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Lexicographic max–min approach for an integrated vendor-managed inventory problem

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ABSTRACT

Simultaneous reductions in inventory of raw materials, work-in-process, and finished items have recently become a major focus in supply chain management. Vendor-managed inventory is a well-known practice in supply chain collaborations, in which manufacturer manages inventory at the retailer and decides about the time and replenishment. In this paper, an integrated vendor-managed inventory model is presented for a two-level supply chain structured as a single capacitated manufacturer at the first level and multiple retailers at the second level. Manufacturer produces different products where demands are assumed decreasing functions of retail prices. In this chain, both the manufacturer and retailers contribute to determine their own decision variables in order to maximize their benefits. While previous research on this topic mainly included a single objective optimization model where the objective was either to minimize total supply chain costs or to maximize total supply chain benefits, in this research a fair profit contract is designed for the manufacturer and the retailers. The problem is first formulated into a bi-objective non-linear mathematical model and then the lexicographic max–min approach is utilized to obtain a fair non-dominated solution. Finally, different test problems are investigated in order to demonstrate the applicability of the proposed methodology and to evaluate the solution obtained.

1. Introduction

Vendor-managed inventory (VMI) is a well-known practice for supply chain collaboration, in which manufacturer manages inventory at the retailer and decides when and how much to replenish. In recent years, there has been an increasing interest in cooperative and non-cooperative relationship between both manufacturer and retailers in the VMI program. For instance, VMI has been adapted to the lean production requirements of manufacturers in automobile manufacturing supply chain management based on information system integration [14]. In order to analyze the supply chain performance improvement, Xu et al. [25] presented a real case study in a Chinese medium-sized aluminum manufacturing company. They showed the VMI strategy could significantly improve the supply chain performance such as reducing customer order cycle time and reducing safety inventory costs.

Although the benefits of VMI to the retailer include reduction of overhead costs and, if consignment stock is adopted, transfer of inventory costs to the manufacturer, the benefits of VMI to manufacturer are not very straightforward [11]. Meanwhile, research works mainly focus on the following three aspects of VMI programs [7]:

1. Investigating the benefits of VMI programs compared with normal supply modes without VMI.
2. Operational decisions in VMI programs.
3. Designing contracts for VMI programs.

The literature related to this paper can be classified into those of the second category.

Yao et al. [27] introduced a model to explore the effects of cooperative supply chain initiatives such as VMI, first developed by Vlist et al. [23]. In this issue, the authors showed that when the shipment sizes from a supplier to a buyer increase, inventory at the supplier goes down and inventory at the buyer goes up. Zhang et al. [33] presented an integrated VMI model for a single-vendor multiple-buyer supply chain problem, where the vendor first purchases and processes raw materials and then delivers

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finished items to multiple buyers. Investment decision, constant production, and demand were considered where the buyers' ordering cycles might be different and that each buyer could replenish more than once in one production cycle.

Impact of the consignment inventory (CI) and VMI policies was studied by Gümüş et al. [8]. The goal was to analyze CI in a two-party supply chain under deterministic demand and to provide some general conditions under which CI creates benefits for the vendor, for the customer, and for the two parties together. Sari [20] presented a comprehensive simulation model representing two popular supply chain initiatives, collaborative planning forecasting replenishment (CPFR) and VMI, in order to select an appropriate collaboration mode in business conditions. Their results showed that benefits of CPFR are always higher than VMI. Besides, an integrated production–inventory model was developed by Zavanella and Zanoni [32], in which a particular VMI policy known as consignment stock (CS) for both the buyer and the supplier was investigated. Yu et al. [29,30] showed how the vendor can take into account the advantage of his information for increasing his own profit by using a Stackelberg game in a VMI system. Yu et al. [28] showed how to analyze the intrinsic evolutionary mechanism of the VMI supply chains by applying the evolutionary game theories. Darwish and Odah [3] developed a model for a supply chain with single vendor and multiple retailers based on VMI, considering capacity constraints by selecting high penalty cost. Almehdawe and Mantin [1] studied supply chains composed of a single capacitated manufacturer and multiple retailers. They formulated a Stackelberg game VMI framework under two scenarios: in the first, the manufacturer is the leader; in the second, one of the retailers acts as the dominant player of the supply chain. In addition, market demand was considered a function of retail price. This model was also extended by Yu et al. [29,30] when advertising investment and pricing come to the picture.

The quaternary policy towards integrated logistics and inventory aspect of the supply chain was proposed by Arora et al. [2]. They considered a supply chain with multiple retailers and distributors, in which all distributors follow a unique policy and the VMI system is used for updating the inventory of the retailers. Yang et al. [26] studied the effects of the distribution centre (DC) in a VMI system comprising one manufacturer, one DC, and n retailers where the system aims to maximize the overall system profit. While Lee and Ren [11] showed the supply chain total cost decreases under VMI, the reduction is larger when there is exchange rate uncertainty compared with the case of no exchange rate uncertainty. They considered a state-dependent (s, S) policy for the supplier. Pasandideh et al. [18] presented a multi-product multi-constraint economic order quantity (EOQ) model under the VMI policy for a supply chain. They developed a genetic algorithm to find the best order quantities and the maximum backorder levels such that the total inventory cost of the supply chain is minimized.

