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A R T I C L E I N F O

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ABSTRACT

We study ex ante information sharing in a supply chain consisting of a downstream retailer and a maketo-stock upstream manufacturer. The retailer has imperfect demand information and may choose to share it with the manufacturer. Based on the information sharing arrangement, the manufacturer makes the wholesale price and the stocking level decisions. Then the retailer decides the order quantity and the manufacturer fulfills the order up to the available stock level. We find that the retailer has an incentive to voluntarily share the information with the make-to-stock manufacturer if the magnitude of demand uncertainty is intermediate. This stands in sharp contrast with the existing studies which show that the retailer never shares information when the manufacturer is make-to-order. Our results highlight the interdependence between the retailer's incentive to share information and the manufacturer's operational and marketing decisions.

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1. Introduction

Modern data collection technologies have led to an explosion in both the scope and volume of customer and market data that are accessible to retailers. In the big data age, information sharing has been drawing heightened attention from practitioners and researchers. In 1990s, Electronic Data Interchange (EDI) was established between Walmart and its suppliers such as Procter & Gamble (P&G) and Johnson and Johnson, to share point-of-sales data. In 2000s, some initiatives such as Collaborative Planning Forecasting and Replenishment (CPFR) and Radio Frequency Identification (RFID) were adopted by Walmart and its suppliers, and this effectively facilitated sharing more accurate information between them. Information sharing through instruments such as EDI reduces the bullwhip effect along the supply chain (e.g. [30,31]) and brings significant value to supply chain partners (e.g. [6,14]).

However, incentives for information sharing are still an issue. Consider a supply chain consisting of a downstream retailer and an upstream manufacturer. It is known in the literature that if the manufacturer makes the wholesale price decision and the retailer makes the order quantity decision, the retailer does not have an incentive to share his private demand information with the manufacturer (e.g. [33,49]). On the other hand, information sharing is widely observed in practice, usually initiated by retailers. Why? According to IHL research, the out-of-stock problem in North America costs approximately \$93 billion annually [27]. This is particularly a problem at the retail end where out-of-stock stubbornly remains at 8%, rising to over 15% during promotional periods [41,15]. Demand Clarity [15], a retail supply chain management consultancy, points out that the key reason for this retail out-ofstock phenomenon is that the retailer, where the information flow starts and the product flow ends, is disconnected with the supplier's production planning. Information sharing can help bridge the downstream retailing and the upstream production to reduce the occurrence of stocking out. Therefore, the retailer's incentive to share information is influenced not only by the manufacturer's wholesale price decision but also by its production or stocking level decision which directly impacts the likelihood of stocking out.

This paper attempts to rethink the retailer's incentive to share demand information in a supply chain by considering the manufacturer's multiple decisions, in contrast to some existing studies where the manufacturer only makes one decision such as price or capacity (e.g. [8,33,19,36]). We focus on ex ante information sharing commitment, where a retailer decides whether to share information before he observes the content of information. If firms agree to share information, they need to set up IT systems for information transmission beforehand (e.g. EDI). Many existing studies in this stream of inquiry investigate a retailer's incentive to share information with a make-to-order manufacturer. That is,







 $[\]ensuremath{\,^{\ensuremath{\ensuremath{^{\ensuremath{\,\times}}}}}$ This manuscript was processed by Associate Editor Ryan.

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the manufacturer produces and delivers the exact amount according to the order from the retailer. However, in some industries that require less customization, like daily necessities (e.g., P&G) and sports dress (e.g., Nike), the manufacturers are make-to-stock in that a certain stock level is built before their retailers place orders. When the manufacturer makes to stock, the incentive for the retailer to share information becomes more complex.

Our research questions include: How do the manufacturer's marketing decision (wholesale price) and operational decision (stock level) together interact with the retailer's information sharing strategy? When does the retailer have an incentive to share information and how does his benefit from information sharing depend on his information precision? How does the manufacturer's production mode (i.e., make-to-stock or make-to-order) impact the retailer's information sharing strategy? Under what conditions does the supply chain as a whole profit from information sharing?

We study these questions in the setting of a two-tier supply chain where the retailer is supplied by a make-to-stock manufacturer. The demand is uncertain and price sensitive. The retailer observes an imperfect signal about the demand. A three stage game is played. First, the retailer decides whether to commit to sharing the demand information with the manufacturer. After that, the retailer observes a demand signal which is revealed to the manufacturer if the retailer has committed to sharing information. Second, the manufacturer chooses her stock level and wholesale price. Finally, the retailer decides the order quantity and the manufacturer delivers the goods subject to the stock level. This model differs from the classical models in the literature on ex ante information sharing in that the information sharing arrangement interacts with both the manufacturer's marketing and operational decisions. On one hand, the retailer may like to conceal information to induce a low wholesale price when the market demand is high. On the other hand, he may like to share information to induce the manufacturer to build a higher stock level which would better serve the high market demand.

We have several interesting findings. We highlight the distinction between a supply chain with a make-to-stock manufacturer and that with a make-to-order manufacturer in terms of information sharing strategies and firms' profitability. The retailer never shares information with a make-to-order manufacturer but may do so with a make-to-stock manufacturer. Information sharing never benefits the supply chain when the manufacturer is maketo-order but may benefit the supply chain when the manufacturer is make-to-stock.

While sharing information has a direct negative impact on the retailer-the informational advantage disappears, it has a positive impact by inducing the manufacturer to build up enough stock for the high demand market. Information sharing has three impacts on the supply chain performance. First, it enables the manufacturer to confidently build enough stock for the high demand market. Second, when the demand is low, it induces a low wholesale price to guarantee a certain level of retail sales. Third, it renders a high wholesale price when the demand is high and reduces the sales. Although the first two impacts benefit the supply chain, the third works to its detriment. When the high demand is not attractive, the first effect is negligible. When the high demand is attractive, the third effect dominates the first two effects. We find that the retailer in balance has an incentive to share information when the magnitude of demand uncertainty is intermediate. Without information sharing, the manufacturer may build up just enough stock for the low demand market.

This paper belongs to the literature on the incentives for precommitment information sharing in supply chains. Li [33] and Zhang [49] show that when an upstream manufacturer serves multiple retailers who have private demand information, no information sharing is the unique equilibrium outcome if the retailers make the order quantity decisions. Li and Zhang [34] show that when the downstream competition is intense, confidentiality triggers the retailers' incentives to share the demand information with the manufacturer, and that a wholesale price contract plus confidentiality can coordinate the supply chain. Ha and Tong [25] incorporate chain-to-chain competition into information sharing, and study how the downstream competition and contract types impact the incentives for information sharing. Gal-Or et al. [19] study the information sharing arrangement in a distribution channel when retailers are asymmetrically informed. They suggest that the manufacturer may choose to share information with only the less-informed retailer rather than with both. Yao et al. [46] consider a supply chain consisting of one supplier and two value-adding heterogeneous retailers. They study a retailer's incentive to share his cost information about the valueadded service with the supplier. Shin and Tunca [40] examine the effect of downstream competition on incentives for demand forecast investments in supply chains. They identify contract schemes to coordinate the supply chain. Kurtuluş et al. [29] study the collaborative demand forecasting combination when both firms in a supply chain can invest to improve the quality of their demand information. Ha et al. [26] study how the incentive for vertical information sharing is impacted by the competition between supply chains, by the production diseconomies, and by the accuracy of demand information. Shang et al. [39] investigate the demand information sharing in a supply chain with two competing manufacturers selling substitutable products through a common retailer. In all these papers, the manufacturers make only marketing decisions, either wholesale price or other contract forms. In contrast, we study the incentives for information sharing, when the upstream manufacturer may adjust both the marketing and the operational decisions (i.e., the wholesale price and the stock level) according to information sharing arrangement. Therefore, our research contributes to the literature in two aspects. First, we show that the countervailing effects of the wholesale price and the stock level on the retailer's profitability may motivate voluntary information sharing that is commonly observed in practice. Second, we show that a manufacturer's mode of production (make-to-stock vs. make-to-order) has a salient effect on a retailer's information sharing strategy.

