These products are sent to the customers.

Table 3 The computational results

Product-related variables					
j	1	2	3	4	5
X_j	350	375	350	350	375
P_j	1,050	1,125	1,050	1,050	1,125
Y_j	700	750	700	700	750
Z_j	350	375	350	350	375
Part-related variables					
i	1	2	3	4	5
E_i	3,600	3,275	3,925	4,325	4,000
G_i	1,202	1,095	1,311	1,445	1,336
R_i	1,199	1,090	1,307	1,440	1,332
Q_{ik} (Units of part i to be purchased from external supplier k)					
i/k	1	2	3	4	5
1	–	–	–	3,607	–
2	–	–	–	–	3,281
3	3,932	–	–	–	–
4	–	4,333	–	–	–
5	–	–	4,008	–	–
F_{il} (Units of part i to be recycled in recycling site l)					
i/l	1	2	3	4	5
1	–	1,199	–	–	–
2	–	–	–	1,090	–
3	–	1,307	–	–	–
4	–	1,440	–	–	–
5	1,332	–	–	–	–

Then, 700 units are returned. 350 units of them go to repair site, and they are used to satisfy demand. Another 350 units are disassembled. Part-related variables also are illustrated in Table 3. For example, 3,600 units of part 1 are divided to 1,199 units of end-of-use parts, 1,202 units of wastes, and 1,199 units of end-of-life parts. The shortage of required parts is purchased from external suppliers. For instance, the manufacturer buys part 1 from supplier 4 because he has suggested the least purchasing cost ($6). The units of recycled parts also are written in Table 3. The part 1 is recycled in recycling site 2 because cost of recycling in this site ($2) is less than the others.

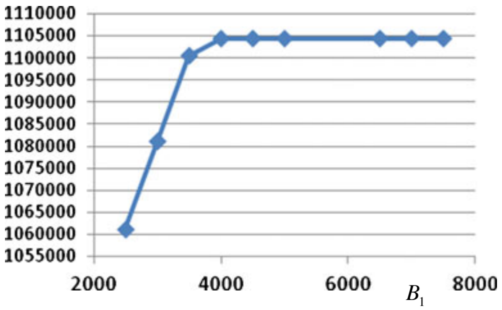

Fig. 2 Sensitivity analysis of the max capacity of disassembly site to dissemble part 1

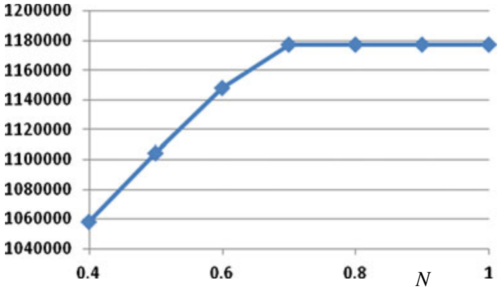

Fig. 3 Sensitivity analysis of N (max percent of total returns)

6 Sensitivity analysis

In order to validate the proposed model, sensitivity analysis is performed. We observed the changes of objective function by varying the capacity of disassembly site for part 1, while the other factors are fixed. Figure 2 shows the result. This analysis illustrates that the maximum objective function can be obtained with a certain capacity of disassembly site (in this example, 4,000). Therefore, in reality, the capacity of disassembly site should be expanded to a specific level. Therefore, the costs of investment will decrease. On the other hand, the effects of change in max percent of total returns (N) are illustrated in Fig. 3. It is undeniable that by increasing the amount of returns, the profit will increase. However, it is noticeable that the value of objective function for $N>0.67$ is fixed. In this situation, the major portion of demand is satisfied by commercial returns. Besides, the rest of demand can be supplied by the parts that are obtained by the end-of-use and end-of-life returns. As a result, the manufacturer does not purchase new parts from external suppliers and there is no purchasing cost. Figure 4 shows the effects of max percent of commercial returns on the objective function. It is obvious that by increasing z, the value of objective function increases because the commercial returns only need some light repairs. In other words, the costs of light repairs are less than the costs of disassembly, recycling, and manufac-