A logistics network design under VMI by considering location, transportation, pricing, and warehouse–retailer inventory replenishment decisions was presented by Shu et al. [22]. Zanoni et al. [31] provided a two-level supply chain model for a single-vendor single-buyer at each level and compared different policies that the vendor might adopt to exploit the advantages offered by the VMI with consignment agreement when the vendor's production process is subject to learning effects.

To summarize, many research works in supply chain environment assume a non-cooperative relation (such as the one in the Stackelberg game) between the manufacturer and the retailers with the manufacturer acting as the leader and the retailers as the followers [1]. In addition, most of the literature on the VMI problem only aim to optimize manufacturer's objectives and do not pay attention to retailers' objectives [29,30,1]. Moreover, there has been little discussion about designing fair contracts in VMI problems so far. Besides, previous research works on this topic mainly included a single objective optimization model where the objective was either to minimize the total cost or to maximize the total benefit. However, this paper presents a two-level supply chain model by assuming a single capacitated manufacturer at the first level and multiple retailers at the second level. This chain is considered integration between the manufacturer and retailers where the manufacturer (vendor) produces multiple products, sells to retailers, and manages the retailers' inventories under VMI. A fair profit contract between the manufacturer and his retailers is adopted in this research. Our motivation of defining a fair profit contract is that both the manufacturer and retailers are able to contribute to determine their optimal decision variables in order to maximize their benefits. In other words, the manufacturer and his retailers maximize their benefits as close to one another as possible. Besides, the demand rate for each product in each local retail market is assumed a decreasing function of the retail price called the Cobb–Douglas demand function. Finally, this paper formulates the problem into a non-linear mathematical model with two-objectives in order to maximize both the manufacturer and retailers' benefit. It is assumed that both the objectives are equally important, and it is needed to find a "fair" non-dominated solution by the lexicographic max–min approach. A fair non-dominated solution is a solution with all normalized objective function values as equal as possible. Following Erkut et al. [6], we discuss the conversion of the original lexicographic max–min problem to a lexicographic maximization problem without using the dual formulation of the LP problem.

The reminder of this paper is organized as follows. Section 2 contains problem description. The mathematical formulation of the problem is given in Section 3. Section 4 discusses the lexicographic max–min approach to solve the problem. The applicability and the performances of the proposed method are demonstrated in Section 5 using some numerical examples. Moreover, sensitivity analyses on the effects of some input parameters on the objective functions are performed in this section. Finally, we conclude the paper with a discussion of possible further research in Section 6.

2. Problem description

Consider a two-level supply chain consisting of a single manufacturer at the first level and multiple retailers at the second. The manufacturer's capacity is finite in producing different products with a fixed production rate. He sells the products to its retailers with a common replenishment cycle. A common replenishment cycle eliminates the influence of the variation of the replenishment cycle as well as backorder rate of every retailer. The manufacturer must sell the products to his retailers at different wholesale prices. Besides, the manufacturer and retailers are operating in distinctive markets with no conflict of interests. Integration is established between the manufacturer and all retailers, in which manufacturer manages inventory at all levels by having access to retailers' inventory as well as his own (i.e. VMI). Moreover, each retailer pays to the manufacturer a cost of ξ_{ic} per unit consumed per time unit to have his inventory managed by the manufacturer. The manufacturer decides on his replenishment cycle of the finished products, wholesale prices, and fraction of backlogging. Retailers' decisions include their retail prices.

2.1. Assumptions

The followings are assumed in this paper:

1. The demand for every retailer and every product is constant over time.

2. The demand function for all retailers and all products is a convex function with respect to its retail price (see the paragraph before Eq. (1) for more details)
3. Lead-time of each product at each level of the supply chain is assumed negligible compared to the common replenishment cycle time.
4. The production setup cost occurs at the beginning of each common replenishment cycle.
5. The setup cost is realized once in every replenishment cycle.
6. Planning horizon is infinite.

