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# Stochastic Petri Net Modeling, Simulation and Analysis of Public Bicycle Sharing Systems

Karim Labadi, Taha Benarbia, Jean-Pierre Barbot, *Member, IEEE*, Samir Hamaci, and Abdelhafid Omari

***Abstract*—Public Bicycle-Sharing Systems (PBSS) have been appearing in more and more cities around the world in the last few years. Although their apparent success as an alternative form of public transportation mode, there are major challenges confronting the operators while few scientific works are available to support such complex dynamical systems to influence their economic viability and operational efficiency. One of the most crucial factors for the success of a PBS system is its ability to ensure that bicycles are available for pick up and vacant berths available for bicycle drop off at every station. In this paper, we develop an original discrete event approach for modelling and performance evaluation of public bicycle-sharing systems by using Petri nets with time, inhibitor arcs and variable arc weights.**

***Note to Practitioners*—To help designers, planners and managers of public bicycle sharing systems (PBSS), performance and optimization models and decision making tools are unavoidable. The recent momentum of PBSS, as a new mobility option in dense urban areas, has not been followed by an intense research. This paper was particularly motivated by the balancing problem studying the repositioning of bicycles among bicycle stations. In this contribution, we deals with a powerful and original Petri net approach suitable for performance modelling and simulation analysis for control and balancing purposes of such complex dynamical systems. The authors believe that this new area of research has significant promise for the future and to influence economic viability and operational efficiency of these new urban transportation systems.**

***Index Terms*—Bike-sharing systems, control, discrete event systems, modelling, performance evaluation, stochastic Petri nets.**

## I. Overview of Bicycle-Sharing Systems

BICYCLE-SHARING Systems (PBSS) are one of the solutions that currently spreading in developed countries to face many public transportation problems, including traffic congestion, air pollution, global oil prices, and global warming. Historically speaking, the world's first public bicycle initiative is believed to be White Bikes, launched in Amsterdam in 1965. Over the recent years, public bicycle-sharing systems have developed from being interesting experiments in urban mobility to mainstream public transport options in numerous big cities in the world particularly in Europe, the birth-continent of this sharing concept (Vélib' in Paris, Bicing in Barcelona, Call-a-bike in Munich, Oybike in London, etc.).

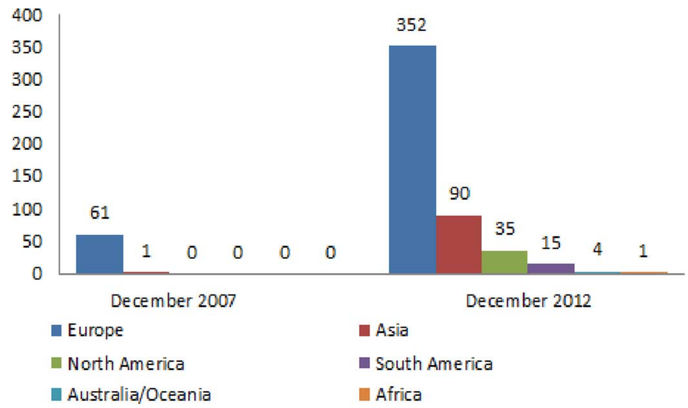


Fig. 1. Global Bike-sharing Services (http://bike-sharing.blogspot.fr/).

Since its inception, bicycle-sharing programs have grown worldwide. In December 2012, there are 493 programs with Europe still leading the pack with 352, followed by Asia (90), North America (35), South America (15), Australia/Oceania (4), and Africa with one (see Fig. 1). A still growing list of cities which provides such green public transportation mode can be found at the Bicycle-sharing world map as shown in Fig. 2.

Nowadays public bicycle-sharing systems are flourishing all over the world. The apparent success of PBSS projects comes with its challenges. For them to become an alternative to more traditional forms of public transport, public bicycle systems need to be carefully designed and efficiently managed. Vogel et al., [49] identify three management and design measures divided into different planning horizons: (*a*) Strategic network design comprising decisions about the location, number and size of stations, (*b*) Tactical incentives for customer based distribution of bicycles and (*c*) Operational provider based repositioning of bicycles. A crucial question for the operational efficiency of a PBS system is its ability to meet the fluctuating demand for bicycles at each station and to provide enough vacant lockers to allow the renters to return the bicycles at their destinations. Indeed, some stations have more demand than others, especially during peak hours. In addition, not surprisingly stations located at the top of hills are chronically empty

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K. Labadi is with the EPMI/ECS-Lab, EPMI, 95092 Cergy-Pontoise Cedex, France (e-mail: k.labadi@epmi.fr).

T. Benarbia is with the IMSI, Institut de maintenance et de Sécurité Industrielle, Université d'Oran, Oran, Algérie (e-mail: taha_05@yahoo.fr).

J.-P. Barbot is with the ECS-Lab, ENSEA, 95014 Cergy-Pontoise Cedex, France (e-mail: barbot@ensea.fr).

S. Hamaci is with the EPMI, 95092 Cergy-Pontoise Cedex, France (e-mail: s.hamaci@epmi.fr).

A. Omari is with the USTO, Université des Sciences et de la Technologie d'Oran, Faculté de Génie Electrique, Département d'Automatique, BP 1505 El-M'naouar, Oran, Algérie (e-mail: o_abdelhafid@hotmail.com).

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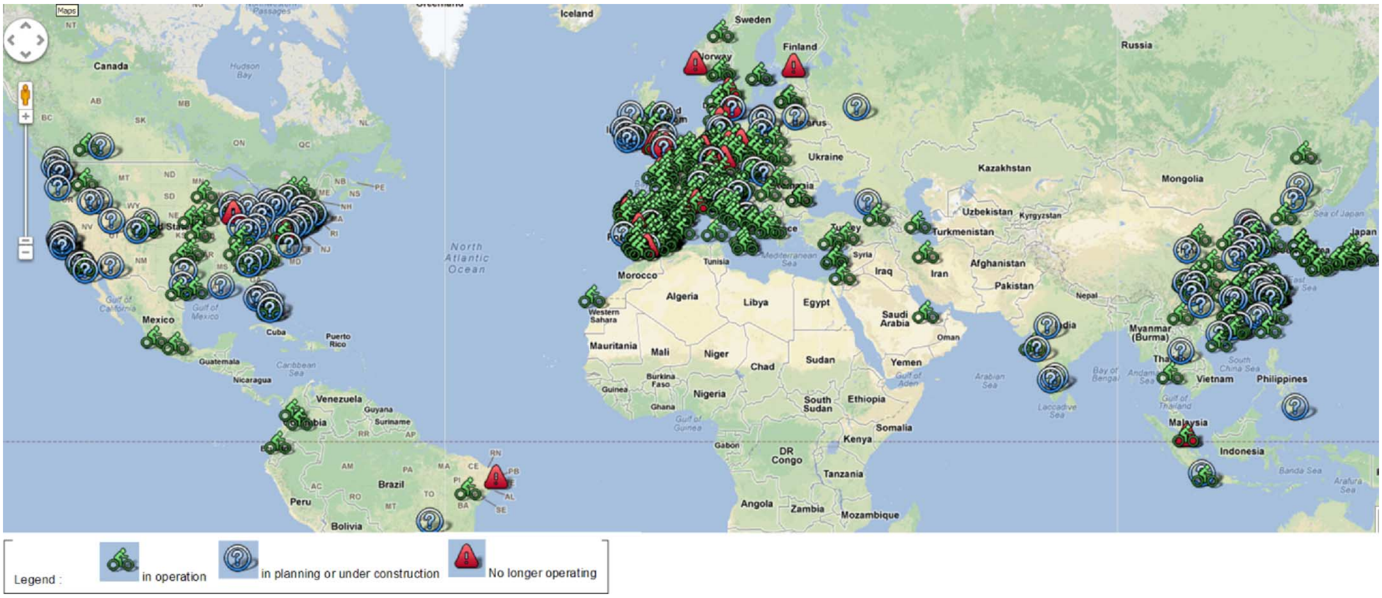


Fig. 2. Bike-sharing world map/date: 15th January 2013 (source http://bike-sharing.blogspot.fr/).

of bicycles, as the customers ride down the hill but do not wish to make the return trip uphill. Bicycles also tend to collect in stations in the city centers and stay there. In some cases, the imbalance is temporary, e.g., high return rate in a suburban train station in the morning and high renting rate in the afternoon. In other cases, the imbalance is persistent, e.g., relatively low return rate in stations located on top of hills. If no action is taken by the service provider they rapidly fill or empty, thus preventing other users from collecting or delivering bicycles. Thus, the system requires constant regulation to constantly re-distribute the bicycles across the stations in the system in order to ensure that users will be able to check out a bicycle to use or find a dock to return the bicycle. The regulation system dispatches a fleet of vehicles which are driven around the stations and move bikes from saturated stations to empty ones.

The rebalancing operation can be carried out in two different modes [49]:

1) *Static rebalancing mode:* the bicycle redistribution operation can be carried out during the night when the usage rate of the PBS system is very low.
2) *Dynamic rebalancing mode:* the bicycle redistribution operation can be carried out during the day when the usage rate of the PBS system is significant.

In this paper, we present an original approach for PBS systems modelling and performance evaluation for control purposes. A modular Petri nets model with variable arc weights (marking dependent weights) is developed as a powerful modelling, analysis and simulation tool of these complex dynamical systems. The resulting dynamic and configurable model is different from the one we have already presented in our previous works [3], [23], [26]. The new Petri net model takes into account new decision parameters and all specific situations that arise during the control and the balancing of the bike stations. To the best of our knowledge, no other work has been undertaken on PBS systems modelling and performance evaluation from a discrete-event point of view. Besides the next section, where a literature review describing some recent existing works, the rest of the paper is dedicated to our original work.

## II. The State of the Art

The recent momentum of public bicycle-sharing systems, as a new mobility option in dense urban areas, has not been followed by an intense research. To help planners and decision makers of such complex transportation systems, modelling, performance analysis and optimization methods and decision making tools are unavoidable.

### *A. Review of Related Works*

Regarding operational issues, the static balancing problem studying the repositioning of bicycles among bicycle stations, where the customer demand is assumed to be negligible, has been recently addressed by some works [4], [40],[8], [16]. Several mathematical formulations of the problem are proposed by Raviv *et al.* [38] and an exact algorithm based on column generation and a suitable pricing algorithm based on dynamic programming are given by Chemla *et al.* [8]. Using estimates of demand for bicycles between stations in each time period, Shu *et al.* [40] develop practical OR model to predict the flow of bicycles to estimate the number of trips supported by the system given an initial allocation of bicycles and the number of docks needed in each station. Also, the viability of a periodic re-distribution of bicycles is examined.

Similarly, from an OR perspective, the bicycle repositioning problem bears great similarities to some other routing problems which have been largely studied in the literature. As an example from this point of view, Forma *et al.* [16] consider the bicycle repositioning problem as a variation of the Pickup and Delivery problem (PDP). Benchimol *et al.* [4] present among other things complexity results, lower bounds, approximation algorithms, and a polynomial algorithm to solve the static balancing problem. In the real situation, the bikes are moving between the stations while the truck tries to keep the stations balanced. Even

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fewer authors deal with this dynamic case. In particular Contrado *et al.* [11] propose a mathematical formulation of the dynamic problem on a space-time network. Some works propose a fixed repositioning time interval [35], [39] or assume redistribution vehicles moving at random from saturated stations to empty ones [17]. A modular soft computing based method for dynamic bike redistribution process is presented by Caggiani and Ottomanelli [7]. In general, the dynamic balancing problem is investigated without focusing on redistribution patterns and time periods [50].