The value of vertical information sharing in supply chains has been substantially studied. In this line of research, manufacturers usually make inventory or capacity or stock level decisions. Representative papers include Bourland et al. [5], Chen [10], Gavirneni et al. [21], Lee et al. [31], Cachon and Fisher [7], Aviv [1–3], Fiala [17], Zhang [48], and Trapero et al. [42]. Chen [11] provides a survey of earlier work. The focus of these studies is the benefits of information sharing on firms' operational improvements such as ordering function, inventory allocation, and inventory cost saving. In contrast to these papers, we focus on the retailer's incentives for sharing private demand information when the manufacturer makes both the wholesale price and the stock level decisions.

Our paper is related to strategic or ex post information disclosure in a vertical channel where a downstream retailer makes the information sharing decision *after* the demand is realized. Guo [22] examines the impacts of the downstream firm's information acquisition and strategic disclosure to the upstream firm. Guo et al. [24] extends Guo [22] by investigating the strategic information sharing in two competing channels. In their models, the upstream manufacturers only make the wholesale price decisions. Chu and Lee [12] study the retailer's strategic disclosure of the demand information when it is costly to share information with the manufacturer who only makes the stock level decision. The aforementioned papers assume truthful information

disclosure between firms. Some recent research studies how firms reach truthful information transmission under a simple wholesale price contract. Ren et al. [38] identify a review strategy profile that supports the truthful-sharing equilibrium when firms have a longterm relationship and interact repeatedly. Özer et al. [36] study how the behavioral factors induce effective cheap-talk information sharing. They determine the role of trust in inducing truthful information sharing in the absence of reputation. Chu et al. [13] find that when the manufacturer makes both the capacity and the wholesale price decisions, truthful information sharing can be achieved by cheap-talk without behavioral or reputation concerns. because of the retailer's countervailing incentives to inflate the demand to increase the capacity level and to deflate it to reduce the wholesale price. While all concern with wholesale price and/or stock level decisions, our research differs from the above studies in that we focus on a retailer's incentives for making information sharing commitment before the information content is revealed to him, whereas they examine a retailer's incentive to strategically share or to tell the truth after the information content becomes known to him.

There is a literature on credibly sharing information in a supply chain by contract design. Cachon and Lariviere [8] investigate how a manufacturer can use a capacity reservation contract to credibly share private demand information with a supplier when the information cannot be verified. Özer and Wei [35] show that a combination of buyback and advance purchase contract induces credible information sharing and coordinates the supply chain. There are some articles studying contract design to coordinate the supply chain, including Cachon and Lariviere [9], Palsule-Desai [37], and Gao et al. [20]. In contrast, we study the incentives for directly sharing information instead of screening or signaling the information indirectly.

Loosely related to our study on information sharing is horizontal information pooling in economics literature. Pioneering work includes Vives [44], Gal-Or [18] and Li [32]. Wu et al. [45] study the ex ante incentives for firms to share their private demand or cost information in a Cournot duopoly with capacity constraints. The focus of this stream of research is the incentive of an oligopoly firm to share information with their competitors. In contrast, we study vertical information sharing in a supply chain.

The rest of this paper is organized as follows. The next section sets up the model. Section 3 presents a base case where the manufacturer makes to order. Section 4 analyzes firms' decisions and profits, and presents the information sharing arrangement. Section 5 examines the impact of information sharing on the supply chain profit and how to achieve information sharing with payment. Section 6 discusses the robustness of the key results. Section 7 concludes. All formal proofs are in the Appendix.

2. Model setup

A supply chain consists of an upstream manufacturer (she) and a downstream retailer (he). The manufacturer sets a wholesale price w and supplies to the retailer subject to the stock level K that she has secured at a unit cost c > 0. The retailer has a constant marginal operating cost, which we normalize to zero. Both firms are risk neutral. The inverse demand function is given by

 $p = \tilde{A} - q$,

where p is the market clearing price and q is the sale quantity set by the retailer. The intercept of the demand function, \tilde{A} , which represents the random market potential, follows a binary distribution

$$\tilde{A} = \begin{cases} H & \text{with probability } r \\ L & \text{with probability } 1 - r \end{cases}$$

where $r \in (0, 1)$, and H and L correspond to the high and the low demand states, respectively, with H > L > c. Two-state demand distributions have often been used in studies on information sharing in supply chains (e.g., [8,25,22]). The expectation of the uncertain market potential is $\overline{A} = rH + (1 - r)L$.

The retailer observes a signal *Y* about \tilde{A} before deciding his order quantity. The signal *Y* has two possible values, *h* and *l*; *h* is the good news indicating a better chance of high demand, and *l* is the bad news. The extent to which *Y* reflects the true demand state \tilde{A} is described by conditional probabilities, $Pr(h|H) = Pr(l|L) = \rho \in [\frac{1}{2}, 1]$. Similar information structure has been employed in the literature to model the imperfect information (e.g., [28,23]). The probabilities of observing the high demand signal *h* and the low demand signal *l* are, respectively,

$$\lambda_h = \Pr(h) = \Pr(H) \cdot \Pr(h|H) + \Pr(L) \cdot \Pr(h|L) = \rho r + (1-r)(1-\rho),$$

$$\lambda_l = \Pr(l) = \Pr(H) \cdot \Pr(l|H) + \Pr(L) \cdot \Pr(l|L) = (1-\rho)r + (1-r)\rho.$$

Clearly, $\lambda_h + \lambda_l = 1$. The Bayesian updated probabilities of the demand states conditional on the signal *Y* are the following:

$$Pr(H|h) = \frac{\rho r}{\rho r + (1-\rho)(1-r)},$$

$$Pr(H|l) = \frac{(1-\rho)r}{\rho(1-r) + (1-\rho)r},$$

$$Pr(L|h) = \frac{(1-\rho)(1-r)}{\rho r + (1-\rho)(1-r)},$$

$$Pr(L|l) = \frac{\rho(1-r)}{\rho(1-r) + (1-\rho)r}.$$

The parameter ρ can be regarded as an indicator of signal precision. Note that both Pr(H|h) and Pr(L|l) are increasing in ρ . When $\rho = 1/2$, the signal is not informative at all and the posterior is the same as the prior, Pr(H|h) = r and Pr(L|l) = 1 - r. When $\rho = 1$, the signal is perfect, Pr(H|h) = 1 and Pr(L|l) = 1.

The expectation of \tilde{A} conditional on the demand signal is

$$\hat{H} = E(\tilde{A}|h) = H \cdot \frac{\rho r}{\rho r + (1-\rho)(1-r)} + L \cdot \frac{(1-\rho)(1-r)}{\rho r + (1-\rho)(1-r)},$$
$$\hat{L} = E(\tilde{A}|l) = H \cdot \frac{(1-\rho)r}{\rho(1-r) + (1-\rho)r} + L \cdot \frac{\rho(1-r)}{\rho(1-r) + (1-\rho)r}.$$

Obviously, $\hat{H} > \bar{A} > \hat{L}$ and $E_Y(E(\tilde{A}|Y)) = \bar{A}$. It can be easily verified that \hat{H} is increasing in ρ while \hat{L} is decreasing in ρ .

We model the sequence of events/decisions as a three-stage game:

- 1. The retailer decides whether to share his demand signal *Y* with the manufacturer.
- 2. The retailer observes *Y*, which is transmitted to the manufacturer if an information sharing arrangement has been made in stage 1. Based on the information received, if any, the manufacturer decides the wholesale price and the stock level, (*w*,*K*).
- 3. Upon learning *w*, the retailer chooses his order quantity *q*. The manufacturer fulfills the order subject to the stock availability. Then the uncertain demand intercept \tilde{A} is realized and the firms collect their revenues.

Note that the manufacturer remains uninformed of \tilde{A} unless the retailer shares the demand signal *Y* with her and that the information sharing decision is made before the retailer observes the demand signal (e.g., [33,49,34,25,40,26]). In other words, our model concerns the ex ante (or precommitment) information sharing, where the formal long-term sharing processes take place according to the precommitted format. If firms agree to share information, they have to set up an IT system for information transmission, such as EDI, which requires precommitment. After the information sharing agreement is reached, the firms may engage in multiple transactions. Therefore, the stock level and the wholesale price decisions are made after the information sharing decision.