2.2. Indices, parameters, and decision variables

The following indices, parameters, and decision variables are used throughout the paper:

Indices
c Index for retailers ($c = 1,2,\ldots,n$)
i Index for product types ($i = 1,2,\ldots,I$)

Input parameters
D_{ic} Retailer c's demand for finished product i
ξ_{ic} Inventory management cost of the finished product i for retailer c (\$/unit/time)
cm Production cost per unit for finished product (\$/unit)
Φ_{ic} Transportation cost per unit of finished product i shipped from the manufacturer to retailer c (\$/unit)
r Production rate of finished products
S_i Setup cost for the common cycle time for product i (\$)
SR_C Fixed order cost paid by the manufacturer to retailer c (\$)
π_{ic} Backorder cost paid by the manufacturer to retailer c for product i (\$/unit/time)
H_i Holding cost at the manufacturer's side (\$/unit/time)
h_{ic} Holding cost paid by the manufacturer at retailer c's side for product i (\$/unit/time)

Decision variables
w_{ic} Wholesale price of the finished product i, provided by the manufacturer to retailer c (\$/unit)
p_{ic} Retail price charged by retailer c for product i (\$/unit)
b_{ic} Fraction of backlogging time of finished product i for retailer c in the common replenishment cycle
C_i Common replenishment cycle time for the finished product i

About the second assumption stated in Section 2.1, the demand faced by each retailer for each product is assumed to follow the Cobb–Douglas demand function characterized by a constant elasticity demand function of the form given in Eq. (1).

$$D_{ic} = k_c p_{ic}^{-e_c} : \quad \forall i = 1,\ldots,I, \quad c = 1,\ldots,n. \tag{1}$$

In which k_c and $e_c > 1$ represent the market scale of retailer c and the demand elasticity of retailer c with respect to its retail price, respectively [1].

3. Mathematical model

The integrated manufacturer–retailers model can be formulated as follows:

$$\max \quad z_1 = \sum_{i=1}^{I}\sum_{c=1}^{n} D_{ic}(w_{ic} - cm - \Phi_{ic}) - \sum_{i=1}^{I}\left[\frac{S_i}{C_i}\right] - \sum_{i=1}^{I} H_i\left[\sum_{c=1}^{n}\left(\frac{D_{ic}^2 C_i^2}{2rC_i}\right)\right] - TC_{VMI}, \tag{2}$$

$$\max \quad z_2 = \sum_{i=1}^{I}\sum_{c=1}^{n} D_{ic}(p_{ic} - w_{ic} - \xi_{ic}), \tag{3}$$

$$\text{s.t.}: \quad TC_{VMI} = \sum_{i=1}^{I}\sum_{c=1}^{n}\left[\frac{SR_c}{C_i}\right] + \sum_{i=1}^{I}\sum_{c=1}^{n} h_{ic}\left(\frac{D_{ic}(1-b_{ic})^2 C_i}{2}\right) + \sum_{i=1}^{I}\sum_{c=1}^{n}\pi_{ic}\left(\frac{D_{ic}b_{ic}^2 C_i}{2}\right) - \sum_{i=1}^{I}\sum_{c=1}^{n}\xi_{ic}D_{ic}, \tag{4}$$

$$\sum_{i=1}^{I}\sum_{c=1}^{n} D_{ic} \leqslant r, \tag{5}$$

$$p_{ic} > w_{ic} + \xi_{ic}, \quad \forall i = 1,\ldots,I, \quad c = 1,\ldots,n, \tag{6}$$

$$0 \leqslant b_{ic} \leqslant 1, \quad \forall i = 1,\ldots,I, \quad c = 1,\ldots,n, \tag{7}$$

$$C_i, w_{ic}, p_{ic} \geqslant 0, \quad \forall i = 1,\ldots,I, \quad c = 1,\ldots,n, \tag{8}$$

It is clear that the above formulation is non-linear with two conflicting objective functions. The first objective function (z_1) that is given in Eq. (2) is the net profit of the manufacturer obtained by the revenue from sale of finished products to retailers at wholesale prices minus the costs including production, transportation, setup, holding, and TC_{VMI}. The second objective function (z_2) that is given in Eq. (3) shows the net profit of all retailers. Eq. (4) represents TC_{VMI} that is defined as the total inventory cost incurred by the manufacturer to manage all retailers' inventory. The inventory costs at each retailer's side are the fixed inventory costs, variable inventory costs, and back-ordering costs. Inequality (5) insures that total demand faced by the manufacturer does not exceed his production capacity. Inequalities (6) show the least acceptable prices in order to assure positive net profits for all retailers. Inequalities (7) are to set limits for the fraction of backlogging rates and inequalities (8) guarantee non-negative values for all decision variables. Fig. 1 shows how one can obtain average inventories in order to derive holding and backorder costs of retailers. Besides, Fig. 2 shows the total inventory of the manufacturer for product i in a common replenishment cycle [29,30].

This model was originally presented by Almehdawe and Mantin [1], where they formulated a Stackelberg game VMI framework with a single objective and only one product. However, a bi-objective optimization model is derived in the current model for several products, in which TC_{VMI} represents the total cost paid by the manufacturer to manage all retailers' inventory. It consists of the difference between all the inventory costs he realizes and the revenue he receives from the retailers to manage their inventory.