Now, about strategic issues, Dell'Olio *et al.* [12] present a complete methodology for the design and implementation of bicycle sharing systems based on demand estimates considering the stations and the fares. Lin *et al.* [28], [29] address the strategic problem of finding optimal stations and the necessary bicycle lanes using mathematical programming techniques. The problem is formulated as a hub location inventory model. Besides OR approaches developed, particularly by using mathematical programming techniques to support decision making in the design and management of bike sharing systems, Data Mining techniques receive attention in academia as well as in practice. Data Mining is particularly suitable to analyze and to predict the dynamics of bike sharing systems. By exploring and analyzing the temporal and geographic human mobility data in an urban area using the amount of available bicycles in the stations of a PBS system, statistical and prediction models can be developed for tactical and operational management of such systems. A case study, presented by Vogel *et al.* [49], shows how Data Mining applied to operational data offers insight into typical usage patterns of bike-sharing systems and is used to forecast bike demand with the aim of supporting and improving strategic and operational planning. The design and management of PBS systems need to have detailed information about the journeys being made in urban areas and the users' characteristics. Barcelona [18], Lyon [5] or London [27] have been analyzed in this sense.

### *B. The Application of Petri Nets*

As noted by Vogel *et al.* [49], although extensive analysis of bicycle data or customer surveys can be applied to predict future bicycle demand at stations, the demand still has to be considered stochastic and not deterministic. Moreover, various points in time have to be incorporated in a suitable mathematical optimization models. Such a stochastic and dynamic model can be computational intractable. In addition, customer behavior cannot be modeled in these mathematical optimization models. Otherwise, public bicycle sharing systems may well be described through a discrete event dynamic model. They can be associated with a space of discrete state (e.g., number of available bicycles in a bike station), and changes of state involved by some discrete events (e.g., return (or departure) of bikes to (from) a bike station by user) over time. In addition to their self-service mode, they are stochastic complex systems and mainly characterized by parallelism, synchronization and concurrency. Among the formalisms used to model the dynamic systems, Petri nets (PN) are widely used in a number of different disciplines including manufacturing, communication, production and logistic systems [47], [48], [51], [42], [34], [25], [24], [9]. Compared to other formalisms, there are several reasons for using Petri nets for discrete event systems modelling and analysis. As graphical and mathematical tools, they provide a uniform environment for modeling and behavioral proprieties and performances analysis, as well as for design of discrete event systems. PN allow for the performance evaluation of the modeled systems. Both deterministic and stochastic measures can be evaluated by using a broad class of PN models incorporating in their definitions deterministic and and/or probabilistic time functions [51].

However, although PN have been widely used in various domains, they played a relatively minor role in modeling and analysis of urban transportation systems. According to some existing works, the modelling of the systems by using PN formalisms can be considered from either a discrete and/or a continuous point of view. Continuous PN for the macroscopic and microscopic traffic flow control are used in [44], [20], while hybrid PN are used in [14] to provide a valuable model of urban networks of signalized intersections. Recently, batch PN with controllable batch speed are used [13] to study a portion of the A12 highway in the Netherlands. From a discrete point of view, generalized stochastic PN are used [6], [1] for modelling and planning of public transportation systems. The two complementary tools, PN and $(\max, +)$ algebra, have been used [36], [37] to deal with the modelling and the performance evaluation of a public transportation system. For the modelling of passenger flows at a transport interchange, as shown in [43], colored PN are able to incorporate some specific parameters and data in the model such as the variation of walking speeds between passengers and the restricted capacity of features of the interchange infrastructure. These are a few works that demonstrate the potential of PN as a tool for modelling and performance analysis of urban transportation systems. However, the applications are generally limited to intersection traffic control [15], [46], [19], [30], [44], [20] and to some studies dealing with the modelling and performance evaluation of urban transportation systems [36], [37], [43]. Our contribution on the field of public bicycle sharing systems is the first one in the Petri nets literature. Unlike our recent works [3], [23], [26], no other studies has been undertaken on the dynamics modelling and performance evaluation of such dynamical systems.

## III. Description of Stochastic Timed Petri Nets With Variable ARC Weights

### *A. Basic Definition of a Petri Net*

In its basic form, a Petri net is a directed bipartite graph with one set of vertices called *places* (drawn as circles) and the other called *transitions* (drawn as bars). Places may contain *tokens* which are drawn as dots. Places and transitions are connected by directed arcs or inhibitor arcs (drawn with a circled head). Arcs may be labeled with integer numbers denoting their weights. A transition is said to be *enabled*, if all of its input places contain at least as many tokens as the weight of the corresponding input arc and all of its inhibitor places contain less tokens than the weight of the corresponding inhibitor arc. A transition *fires* by removing from each input place as many tokens as the weight of the corresponding input arc, and by adding to each output place

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as many tokens as the weight of the corresponding output arc. Formally:

$$\mathrm{PN} = (P, T, Pre, Post, Inhib, M_0), \text{ where :}$$

$\mathrm{P} = \{\mathrm{p}_1, \mathrm{p}_2, \dots, \mathrm{p}_n\}$ is a finite and non-empty set of places; $\mathrm{T} = \{\mathrm{t}_1, \mathrm{t}_2, \dots, \mathrm{t}_\mathrm{m}\}$ is a finite and non-empty set of transitions; Pre: $(\mathrm{P} \times \mathrm{T}) \rightarrow \mathrm{N}$ is an input function that defines directed weighted arcs from places to transitions, where $N$ is a set of nonnegative integers; Post: $(\mathrm{P} \times \mathrm{T}) \rightarrow \mathrm{N}$ is an output function which defines directed weighted arcs from transitions to places; *Inhib*: $(P \times T) \rightarrow N$ is an inhibition function that defines inhibitor weighted arcs (circle-headed arcs), and $M_0$ represents the initial marking (initial distribution of the tokens in the places). The set of input places and the set of output places of a transition $t_j$ are denoted by ${}^\bullet t_j$ and $t_j^\bullet$, respectively, and ${}^\circ t_j$ represents the set of places connected with $t_j$ by inhibitor arcs. The weights of an input arc and of an output arc are respectively denoted by $Pre(p_i, t_j)$ and $Post(p_i, t_j)$, and the weights of an inhibitor arc is denoted by $Inhib(p_i, t_j)$.

- A transition $t_j$ is said to be enabled at a marking $M$ if and only if:

$$\forall p_i \in {}^\bullet t_j, M(p_i) \geq \mathrm{Pre}(p_i, t_j) \quad (1)$$
$$\forall p_i \in {}^\circ t_j, M(p_i) < Inhib(p_i, t_j). \quad (2)$$

- A firing of an enabled transition $t_j$ results in a new marking $M'$:

$$\forall p_i \in P, M'(p_i) = M(p_i) - \mathrm{Pre}(p_i, t_j) + Post(p_i, t_j). \quad (3)$$

### *B. Time and Variable Weights*

The classical definition of a PN assumes that the weights of the input and output arcs are constant but, at times, the reality being modeled behave differently: a given event might require removing from a place, or adding to a place, a number of tokens which varies according to the marking of the net. The concept of marking dependent weights is used in our previous works [24],[9] for modelling of control and monitoring operations of inventory control systems and supply chain networks. Similarly, by incorporating basic Petri nets with the inhibitor arcs, and marking dependent weights, control and rebalancing operations of bike sharing systems can be easily described in the model (Section IV). Therefore, we consider that for any arc $(i, j)$, its weight $w(i, j)$, assumed to take a positive value, is now defined as an affine function of the marking $M$ with integer coefficients $\alpha_i$.

$$w(i, j) = \alpha_0 + \alpha_1 M(p_1) + \alpha_2 M(p_2) + \cdots + \alpha_n M(p_n). \quad (4)$$

As an example, we consider an inventory system with continuous review $(\mathrm{s}, \mathrm{S})$ policy with $\mathrm{s} = 5$ and $\mathrm{S} = 20$. For this system, when the inventory position (p2) drops below a given reorder point 5, an order with quantity $20 - \mathrm{M(p2)}$ will be placed to raise the inventory position to the fixed order-to-level 20. As shown in Fig. 3, the different operations of the system are modeled by using a set of transitions: generation of replenishment orders $(t3)$; inventory replenishment $(t2)$; and order delivery $(t1)$. In the model, the weights of the arcs $(t3, p2)$, $(t3, p3)$ are variable and depend on the current marking of the model. According to the current marking, $M_i = (2, 2, 0)^T$, shown in Fig. 3, the transition $t_3$ is enabled since the following condition is satisfied:

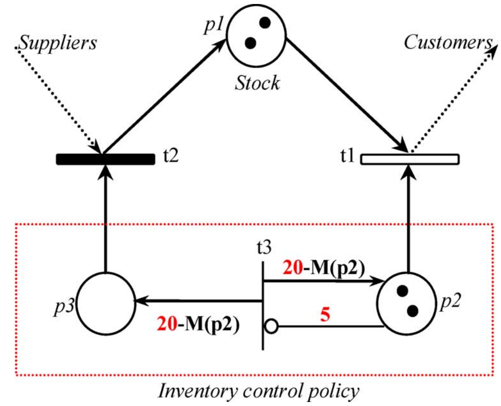


Fig. 3. Petri net with arc weights depending on its marking.

$$M(p_2) = 2 < Inhib(p_2, t_3) = 5.$$

After the firing of $t_3$, $20 - M(p_2) = 20 - 2 = 18$ tokens are added into the places $p_2$ and $p_3$. In other words, the firing of the transition $t_3$ from the initial marking $M_i$ leads to a new marking $M_f = (2, 20, 18)^T$.

The original Petri nets do not convey any notion of time. With this class of nets, it is only possible to describe the logical structure of the modeled system, but not its time evolution. Naturally, the timing concept is necessary to the temporal performance evaluation of many systems including bike-sharing systems considered in this paper. Since transitions are often used to model events or activities (e.g., control, order, transportation, etc.), transition enabling duration corresponds to activity execution and transition firing corresponds to activity completion. Hence, a timing concept is included into the PN formalism. Generally, two types of transitions can be used to model a discrete event system: (1) immediate transitions, with zero firing delay and (2) timed transitions with deterministic or stochastic firing delay. In graphical representation, *immediate, deterministic, and stochastic* transitions are represented by thin bars, filled rectangles, and empty rectangles, respectively. For readers not familiar with PN formalisms, and their several applications and associated analysis techniques [51], more details dealing with stochastic timed Petri nets such as SPN, GSPN, and DSPN models can be found in [10], [31]–[33].

## IV. Petri Net Modelling Approach of a PBS System

Consider a PBS system with $N$ stations denoted by $S = \{S_1, S_2, \dots, S_N\}$. Each station $S_i \in S$ is equipped with $C_i$ bike stands (the capacity of a station $S_i$). The system requires a constant control which consists in transporting bicycles from stations having excess of bicycles to stations that may run out of bicycles soon. In the general way, the main objective of the control system, performed by using redistribution vehicles, is to maintain $R_i$ (*reorder point*) bicycles per station $\mathrm{S}_i$ to ensure bicycles are available for pick up and thus $(C_i - R_i)$ vacant berths available for bicycle drop off at every station.