3. Base case: make-to-order manufacturer

We first analyze the case when the manufacturer makes to order. In this case, the manufacturer only makes the wholesale price decision and does not have the stock level constraint.

Throughout the paper, we impose a simplifying assumption on problem parameters to the effect that, for make-to-order manufacturing, the equilibrium retail sales and prices are strictly positive under the low demand condition. This ensures that the manufacturer's decision (*w*,*K*) and the retailer's decision *q* are all interior points. Similar assumptions are commonly made in the information sharing literature (e.g., [44,32,33,19,34]). In fact, various models in accounting, economics, and operations management literature use similar (implicit) assumptions to avoid dealing with the boundary solutions. Specifically we assume $\hat{L} > \frac{1}{2}(\overline{A} + c)$, i.e., $\lambda_h(\hat{H} - \hat{L}) < \hat{L} - c$. This condition can be equivalently expressed as $H - L < \eta(r, \rho, L, c)$, where

$$\eta = \begin{cases} \infty & \text{if } \rho \leq \frac{2-r}{3-2r} \\ \frac{(r+\rho-2r\rho)}{r((3-2r)\rho-(2-r))}(L-c) & \text{if } \rho > \frac{2-r}{3-2r}. \end{cases}$$

The standard deviation of the demand is $\sqrt{r(1-r)}(H-L)$, and H-L can be used to measure the market dispersion. Everything else fixed, a greater *H* means larger dispersion of the market demand. Thus the above condition essentially requires that the market dispersion is not too large.

3.1. Retailer's order quantity decision

In the third stage of the game, given the manufacturer's wholesale price *w*, the retailer chooses his order quantity *q* to maximize his expected profit conditional on *Y*, $q \cdot (E(\tilde{A}|Y) - q - w)$, which leads to his optimal order quantity,

$$q^{o}(w,Y) = \frac{E(\tilde{A}|Y) - w}{2},$$

where the superscript *o* represents the make-to-order. Note that the retailer makes his quantity decision after he observes signal Y about the true market potential \tilde{A} but *before* \tilde{A} is realized.

3.2. Manufacturer's wholesale price decision

In the second stage of the game, the manufacturer sets her wholesale price *w* in anticipation of the retailer's order quantity. There are two possible scenarios for the manufacturer, depending on the first-stage information sharing arrangement. We discuss the two scenarios separately.

3.2.1. With information sharing

The manufacturer maximizes her profit, $(w-c) \cdot q^o(w, Y)$, based on the demand signal *Y*. This leads to the manufacturer's optimal wholesale price

$$w_S^o(Y) = \frac{E(\tilde{A}|Y) + c}{2},$$

and the retailer's equilibrium quantity decision

$$q_{S}^{o}(Y) = q^{o}(w_{S}^{o}(Y), Y) = \frac{E(\tilde{A}|Y) - c}{4},$$

where the subscript *S* denotes the sharing information. Ex ante expected profits for the manufacturer and retailer are given by

$$\Pi_{S}^{0} = \frac{1}{8}\lambda_{h}(\hat{H} - c)^{2} + \frac{1}{8}\lambda_{l}(\hat{L} - c)^{2} \text{ and}$$

$$\pi_{S}^{0} = \frac{1}{16}\lambda_{h}(\hat{H} - c)^{2} + \frac{1}{16}\lambda_{l}(\hat{L} - c)^{2}.$$

3.2.2. Without information sharing

The manufacturer makes her wholesale price decision without knowing the demand signal. Her expectation of the order quantity is $E_Y[q^o(w, Y)] = (\overline{A} - w)/2$ and she maximizes her expected profit $(w - c) E_Y[q^o(w, Y)]$. This leads to her optimal wholesale price

$$w_N^o = \frac{\overline{A} + c}{2},$$

and the retailer's quantity decision

$$q_N^o(Y) = q^o(w_N^o, Y) = \frac{2E(\bar{A}|Y) - \bar{A} - c}{4},$$

where the subscript *N* denotes the *n*ot sharing information. Note that the assumption $H-L < \eta$ mentioned earlier is equivalent to $\hat{L} > w_N^0$. Ex ante expected profits for the manufacturer and retailer are given by

$$\Pi_{N}^{0} = \frac{(A-c)^{2}}{8} \text{ and } \\ \pi_{N}^{0} = \lambda_{h} \frac{(2\hat{H} - \overline{A} - c)^{2}}{16} + \lambda_{l} \frac{(2\hat{L} - \overline{A} - c)^{2}}{16}$$

3.3. Information sharing arrangement

In the first stage of the game, the retailer decides whether or not to share his demand signal to the manufacturer by comparing his ex ante expected profits between the two scenarios.

Lemma 1. $\pi_S^0 \le \pi_N^0$, i.e., sharing information always hurts the retailer.

Therefore, no information sharing will be shared voluntarily.

Lemma 2. $\Pi_S^o + \pi_S^o \le \Pi_N^o + \pi_N^o$. Sharing information always hurts the supply chain.

Therefore, no information sharing can be arranged through payment from the manufacturer to the retailer.

Lemmas 1 and 2 agree with the existing studies under similar settings in the information sharing literature (e.g. [33]); that is, the retailer does not share information with the make-to-order manufacturer, and information sharing hurts the supply chain. The following lemma characterizes how the loss from information sharing changes with the signal precision.

Lemma 3. $\pi_N^o - \pi_S^o$ and $\Pi_N^o + \pi_N^o - (\Pi_S^o + \pi_S^o)$ are both increasing in ρ . The retailer and the supply chain lose more from sharing information *if* the signal is more precise.

With information sharing, the manufacturer can adjust the wholesale price according to the updated demand information, and take away more profit of the supply chain. She can do so more effectively as the demand information becomes more precise. That is, information sharing aggravates the double marginalization problem and hurts the supply chain. This effect becomes more severe when the shared demand information is more accurate.

4. Make-to-stock manufacturer

We now analyze the case when the manufacturer makes to stock. In this case, the manufacturer makes both the wholesale price and the stock level decisions. She then fulfills the retailer's order up to her available stock.

4.1. Retailer's order quantity decision

In the third stage of the game, given the manufacturer's wholesale price w and stock level K, the retailer chooses his order quantity q based on his demand signal Y. He maximizes his profit

$$\pi(q, Y) = q \cdot (E(A|Y) - q - w),$$

subject to the stock constraint $0 \le q \le K$. His sales at the optimal order quantity is

$$q^{s}(w, K, Y) = \min\left[\frac{E(\tilde{A}|Y) - w}{2}, K\right],$$
(1)

where the superscript *s* denotes the make-to-stock.

4.2. Manufacturer's wholesale price and stock level decisions

In the second stage of the game, the manufacturer sets her wholesale price *w* and stock level *K* in anticipation of retailer's order quantity. We assume that the salvage value of the leftover stock is zero. This simplifying assumption does not change our main findings. There are two possible scenarios for the manufacturer, depending on the first-stage information sharing arrangement. We discuss the two scenarios separately.