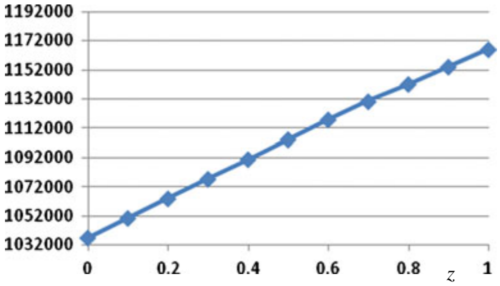

Fig. 4 Sensitivity analysis of z (max percent of commercial returns)

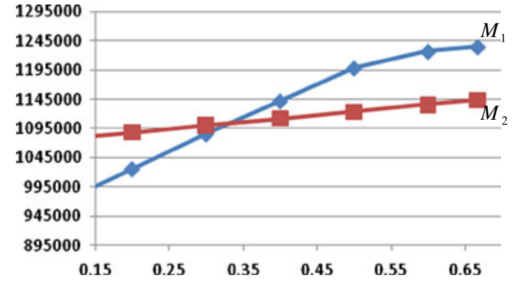

Fig. 5 Sensitivity analysis of M_1 (max percent of end-of-use returns), M_2 (max percent of end-of-life returns)

turing new products. Therefore, the manufacturer prefers to have commercial returns as much as possible. Similar effects have been observed in Fig. 5 for M_1 (max percent of end-of-use returns), and M_2 (max percent of end-of-life returns).

7 Extended model

In this section, it is supposed that remanufactured products are sent to the secondary market. This process may happen because of the lower quality of remanufactured products. The secondary market may be another country. The manufacturer has to satisfy the demand of same and secondary markets. The shortage of products in the secondary market should be supplied by new products. The manufacturer is interested to know how much should be produced to satisfy the demands of same and secondary markets. The new variables and parameters are written in Table 4. Other parameters are as same as Table 2.

The objective function and some constraints are similar to the proposed model. Constraints (23) ensure that the number of manufactured parts for the secondary market is equal to the number of recycled parts and the number of purchased parts from external suppliers (for the secondary market), and the number of end-of-use parts. Constraints (24) represent that the number of manufactured parts for the same market is equal to the number of purchased parts from external suppliers for the same market. Furthermore, constraints (25) and (26) are related to the demand. Constraints (27) show the maximum percent of total returned products. Constraints (28) ensure that the summation of parts of same and secondary markets is equal to the total parts. In the same order, constraints (29) are designed for products. Constraints (30) and (31) are related to decision variables. The extended model is solved by GAMS. The results of sensitivity analyses are illustrated in Figs. 6, 7, 8, and 9. Sensitivity analysis for the max capacity of disassembly site to dissemble part 1 (Fig. 6) shows that there is a certain maximum capacity of disassembly site. These results are useful for managers because they can prevent additional costs in remanufacturing network configuration.

Table 4 Additional variables and parameters for the secondary market

Variables		Parameters	
PA_j	Units of product j to be produced for the same market	DA_j	Demand for product j in the same market
PE_j	Units of product j to be produced for the secondary market	DE_j	Demand for product j in the secondary market
QA_{ik}	Units of part i to be purchased from external supplier k for the same market		
QE_{ik}	Units of part i to be purchased from external supplier k for the secondary market		

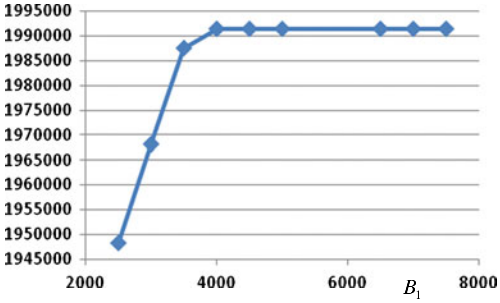

Fig. 6 Sensitivity analysis of the max capacity of disassembly site to dissemble part 1 (secondary market)

(1)