4. The lexicographic max–min approach

A brief discussion on the lexicographic max–min (LMM) as a refinement of the standard max–min approach along with the rationale behind its use for the integrated vendor managed inventory problem at hand is presented in this section. This approach

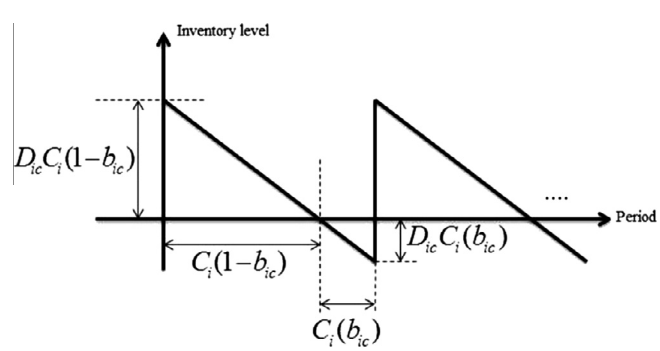

Fig. 1. Inventory level of retailer c for product i per common replenishment cycle.

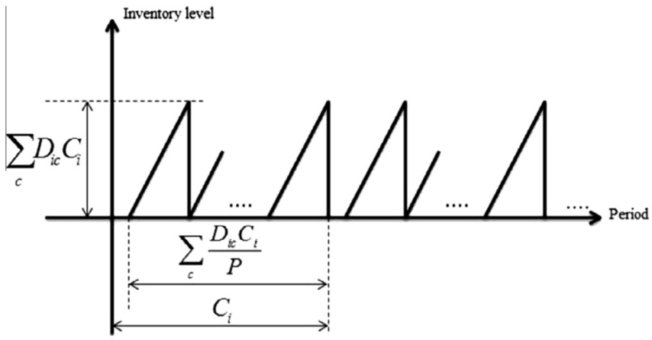

Fig. 2. Inventory level of the manufacturer for product i per common replenishment cycle.

can be utilized to simultaneously maximize the smallest manufacturer's profit and the smallest retailers' profit as close to one another as possible. Taking advantage of this approach enables one to design a fair profit contract between the manufacturer and the retailers, i.e., this approach can create a good win–win scenario for both the manufacture and its retailers in the supply chain. LMM is adopted in this research because it provides solutions satisfying fairness and efficiency properties [19]. Another advantage of the LMM approach is that it usually works on a lower dimension than the domain set, which may simplify the analysis [19].

The lexicographic max–min method that first was introduced by Dresher [4] was later refined to the formal nucleolus definition by Schmeidler [21]. It has been applied for multi-period resource allocation [9], linear multiple criteria problems [12], fair bandwidth allocation in computer networks [19], water resource allocation [24], and waste management [6]. Moreover, the LMM solution is known in the game theory as the nucleolus of a matrix game [12]. It is also called lexicographic max-ordering [5] and lexicographic centers in location [16].

The standard lexicographic approach to find max–min solutions selects a unique set of outcomes which may be a non-unique solution in the decision space but all the solutions have exactly the same distribution of outcomes [15]. Although the lexicographic leads to a Pareto efficient solution [13], this solution is not fair since it is first necessary to establish a strict precedence among all utility functions [19]. Therefore, this simple lexicographic approach is not applicable to our research. That is why the LMM approach is taken in this research to find a fair solution for the integrated manufacturer–retailers problem modeled in Section 3.

Let $f_i(x)$ $(i = 1,2,\ldots,P)$: $D \rightarrow R$ be an objective function to be optimized, where x is a feasible solution and D is a set of feasible solutions. A multi-objective optimization (maximization for instance) problem, is formulated as follows:

$$\begin{aligned} &\text{Max} \quad f_1(x), f_2(x), \ldots, f_P(x), \\ &\text{s.t.}: \quad x \in D. \end{aligned} \tag{9}$$

In these problems, a set of solutions called Pareto optimal is desired. To find the Pareto optimal solution the following order relation is first defined:

$$x \succ y \Longleftrightarrow f_i(x) \geqslant f_i(y), \quad (\forall i = 1,2,\ldots,P) \wedge f_i(x) > f_i(y), \quad (\exists i = 1,2,\ldots,P). \tag{10}$$

If the above relation holds between x and y, the solution x is said to dominate y. Then, the Pareto optimal solution is defined as follows: A solution $x^* \in D$ is said to be a Pareto optimal solution if there is no solution $x \in D$ such that $x^* \prec x$ [10]. The concept of a "fair" efficient solution is a refinement of the Pareto optimality. The fair solution is a solution with all normalized objective function values as close to one another as possible [15]. An alternative approach depends on the so-called the max–min solution concept, where the worst performance is maximized:

$$\text{Max}\left\{ \min_{i=1,2,\ldots,P} f_i(x) : x \in Q \right\}. \tag{11}$$

It should be noted that the optimal solution set of the max–min problem (11) always contains an efficient solution of the original multi-objective problem given in (9). The max–min solution concept depends on optimization of the worst outcome, and it is regarded as maintaining equity as described by the following theorem [17].