4.2.1. With information sharing

The manufacturer maximizes her profit, $w \cdot q^s(w, K, Y) - c \cdot K$, based on the demand signal *Y*. This leads to the manufacturer's optimal wholesale price and stock level

$$w_{S}^{s}(Y) = \frac{E(\tilde{A}|Y) + c}{2}, \quad \text{and} \quad K_{S}^{s}(Y) = \frac{E(\tilde{A}|Y) - c}{4}, \tag{2}$$

and the retailer's equilibrium order quantity

$$q_{S}^{s}(Y) = q^{s}(w_{S}^{s}(Y), K_{S}^{s}(Y), Y) = K_{S}^{s}(Y).$$
(3)

Note that $w_{S}^{s}(Y) = w_{S}^{o}(Y)$ and $q_{S}^{s}(Y) = q_{S}^{o}(Y)$. Having the same demand information, a make-to-stock manufacturer precisely anticipates the retailer's order quantity and sets up a stock level such that it just fulfills the retailer's order. Thus, if information is shared, firms' equilibrium decisions are the same whether the manufacturer is make-to-stock or make-to-order. The expected profits for the manufacturer and the retailer are, respectively,

$$\Pi_{S}^{s} = \frac{1}{8}\lambda_{h}(\hat{H} - c)^{2} + \frac{1}{8}\lambda_{l}(\hat{L} - c)^{2},$$

and

 $\pi_{\rm S}^{\rm s} = \frac{1}{16} \lambda_h (\hat{H} - c)^2 + \frac{1}{16} \lambda_l (\hat{L} - c)^2.$

Lemma 4. π_{S}^{s} and Π_{S}^{s} are both increasing in ρ . When information is shared, a more precise signal benefits both firms.

4.2.2. Without information sharing

The manufacturer makes her wholesale price and stock level decisions without knowing the demand signal. She chooses (w,K) to maximize her expected profit:

$$\Pi_N(w,K) = w \cdot E_Y[q^s(w,K,Y)] - c \cdot K.$$
(4)

Her problem is similar to that of a price-setting newsvendor.

We now characterize the manufacturer's optimal decisions. Let

$$\delta(r, \rho, L, c) = \frac{2(r+\rho-2r\rho)}{r(1+r-2r\rho)} \left(\frac{1+r+\rho-2r\rho}{1-r-\rho+2r\rho}c - L\right)^+.$$

Proposition 1. The manufacturer's optimal wholesale price $w_{N_i}^s$, stock level $K_{N_i}^s$, and expected profit $\Pi_{N_i}^s$, are as follows:

(i) If
$$H - L \le \delta$$
, then $w_N^s = \frac{1}{2}(\hat{L} + c)$, $K_N^s = \frac{1}{4}(\hat{L} - c)$, and $\Pi_N^s = \frac{1}{8}(\hat{L} - c)^2$;
(ii) If $H - L > \delta$, then $w_N^s = \frac{1}{2}(\overline{A} + c)$, $K_N^s = \frac{1}{4}(2\hat{H} - \overline{A} - c)$, and $\Pi_N^s = \frac{1}{8}(\overline{A}^2 - c^2) - \frac{1}{4}(2\hat{H} - \overline{A} - c)c$.

Note that $\hat{L} > w_N^s$ for $H - L > \delta$ if and only if $H - L < \eta$. Note also that $\eta \ge \delta$ if and only if $c \le c_{\max}$ where

$$c_{\max} = \begin{cases} L & \text{if } \rho \leq \frac{2-r}{3-2r} \\ \frac{3(2\rho-1)(1-r)(1-r-\rho+2r\rho)}{2(2r-1)(r-3)\rho^2 + (11r-4r^2+1)\rho + (r+1)(r-3)}L & \text{if } \rho > \frac{2-r}{3-2r}. \end{cases}$$

It can be shown that $c_{\max} \le L$. As we have assumed $H - L \le \eta$ throughout the paper, we will not mention this condition again unless required by exposition clarity. For example, in Proposition 1, $H - L \le \delta$ means $H - L \le \min[\delta, \eta]$ and $H - L > \delta$ means $\delta < H - L < \eta$. In case the unit production cost $c_{\max} < c \le L$ (i.e., $\delta > \eta$) the manufacturer's optimal decisions are always given by part (i) of Proposition 1.

Given (w_N^s, K_N^s) , the retailer's order quantity is

$$q_N^{s}(Y) = \begin{cases} \frac{1}{4}(\hat{L}-c) & \text{if } H-L \le \delta\\ \frac{1}{4}(2E(\tilde{A}|Y)-\overline{A}-c) & \text{if } H-L > \delta. \end{cases}$$

The retailer's decision depends on (w_N^s, K_N^s) . Fixing r, ρ, L , and c, a greater H means greater market dispersion. When the market dispersion is small, the retailer orders the same quantity for the high and low signals, both equal to the manufacturer's stock level, $q_N^s(l) = q_N^s(h) = K_N^s$. When the market dispersion is large, the retailer orders less for the low demand signal than for the high demand signal, i.e., $q_N^s(l) < q_N^s(h) = K_N^s$.

Remark 1. When the market dispersion is small, the manufacturer builds up a relatively low stock level such that even the low demand order quantity uses up the stock. We dub this as the manufacturer's *conservative strategy*. When the market dispersion is large, the high demand market is very attractive and the manufacturer builds up enough stock to fulfill the high demand order quantity, and it is more than the low demand order quantity. We dub this as the manufacturer's *ambitious strategy*. Note that the threshold δ is increasing in *c*. As the unit production cost *c* increases (fixing ρ , *r*, *H*, *L*), the situation may change from $H-L > \delta$ to $H-L \leq \delta$, and the manufacturer may switch from the ambitious to the conservative strategy.

Corollary 1. The optimal stock level K_N^s decreases in c.

Corollary 2. If $H-L \leq \delta(r, \rho, L, c_{\max})$, the manufacturer's optimal wholesale price w_N^s increases in *c*, with the exception of a one-time shift downward at \hat{c} where \hat{c} is uniquely determined by $H-L = \delta(r, \rho, L, \hat{c})$.

Surprisingly, the manufacturer's stock level and her wholesale price may *both* decrease as *c* increases (Corollaries 1 and 2). It may seem intuitive that when the manufacturer has less stock, she should raise her wholesale price to increase the margin. However, when the demand is discrete, the manufacturer may sharply switch from a more optimistic strategy that meets the high demand order to a more pessimistic strategy that meets only the low demand states. When such a switch is made, an increase in wholesale price is not optimal because it will cause leftover even for the low demand states. Therefore, w_N^S and K_N^S jump downwards at the same time when *c* crosses a certain threshold, as Fig. 1 shows. Although we have used a two-point distribution, we believe that similar result can be obtained for some other discrete



Parameter: $H = 10, L = 6, r = 1/2, \rho = 4/5$

Fig. 1. The optimal w_N^s and K_N^s for different *c* values without information sharing.

distributions. It is interesting to compare the result here with the existing studies on price-setting newsvendor, where the price always changes in the opposite direction with the stock level under certain conditions (e.g., [47,16,43]).

We next analyze how the firms' expected profits, when no information is shared, vary with the precision of the retailer's demand signal.

Corollary 3. The manufacturer's optimal expected profit Π_N^s is decreasing in the signal precision ρ .

Intuitively, when the retailer has a more precise signal but does not share it, the manufacturer suffers from greater informational disadvantage.

The retailer's expected profit is given by

$$\pi_{N}^{s} = \begin{cases} \frac{1}{16} (\hat{L} - c)^{2} + \frac{1}{4} \lambda_{h} (\hat{H} - \hat{L}) (\hat{L} - c) & \text{if } H - L \leq \delta \\ \frac{1}{16} \lambda_{h} (2\hat{H} - \overline{A} - c)^{2} + \frac{1}{16} \lambda_{l} (2\hat{L} - \overline{A} - c)^{2} & \text{if } H - L > \delta. \end{cases}$$

We can show that π_N^s is increasing in ρ for $H-L > \delta$, i.e., when the manufacturer adopts the ambitious strategy, the retailer earns higher profit with a more precise demand signal. However, because a change in ρ may change the manufacturer's strategy, a more precise demand signal may hurt the retailer under certain conditions.

Corollary 4. (i) π_{N}^{s} is increasing in ρ when ρ is close to $\frac{1}{2}$; (ii) π_{N}^{s} is decreasing in ρ when ρ is close to 1 if and only if $(L-c)/2r < H-L < \delta(r, 1, L, c)$.