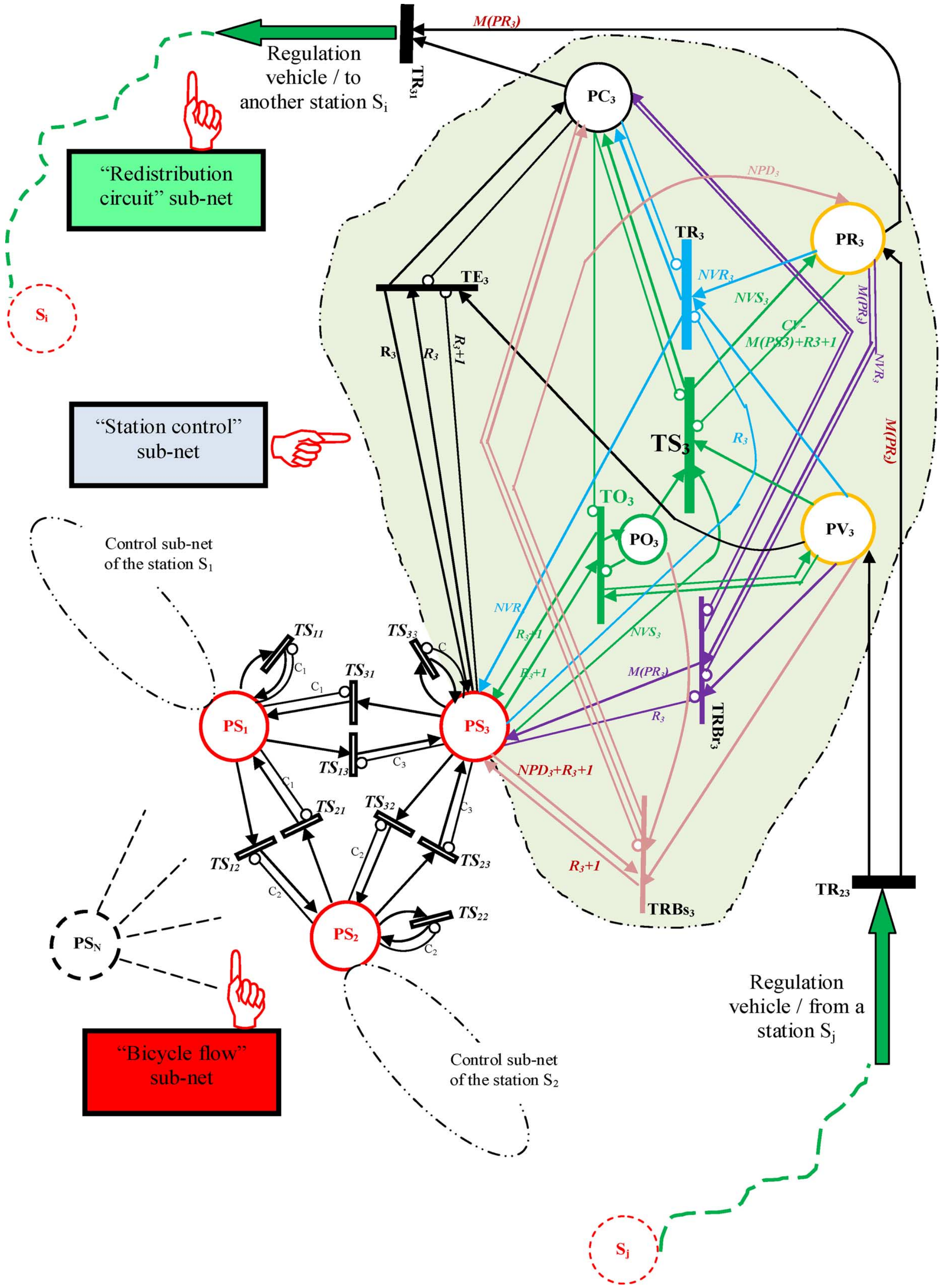


Fig. 4. Petri net model of a bike sharing system.

Compared to our previous works, the model presented in Fig. 4 takes into account new decision parameters of the network, specified in Table II, and all specific situations that arise during the control and the balancing of the bike stations, summarized in Table IV. Fig. 4 and Tables I – III allow readers to quickly gain an understanding of the Petri net model.

A closer look at this model shows three subnets (modules) representing three different functions indicated as follows: (*1*) the "station control" subnet; (*2*) the "bicycle flow" subnet; and (*3*) the "redistribution circuit" subnet. The main function of each subnet is described in Section V. Thanks to the modularity of the developed model, the Petri net representing a PBS system with N stations contains:

$$\begin{cases} 5 * N & \text{Places} \\ N^2 + 7 * N & \text{Transitions} \\ 2 * N^2 + 32 * N & \text{Classic arcs} \\ N^2 + 12 * N & \text{Inhibitor arcs .} \end{cases}$$

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TABLE I
INTERPRETATION OF PLACES OF THE MODEL

| | |
|---|---|
| $PS_i$ | Represents a station $S_i$. Its marking $M(PS_i)$ corresponds to the number of bicycles available in the station $S_i$. |
| $PR_i$ | Represents a redistribution vehicle. Its marking, $M(PR_i)$, corresponds to the number of bicycles available in its trailer. |
| $PO_i$ | Specify whether the number of bicycles in a station $S_i$ is greater than the reorder point $R_i$. It is the case when $M(PO_i) = 1$. |
| $PC_i$ | Specify the end of the balancing of a station $S_i$. $M(PC_i) = 1$ means that the balancing of $S_i$ is completed and the redistribution vehicle is liberated to go to the next station. |
| $PV_i$ | Used to indicate the arrival of a redistribution vehicle at a station $S_i$ for a balancing operation. It is the case when $M(PV_i) =1$. |

TABLE II
INTERPRETATION OF PARAMETERS OF THE MODEL

| | |
|---|---|
| $C_i$ | Capacity of each station $S_i$. $C_i$ corresponds to the maximal number of bike stands in a station $S_i$. |
| $R_i$ | Reorder point of a station $S_i$. The decision to adding or removing bicycles into (from) the station is performed according to $R_i$. |
| $CV_i$ | Capacity of the regulation vehicle (maximal number of bicycles which can be transported by the redistribution vehicle) |
| $NPD_i$ | Number of available places in the regulation vehicle. Formally: $NPD_i = CV_i - M(PR_i)$. |
| $NVR_i$ | Number of bicycles to be added into a station $S_i$. Formally: $NVR_i = R_i - M(PS_i)$ |
| $NVS_i$ | Number of bicycles to be removed from a station $S_i$. Formally: $NVS_i = M(PS_i) - R_i$ |

TABLE III
INTERPRETATION OF TRANSITIONS OF THE MODEL

| | |
|---|---|
| $TS_{ij}$ | Used to model the bicycle flow between the different stations (displacement of a bicycle from a station $S_i$ to another $S_j$. |
| $TR_{ij}$ | Used to model the displacement of the redistribution vehicle from a station $S_i$ to the next station $S_j$ ($i \neq j$). |
| $TR_i$ | Test and add (if necessary) bicycles into a station $S_i$ if the number of bicycles available in this station is less than the reorder point $R_i$. The number of the bicycles to be added into the station is: $NVR_i = R_i - M(PS_i)$. |
| $TO_i$ | Test if the number of bicycles available in a station $S_i$ is greater than the reorder point $R_i$. |
| $TS_i$ | Test and remove bicycles from a station $S_i$ if the number of bicycles available in this station is greater than the reorder point $R_i$. The number of the bicycles to be removed from the station is: $NVS_i = M(PS_i) - R_i$. |
| $TE_i$ | Test if the number of bicycles available in a station $S_i$ is equal to the reorder point $R_i$. |
| $TRBr_i$ | Test if the number of bicycles $NVR_i$ to be added into the station $S_i$ is greater than the number of available bicycles $M(PR_i)$ in the regulation vehicle. |
| $TRBs_i$ | Test if the number of bicycles $NVS_i$ to be removed from the station $S_i$ is less than the number of available places $NPD_i = NPD_i = CV_i - M(PR_i)$ in the regulation vehicle. |

More clearly, we need to:

- $(5*\mathrm{N})$ places denoted by $\mathrm{PS}_i, \mathrm{PC}_i, \mathrm{PR}_i, \mathrm{PV}_i, \mathrm{PO}_i$ ($\mathrm{i} = 1$ to N) described in Table I.
- $(\mathrm{N}^2+7*\mathrm{N})$ transitions described in Table III. They include:
  - $(\mathrm{N}^2)$ transitions denoted by $\mathrm{TS}_{ij}$ and $\mathrm{TS}_{\mathrm{ii}}$ to represent the "bicycle flow" between the stations;
  - $(6 * \mathrm{N})$ transitions denoted by $\mathrm{TR}_i$, $\mathrm{TS}_i$, $\mathrm{TE}_i$, $\mathrm{TO}_i$, $\mathrm{TRBr}_i$, and $\mathrm{TRBs}_i$ to represent the different control operations of each station;
  - (N) transitions denoted by $\mathrm{TR}_{ij}$ to represent the displacement of the redistribution vehicle from a station $\mathrm{S}_i$ to another $\mathrm{S}_j$.

TABLE IV
FORMAL DEFINITION OF BALANCING OPERATIONS

| Operational Functions | Transition | Main condition (Enabling condition of the transition) | Action (New marking of the station after the firing of the transition) |
|---|---|---|---|
| "Add bicycles to a station" operation | $TR_i$ | $M(PS_i) < R_i$ | $M'(PS_i) = M(PS_i) + NVR_i = R_i$ |
| "Remove bicycles from a station" operation | $TS_i$ | $M(PS_i) > R_i$ | $M'(PS_i) = M(PS_i) - NVS_i = R_i$ |
| "No action operation" | $TE_i$ | $M(PS_i) = R_i$ | $M'(PS_i) = M(PS_i) = R_i$ |
| "Add minimal bicycles to a station" operation | $TRBr_i$ | $NVR_i > M(PR_i)$ | $M'(PS_i) = M(PS_i) + Min\left[NVR_i, M(PR_i)\right]$ |
| "Remove minimal bicycles from a station" operation | $TRBs_i$ | $NVS_i > NPD_i$ | $M'(PS_i) = M(PS_i) - Min\left[NVS_i, NPD_i\right]$ |

- $(2*\mathrm{N}^2+32*\mathrm{N})$ classic and $(\mathrm{N}^2+12*\mathrm{N})$ inhibitor arcs where:
  - Classic arcs include $(2*\mathrm{N}^2)$, $(28*\mathrm{N})$ and $(4*\mathrm{N})$ arcs for the "bicycle flow", "station control" and "redistribution circuit" subnets, respectively;
  - $(\mathrm{N}^2)$ inhibitor arcs are used between places $\mathrm{PS}_i$ and transitions $\mathrm{TS}_{ij}$ to respect the capacity of each station ("bicycle flow" subnet) and $(12*\mathrm{N})$ others are needed to specify some control operations performed by the "station control" subnets.