Subject to

(3), (4), (5), (6), (7), (8), (9), (10), (12), (13), (14), (15), (16), (18), (19), (20)

$$\sum_{j=1}^{J} q_{ij}PE_j = \sum_{l=1}^{L} F_{il} + \sum_{k=1}^{K} QE_{ik} + R_i \quad \forall i \tag{23}$$

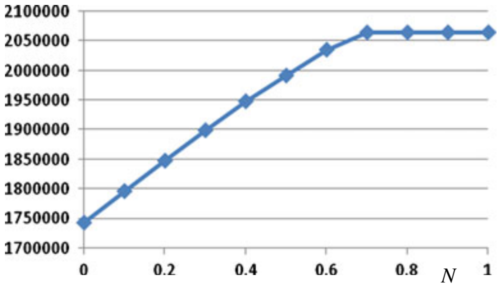

Fig. 7 Sensitivity analysis of N (max percent of total returns), secondary market

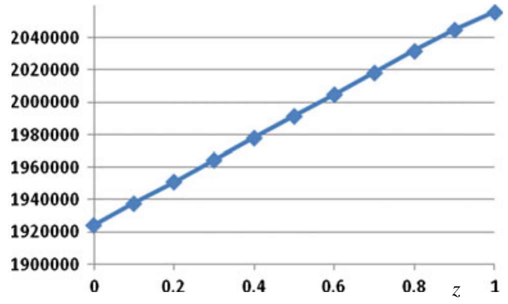

Fig. 8 Sensitivity analysis of *z* (max percent of commercial returns), secondary market

$$\sum_{j=1}^{J} q_{ij} PA_j = \sum_{k=1}^{K} QA_{ik} \quad \forall i \tag{24}$$

$$PA_j = DA_j \quad \forall j \tag{25}$$

$$PE_j + X_j = DE_j \quad \forall j \tag{26}$$

$$Y_j \leq NPA_j \quad \forall j \tag{27}$$

$$QA_{ik} + QE_{ik} = Q_{ik} \quad \forall i, k \tag{28}$$

$$PA_j + PE_j = P_j \quad \forall j \tag{29}$$

$$U_{il}, V_j, W_j \in \{0, 1\} \quad \forall i, j, l \tag{30}$$

$$P_j, Z_j, Q_{ik}, E_i, F_{il}, G_i, R_i, Y_j, X_j, QA_{ik}, QE_{ik}, PA_j, PE_j \geq 0 \quad \forall i, j, k, l \tag{31}$$

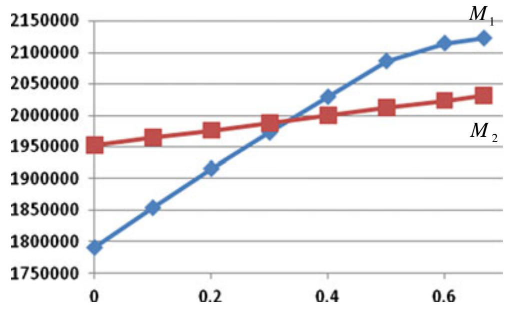

Fig. 9 Sensitivity analysis of M_1 (max percent of end-of-use returns), M_2 (max percent of end-of-life returns), secondary market

8 Conclusions

We proposed a closed-loop supply chain that consists of manufacturer, collection, repair, disassembly, recycling, and disposal sites. The major contribution of this research lies in designing and solving a general network based on return–recovery pairs and product life cycle. We proposed a novel mathematical model to optimize the closed-loop network. The mixed-integer linear programming model determines the units of products to be produced, disassembled, and repaired. In addition, it determines units of parts to be purchased from external suppliers, units of parts to be disassembled, recycled, and disposed while maximizing the profit. The model is solved by GAMS. We also developed the model for situation that remanufactured products are sent to a secondary market. A numerical example is performed to analyze the results. Furthermore, sensitivity analysis is utilized to validate the models. The results of our paper indicate that the manufacturer should take into account key factors such as production capacity, demand, supplier's capacity, end-of-life, end-of-use, and commercial returns. One of the insights from our study is that the maximum objective function can be obtained with a certain capacity of disassembly site. Therefore, managers can decrease the costs of investment. We also observed that the value of objective function for primary market is more than the extended model which is formed of primary and secondary markets because the manufacturer needs to purchase fewer parts from external suppliers. This result is obtained when total demands are equal.