As we have mentioned, when the manufacturer adopts the ambitious strategy (i.e., $H - L > \delta$), the retailer's better knowledge of the demand brings him greater profit. When the manufacturer uses the conservative strategy (i.e., $H - L \le \delta$), by Proposition 1, her decision is determined by \hat{L} (which is decreasing in ρ). An increase in ρ has three effects on the retailer. First, the wholesale price is lower, which is positive to the retailer; second, the stock level is smaller, which is negative to the retailer; third, the retailer is better informed. Without information sharing, the retailer has an informational advantage which has a greater impact on his profitability when his demand signal is more accurate and the market uncertainty is larger. However, because the manufacturer's stock level just meets the low demand under the conservative strategy, the retailer suffers from lost sales if the demand is high. Thus the second effect becomes very significant when the signal is already very accurate (ρ close to 1) and the high demand is appropriately attractive (H-L > (L-c)/2r). Corollary 4 shows that, when information is not shared, the retailer's more accurate demand signal does *not* necessarily translate into higher profits for him, in contrast to the case when information is shared (Lemma 4).

4.3. Information sharing arrangement

Proposition 2. $\Pi_{S}^{s} > \Pi_{N}^{s}$, *i.e.*, *information sharing always makes the manufacturer better off.*

Information sharing enables the manufacturer to adjust wholesale price and stock level decisions according to updated demand information.

Proposition 3. Sharing information is a dominant strategy for the retailer if and only if $\rho > \hat{\rho}$ and $\tau < H - L < \delta$, where

$$\tau = \frac{2(r+\rho-2r\rho)(1-r-\rho+2r\rho)}{r(6\rho-2\rho^2-3-(2\rho-1)(3-2\rho)r)}(L-c)$$

and $\hat{\rho} = \hat{\rho}(r) = (4r - 3 + \sqrt{4r^2 - 6r + 3})/2(2r - 1) > (3 - \sqrt{3})/2.$

Note that $\tau < \eta$ if and only if $\rho \le (2-r)/(3-2r)$ or $\rho > (2-r)/(3-2r)$ and $r < \frac{1}{2}$. When the manufacturer makes to stock, the retailer has an incentive to share information when the demand signal is precise enough and the magnitude of the uncertainty in demand (indicated by the market dispersion H-L) is intermediate. We note that $\rho > \hat{\rho}$ holds only when $\rho > \frac{3-\sqrt{3}}{2}$.

Whether the retailer is willing to share information depends on the manufacturer's strategy when information is not shared, which in turn depends on the market dispersion.

When the market dispersion is large $(H-L \ge \delta)$, the high demand is very attractive and the manufacturer adopts the ambitious strategy (Remark 1). In this case, the retailer is always hurt by sharing information, because the manufacturer takes away most of the earnings from the high demand market.

When the market dispersion is small (i.e., $H-L < \delta$), the manufacturer adopts the conservative strategy (Remark 1). That is, without sharing information, the retailer risks stocking out. Information sharing has two countervailing effects on the retailer. The upside is that it eliminates stock-out. The downside is that it induces a higher wholesale price for the high demand market. The net effect is negative if the market dispersion is too small (i.e., $H-L \le \tau$), as meeting the high demand does not add much value and the retailer would like to conceal his demand signal to keep his informational advantage. But if the market dispersion is intermediate (i.e., $\tau < H < \delta$), the positive side dominates the negative as meeting the high demand brings greater value.

Remark 2. $\tau(r, \rho, L, c)$ is decreasing in *c* while $\delta(r, \rho, L, c)$ is increasing in *c*. That is, as the unit production cost *c* increases, the retailer may switch from not sharing information to sharing information.

Corollary 5. *If the retailer has perfect demand information, i.e.,* $\rho = 1$, *he has an incentive to share information with a make-to-stock manufacturer if and only if* $2(L-c) < H-L < 2((2-r)c-rL)/r^2$.

Apparently, the manufacturer's mode of production (make-toorder or make-to-stock) has a salient effect on the retailer's information sharing decision. When a manufacturer makes to order (and the retailer's order is always met in full), the wholesale price is the key driver in the retailer's information sharing decision and he never has an incentive to share information with the manufacturer (Lemma 1). However, when the manufacturer has to decide on both the wholesale price and the stock level, the retailer may like to share the demand information to avoid stock out at the high demand even though he has to pay a high wholesale price.

The retailer does not share information with the make-to-stock manufacturer when his signal is of very low quality (Proposition 3). But if the retailer has an incentive to share the information, how does the benefit of information sharing vary with the signal precision? It turns out that the benefit does not have clear monotonicity with respect to the signal precision ρ . Nevertheless, we can show that a more accurate demand signal makes information sharing more valuable to the retailer if the signal is already very informative.

Proposition 4. The retailer's benefit from information sharing, $\pi_{s}^{s} - \pi_{N}^{s}$, is increasing in the signal precision ρ when ρ is close to 1.

Proposition 4 again shows the significant impact of the manufacturer's production mode on the information sharing arrangement. When facing a make-to-order manufacturer, sharing information hurts the retailer more as the signal becomes more precise (Lemma 3). In contrast, when facing a make-to-stock manufacturer, sharing information may benefit the retailer more as the signal becomes more precise (Proposition 4).

5. Information sharing through payment

By Proposition 2 the manufacturer always benefits from information, but by Proposition 3 information sharing may not be achieved. In this section, we examine whether the manufacturer can pay the retailer for his private information and make both firms better off. Suppose that the manufacturer can decide whether to invest in information sharing in the first stage. Specifically, in the first stage, the manufacturer may negotiate with the retailer to buy information from him and commit to him a lump sum payment. Such payment arrangement is possible only if the supply chain is better off by information sharing. Otherwise, the manufacturer would not have enough gain to compensate the retailer for his loss because of information sharing.

Proposition 5. The supply chain profit is higher with information sharing than without, if and only if $H-L \le \max[\delta, \zeta]$ where $\zeta(r, \rho, L, c) = (8(r+\rho-2r\rho)/r(2\rho-1)(1-r))c$.

Note that $\eta > \zeta$ if and only if $\rho \le (2-r)/(3-2r)$ or $\rho > (2-r)/(3-2r)$ and $c < ((2\rho - 1)(1-r)/(26\rho - 17 - (2\rho - 1)9r))L$. Information sharing never benefits the supply chain when the manufacturer is make-to-order (Lemma 2) but may benefit the supply chain when the manufacturer is make-to-stock. Information sharing has three effects on the supply chain performance. First, it enables the manufacturer to build up enough stock when the demand is high (i.e., it avoids the manufacturer using the conservative strategy). Second, it enables the manufacturer to set a low wholesale price to

guarantee a certain level of sales when the demand is low. Third, it causes a high wholesale price for a high demand, reduces the order quantity, and aggravates the double marginalization problem. The first two effects are positive to the supply chain, while the last effect is negative. When the high demand is not attractive at all, the first effect is negligible. When the high demand is attractive, the third effect dominates the first two positive effects.

We observe that δ and ζ are both increasing in *c*. Thus, as the manufacturer's unit production cost increases, the impact of sharing information may change over from harming the supply chain to benefiting it.

Immediately following Proposition 5, we have the corollary below regarding the payment arrangement to achieve information sharing.

Corollary 6. If $H-L \le \max[\delta, \zeta]$, there exists a payment χ such that the manufacturer pays the retailer χ and information sharing is achieved. Both the manufacturer and the retailer benefit from this payment-for-information arrangement.

6. Discussion

Our model has assumed a binary distribution for the uncertain market potential and the signal. A natural question is whether our results still hold for more general distributions. Assume that the market potential \tilde{A} follows a continuous distribution with mean μ and variance σ^2 . The retailer observes a signal Y about \tilde{A} . The joint probability distribution of (\tilde{A}, Y) satisfies two conditions: (C1) $E(Y|\tilde{A}) = \tilde{A}$, that is, the signal Y is an unbiased estimator of \tilde{A} ; and (C2) $E(\tilde{A}|Y) = a + kY$, where *a* and *k* are constants. The information structure implied by conditions (C1) and (C2) is general enough to include a variety of Bayesian updating structure with conjugate prior-posterior pairs such as the Normal-Normal, the Gamma-Poisson, and the Beta-Binomial pairs. The absolute precision of signal Y is $u = 1/E[Var(Y|\tilde{A})]$ and $t = \sigma^2/E[Var(Y|\tilde{A})]$ is the precision of Y relative to the market fluctuation. It can be shown that $E(\tilde{A}|Y) = (1/(1+t))\mu + (t/(1+t))Y$ and $E_Y[E(\tilde{A}|Y)]^2 =$ $(t/(1+t))\sigma^2 + \mu^2$.