Theorem 1. *If exists a non-dominated outcome vector $\bar{y} \in Y$ with perfect equity $\bar{y}_1 = \bar{y}_2 = \cdots = \bar{y}_P$ then $\bar{y}$ is the unique optimal fair solution of the max–min problem.*

$$\text{Max}\left\{ \min_{i=1,2,\ldots,P} y_i : y \in Y \right\}. \tag{12}$$

Note that the standard max–min approach depends on minimization of $\bar{y}_1$ and it ignores $\bar{y}_j$ for $j \geqslant 2$. It is a reason why the Standard max–mix approach is, in general, too crude to satisfy the Pareto optimality principle [15]. Therefore, we solve a LMM problem as a refinement of this max–min problem.

Let $\Theta(a) = (a\langle_1\rangle, a\langle_2\rangle, \ldots, a\langle_P\rangle)$ be a vector obtained from a by rearranging its components in non-decreasing order and Θ: $R^P \rightarrow R^P$ a map, which orders the components of vectors in a non-decreasing order. That means $a\langle_1\rangle \leqslant a\langle_2\rangle \leqslant \cdots \leqslant a\langle_P\rangle$ where a_i is the ith component of $\Theta(a)$. Comparing lexicographically such ordered vectors f one gets the so-called lex-max order. Therefore, LMM problem is:

$$lex\ \max\{\Theta(f)\} = lex\ \max\{(f_{\langle 1 \rangle}, f_{\langle 2 \rangle}, \ldots, f_{\langle P \rangle}) : \ f \in A\}, \tag{13}$$

where $A = \{f: f = f(x), x \in D\}$.

Theorem 2. *An optimal solution of the problem (13) is also the optimal solution of the problem (12).*

In problem (13), in addition to maximize the worst (smallest) outcome, we also maximize the second smallest outcome (provided that the smallest one remains as large as possible), maximize the third smallest (provided that the two smallest remain as large as possible), and so on. The LMM solutions satisfy the principles of Pareto-optimality (efficiency) and perfect equity as described by the following theorem [6].

Theorem 3. *$x^* \in D$ is Pareto-optimal with perfect equity $f_1(x^*) = f_2(x^*) = \cdots = f_P(x^*)$, it is an optimal fair solution of the optimization problem (13).*

However, problem (13) is not a standard mathematical program. In following, we describe an approach to transfer the LMM problem (13) to a lexicographic maximization problem. Let $\eta_i(y) = \sum_{j=1}^{i} y_{\langle j \rangle}$ be cumulated criteria expressing, respectively the worst (smallest) outcome, the total of the two worst outcomes, the total of the three worst outcomes, etc. [17]. Therefore, for any given vector f, the cumulated ordered value $\eta_i(f)$ can be found as the optimal value of the following LP problem:

$$\eta_i(f) = \max \quad \sum_{j=1}^{P} f_j a_{ij}, \tag{14}$$

$$\text{s.t.}: \quad \sum_{j=1}^{P} a_{ij} = i, \quad \forall j = 1,\ldots,P; \tag{15}$$

$$a_{ij} \in \{0,1\}, \quad \forall j = 1,\ldots,P; \tag{16}$$

where a_{ij} is a binary variable and can be relaxed to a continuous variable, i.e., $0 \leqslant a_{ij} \leqslant 1$. Therefore, by using the above model, lexicographic max–min problem converts to a lexicographic maximization problem. By solving the above model, one can find a fair non-dominated solution for the integrated manufacturer–retailers problem at hand. Numerical examples are provided in the next section to demonstrate this approach.

5. Performance evaluation and comparison

The aim of this section is to demonstrate the applicability and to assess the performances of the proposed model using hypothetical examples having randomly generated data. These examples are solved using GAMS 23.5 software and making use of the CONOPT3 solver on an Intel(R), core(TM) i7, 3.23 GHz lap top with 512 Mb RAM.

The case study is adapted from Almehdawe and Mantin [1]and Yu et al. [28–30] and is adjusted to suit this research. Consider a supply chain consider a supply chain consisting of a single manufacturer and three retailers. The manufacturer produces two products. The retailers are different in terms of market size, characterized by the market scale k_c, and customer's sensitivity to price changes, characterized by the price elasticity e_c. Random examples are generated according to the information provided in Table 1, where the term "U" implies a uniform distribution. Some of these ranges are selected based on the work by Almehdawe and Mantin [1].

