Advances in Teaching the Time Value of Money

Terrance Jalbert, University of Hawaii at Hilo
Mercedes Jalbert, Jayco Travel and Tours
Canri Chan, Monterey Institute of International Studies

ABSTRACT

Students frequently experience difficulty in identifying the appropriate time value of money (TVM) technique to apply to a TVM problem. This paper offers a modification of a recently published TVM technique developed by Jalbert (2002) in order to help students understand and solve TVM problems. The modified technique developed here simplifies the previously developed technique by reducing the number of questions that students must examine to arrive at the appropriate TVM technique for a problem. In addition, the technique developed here eliminates the need for students to learn annuity techniques altogether. As such, the technique developed here specifically intends to meet the needs of students who experience difficulty in understanding annuities. Modified visual aids are provided to assist students in selecting correct techniques. By using these techniques, students will find it much easier to identify appropriate TVM techniques.

INTRODUCTION

Many techniques have been developed for presenting the time value of money (TVM). Despite this considerable effort on the part of instructors, students frequently experience difficulty in identifying the appropriate technique to solve a specific TVM problem (Eddy and Swanson, 1996). However, it is well known that a pedagogy, which works well with one audience, does not necessarily work well with another (Bloom, 1956). Thus, the development of new and different techniques that appeal to various audiences may prove beneficial. Students nearly unanimously experience difficulty in identifying the appropriate technique to apply to TVM problems. While the TVM issue is complex, some of the difficulty can be attributed to the approach that finance texts take to the issue. This contention is confirmed by Eddy and Swanson who argue that instructors do not sufficiently develop a frame of reference which begins with simple learning objectives focused on individual topics and progresses to higher levels of understanding (Eddy and Swanson, 1996). Jalbert (2002) surveys seven popular text books and identifies several areas of concern regarding the TVM presentations in these books. Jalbert (2002) addresses these concerns by providing precise definitions and visual aids.

 Specifically, the method developed in Jalbert (2002) attempted to appeal to students who benefit from precise definitions and visual aids. In this paper, we modify that technique to make it simpler and easier to utilize. The technique provides instructors a new tool in their arsenal to teach students TVM concepts. The paper begins by reviewing the technique proposed in Jalbert (2002). The paper then offers a simplification to that technique that makes it simpler to use. A summary of the technique developed in Jalbert (2002) is presented here.

THE JALBERT TECHNIQUE

The method developed in Jalbert (2002) requires students to answer four questions to determine which technique should be used to solve a time value of money problem. By answering these four questions, the appropriate TVM technique for any basic problem can be identified. The four questions as outlined in Jalbert (2002) are: 1) Is there a series of cash flows? 2) Is the number of cash flows limited? 3) Is there equal time spacing between each of the cash flows? and 4) Is the dollar amount of each cash flow equal?

The first characteristic examines whether there is a single cash flow or a series of cash flows. A single cash flow implies that there is only one cash flow, whereas a series of cash flows implies that there is more than one cash flow. An investment that promises to pay $100 per year, for example, at the end of each of the following three years is a series of cash flows. The investment provides three cash flows of $100 each. Alternatively, an investment that

promises to pay a single amount of $1,000, five years from today is not a series of cash flows. If a problem involves a single sum, it is not necessary to examine the remaining characteristics. The appropriate technique is to compute the present or future value of a single sum. Techniques for computing the present and future values of single sums are well known and are not discussed here. If a problem involves a series of cash flows, the remaining three characteristics must be examined to determine the appropriate TVM technique to apply to the problem.

The second characteristic examines whether there are a limited number of cash flows or if the cash flows continue into infinity. For example, an investment that promises to pay a $100 cash flow on January 1st of each of the next three years has a limited number of cash flows. An investment that pays a $100 cash flow on January 1st each year into infinity has an infinite number of cash flows. The third characteristic examines if there is an equal amount of time between each of the cash flows. This means that the cash flows must occur at some constant time interval such as one cash flow per month, or one cash flow per year. An investment that promises to pay us $100 on January 1st of each of the next three years has equal time spacing. There is a one-year equal time interval between each of the cash flows. On the other hand, an investment that promises to pay $100 on January 1st of the next year, nothing on January 1st of the second year, $100 on January 1st of the third year, and $100 on January 1st of the fourth year does not have equal time spacing. In this case, the interval between the first two cash flows is two years while the interval between the third and fourth is one year.

Finally, The fourth characteristic examines if each cash flow is an equal dollar amount or varying dollar amounts. An investment that promises to pay $100 per year at the end of each of the next three years has equal dollar amounts as each cash flow is for $100. An investment that promises to pay $100 the first year and $200 the second year does not have equal dollar amounts. While there are a series of cash flows, the cash flows are not for the same dollar amount. By examining these four characteristics, the appropriate technique to apply to any TVM problem can be identified.