Many research directions still require intensive research. Uncertainty is one of the important problems in supply chain management. It is worthwhile to take into account uncertainty of parameters such as demand and return. Besides, the proposed model is designed for a single period. The model can be extended to consider multiple periods. In this condition, the inventory level of *t* is different from *t*-1. In addition, beginning inventory should be taken into account. In the proposed model, recycling and end-of-use returns and new parts were used to manufacture new products. The price of reused products is a function of other factors such as demand, manufacturing process, and environmentally concerns particularly for products that have short life cycle. Determining the price of remanufactured parts based on the market demand can be a subject of future research. Moreover, it is hard to solve the model, when the numbers of variables and constraints increase. In this situation, heuristics algorithms such as Genetic Algorithm and Scatter Search can be useful.

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Appendix

Table 5 Product-related parameters

j	1	2	3	4	5
S_j	150	200	220	230	250
a_j	1	2	2	2	3
H_j	2.5	2.5	3.5	2.5	3.5
y_j	30	35	30	30	35
e_j	1	2	1	1	1
C_j	9,000	10,000	8,500	10,000	9,500
D_j	1,400	1,500	1,400	1,400	1,500
c_j	4	5.5	2.5	3.5	3.5
d_j	1	2	1	2	1
f_j	5	5	4	5	4
g_j	5	5	4	5	4

Table 6 Part-related parameters

i	1	2	3	4	5
B_i	9,000	10,000	8,500	10,000	9,500
h_i	4	5.5	2.5	3.5	3.5
m_i	3	4	4	4	3
r_i	1	1	1	1	1

Table 7 q_{ij} (the usage of part i per unit of product j)

i/j	1	2	3	4	5
1	2	1	3	1	3
2	1	3	2	1	2
3	3	2	1	4	1
4	2	1	2	3	4
5	1	3	2	2	3

Table 8 Recycling site-related parameters

n_{il} (Unit recycling cost for part i in recycling site l)					
i/l	1	2	3	4	5
1	3	2	3	3	4
2	4	4	3	2	4
3	4	3	4	3	4
4	4	3	3	4	3
5	3	3	4	4	4
o_{il} (Set-up cost of recycling site l for part i)					
i/l	1	2	3	4	5
1	4	5	4	4	4
2	4	4	4	4	5
3	5	5	4	5	5
4	4	5	5	5	5
5	4	4	4	5	4
s_{il} (Resource usage to recycle one unit of part i in recycling site l)					
i/l	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1
O_{il} (Max capacity of recycling site l to recycle part i)					
i/l	1	2	3	4	5
1	9,000	10,000	8,500	10,000	9,500
2	10,000	9,000	8,500	10,000	9,500
3	9,000	10,000	8,000	9,500	10,000
4	8,500	9,000	10,000	9,500	8,500
5	9,000	9,500	10,000	9,000	8,500

Table 9 Supplier-related parameters

p_{ik} (The cost of purchasing part i from external supplier k)					
i/k	1	2	3	4	5
1	8	8	12	6	15
2	10	15	8	10	5
3	5	7	14	9	8
4	9	5	10	13	8
5	12	9	5	7	6
b_{ik} (Internal resource usage of supplier k to produce one unit of part i)					
i/k	1	2	3	4	5
1	1.5	2	3	1	3
2	2	1	1	3	1
3	2	1.5	1	3	2.5
4	1.5	3	2.5	2	3
5	3	2	3	2	1.5
T_k (The capacity of supplier k)					
k	1	2	3	4	5
T_k	100,000	75,000	90,000	60,000	125,000

Table 10 A (Max capacity of manufacturer plant), M_1 (max percent of end-of-use returns), M_2 (max percent of end-of-life returns), N (max percent of total returns), z (max percent of commercial returns), t (max number of recycling sites)

A	200,000	N	0.5
M_1	0.333	z	0.5
M_2	0.333	t	6

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