Hereafter a formal description of the function of each subnet and the dynamic behavior of the Petri net model are developed.

## V. MODULAR DESCRIPTION OF THE MODEL

### A. The "Bicycle Flow" Subnet

A public bicycle system is a bank of bicycles which are continuously used by users to travel from a given station to another. Thus, each bicycle of the system can be taken out from any station and returned to the same or any other station, provided that there is an available locking berth. The subnet representing the displacements of the bicycles (bicycle flow) between the different stations of the system is represented by the places $PS_i$, the transitions $TS_{ij}$ and all of the corresponding arcs as shown in Fig. 5.

- As shown in the subnet, each station $S_i$ of the PBS system is modeled by using a place denoted by $PS_i$. The current available bicycles in the station $S_i$ is given by the current marking $M(PS_i)$ which corresponds to the number of available tokens in the place $PS_i$.
- The bicycle flow is represented, in terms of the Petri net formalism, by the multiple token displacements from any place to the same or any other place by firing transitions denoted by $TS_{ij}$ (possibly $TS_{\mathrm{ii}}$) connecting the different places of the subnet.

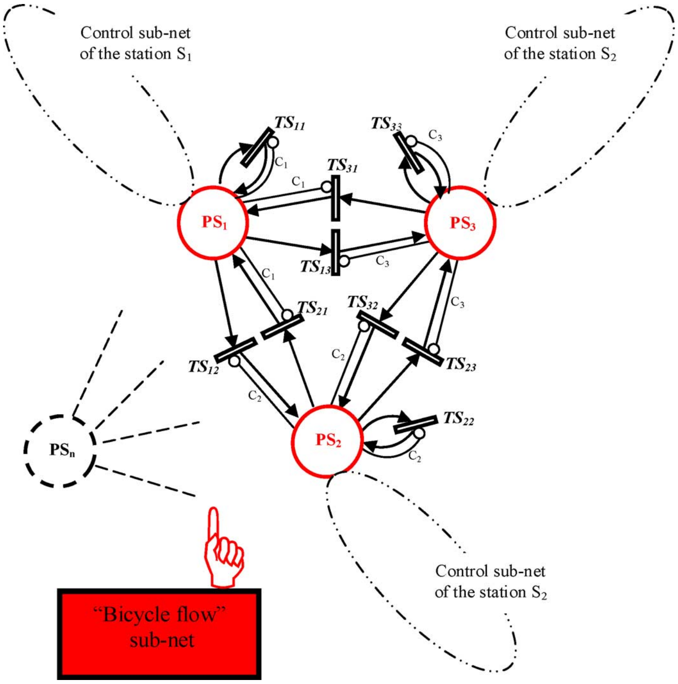


Fig. 5. The "bicycle flow" subnet.

- Each station $S_i$ is equipped with $C_i$ bicycle stands. It is the capacity of each place $PS_i$ in the subnet. The parameter $C_i$ represents the weight of the inhibitor arcs connecting the places $PS_i$ with the transition $TS_{ij}$. The inhibitor arcs are used in order to respect the condition that a bicycle can be returned (by users) to a given station if and only if there is at least one empty bicycle stand.

In real situation, the delays associated to the displacements of users; the frequentation rates of each station by users; . . . are not deterministic parameters. Thus, according to the stochastic behavior of the bicycles between the different stations, the transitions $TS_{ij}$ of the "bicycle flow" subnet are considered to be subject to stochastic distributions.

Similar to recent studies using real data[41], [40], we consider exponential distributions by assuming that bicycle flow process is a Poisson process.

### B. The "Station Control" Subnet

The subnet representing the control function of each station $S_i$ is represented in Fig. 6. As shown in the model represented in Fig. 4, the considered subnet is duplicated for each station $S_i$. We recall that, the main objective of the control function is to rebalance bicycles between stations that are emptying out and those that are filling.

The control function of the system is performed by using five places denoted by $PS_i$, $PC_i$, $PO_i$, $PV_i$, $PR_i$, six transitions denoted by $TE_i$ $TR_i$, $TS_i$, $TO_i$, $TRBr_i$, and $TRBs_i$ and all of the corresponding arcs. All of these components are interpreted in Tables I and III (Section IV).

When the redistribution vehicle arrives at a bike station $S_i$, indicated by $M(PV_i) = 1$, the balancing operation of the station is performed according to the current number of bicycles

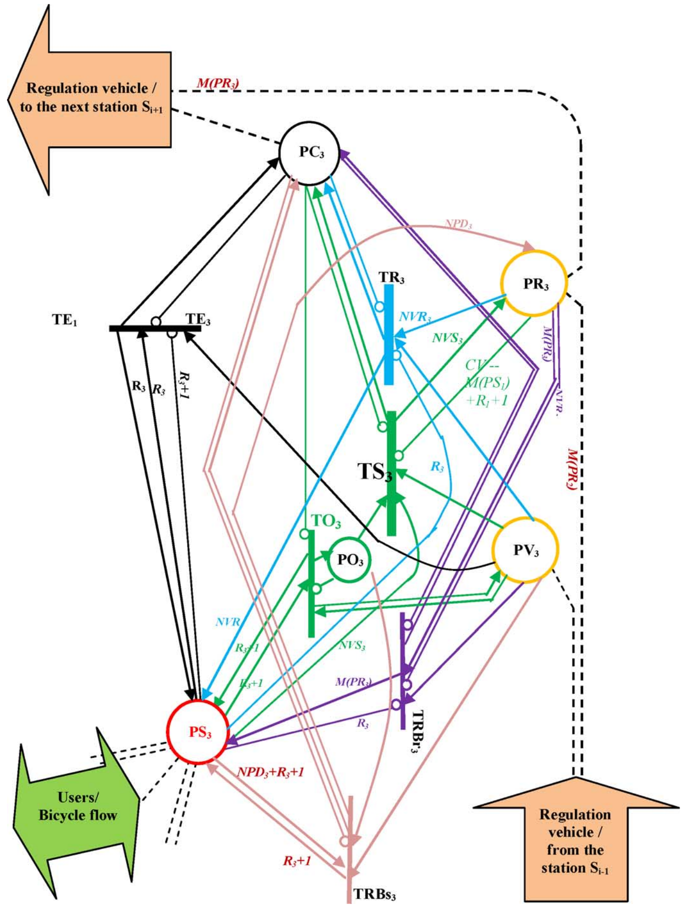


Fig. 6. The "station control" subnet of a station $S_i$.

available in this station, $M(PS_i)$, and in the regulation vehicle $M(PR_i)$. The different decisions to be made, by using the transitions cited above, are summarized in Table IV and then formally described thereafter.

Before a formal description of the dynamic behavior of the "station control" subnet, the temporal specifications of its transitions are detailed. As noted above, six transitions, $TE_i$, $TR_i$, $TS_i$, $TO_i$, $TRBs_i$ and $TRBr_i$ are used to model specific control operations (discrete events) or some logical conditions of the system. The designation of each transition is specified in Table III. According to the dynamic behavior of the system:

- The transitions $TE_i$ and $TO_i$ may be considered as immediate transitions (with zero delay) since they are used to verify some logical conditions of the system. Indeed, we assume that to test and to indicate that the current number of available bicycles in a station $S_i$ is equal to the reorder point $R_i$ (i.e., $M(PS_i) = R_i$) require zero delay.
- On the other hand, the transitions $TS_i$, $TR_i$, $TRBS_i$ and $TRBr_i$ may be considered as stochastic transitions due to the nature of the actions (operations) that they represent. Naturally, "To remove" (or "To add") bicycles from (to) a station $S_i$ require times to be accomplished. In real situation, this execution delay is not deterministic. It depends on the (stochastic) number of bicycles to be added to (or removed from) the system and also on the swiftness of the controllers.

- Finally, it is important to note that the "control station" subnet is without firing conflicts. In fact, the enabling conditions of the transitions cannot be satisfied at the same time. For example, the main enabling conditions of the transitions of $TS_i, TR_i$ and $TE_i$, are respectively the equations (*i*) $M(PS_i) > R_i$; (*ii*) $M(PS_i) < R_i$; (*iii*) $M(PS_i) = R_i$ which cannot be satisfied at the same time.

Now, in line with Table IV, five cases are developed in the following to describe in a formal way the different repositioning activities which can be performed by the subnet representing the control function of the system.

In all cases, we consider that: (a) The reorder regulation point $R_3$ is fixed to 10; (b) the capacity of the station $S_3$, $C_3$, is fixed to 15; (c) the capacity of the regulation vehicle, $PV_3$, is fixed to 20. Initially, in the different cases, we consider that (d) $M(PV_3) = 1$ indicating that the regulation vehicle has arrived at the station $S_3$; and (e) $M(PC_3) = 0$ indicating that the control of the station has not yet occurred. In addition to other enabling conditions of the transitions in each case, (d) and (e) are common to all the different transitions which represent the different repositioning activities. For each case, it will be indicated the number of available bicycles in the regulation vehicle, i.e., $M(PR_3)$, and the number of available bicycles in the station $S_3$, $M(PS_3)$, when this station is under the control ($M(PV_3) = 1$).

• ***Case 1—*** *Addition of $NVR_i$ bicycles to a station $S_i$ operation when the number of available bicycles in the regulation vehicle is sufficient: $NVR_i \leq M(PR_i)$*

In this case, consider that there are 7 available bicycles in the station $S_3$ (*i.e.,* $M(PS_3) = 7$) and 15 available bicycles in the regulation vehicle (*i.e.,* $M(PR_3) = 15$). Here, the number of bicycles to be added to the station is:

$$NVR_3 = R_3 - M(PS_3) = 10 - 7 = 3.$$

This operation is possible since there are $M(PR_3) = 15$ available bicycles in the regulation vehicle. It will be performed by the transition $TR_3$ as follows.

$$M(PS_3) = 7 < R_3 = 10$$
$$M(PR_3) = 15 \geq NVR_3 = 10 - 7 = 3.$$

According to the above equations, the number of available bicycles in the station $S_3$ is less than the reorder point $R_3$ and there are enough bicycles in the vehicle to balance the station at the fixed level $R_3$. Consequently, only the transition $TR_3$ is enabled and after its firing, $NVR_3 = M(PS_3) - R_3 = 10 - 7 = 3$ bicycles will be removed from the vehicle and then added to the station $S_3$:

$$M'(PR_3) = M(PR_3) - NVR_3 = 15 - (10 - 7) = 12$$
$$M'(PS_3) = M(PS_3) + NVR_3 = 7 + (10 - 7) = 10.$$

At the same time, after the firing of the transition $TR_3$, the end of the repositioning activities and the control of the station $S_3$, will be indicated with the new marking $M'(PC_3) = 1$, $M'(PV_3) = 0$. The vehicle leaves the station to go to another station.