Jalbert (2002) shows the appropriate TVM technique by answering these questions. Specifically, he develops a flow chart to identify the appropriate technique. The flow chart as developed in Jalbert (2002) appears here as Figure 1. Jalbert (2002) also offers a second visual aid for students to select appropriate TVM techniques. The table lists a summary of each characteristic for each technique. Jalbert's table appears here as Table 1.

One of the challenges of teaching the TVM has to do with students who experience problems in understanding the annuity techniques, distinguishing between an annuity due and an ordinary annuity, or selecting techniques to solve problems. This issue is compounded when students are required to deal with a deferred annuity. Jalbert (2002) addresses these issues by developing a figure that can be used to select the appropriate annuity technique.

Figure 2 shows the method as suggested by Jalbert (2002) suggested for identifying the appropriate annuity technique. Figure 2 shows that the value of any annuity can be computed at four different points on the time line: 1) one time period before the first cash flow (present value of an ordinary annuity, PVOA), 2) at the time of the first cash flow (present value of an annuity due, PVAD), 3) at the time of the last cash flow (future value of an ordinary annuity, FVOA), and 4) one time period following the last cash flow (future value of an annuity due, FVAD). Jalbert (2002) points out that letters are intentionally used in the figure to help students generalize the illustration to any time period. While not noted in Jalbert (2002), the techniques developed there are limited to situations where the interest rate remains fixed over the entire time period in question.
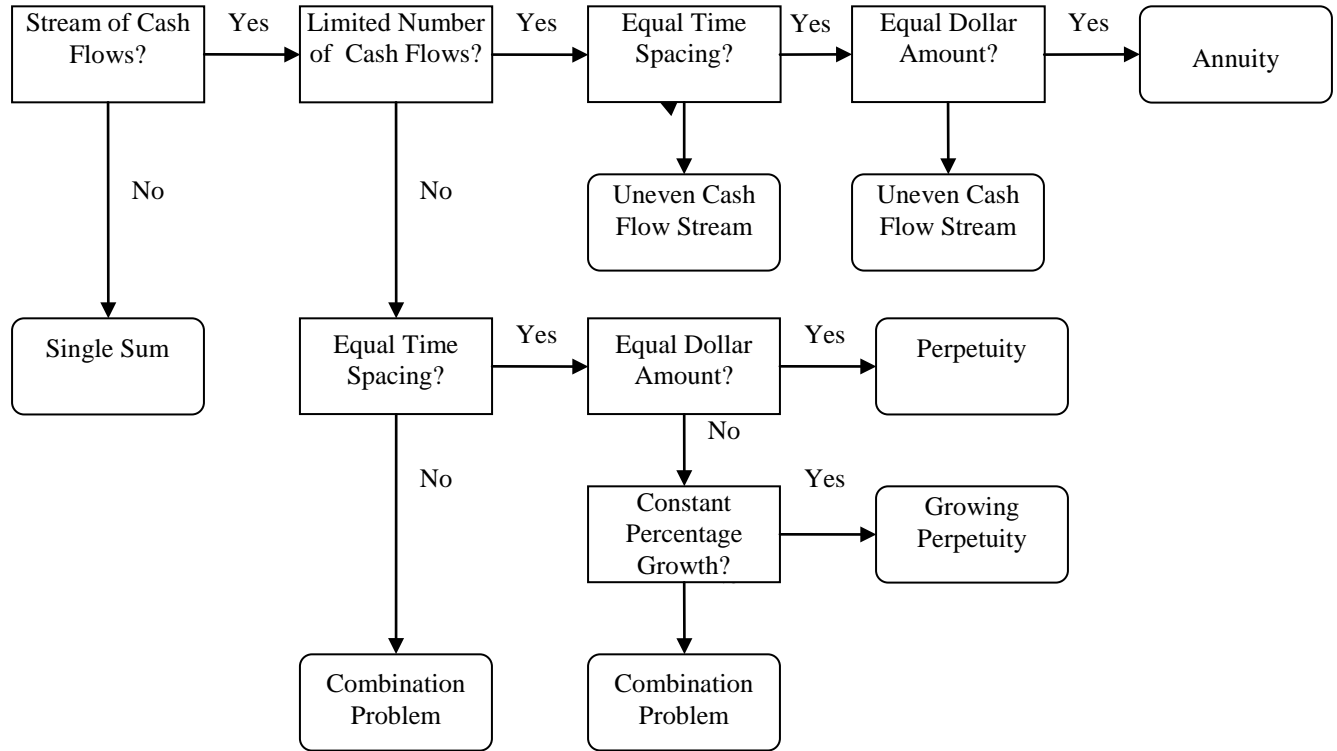
FIGURE 1: TECHNIQUE IDENTIFIER

```
+------------------+   Yes  +------------------+   Yes  +----------------+   Yes  +----------------+   Yes  +--------------+
| Stream of Cash   |------->| Limited Number   |------->| Equal Time     |------->| Equal Dollar   |------->|   Annuity    |
| Flows?           |        | of  Cash Flows?  |        | Spacing?       |        | Amount?        |        |              |
+------------------+        +------------------+        +----------------+        +----------------+        +--------------+
        |                           |                          |                         |
        | No                        | No                       v                         v
        |                           |                  +----------------+        +----------------+
        |                           |                  | Uneven Cash    |        | Uneven Cash    |
        |                           |                  | Flow Stream    |        | Flow Stream    |
        v                           v                  +----------------+        +----------------+
+------------------+        +------------------+   Yes  +----------------+   Yes  +--------------+
|   Single Sum     |        | Equal Time       |------->| Equal Dollar   |------->|  Perpetuity  |
|                  |        | Spacing?         |        | Amount?        |        |              |
+------------------+        +------------------+        +----------------+        +--------------+
                                   |                          | No
                                   | No                       v
                                   |                  +----------------+   Yes  +--------------+
                                   |                  | Constant       |------->|  Growing     |
                                   |                  | Percentage     |        |  Perpetuity  |
                                   |                  | Growth?        |        |              |
                                   v                  +----------------+        +--------------+
                            +------------------+              |
                            | Combination      |              v
                            | Problem          |      +----------------+
                            +------------------+      | Combination    |
                                                      | Problem        |
                                                      +----------------+
```