Under such a structure of imperfect information, the retailer never shares information with a make-to-order manufacturer (e.g., [33]). We only need consider the case when the manufacturer makes to stock. Given the manufacturer's wholesale price w and stock level K, the retailer's optimal order quantity is given by (1).

If the signal *Y* is shared, the manufacturer's optimal wholesale price and stock level are given by (2), which leads to the retailer's optimal order quantity in equilibrium (3). The expected profits for the manufacturer and the retailer are, respectively,

$$\Pi_{S}^{s} = \frac{1}{8} E_{Y}[(E(\tilde{A}|Y) - c)^{2}] = \frac{1}{8} \left[(\mu - c)^{2} + \frac{t}{1 + t} \sigma^{2} \right]$$

and

$$\pi_{S}^{s} = \frac{1}{16} E_{Y}[(E(\tilde{A}|Y) - c)^{2}] = \frac{1}{16} \left[(\mu - c)^{2} + \frac{t}{1 + t} \sigma^{2} \right]$$

If the signal Y is not shared, the manufacturer chooses (*w*,*K*) to maximize her expected profit (4). For a variety of conjugate pairs and parameters, we numerically calculate the optimal (w_N^s, K_N^s) , and the manufacturer's ex ante profit Π_N^s . Then we use (w_N^s, K_N^s) to obtain the retailer's ex ante profit:

$$\pi_N^{s} = E_Y[\pi(Y)] = E_Y[q^{s}(w_N^{s}, K_N^{s}, Y) \cdot (E(A|Y) - q^{s}(w_N^{s}, K_N^{s}, Y) - w_N^{s})].$$

Fig. 2 shows when the retailer benefits from sharing information with the make-to-stock manufacturer. The graphs are constructed for the Gamma-Poisson conjugate pair where the mean demand is fixed at $\mu = 10$. The vertical axis is the unit production



Parameter: $\mu = 10$, Gamma-Poisson

Fig. 2. The retailer's information sharing arrangement.



Fig. 3. The optimal w_N^s and K_N^s for different *c* values without information sharing.

cost *c*, while the horizontal axis in the left (right) graph is the standard deviation of the demand σ (the signal precision *t*). We assume that σ is small relative to μ . Note that larger σ implies greater *t*. In the graphs, the shaded areas are where the retailer has incentives to share information. Thus, Fig. 2 shows that the retailer voluntarily shares information when the demand variance is reasonably large (the left graph) and when the signal precision is relatively high (the right graph). These results are parallel to Proposition 3. We also observe that when the unit production cost *c* increases, the retailer may switch from not sharing information. This is parallel to Remark 2.

Fig. 3 is based on Normal–Normal conjugate pair, with $\mu = 10$, $\sigma = 2$, and t=4. It shows that when *c* increases, the manufacturer's optimal wholesale price w_N^s and stock level K_N^s always vary in the opposite direction, which is different from Corollary 2. This is because the demand follows a continuous distribution. In particular, Fig. 3 contrasts with Fig. 1 in which the demand follows a discrete distribution and the manufacturer's optimal wholesale price and stock level jump downwards at the same time when crossing a certain unit cost.

We have also assumed that Pr(h|H) = Pr(l|L). This is for ease of exposition. In fact, our results holds qualitatively if we relax this assumption. Specifically, if $Pr(h|H) \neq Pr(l|L)$ then either Pr(h|H) (fixing Pr(l|L)) or Pr(l|L) (fixing Pr(h|H)) can be an indicator of the precision of the demand signal (e.g., [4]). We numerically find that, for example, the retailer has an incentive to share the demand information with the make-to-stock manufacturer when the demand signal is precise and the magnitude of the uncertainty in demand is intermediate.

Following most extant studies on ex ante information sharing, we have assumed that the retailer places an order before the demand state is completely realized. If the retailer orders after the demand uncertainty is resolved, there is still information asymmetry between the manufacturer and the retailer even if an information sharing link is established. Therefore, the analysis for the make-toorder case (the make-to-stock case), both when information is shared and when information is not shared, should be similar to Section 3.2.2 (Section 4.2.2) where information asymmetry exists. Specifically, for the make-to-order case with information sharing, the solutions are obtained by simply replacing in Section 3.2.2 the manufacturer's expectation of the demand $E(\tilde{A})$ with the updated expectation $E(\tilde{A}|Y)$, and the retailer's expectation of the demand $E(\tilde{A}|Y)$ with the realization of the demand \tilde{A} ; for make-to-order case without information sharing, the solutions are obtained by considering the perfect information extreme case ($\rho = 1$) in Section 3.2.2. Similarly, the make-to-stock case analysis can be easily conducted by making corresponding changes in Section 4.2.2.

7. Conclusion

This paper studies information sharing in a supply chain when the upstream manufacturer makes both the wholesale price and the stock level decisions. The analyses highlight the interdependence between retailer's information sharing strategy and the manufacturer's operational and marketing decisions. The results have important managerial insights into information sharing arrangement and firms' profitability. Unlike the classical information sharing arrangement where the manufacturer only makes the pricing decision, we show that the retailer may change over his strategy when the manufacturer makes both the pricing and the stock decisions. When the magnitude of the uncertainty in demand is intermediate, the retailer voluntarily share information with the make-to-stock manufacturer. This helps him avoiding the manufacturer's conservative strategy where only very limited stock is secured.

A more precise signal hurts the retailer more if he shares it with a make-to-order manufacturer, but it may benefit him more if he shares it with a make-to-stock manufacturer.

If the manufacturer only makes pricing decision, sharing information can never make the supply chain better off. In contrast, if the manufacturer makes both the pricing and the stock decisions, the supply chain can benefit from information sharing. This is because information sharing helps the supply chain build enough stock to avoid missing an optimistic demand market.

As for future studies, it would be interesting to incorporate stock constraints into information sharing under different channel structures, (e.g., chain-to-chain competition or competition among multiple retailers), and explore how the information sharing strategy will change with competition. Another possible direction of future research is to consider bilateral information exchange between a manufacturer and a retailer, where both firms may possess imperfect demand signals. We have assumed that the retailer can obtain the information at no cost. In reality, acquiring information can be costly, and thus it would be interesting to investigate how information acquisition cost affects the information sharing arrangement.

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Appendix A. Proofs

Proof of Lemma 1. The difference between the retailer's ex ante profit with information sharing and that without is

$$\pi_{S}^{0} - \pi_{N}^{0} = -\frac{3}{16} \frac{r^{2}(2\rho - 1)^{2}(1 - r)^{2}((H - L)^{2})}{(r + \rho - 2r\rho)(1 - r - \rho + 2r\rho)}.$$

Hence, $\pi_{S}^{0} \leq \pi_{N}^{0}$, where the strict inequality holds for $\rho > \frac{1}{2}$.