• ***Case 2—*** *Remove $NVS_i$ bicycles from a station $S_i$ operation when the number of available places in the regulation vehicle: $NPD_i \geq NVS_i$*

In this case, consider that there are 15 available bicycles in the station $S_3$ (*i.e.,* $M(PS_3) = 15$) and 12 available bicycles in the regulation vehicle (*i.e.,* $M(PR_3) = 12$). Here, the number $NVS_3$ of bicycles to be removed from the station and the number $NPD_3$ of available places in the vehicle are given as follows:

$$NVS_3 = M(PS_3) - R_3 = 15 - 10 = 5.$$
$$NPD_3 = CV_3 - M(PR_3) = 20 - 12 = 8 > 5.$$

In this case, the repositioning operation will be performed by the transitions $TO_3$ and $TS_3$. Firstly, the immediate transition $TO_3$ is enabled since:

$$M(PS_3) = 15 \geq R_3 + 1 = 11 \ (i.e., M(PS_3) > 10).$$

Then, after the firing of the transition $TO_i$, a token will be placed in the place $PO_3$ indicating that the current number of the available bicycles in the station $S_3$ is greater than the reorder regulation point $R_3$ ($M(PS_3) > 10$). With this indication, the transition $TS_3$ is systematically enabled, and its firing removes the excess of bicycles, $NVS_3$, from this station. The new markings of the concerned places representing the station and the vehicle are as follows:

$$M'(PS_3) = M(PS_3) - NVS_3 = 15 - (15 - 10) = 10$$
$$M'(PR_3) = M(PR_3) + NVS_3 = 12 + (15 - 10) = 17.$$

Similarly to the case 1, after the firing of the transition $TS_3$, $M'(PC_3) = 1$, $M'(PV_3) = 0$ indicating the end of the repositioning activities and the control of the station $S_3$.

• ***Case 3—*** *No repositioning operation when the number of the available bicycles in the controlled station is equal to $R_i$: $M(PS_i) = R_i$*

Contrarily to the two previous actions corresponding "*to remove $NVS_i$*" (or "*to add $NVR_i$*") bicycles from (to) the station $S_i$, the "*no repositioning operation*" will be performed when the current number of bicycles in the controlled station $S_i$ is equal to the reorder point $R_i$. Testing that $M(PS_i) = R_i$ is made by the transition $TE_i$ with its corresponding arcs connecting the places $PS_i$ and $PC_i$ with this transition. Naturally, the firing of $TE_i$ will not change the marking of the place $PS_i$ which represents the number of bicycles in the controlled station $S_i$. Similarly to the two previous functions, after the firing of $TE_i$, $M'(PC_3) = 1$, $M'(PV_3) = 0$ indicating the end of the control operation of the station $S_i$ and then the vehicle is moving to another station $S_j$ by firing a transition $TR_{ij}$.

***Case 4—*** *Min-Addition of bicycles to a station $S_i$ operation when the number of available bicycles in the regulation vehicle is not sufficient: $NVR_i > M(PR_i)$*

Consider that when the regulation vehicle arrives at the station $S_3$, there are two available bicycles in this station (*i.e.,* $M(PS3) = 2$) and five available bicycles in the regulation vehicle (*i.e.,* $M(PR_3) = 5$). Normally, the number of bicycles to be added to the station is:

$$NVR_3 = R_3 - M(PS_3) = 10 - 2 = 8.$$

But, this action is not possible in this case, because the number of available bicycles in the vehicle is not sufficient. In other words, we have:

$$NVR_3 = 8 > M(PR_3) = 5.$$

Contrary to the case 1, the transition $TR_3$ is not enabled. However, the transition $TRBr_3$ becomes enabled since all the conditions are satisfied. Particularly, we have:

$$M(PS_3) = 2 < R_3 = 10.$$
$$M(PR_3) = 5 < NVR_3 = (R_3 - M(PS_3)) = 8.$$

After the firing of the transition $TRBr_3$, the rest of available bicycles (*i.e.,* $M(PR_3)$) in the vehicle will be deposited in the station $S_3$. Formally, the new markings are as follows:

$$M'(PS_3) = M(PS_3) - M(PR_3) = 2 + 5 = 7.$$
$$M'(PR_3) = M(PR_3) - M(PR_3) = 5 - 5 = 0.$$

Similarly to the previous cases, after the firing of the transition $TRBr_3$, $M'(PC_3) = 1$, $M'(PV_3) = 0$ indicating the end of the repositioning activities and the control of the station $S_3$. In this case, the vehicle leaves the station with an empty trailer.

***Case 5***— *Min-Remove bicycles from a station $S_i$ operation when the number of available places in the regulation vehicle is not sufficient: $NPD_i < NVS_i$*

Now, as the final case, consider that there are 15 available bicycles in the station $S_3$ (*i.e.,* $M(PS_3) = 15$) while the regulation vehicle arrives at this station with 18 bicycles in its trailer (*i.e.,* $M(PR_3) = 18$). In principle, the number of bicycles to be removed from the station is:

$$NVS_3 = M(PS_3) - R_3 = 15 - 10 = 5.$$

But, the number of available places in the vehicle is not sufficient since: $NPD_3 = CV_3 - M(PR_3) = 20 - 18 = 2$.

Compared to the case 2, the number of the bicycles which can be removed from the station $S_3$ is limited only to $NPD_3 = 2$.

Firstly, according to the subnet model, by firing the immediate transition $TO_i$, a token will be placed in the place $PO_3$ indicating that the current number of available bicycles in the station $S_3$ is greater than the reorder regulation point $R_3$ ($M(PS_3) > 10$). With this indication, the transition $TRBs_3$ is systematically enabled since all the conditions are satisfied. Particularly, we have:

$$M(PO_3) = 1 \geq 1.$$
$$M(PS_3) = 15 \geq NPD_3 + R_3 + 1 = 13.$$

Precisely, The first equation indicates that $M(PS_3) > R_3$ and the second equation signify exactly that $NPD_3 < NVS_3$.

After the firing of the transition $TRBs_3$, only two bicycles ($NPD_3$) will be removed from the station $S_3$ and deposited in the vehicle. According to the subnet, the new markings of the concerned places are as follows:

$$M'(PS_3) = M(PS_3) - [NPD_3 + R_3 + 1] - (R_3 + 1)$$
$$M'(PS_3) = M(PS_3) - NPD_3 = 15 - 2 = 13.$$
$$M'(PR_3) = M(PR_3) + NPD_3 = 18 + 2 = 20.$$

Obviously, $M'(PC_3) = 1$, $M'(PV_3) = 0$ indicating the end of the repositioning activities and the control of the station $S_3$. In this case, the vehicle leaves the station with a full trailer.

### *C. The "Redistribution Circuit" Subnet*

As shown previously, the PBS system requires constant control which consists in transporting bicycles from stations having excess of bicycles to stations that may run out of bicycles soon. In our model, we consider that the vehicle(s) used to rebalance bicycles between stations visits successively the stations $S_1, S_2, \ldots, S_N$. The subnet representing this function is described in the following (see Fig. 4).

- As can be seen in Fig. 4, the places denoted by $PR_i$, $PV_i$ ($i = 1, 2, \ldots, N$), the transitions denoted by $TR_{ij}$ ($i \neq j$ and $i, j = 1, 2, \ldots, N$) and all of the corresponding arcs form a closed path. The resulting subnet represents a circuit which the redistribution vehicle follows in order to visit and to control successively the different stations of the network.
- When the redistribution vehicle arrives at a given station $S_i$, the marking of the place $PV_i$ is equal to 1 (i.e., $M(PV_i) = 1$), and the marking of the place $PR_i$ (i.e., $M(PR_i)$) indicates the current available bicycles in the vehicle. The displacement of the vehicle from a station $S_i$ to another station $S_j$ is modeled by the transition $TR_{ij}$.
- Obviously, the circuit is connected to the control subnet of the system. Indeed, the places $PR_i$ and $PV_i$ are connected to the transitions $TE_i$, $TR_i$, $TS_i$, $TO_i$, $TRBr_i$, and $TRBs_i$ in order to execute the control function of the station $S_i$. The connection is made by the corresponding arcs. Now, when the control of a given station $S_i$ is finished, which will be indicated by $M(PC_i) = 1$, the redistribution vehicle leaves the station $S_i$ and goes to the next section $S_j$ by firing the transition $TR_{ij}$.

In the practice, the displacement delay from a station $S_i$ to another station $S_j$ is not deterministic. Thus, the transitions $TR_{ij}$ must be considered as stochastic transitions in the Petri model. As illustrated in Fig. 7, for a large bicycle-sharing system with $N$ stations implemented in a big city, the regulation of the stations can be performed by using several redistribution vehicles distributed over different districts of the city. In our Petri net model, it is easy to represent this situation with different circuit for each redistribution vehicle.

As illustrated in Fig. 7, a place $P_D$ (regulation center) and two transitions $T_R$ (regulation start) and $T_D$ (regulation end) can be integrated in the redistribution circuit allowing modeling the return of the regulation vehicle to the control and managing center (park) by firing the transition $T_D$ at the end of each regulation operation of the system. Furthermore, we can consider two different rebalancing modes of the stations:

(1) — Periodic rebalancing mode: In this case, the firing delay associated to the transition $T_D$ represents the frequency regulation of the stations (taking into account the regulation vehicle's displacement delay from the regulation center to the first station to be controlled). (2) — Continuous rebalancing mode: Unlike the periodic mode, the firing delay of the transition $T_D$ represents only the regulation vehicle's displacement delay from the regulation center to the first station to be controlled.

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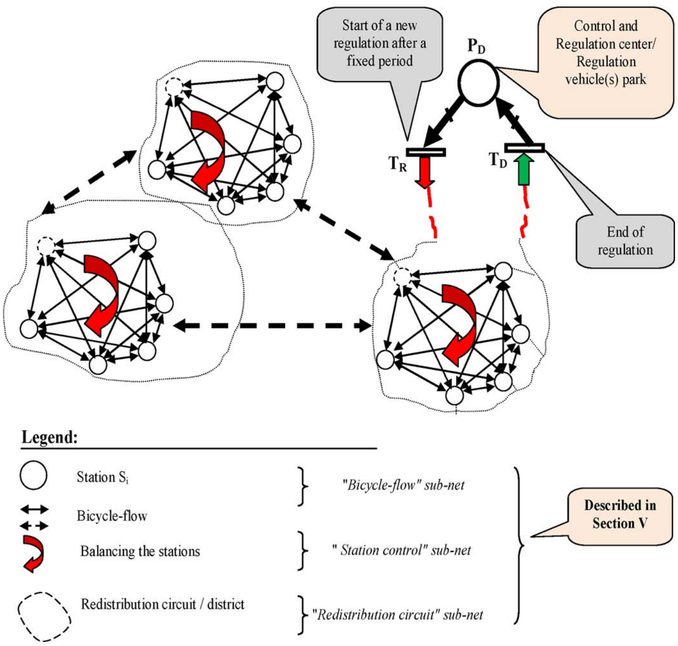


Fig. 7. Illustration of the balancing of the stations by using more than one regulation vehicle.

More generally, the rebalancing of public bicycle sharing systems is performed in a periodic way [41]. In addition, and as detailed in Section II, the rebalancing operation can be carried out in two different modes: static or dynamic.