For a randomly generated problem, the LMM approach is employed to determine which Pareto-optimal solution to be implemented in order to get a fair trade-off between manufacturer and retailers' profits. Certainly, the lexicographic optimization may also be treated as a sequential (hierarchical) optimization process where first $g_1(x)$ is maximized on the entire feasible set, next $g_2(x)$ is maximized on the optimal set, and so on. This may be implemented as shown in the following standard sequential algorithm with predefined objective functions ($lex\ \max\{(g_1(x),\ldots, g_m(x))\colon x \in D\}$), in which $g_s(x) = \sum_{j=1}^{P} f_j a_{sj}$:

Step 0: Set $s := 1$.
Step 1: Solve problem Q_s defined as:

$$\max_{x\in D}\{\vartheta_s; \vartheta_s \leqslant g_s(x), \quad \vartheta_j^0 \leqslant g_j(x) \quad \forall j < s\}$$

and denote the optimal solution by (x^0, ϑ^0)
Step 2: If $s = m$, then STOP (x^0 is the optimal solution).
Otherwise, set $s := s + 1$ and go the **Step 1**

For example, for $s = 1$, we build the first problem (Q_1) with the objective $\vartheta_1; \vartheta_1 \leqslant \sum_{j=1}^{P} f_j a_{1j}$ being maximized and constraints shown in Eqs. (15) and (16). For the next iterations ($s > 1$), the problem Q_s is built by adding new constraints $\vartheta_{s-1}^0 \leqslant \sum_{j=1}^{P} f_j a_{s-1,j}$ where ϑ_{s-1}^0 is the optimal objective value of the problem Q_{s-1} and $\sum_{j=1}^{P} a_{s-1,j} = s - 1$ and so on. For the integrated manufacturer–retailers problem at hand, inequalities (4)–(8) are considered in each iteration. The objective values obtained by both the max–min and the LMM methods along with their CPU times of reaching the solutions are given in Table 2.

The results in Table 2 indicate the equality of both objectives in all cases, which means they are close to their maximum values equally. This is consistent with what was stated in Theorem 3. In other words, fair non-dominated solutions with all normalized objectives as equal as possible are obtained. Besides, LMM provides better solutions with less CPU times compared to the ones of the max–min method. In the next subsection, sensitivity analyses are performed to investigate the effects of market scale and demand elasticity on manufacturer and retailers' profits.

Table 1
Test problem generation.

Parameter	Value
(k_1, k_2, k_3)	(3000, 2000, 2000)
(e_1, e_2, e_3)	(1.2, 1.3, 1.5)
ξ_{ic}	$\sim U(1.2, 2.4)$
cm	4
Φ_{ic}	3
P	1000
S_i	$\sim U(10, 30)$
SR_C	$\sim U(20, 40)$
π_{ic}	$\sim U(150, 200)$
H_i	$\sim U(2, 5)$
h_{ic}	$\sim U(0.5, 3)$

5.1. Sensitivity analyses

Since k_c and e_c in the Cobb–Douglas demand function presented in Eq. (1) represent market scale and demand elasticity of retailer c with respect to retail price, respectively, an initial value and twelve variations from the initial value are considered to observe the effects of these changes on manufacturer's profit (z_1) and all retailers' profits (z_2). The LMM solutions for wholesale prices, retail prices, fraction backlogged, and replenishment cycles (the decision variables) based on the initial values is summarized in Table 3. The results in Table 3 show that retailer 1 with high market scale and low demand elasticity earns high retail price and demand for the products.

Furthermore, an instance in the sensitivity analyses shows that when the market scale of retailer 1 (k_1) increases from 3000 to 3500, z_1 and z_2 in the LMM and the max–min method increase from \$3755.520 to \$4140.088 and \$1864.479 to \$2046.598, respectively. Moreover, when this parameter decreases from 3000 to 2500, z_1 and z_2 in the LMM and the max–min method decrease to \$3371.065 and \$1487.486, respectively. In addition, when the demand elasticity faced by retailer 3 increases from 1.5, as its initial value, to 3, the LMM and the max–min solutions decrease from \$3755.520 to \$2725.190 and \$1864.479 to \$968.174, respectively. Based on the above sensitivity analyses (and the other sensitivity analyses not shown here) it can be seen that while both methods provide fair non-dominated solutions in all cases, LMM provides better solutions in terms of the objective functions than the ones of the max–min method with almost equal CPU times.

5.2. Comparison

In order to assess the performance of the proposed methodology and compare it to the one of the max–min method, different test problems with different numbers of retailers and finished products are considered in this section, where in all problems $k_c = 2000$ and $e_c = 1.5$. The results obtained using the max–min and the LMM methods are summarized in Table 4. Besides, the values of z_1 obtained by LMM and max–min methods for three, five, and seven finished products are shown in Figs. 3–5, respectively.