Proof of Lemma 2. Taking the difference between the supply chain's ex ante profits with and without information sharing, we have

$$\Pi_{S}^{0} + \pi_{S}^{0} - (\Pi_{N}^{0} + \pi_{N}^{0}) = -\frac{1}{16} \frac{r^{2}(2\rho - 1)^{2}(r - 1)^{2}(H - L)^{2}}{(r + \rho - 2r\rho)(1 - r - \rho + 2r\rho)^{2}}$$

Hence, $\Pi_S^o + \pi_S^o \le \Pi_N^o + \pi_N^o$, where the strict inequality holds for $\rho > \frac{1}{2}$. \Box

Proof of Lemma 3. Taking the first order derivative on $π_N^o - \pi_S^o$ with respect to *ρ*, we have

$$\frac{d}{d\rho}(\pi_N^0 - \pi_S^0) = \frac{3}{16} \frac{r^2(H - L)^2(2\rho - 1)(1 - r)^2}{(r + \rho - 2r\rho)^2(1 - r - \rho + 2r\rho)^2} > 0.$$

Taking the first order derivative on $\Pi_N^0 + \pi_N^0 - (\Pi_S^0 + \pi_S^0)$ with respect to ρ , we have

$$\frac{d}{d\rho}[\Pi_N^o + \pi_N^o - (\Pi_S^o + \pi_S^o)] = \frac{1}{16} \frac{r^2 (H-L)^2 (2\rho-1)(r-1)^2}{(r+\rho-2r\rho)^2 (1-r-\rho+2r\rho)^2} > 0. \quad \Box$$

Proof of Proposition 1. The manufacturer's expectation of the order quantity is

$$E_{Y}[q^{s}(w, K, Y)] = E_{Y}\left\{\min\left[\frac{E(\tilde{A}|Y) - w}{2}, K\right]\right\}$$
$$= \begin{cases} K & \text{if } \frac{\hat{L} - w}{2} \ge K\\ \frac{\overline{A} - w}{2} & \text{if } \frac{\hat{H} - w}{2} \le K\\ \lambda_{l} \frac{\hat{L} - w}{2} + \lambda_{h} K & \text{if } \frac{\hat{L} - w}{2} < K < \frac{\hat{H} - w}{2}. \end{cases}$$

She maximizes her expected profit

 $\Pi_N(w, K) = w \cdot E_Y[q^s(w, K, Y)] - c \cdot K.$

If the manufacturer sets $(\hat{L} - w)/2 \ge K$, her expected profit becomes

 $\Pi_N(w,K) = (w-c) \cdot K,$

which leads to the optimal solution $w_1 = \frac{1}{2}(\hat{L} + c)$ and $K_1 = \frac{1}{4}(\hat{L} - c)$. The manufacturer's corresponding expected profit for segment $(\hat{L} - w)/2 \ge K$ is $\Pi_{N1} = \frac{1}{8}(\hat{L} - c)^2$.

If the manufacturer sets $(\hat{H} - w)/2 \le K$, her expected profit becomes

$$\Pi_N(w,K) = w \cdot \frac{\overline{A} - w}{2} - c \cdot K,$$

which leads to the optimal solution $w_2 = \frac{1}{2}(\overline{A} + c)$ and $K_2 = \frac{1}{4}(2\hat{H} - \overline{A} - c)$. The manufacturer's corresponding expected profit for segment $(\hat{H} - w)/2 \le K$ is $\Pi_{N2} = \frac{1}{8}(\overline{A}^2 - c^2) - \frac{1}{4}(2\hat{H} - \overline{A} - c)c$.

If the manufacturer sets $(\hat{L} - w)/2 < K < (\hat{H} - w)/2$, her expected profit becomes

$$\Pi_N(w,K) = (w\lambda_h - c)K + \lambda_l w \frac{\hat{L} - w}{2}.$$

If she sets $w \ge c/\lambda_h$, then the optimal solution is $w_3 = \max\left\{\frac{1}{2}(\overline{A}+c), c/\lambda_h\right\}$ and $K_3 = \min\left\{\frac{1}{4}(2\hat{H}-\overline{A}-c), \frac{1}{2}(\hat{H}-c/\lambda_h)\right\}$. The manufacturer's corresponding expected profit $\Pi_{N3} < \Pi_{N2}$. If she sets $w < c/\lambda_h$, then the optimal solution is $w_4 = \frac{1}{2}(\hat{L}+c)$ and $K_4 = \frac{1}{4}(\hat{L}-c)$. The manufacturer's corresponding expected profit is $\Pi_{N4} = \frac{1}{8}(\hat{L}-c)^2$.

Thus, the optimal wholesale price and stock level can be obtained by comparing the expected profits for the two segments $(\hat{L} - w)/2 \ge K$ and $(\hat{H} - w)/2 \le K$.

We can show that $\frac{1}{8}(\hat{L}-c)^2 > \frac{1}{8}(\overline{A}^2-c^2) - \frac{1}{4}(2\hat{H}-\overline{A}-c)c$ if and only if $H-L < \delta(r,\rho,L,c) = (2(r+\rho-2r\rho)/r(1+r-2r\rho))(((1+r+\rho-2r\rho))/(1-r-\rho+2r\rho))c-L)^+$. Therefore, if $H-L \le \delta$, then the manufacturer's optimal decisions are $w_N^s = \frac{1}{2}(\hat{L}+c)$ and $K_N^s = \frac{1}{4}(\hat{L}-c)$, and the corresponding optimal expected profit is $\Pi_N^s = \frac{1}{8}(\hat{L}-c)^2$; if $H-L > \delta$, then the manufacturer's optimal decisions are $w_N^s = \frac{1}{2}(\overline{A}+c)$ and $K_N^s = \frac{1}{4}(2\hat{H}-\overline{A}-c)$, and the corresponding optimal expected profit is $\Pi^s = \frac{1}{4}(2\hat{H}-\overline{A}-c) = \frac{1}{4}(2\hat{H}-\overline{A}-c)c$

ing optimal expected profit is $\Pi_N^s = \frac{1}{8}(\overline{A}^2 - c^2) - \frac{1}{4}(2\hat{H} - \overline{A} - c)c$. It can be easily verified that $\frac{1}{2}(\overline{A} + c) < \hat{L}$ if and only if $\lambda_h(\hat{H} - \hat{L}) < \hat{L} - c$, i.e., $H - L < \eta$. \Box

Proof of Corollary 2. When *c* equals to zero, δ is zero and $H-L > \delta(r, \rho, L, 0)$. When *c* equals to c_{\max} , $H-L \le \delta(r, \rho, L, c_{\max})$. Recall that δ is increasing in *c*. Thus there is a unique *c* such that $H-L = \delta(r, \rho, L, c)$. When *c* increases, the optimal solution jumps

from the case with $H-L > \delta$ to the case with $H-L \le \delta$, and the optimal wholesale price jumps downwards from $\frac{1}{2}(\overline{A}+c)$ to $\frac{1}{2}(\hat{L}+c)$. It is easy to verify that $\frac{1}{2}(\overline{A}+c) > \frac{1}{2}(\hat{L}+c)$ for all $\rho > \frac{1}{2}$. \Box

Proof of Corollary 3. If $H-L \le \delta$, it is easy to show that Π_N^s is decreasing in ρ because $d\hat{L}/d\rho = -r(H-L)(1-r)/(r+\rho-2r\rho)^2 < 0$. If $H-L > \delta$, we have $(d/d\rho)\Pi_N^s = -cr(H-L)(1-r)/2(1-r-\rho+2r\rho)^2 < 0$. Note also that at $H-L = \delta$ we have $\frac{1}{8}(\hat{L}-c)^2 = \frac{1}{8}(\overline{A}^2-c^2) - \frac{1}{4}(2\hat{H}-\overline{A}-c)c$, and the result follows. \Box

Proof of Corollary 4. It can be easily shown that if $H - L > \delta$,

$$\frac{d}{d\rho}\pi_N^{\rm s} = \frac{r^2(H-L)^2(2\rho-1)(1-r)^2}{4(r+\rho-2r\rho)^2(1-r-\rho+2r\rho)^2} > 0.$$

We only need to consider the case $H - L \le \delta$. If $H - L \le \delta$, we have

$$\lim_{\rho \to 1/2} \left(\frac{d}{d\rho} \pi_N^{\rm s} \right) = \frac{r(H-L)(1-r)(L-c+r(H-L))}{2} > 0$$

and

$$\lim_{\rho \to 1} \left(\frac{d}{d\rho} \pi_N^s \right) = \frac{r(H - L)(L - c - 2Hr + 2Lr)}{8(1 - r)}$$

which is negative if and only if H-L > (L-c)/2r. Note that $\eta(r, \frac{1}{2}, L, c) = \infty$ and $\eta(r, 1, L, c) = (L-c)/r$. When ρ is close to 1, the condition $H-L \le \delta(r, \rho, L, c)$ becomes $H-L \le \delta(r, 1, L, c)$.