## VI. Discrete Event Simulation and Performance Evaluation

Performance modelling and evaluation constitute an important aspect of the design and the operational management of bicycle-sharing systems. Discrete event systems are commonplace, and discrete-state models are normally used to study their behavior. Particularly, Petri nets provide a convenient and concise method of describing these systems. However, real systems are large and complex. Consequently, the underlying state space of the corresponding Petri nets models tends to be extremely large. On the other hand, the using of some stochastic transitions in our PN model generates a stochastic marking (state) process. As the model complexity increases, the use of analytical methods for analysis and performance evaluation becomes harder for many real-life applications. Numeric-analytic methods or discrete event simulation are typically used in such contexts.

A comprehensive and up-to-date database of currently used tools for Petri nets is presented in the Petri Nets World web site: http://www.informatik.uni-hamburg.de/TGI/PetriNets/. These tools support a variety of platforms (windows, unix, etc.) and different families of Petri nets such as ordinary Petri nets, colored Petri nets, object-oriented Petri nets, hierarchical Petri nets, and stochastic Petri nets. The features these tools support vary widely: some support graphical editing but not graphical animation or simulation, some have scripting capabilities, some generate code, and a few support the interchange file format for Petri net models that has been developed as part of an on-going standardization process.

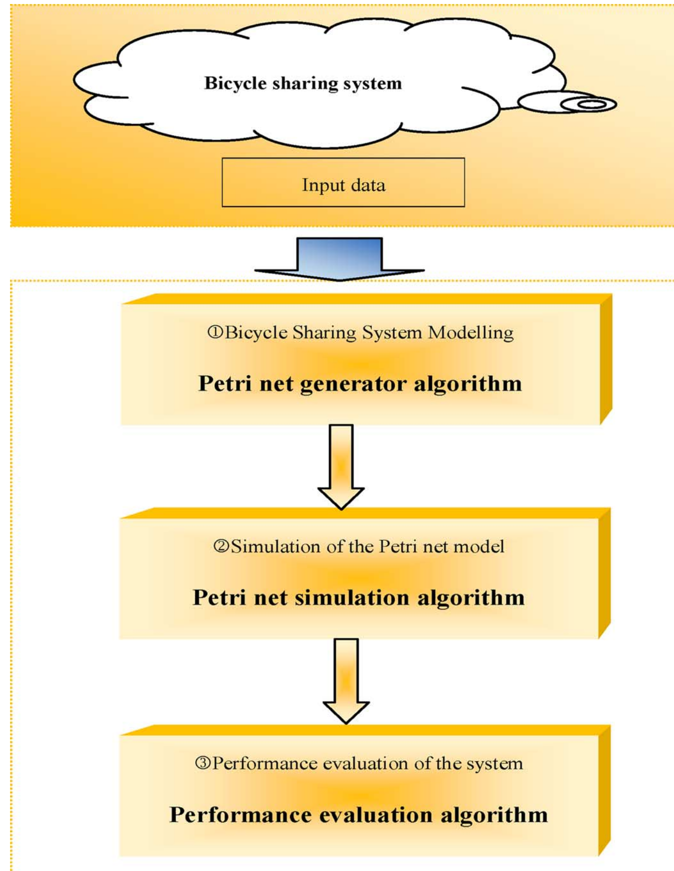


Fig. 8. General architecture of the simulation process.

### A. Discrete-Event Simulation Tool

Despite the many free software tools available for Petri nets simulation and analysis, we had to develop our specific simulation tool. In fact, the "marking dependent weights" concept, which is an important feature to model public bicycle systems, does not exist anywhere in the existing stochastic Petri nets simulation tools.

The general process of our discrete event simulation tool of Petri nets (with deterministic and/or stochastic time and variable marking dependent weights) is shown in Fig. 8. The design of the process was broken up into four distinct modules according to their different functions described as follows:

1) *Petri net generator module:* This part of the process is specifically developed to quickly generate the Petri net model of a bicycle sharing system according to the model developed in this paper. Thanks to the modularity of our model, an algorithm is developed to automatically generate the corresponding model for N stations.
2) *Petri net simulation module:* The general simulation process is shown in Fig. 9. Each cycle represents the firing of one transition (event execution) according to the current marking of the Petri net (state of the system). A new marking is generated after each firing and the simulation time is increased according to the type of transition fired and the time required for its firing.
3) *Performance evaluation module:* In the discrete-event simulation module, each time a transition fires, the above

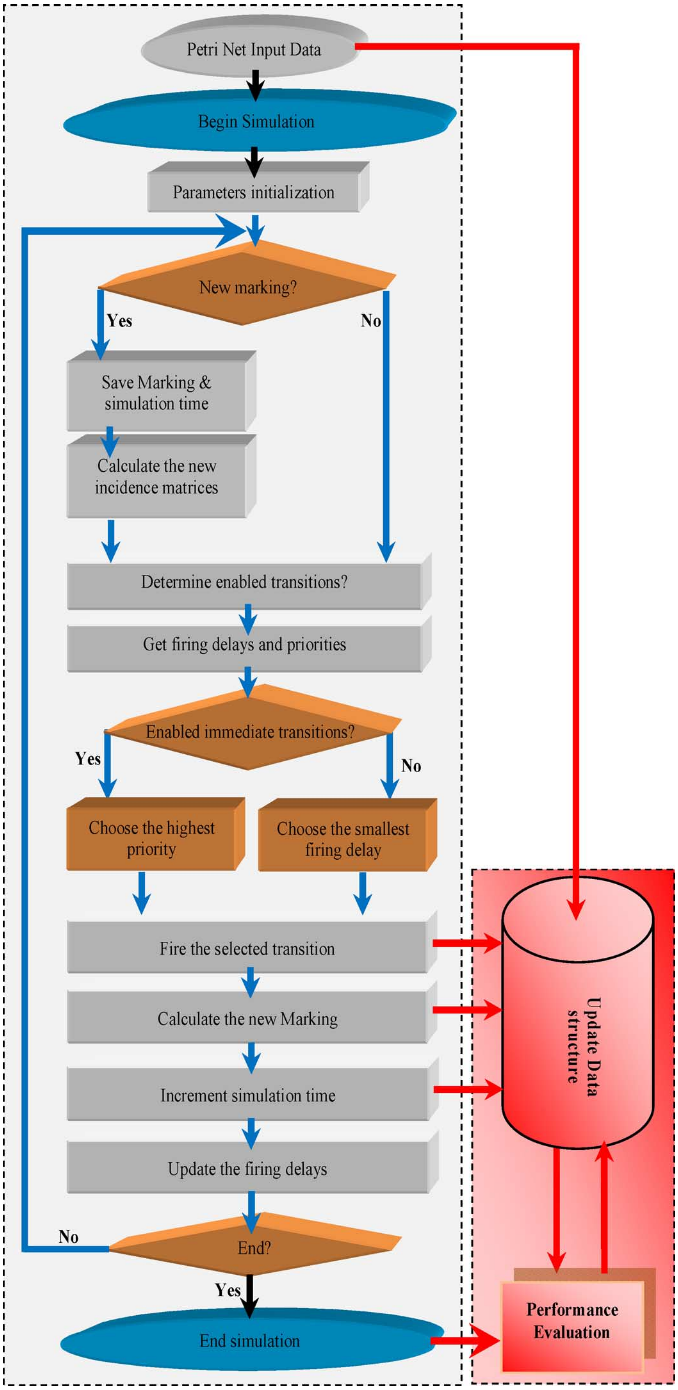


Fig. 9. The PN simulation algorithm.

changes are monitored and stored in suitably developed data structures. At the end of the simulation, performances measures are computed by looking at the values contained in these data structures.

The following two sub-sections are dedicated to a technically detailed description of the simulation and the performance evaluation modules.

### *B. Discrete-Event Simulation Technique*

An organizational chart of our simulation technique is represented in Fig. 9. Each cycle represents the firing of one transition and the simulation time is increased according to the type of transition fired and the time required for its firing. The simulation module is executed in line with the following rules:

Each immediate transition in the Petri net fires in zero delay whereas each timed transition fires after either a deterministic or a stochastic distributed firing delay. When both timed and immediate transitions are enabled, only immediate transitions can be fired because timed transitions are assumed to have a firing priority lower than immediate transitions. When immediate transitions with different priorities are enabled, only those with the highest priority can be fired. When several immediate transitions with the same highest priority are enabled, a choice needs to be made to determine which one among these transitions will fire. The choice may be made in a probabilistic way by assigning appropriate firing weights to the conflicting transitions in order to derive the probability in which a particular transition fires [10]. When some timed transitions are enabled at a marking, the transition with the minimum firing delay causes the marking change. This is called the race condition [32].

An important issue that arises in the firing of timed transitions is how to manage the timing of all enabled transitions that are not fired. Different policies can be adopted to link the past history of these transitions to their future evolution considering various ways of retaining the memory of the times already spent on activities. This concerns the memory policy of transitions. We use two basic memory mechanisms for a timed transition at any instant of marking change [32]: (*a*) Continuing: the timing associated with the transition holds its current value and continues later on the count-down, (*b*) Restarting: the timing associated with the transition is restarted, *i.e.*; its current value is discarded and a new value will be generated when the transition is newly enabled.

*Remark:* Note that, the Petri net model developed in this article is without conflicting situations. Thus, the priority concept is not useful in this case.

### *C. Performance Evaluation and Analysis*

During the simulation phase, each time a transition fires, the state changes are monitored and stored in suitably developed data structures. In terms of the Petri net model, when a transition fires, several changes occur such as: the marking of the places (the input places of the fired transition lose tokens, the output places of the fired transition gain tokens); the simulation time advances by the firing delay of the fired transition; the total number of times this transition fires gets incremented; and so on. At the end of the simulation, the stored data during the simulation phase are used to compute performance measures of the modeled system.

Most performance measures of a modeled system as a stochastic Petri net can be defined based on the steady-state distribution of the model which can be computed by the formula (5). Having the steady-state probabilities, several performance measures of the system can be formulated according to the marking of the palaces and/or the transition parameters. In the following, some of performance indices are specifically formulated for performance analysis of a bicycle-sharing system.

- *Steady state probabilities of states of the system:* Let $M_k$ be a reachable marking in the net. The steady state probability, denoted by $StateProb(M_k)$, of $M_k$ is given by:

$$StateProb\left[M_k\right] = \frac{MTV\left[M_k\right]}{TSIM} \quad (5)$$

where $TSIM$ is the total simulation time and $MTV[M_k]$ gives the total time for which the system has spent in marking $M_k$.

- *Probability of an empty and/or a saturated station:* In the case of a bicycle sharing system, it is very important to evaluate the probability of having an empty station (i.e, $M(PS_i) = 0$), a saturated station (i.e, $M(PS_i) = C_i$) or more generally, exactly $N$ bicycles in the station $S_i$. Formally, as each station is represented by a place $PS_i$ in the Petri net model, then:

$$Prob\left[M(PS_i) = Nb\right] = \frac{\sum\limits_{k|M_k(PS_i)=Nb} MTV\left[M_k\right]}{TSIM}. \quad (6)$$

Where $k \mid M_k(PS_i) = Nb$ signifies that we sum all the times ($MTV[M_k]$) where the marking of the place $PS_i$, representing a station $S_i$, is equal to $Nb$.

  - $Nb = 0$: to evaluate the probability of having an empty station $S_i$. (Without any bicycle).
  - $Nb = C_i$: to evaluate the probability of having a saturated station $S_i$ ($C_i$ represents the capacity of the station).