Paired samples t-test is a useful tool to test the null hypothesis that the mean of the values of z_1 obtained by LMM in all test problems is greater the one obtained by the max–min method. The paired samples t-test (one-tailed) succeeded to reveal a statistically significant difference between the means (LMM Mean = 18,158, LMM Std = 9672, max–min mean = 7255, max–min Std = 3753). In this case, the t-statistic is 8.167, which is far greater than the upper 5% critical point of a t-student distribution with 29 degrees of freedom (1.699). In other words, the LMM method provides better quality solutions in terms of the first objective function. Similarly, this test is implemented for the

Table 2
Solution comparisons of the max–min and the LMM methods.

Setting	Max–min			LMM		
	CPU time (s)	z_1 ($)	z_2 ($)	CPU time (s)	z_1 ($)	z_2 ($)
Initial value	0.726	1846.479	1846.479	0.109	3755.520	3755.520
$k_1 = 3500$	0.156	2046.598	2046.598	0.060	4140.088	4140.088
$k_1 = 2500$	0.11	1487.486	1487.486	0.043	3371.065	3371.065
$k_2 = 4000$	0.125	2156.573	2156.573	0.116	4802.181	4802.181
$k_2 = 1600$	0.113	1622.343	1622.343	0.049	3547.036	3547.036
$k_3 = 1700$	0.287	946.196	946.196	0.103	3674.720	3674.720
$k_3 = 3000$	0.101	1503.574	1503.574	0.073	4025.292	4025.292
$e_1 = 1.7$	0.173	892.094	892.094	0.046	1966.127	1966.127
$e_1 = 2.1$	0.131	731.673	731.673	0.036	1559.012	1559.012
$e_2 = 2$	0.161	1049.722	1049.722	0.098	2832.049	2832.049
$e_2 = 3$	0.109	1164.608	1164.608	0.048	2725.190	2725.190
$e_3 = 3$	0.193	968.174	968.174	0.104	3227.016	3227.016
$e_3 = 2.1$	0.124	1530.647	1530.647	0.042	3310.163	3310.163

Table 3
Results of the LMM method based on the initial values.

Retailer	Product 1				Product 2			
	w ($/unit)	p ($/unit)	b (rate)	D (unit)	w ($/unit)	p ($/unit)	b (rate)	D (unit)
1	32.622	46.683	0.006	29.794	0.548	48.973	0.007	28.131
2	36.997	38.628	0.013	17.300	9.059	35.064	0.006	19.621
3	2.047	25.176	0.012	15.833	6.318	23.827	0.006	17.195
Replenishment cycle (time)	1.435				1.582			

Table 4
Summary of test results.

# Of retailers	# Of products	Max–min			LMM		
		CPU time (s)	z_1 ($)	z_2 ($)	CPU time (s)	z_1 ($)	z_2 ($)
$n = 5$	3	0.431	1393.199	1393.199	0.157	3664.546	3664.546
	5	0.719	3081.036	3081.036	0.268	6173.579	6173.579
	7	1.229	4257.444	4257.444	0.689	8731.821	8731.821
$n = 7$	3	0.307	2544.350	2544.350	0.272	5158.398	5158.398
	5	0.799	3839.617	3839.617	0.361	8717.870	8717.870
	7	0.965	6014.480	6014.480	0.592	12186.242	12186.242
$n = 9$	3	0.749	3193.467	3193.467	0.129	6724.740	6724.740
	5	0.951	5464.342	5464.342	0.548	11193.243	11193.243
	7	1.720	7526.157	7526.157	0.865	15579.723	15579.723
$n = 11$	3	0.927	3460.369	3460.369	0.457	8207.984	8207.984
	5	1.950	6558.041	6558.041	0.712	13726.278	13726.278
	7	2.264	9069.026	9069.026	1.578	19001.325	19001.325
$n = 15$	3	1.027	5439.454	5439.454	0.623	11198.937	11198.937
	5	2.138	6960.869	6960.869	1.003	18544.075	18544.075
	7	3.294	11581.021	11581.021	2.559	25081.856	25081.856
$n = 17$	3	0.887	3864.780	3864.780	0.450	12732.243	12732.243
	5	2.200	9622.528	9622.528	3.462	20941.679	20941.679
	7	2.990	12486.476	12486.476	2.929	28029.860	28029.860
$n = 19$	3	3.213	6356.327	6356.327	2.914	14177.165	14177.165
	5	2.704	10960.295	10960.295	2.215	22967.203	22967.203
	7	2.244	9736.812	9736.812	3.544	30870.568	30870.568
$n = 21$	3	1.138	7775.415	7775.415	1.172	15619.549	15619.549
	5	4.301	12314.367	12314.367	2.666	25077.977	25077.977
	7	6.838	16459.280	16459.280	5.355	33463.904	33463.904
$n = 23$	3	3.045	6349.733	6349.733	1.887	17117.966	17117.966
	5	2.518	9904.490	9904.490	3.723	27156.413	27156.413
	7	7.139	14047.044	14047.044	9.62	36101.473	36101.473
$n = 25$	3	1.862	8470.457	8470.457	1.390	18564.017	18564.017
	5	2.674	2799.373	2799.373	2.243	29352.424	29352.424
	7	3.170	6126.508	6126.508	8.149	38686.014	38686.014

means of z_2 obtained by both methods and the same conclusion has been made. Moreover, a paired samples t-test is designed to determine if the mean CPU times are different. In this case, the t-statistic based on (LMM mean = 2.08, LMM Std = 2.27, max–min Mean = 2.21, max–min Std = 1.64) becomes 0.544 with a *p-value* = 0.591 that shows no significant statistical difference. In