Proof of Proposition 2. If the retailer shares information, the manufacturer's ex ante expected profit is

$$\Pi_{S}^{s} = \frac{1}{8}\lambda_{h}(\hat{H} - c)^{2} + \frac{1}{8}\lambda_{l}(\hat{L} - c)^{2},$$

while if the retailer does not share information, the manufacturer's ex ante expected profit is

$$\Pi_{N}^{s} = \begin{cases} \frac{1}{8} (\hat{L} - c)^{2} & H - L \le \delta \\ \frac{1}{8} (\overline{A}^{2} - c^{2}) - \frac{1}{4} (2\hat{H} - \overline{A} - c)c & H - L > \delta \end{cases}$$

If $H-L \le \delta$, obviously we have $\Pi_S^s > \Pi_N^s$. If $H-L > \delta$, the difference of the profits is $\Pi_S^s - \Pi_N^s = \frac{1}{8}(1-\lambda_h)(\hat{H}-\hat{L})(4c+\lambda_h(\hat{H}-\hat{L})) > 0.$

Proof of Proposition 3. If the retailer shares information in the first stage, his ex ante expected profit is

$$\pi_{S}^{s} = \frac{1}{16} \lambda_{h} (\hat{H} - c)^{2} + \frac{1}{16} \lambda_{l} (\hat{L} - c)^{2};$$

if he does not share information in the first stage, his ex ante expected profit is

$$\pi_{N}^{s} = \begin{cases} \frac{1}{16} (\hat{L} - c)^{2} + \frac{1}{4} \lambda_{h} (\hat{H} - \hat{L}) (\hat{L} - c) & \text{if } H - L \leq \delta \\ \frac{1}{16} \lambda_{h} (2\hat{H} - \overline{A} - c)^{2} + \frac{1}{16} \lambda_{l} (2\hat{L} - \overline{A} - c)^{2} & \text{if } H - L > \delta. \end{cases}$$

We can show that when $H-L > \delta$, $\pi_S^s < \pi_N^s$ always holds. In this case, no information sharing is the unique equilibrium outcome. When $H-L \le \delta$, we have

$$\pi_{S}^{s} - \pi_{N}^{s} = \frac{1}{16} \lambda_{h} (\hat{H} - \hat{L}) (2c + \hat{H} - 3\hat{L}).$$

Therefore, sharing information is the unique equilibrium outcome if and only if $\hat{H} > 3\hat{L} - 2c$.

Let $M_2 = r((2\rho - 1)(2\rho - 3)r + 6\rho - 2\rho^2 - 3)H - (1 - r)((4\rho^2 - 1)r + 2\rho(1 - \rho))L + 2c(r + \rho - 2r\rho)(1 - r - \rho + 2r\rho)$. We have $\hat{H} > 3\hat{L} - 2c$ if and only if $M_2 > 0$. If $\rho > \hat{\rho}(r) = (4r - 3 + \sqrt{4r^2 - 6r + 3})/2(2r - 1)$, then $M_2 > 0$ if and only if $H - L > \tau$ $(r, \rho, L, c) = (2(r + \rho - 2r\rho))((1 - r - \rho + 2r\rho)/(-(2\rho - 1)(3 - 2\rho)r + (6\rho - 2\rho^2 - 3))r)(L - c)$; if $\rho < \hat{\rho}(r)$, then $M_2 > 0$ if and only if $H - L < \tau$; if $\rho = \hat{\rho}(r)$, then $M_2 < 0$ for all r < 1 and L > c.

Note that for $\rho > \hat{\rho}(r)$ we have $\tau > 0$, and that for $\rho < \hat{\rho}(r)$ we have $\tau < 0$. Therefore, $\hat{H} > 3\hat{L} - 2c$ if and only if $\rho > \hat{\rho}(r)$ and $H - L > \tau$. Noting $\hat{\rho}(r)$ increasing in r, we have $\hat{\rho}(r) > \hat{\rho}(0) = \frac{3 - \sqrt{3}}{2}$. It can be shown that $\tau < \eta$ if and only if $\rho \le (2 - r)/(3 - 2r)$ or $\rho > (2 - r)/(3 - 2r)$ and $r < \frac{1}{2}$. When $\rho > \hat{\rho}$, $\tau < \delta$ holds if and only if $c > \tilde{c}$, where $\tilde{c} = ((2\rho - 1)(r - 1)(2 - r - \rho + 2r\rho)(1 - r - \rho + 2r\rho)/(2(r + 1)(2r - 1)^2\rho^3 - (3r + 5)(2r - 1)^2\rho^2 + (6r^3 + 8r^2 - 9r - 1)\rho + (1 - r)(r + 2)(r + 1)))L$. It can be shown that $((1 - r - \rho + 2r\rho)/(1 + r + \rho - 2r\rho))L < \tilde{c} < L$.

Proof of Proposition 4. From the proof of Proposition 3, we have

$$\pi_{S}^{s} - \pi_{N}^{s} = \frac{1}{16} \lambda_{h} (\hat{H} - \hat{L}) (2c + \hat{H} - 3\hat{L}).$$

Take the first order derivative with respect to ρ , and derive the limit when ρ goes to 1

$$\lim_{\rho \to 1} \left(\frac{d}{d\rho} (\pi_{\rm S}^{\rm S} - \pi_{\rm N}^{\rm S}) \right) = \frac{(H - L)((1 - r + 4r^2)(H - L) - 2r(L - c))}{16(1 - r)}$$

Note that by Corollary 5, $\pi_S^s > \pi_N^s$ if and only if $2(L-c) < H-L < 2((2-r)c-rL)/r^2$. Thus the numerator is positive, i.e., $(1-r+4r^2)(H-L)-2r(L-c) > 2(1-2r+4r^2)(L-c) > 0$.

Proof of Proposition 5. If the retailer shares information, the ex ante expected profit of the supply chain is

$$\pi_{S}^{s} + \Pi_{S}^{s} = \frac{3}{16} \lambda_{h} (\hat{H} - c)^{2} + \frac{3}{16} \lambda_{l} (\hat{L} - c)^{2};$$

if the retailer does not share information, the ex ante expected profit of the supply chain is

$$\pi_{N}^{s} + \Pi_{N}^{s} = \begin{cases} \frac{3}{16} (\hat{L} - c)^{2} + \frac{1}{4} \lambda_{h} (\hat{H} - \hat{L}) (\hat{L} - c) & \text{if } H - L \le \delta \\ \frac{\lambda_{h} (2\hat{H} - \overline{A} - c)^{2}}{16} + \frac{\lambda_{l} (2\hat{L} - \overline{A} - c)^{2}}{16} + \frac{\overline{A}^{2} - c^{2}}{8} - \frac{c(2\hat{H} - \overline{A} - c)}{4} & \text{if } H - L > \delta \end{cases}$$

If $H-L \le \delta$, it is easy to verify that $\pi_{S}^{s} + \Pi_{S}^{s} > \pi_{N}^{s} + \Pi_{N}^{s}$ for all H > L. If $H-L > \delta$, it is easy to verify that $\pi_{S}^{s} + \Pi_{S}^{s} > \pi_{N}^{s} + \Pi_{N}^{s}$ if and only if $H-L < \zeta(r, \rho, L, c) = (8(r+\rho-2r\rho)/r(2\rho-1)(1-r))c$. Note that $\eta > \zeta$ if and only if $\rho \le (2-r)/(3-2r)$ or $\rho > (2-r)/(3-2r)$ and $c < ((2\rho-1)(1-r)/(26\rho-17-(2\rho-1)9r))L$. It can be shown that $0 < (2\rho-1)(1-r)/(26\rho-17-(2\rho-1)9r) < 1$ for $\rho > (2-r)/(3-2r)$. Note also that $\zeta > \delta$ if and only if $c < ((2\rho-1)(1-r)(1-r-\rho+2r\rho)/(5\rho+20r^{2}\rho^{2}+7r\rho+2\rho^{2}-14r\rho^{2}-20r^{2}\rho+5r^{2}-5)^{+})L$.

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