    It is also interesting to compute the probability of having one of the undesirable situations (empty or saturated station). In this case, the required probability is the sum of the probability of each situation.

- *Average number of bicycles in the stations:* Having the steady state probability of each state (marking) of the system, the average number of tokens in a place $p_i$ can be computed. Particularly, in the case of the model of a bicycle sharing system, this performance measure can be used to compute the average number of bicycles in each station $S_i$, $AverageNB(S_i)$, by calculating the average marking of the corresponding places $PS_i$. Formally:

$$AverageNB\left[S_i\right] = \sum_{k|M_k \in M^*} M_k\left[PS_i\right] \cdot StateProb\left[M_k\right] \quad (7)$$

where $k \mid M_k \in M*$ signifies that over the summation, we consider all of the reachable markings of the Petri net generated by simulation.

## VII. Configuration and Application of the Model

By using the discrete-event simulation technique described previously, the main objective of this last section is to demonstrate the relevance of the Petri net model of a bicycle sharing system developed formally in Sections IV and V. Here, the dynamic behavior of the model will be analyzed through several simulations in line with different configurations namely: (a) *dynamic case with regulation*, (b) *dynamic case without regulation*, and (c) *static case*. Indeed, as we shall see in this section, our model can be parameterized and configured in several modes according to real situations. We conduct our application using data from VELITUL (Fig. 10), a real self service bicycle system of LAVAL agglomeration (France) (http://www.tul-aval.fr/) of 9 stations with about 100 self-service bicycles. For this application, our Petri net model is parameterized according to real data given in Tables V – VIII. Note that Table VII is obtained by using Google map application to estimate distances and average displacement times of bicycles between the different stations. The data in Table VIII (average time between two successive departures of bicycles from a station to another) are estimated for the June/July 2013 period. However, the reorder points ($R_i$) given in Table VI are arbitrarily chosen for

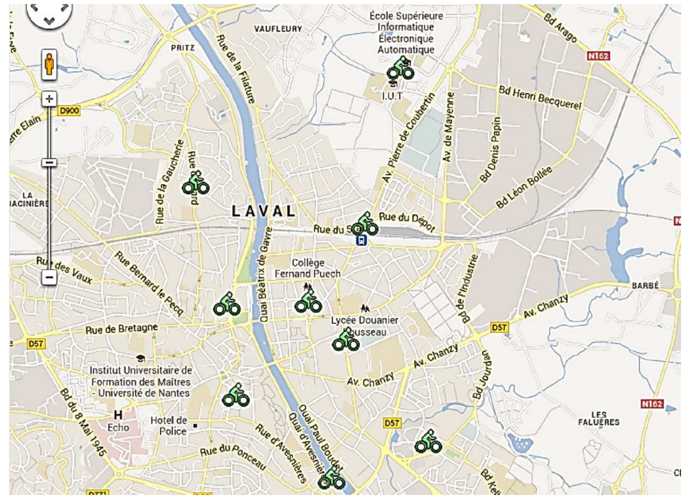


Fig. 10. Distribution of VELITUL's stations ≪ Google map ≫.

TABLE V
Designation of the Stations and Their Capacities

| Stations ($S_i$) | Designations | Capacities ($C_i$) |
|---|---|---|
| **S1** | GARE TUL | 18 |
| **S2** | PRÉFECTURE | 14 |
| **S3** | FELIX GRAT | 16 |
| **S4** | GARE SNCF | 18 |
| **S5** | BIBLIOTHÈQUE | 14 |
| **S6** | AVESNIERES | 14 |
| **S7** | HILARD | 12 |
| **S8** | CENTRE UNIVERSITAIRE | 16 |
| **S9** | CITÉ ADMINISTRATIVE | 14 |

TABLE VI
Designation of the Stations and Their Capacities

| **Stations** | **S1** | **S2** | **S3** | **S4** | **S5** | **S6** | **S7** | **S8** | **S9** |
|---|---|---|---|---|---|---|---|---|---|
| **Initial states** | $M_0(PS_1) = 10$ | $M_0(PS_2) = 08$ | $M_0(PS_3) = 07$ | $M_0(PS_4) = 17$ | $M_0(PS_5) = 07$ | $M_0(PS_6) = 03$ | $M_0(PS_7) = 07$ | $M_0(PS_8) = 10$ | $M_0(PS_9) = 04$ |
| **Capacities** | C1 = 18 | C2 = 14 | C3 = 16 | C4 = 18 | C5 = 14 | C6 = 14 | C7 = 12 | C8 = 16 | C9 = 14 |
| **Reorder point** | R1 = 10 | R2 = 08 | R3 = 10 | R4 = 10 | R5 = 08 | R6 = 08 | R7 = 07 | R8 = 10 | R9 = 08 |

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TABLE VII
THE DISTANCE AND TIME MATRIX FROM STATION TO STATION

| | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 |
|---|---|---|---|---|---|---|---|---|---|
| S1 | - | 700 m (5mn) | 1.2km (7mn) | 1.6km (9mn) | 1km (6mn) | 1.5km (8mn) | 1.1km (7mn) | 2.7km (16mn) | 2km (13mn) |
| S2 | 700 m (5mn) | - | 500m (2mn) | 650m (4mn) | 1.2km (8mn) | 2km (8mn) | 2.5km (11mn) | 2.1km (11mn) | 1.8km (9mn) |
| S3 | 1.5km (7mn) | 500m (2mn) | - | 850m (4mn) | 1.3km (9mn) | 1.4km (6mn) | 3km (14mn) | 2.3km (11mn) | 1.3km (6mn) |
| S4 | 1.6km (9mn) | 650m (4mn) | 850m (4mn) | - | 2.1km (11mn) | 2.5km (10mn) | 2.5km (11mn) | 1.4km (7mn) | 2.2km (10mn) |
| S5 | 1km (6mn) | 1.2km (8mn) | 1.3km (9mn) | 2.1km (11mn) | - | 1.6km (7mn) | 2.1km (12mn) | 3.7km (16mn) | 2.1km (12mn) |
| S6 | 1.5km (8mn) | 2km (8mn) | 1.4km (6mn) | 2.5km (10mn) | 1.6km (7mn) | - | 3.5km (17mn) | 3.8km (18mn) | 1.1km (6mn) |
| S7 | 1.1km (7mn) | 2.5km (11mn) | 3km (14mn) | 2.5km (11mn) | 2.1km (12mn) | 3.5km (17mn) | - | 2.6km (14mn) | 3.1km (17mn) |
| S8 | 2.7km (16mn) | 2.1km (11mn) | 2.3km (11mn) | 1.4km (7mn) | 3.7km (16mn) | 3.8km (18mn) | 2.6km (14mn) | - | 2.5km (14mn) |
| S9 | 2km (13mn) | 1.8km (9mn) | 1.3km (6mn) | 2.2km (10mn) | 2.1km (12mn) | 1.1km (6mn) | 3.1km (17mn) | 2.5km (14mn) | - |

TABLE VIII
BICYCLE FLOW (AVERAGE TIME BETWEEN TWO SUCCESSIVE DEPARTURES)

| ↱→ | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 |
|---|---|---|---|---|---|---|---|---|---|
| S1 | 380mn | 160mn | 170mn | 250mn | 145mn | 180mn | 150mn | 190mn | 175mn |
| S2 | 175mn | 235mn | 240mn | 155mn | 120mn | 160mn | 170mn | 150mn | 140mn |
| S3 | 175mn | 255mn | 400mn | 175mn | 130mn | 240mn | 340mn | 140mn | 270mn |
| S4 | 140mn | 40mn | 160mn | 350mn | 240mn | 150mn | 240mn | 145mn | 125mn |
| S5 | 270mn | 140mn | 240mn | 155mn | 530mn | 340mn | 185mn | 220mn | 175mn |
| S6 | 150mn | 145mn | 150mn | 225mn | 140mn | 400mn | 160mn | 170mn | 150mn |
| S7 | 140mn | 220mn | 340mn | 120mn | 145mn | 115mn | 540mn | 40mn | 170mn |
| S8 | 145mn | 155mn | 120mn | 160mn | 220mn | 110mn | 135mn | 360mn | 145mn |
| S9 | 175mn | 130mn | 240mn | 340mn | 140mn | 270mn | 175mn | 130mn | 515mn |

illustrative purposes of the dynamic behavior of the model for different configurations. It is also, useful to indicate that the regulation vehicle's capacity is fixed to 20 and the simulation results are given in the case of one regulation per day.

### *A. Dynamic Case With Regulation*

Firstly, let us consider the dynamic case with regulation. That is, stations are available for users and the system is under control to rebalance bicycles between stations that are emptying out and those that are filling up (Dynamic rebalancing mode). Thanks to our Petri net simulation tool, behaviors (number of available bicycles/ time of day) of the stations can be observed. The case of four stations ("Gare Tule", "Felix Grat", "Bibliothèque", "Hilard") are represented in Fig. 11 and some performances are given in Table IX.

As discussed in Section VII, our Petri net model can be parameterized according to different decision parameters, including regulation parameters $R_i$ (Table VI) and regulation frequency of the stations. Thus, the performance of the system (Table IX) depends on the chosen values of these parameters. For optimal regulation of the system, our modeling and performance evaluation approach can be coupled with an optimization technique (Genetic Algorithm, for example) in order to search optimal decision and management parameters of the system [21].

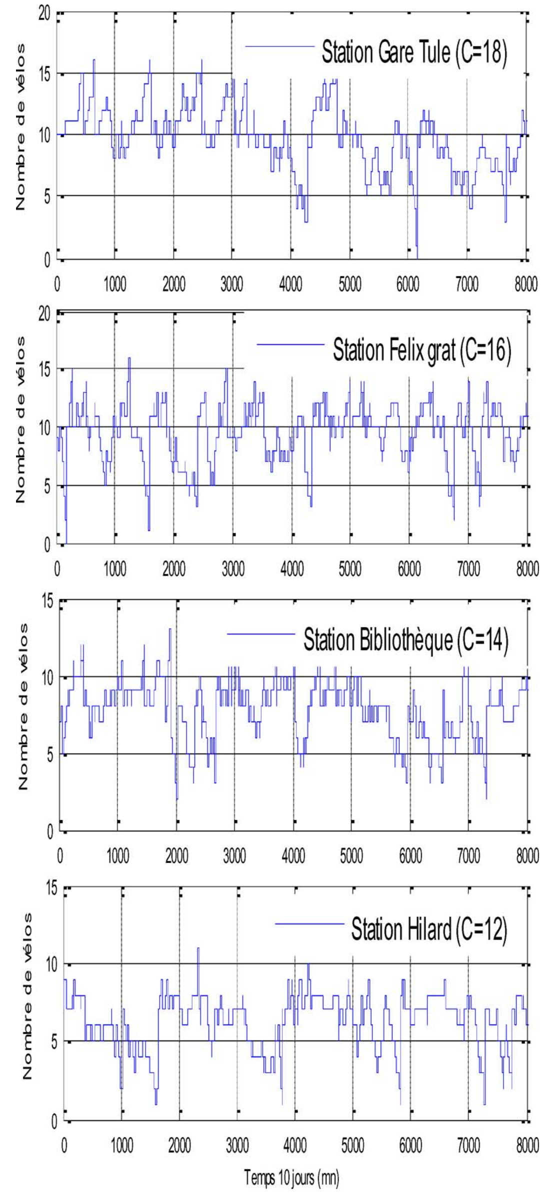


Fig. 11. Behavior of stations (dynamic case with regulation).