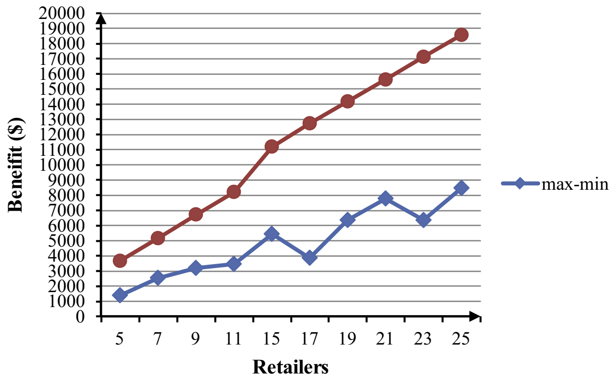

Fig. 3. Values of z_1 obtained by the two methods for three products.

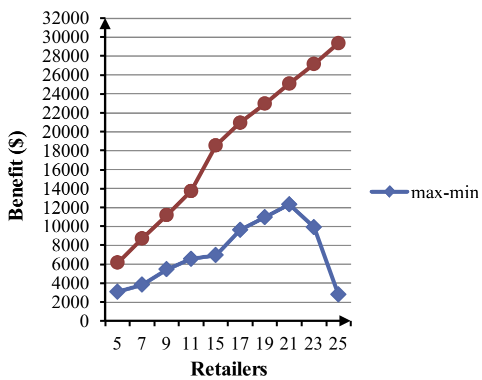

Fig. 4. Values of z_1 obtained the methods for five products.

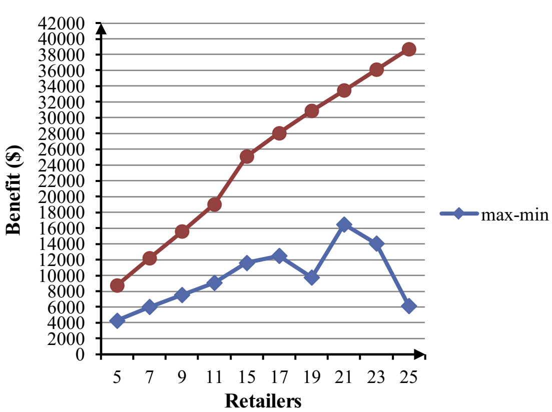

Fig. 5. Values of z_1 obtained by the methods for seven products.

other words, the methods have the same required mean CPU time statistically. In addition, the comparisons in terms of the objective values and CPU times reveal that, increasing the number of retailers, increases z_1 and z_2 and CPU times simultaneously. Nevertheless, it can be seen that fair solutions that are obtained by the LMM method provide better manufacturer and retailers' benefits than the ones obtained by the max–min method.

6. Conclusion and future research

This paper proposed a bi-objective mathematical model for a VMI supply chain problem with a single manufacturer and several retailers. The formulation was shown to be a non-linear mathematical model that would maximize both the manufacturer and retailers' profits. The application of the proposed model was specialized in a case with the Cobb–Douglas demand function. Moreover, a fair profit contract between the manufacturer and its retailers was adopted. Our purpose of the fair profit contract was based on the assumption that both the manufacturer and retailers would contribute to determine their optimal decision variables in order to maximize their benefits. Then, the bi-objective problem was formulated as a lexicographic max–min problem in order to find a fair non-dominated solution, a solution with all normalized objectives as equal as possible. In addition, this paper discussed how to replace the original lexicographic max–min problem with the lexicographic maximum problem. Finally, the result obtained using the lexicographic maximum problem was compared to the one of the max–min method in terms of the objective functions and the required CPU time. Based on some sensitivity analyses, we showed that while both methods provide fair non-dominated solutions for all cases, LMM provides better objective function values than the ones of the max–min method with almost equal CPU times. This conclusion was made using paired samples t-tests to compare equalities of the means of the objective functions and CPU time. In other words, computational results showed that while the two methods had no statistical significant difference in the mean CPU time, the proposed method was superior to the max–min method in terms of the two objective functions in 30 test problems. For future work extensions, the followings are recommended:

- Multi-period setting can be considered.
- Variation in the common replenishment cycle time can be assumed.
- Competition among retailers can be modeled.
- A real-world application of the proposed approach is recommended.
- It is worth utilizing simulation tools in the VMI problem with regard to fair profit contract.

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