TABLE IX
PERFORMANCE EVALUATION (CASE WITH REGULATION)

| Stations | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 |
|---|---|---|---|---|---|---|---|---|---|
| % of time "Empty" | 0.138 | 0.32 | 0.0341 | 0.425 | 1.047 | 0.110 | 0.071 | 0.120 | 0.660 |
| % of time "Full" | 0.000 | 0.067 | 0.620 | 0.094 | 0.250 | 0.092 | 0.011 | 0.260 | 0.210 |
| Average available bicycles | 8.460 | 7.050 | 9.210 | 8.42 | 7.260 | 6.80 | 6.190 | 9.38 | 6.98 |

### *B. Dynamic Case Without Regulation*

Unlike the first case, here, we consider that the regulation system is unavailable. So, the stations remain operational for users but without any control of the bike stations. According to our Petri net model, this configuration is obtained when the

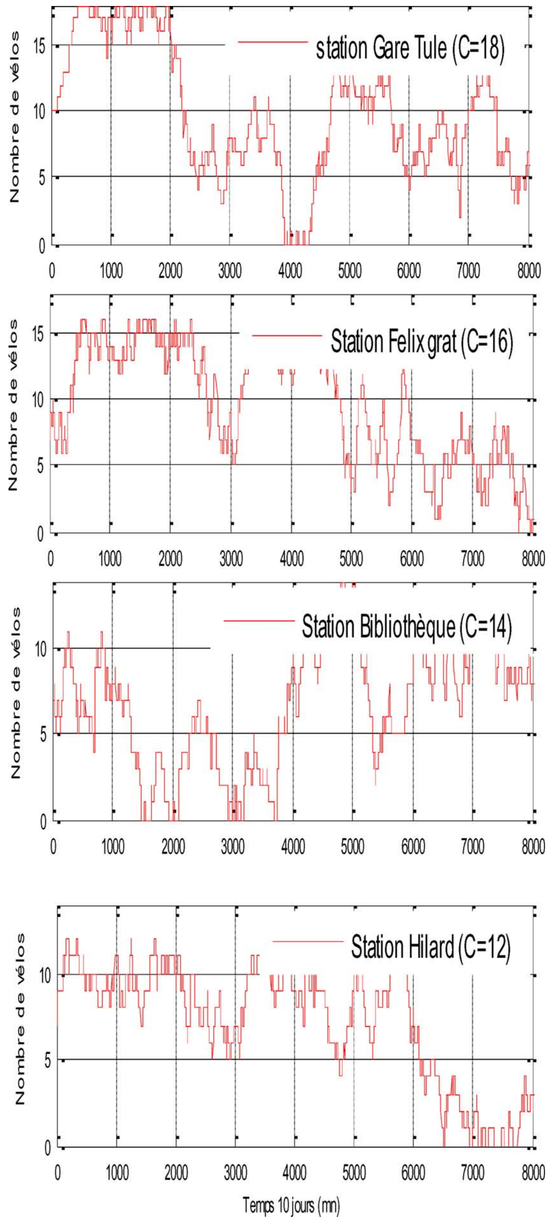


Fig. 12. Behavior of stations (dynamic case without regulation).

initial marking of all the places $PV_i$ and/or $PR_i$ is equal to zero. This configuration deactivates the redistribution vehicle path and the whole control function of each station. In fact, all the transitions ($TE_i$, $TR_i$, $TS_i$, $TO_i$, $TRBr_i$, and $TRBs_i$) of the "control station" subnet and these of the "redistribution circuit" subnet are always not enabled. For this configuration, the behaviors of the considered stations are illustrated in Fig. 12. Compared to the first case, we observe several critical situations confirmed in Table X.

### C. Static Case (During Night)

Finally, this third case represents the behavior of the system when the frequentation of the stations is very low (functioning of the system during night, for example). In terms of the Petri net model, this situation can be simulated by increasing considerably the transition firing delays of the transitions $TS_{ij}$ which represent the displacements of bicycles between the different stations. The corresponding behavior of the stations in this case is illustrated in Fig. 13 where we observe very few bicycles used throughout the night.

TABLE X
PERFORMANCE EVALUATION (CASE WITHOUT REGULATION)

| Stations | S1 | S2 | S3 | S4 | S5 | S6 | S7 | S8 | S9 |
|---|---|---|---|---|---|---|---|---|---|
| % of time "Empty" | 13.70 | 12.12 | 02.23 | 10.22 | 03.37 | 04.30 | 01.092 | 01.98 | 14.79 |
| % of time "Full" | 03.39 | 01.13 | 10.96 | 04.56 | 01.44 | 00.78 | 01.82 | 12.78 | 01.59 |
| Average available bicycles | 6.01 | 4.86 | 11.39 | 7.58 | 8.96 | 5.97 | 5.77 | 10.79 | 4.87 |

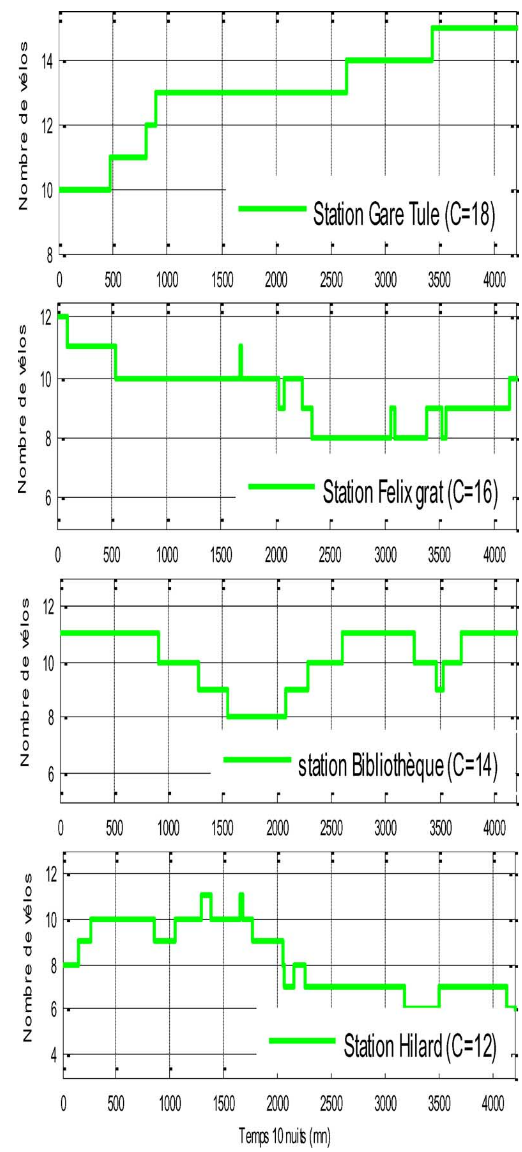


Fig. 13. Behavior of stations (Static case, during night).

## VIII. CONCLUSION

This work drives from our larger project on dynamic modelling, simulation and performance optimization of public bicycle-sharing systems. In this article, an original discrete-event approach is developed for modelling, control, performance evaluation and simulation of such very complex systems. Our approach is the first one in the literature dedicated to this urban transportation mode by using stochastic Petri nets with variable arc weights as a powerful tool for analysis and simulation.

After the first part of this paper (Sections I and II), dedicated to the general presentation and literature review of public bicycle sharing systems research, we presented our new model taking into account new decision parameters and all specific control operations required to re-distribute the bicycles across the stations in order to ensure that users will be able to check out a bicycle to use or find a dock to return the bicycle. Thereafter, the discrete-event simulation tool and the performance evaluation method developed in this project are formally presented and illustrated by some simulations results.

Our modelling and performance evaluation approach is recently completed by an optimization method applied to Critolib, a real self service bicycle system of Creteil city, France [21]. Future works on optimization would be useful to influence economic viability and operational efficiency of these new transportation systems.

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**Karim Labadi** received the M.S. degree in applied automation and informatics from the IRCCyN, "Ecole Centrale de Nantes (ECN)", France, in 2002, and the Ph.D. degree in industrial engineering and optimization from the University of Technology of Troyes (UTT), France, in 2005.

He is currently an Assistant Professor at the School of Electricity, Production and Industrial Management (EPMI) and member of the "Equipe Commande des Systèmes" (ECS-Lab, EA.3649) at the "Ecole Nationale Supérieure d'Electronique et ses Applications" (ENSEA), Cergy-Pontoise, France, since 2006. His research interests includes Petri nets and their applications to modelling, control and performance optimization of stochastic discrete event systems such as production, transportation and logistic systems.

**Taha Benarbia** received the Bachelors degree in electrical engineering and the Masters degree in automation and control from the ENSET, "Ecole Normale Supérieure d'Enseignement Technologique d'Oran", Algeria in 2005 and 2009. He received the Ph.D. degree in electrical engineering from the "Université des Sciences et de la Technologie d'Oran" (USTO Oran, Algérie) and the Cergy-Pontoise University (UCP), France in 2013.

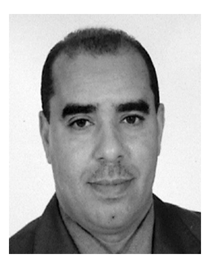
He is currently an Associate Professor at the University of Oran, Algérie. His recent research interests include modelling and performance evaluation of continous and/or discrete systems such as electrical systems and transportation systems.

**Jean-Pierre Barbot** (M'93–SM'10) graduated from the Ecole Normale Supérieure of Cachan, and received the Ph.D. degree in automatic control from the University of Paris XI, France, in 1989 and the HDR in 1997.

He is currently a member of the Equipe Commande des Systèmes (ECS-Lab, EA 3649) at the "Ecole Nationale Supérieure d'Electronique et ses Applications" (ENSEA), Cergy-Pontoise France and member of the EPI Non-A, INRIA Lille, Nord Europe. His main research activities deal with sliding mode control and observation, system under sampling, hybrid system, time-delay system, synchronization of chaotic system and normal form. His main application domains are electrical machine, power converter, cryptography and mechatronic.

**Samir Hamaci** received the Ph.D. degree in automatic degrees from the University of Angers in 2005.

He is currently an Assistant Professor at the School of Electricity, Production and Industrial Management (EPMI) in France. His research interests include modelling, performance analysis, control and resources optimisation of dynamic systems modelled by discrete and hybrid Petri nets in dioid algebra. He has published research papers at national and international journals, conference proceedings as well as chapters of books, one of which has been edited by Kluwer Academic Publishers.



**Abdelhafid Omari** received the M.Sc. degree from the University of Sciences and Technology of Oran, Algeria, in 1992, and the Ph.D. degree in electrical engineering from the University of Electro-communications, Tokyo, Japan, in 2001.

He has worked in academia and in industry. His industrial experience is mainly in the field of mechatronics and robotics. He is currently with the Department of Automatic Control, Faculty of Electrical Engineering, University of Sciences and Technology of Oran, Algeria. His research and teaching interests include linear and nonlinear control methods, robust control design, and intelligent mechatronic systems.