TABLE 1: CLASSIFICATION OF CASH FLOWS

	Single Sum	Annuity	Perpetuity	Growing Perpetuity	Uneven Cash Flow Stream
Series of Cash Flows	No	Yes	Yes	Yes	Yes
Limited Number of Cash Flows	Yes	Yes	No	No	Yes
Each Cash Flow is for an Equal $ Amount	N/A	Yes	Yes	No	Yes orNo
Equal Time Interval Between Cash Flows	N/A	Yes	Yes	Yes	Yes or No

FIGURE 2: ANNUITY TECHNIQUE IDENTIFIER

```
A        B         C         D         E         F         G         H         I

|---------|---------|---------|---------|---------|---------|---------|---------|            Fig. 3

              First            Middle           Last
              Cash Flow        Cash Flows       Cash Flow

         PV       PV                        FV       FV
         OA       AD                        OA       AD
Mode     END      BEG                       END      BEG
```
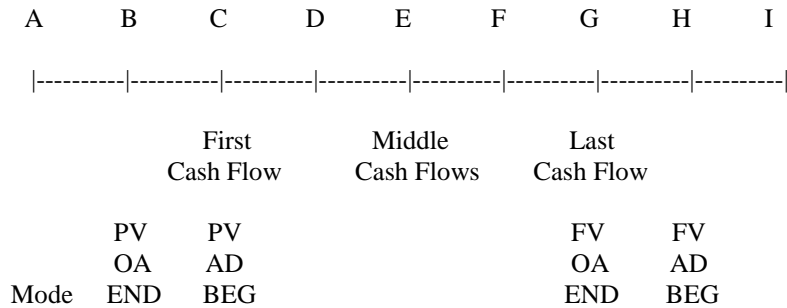
THE JALBERT, JALBERT AND CHAN TECHNIQUE

While the technique developed in Jalbert (2002) is useful for many students, alternate versions can be developed. By modifying the Jalbert technique, students may find it easier to apply an appropriate technique to solve TVM problems. Specifically, the Jalbert Technique is problematic for students who have a difficult time understanding annuity techniques. In this paper, we modify the techniques developed by Jalbert (2002) by eliminating the annuity technique from the analysis. Rather than incorporating annuities directly into the analysis, annuities are treated as a special case of an uneven cash flow problem. The modified technique is a substantial simplification of the original technique. The number of questions that must be asked by students to identify the appropriate technique necessary to solve a problem, is reduced. Moreover, the new technique eliminates the need for students to learn the annuity technique altogether. While the new technique is substantially simpler, there is minimal loss of functionality. The cash flow computations can be completed easily using the cash flow worksheet that is available in most financial calculators. As such, both techniques are equally able to solve most time value of money technique. The resulting modified figure for selecting the appropriate TVM technique to use to solve a problem is presented in Figure 2. The extent of the simplification is readily apparent when Figures 1 and 2 are compared.

FIGURE 3: MODIFIED FIGURE FOR SELECTING THE CORRECT TIME VALUE OF MONEY TECHNIQUE
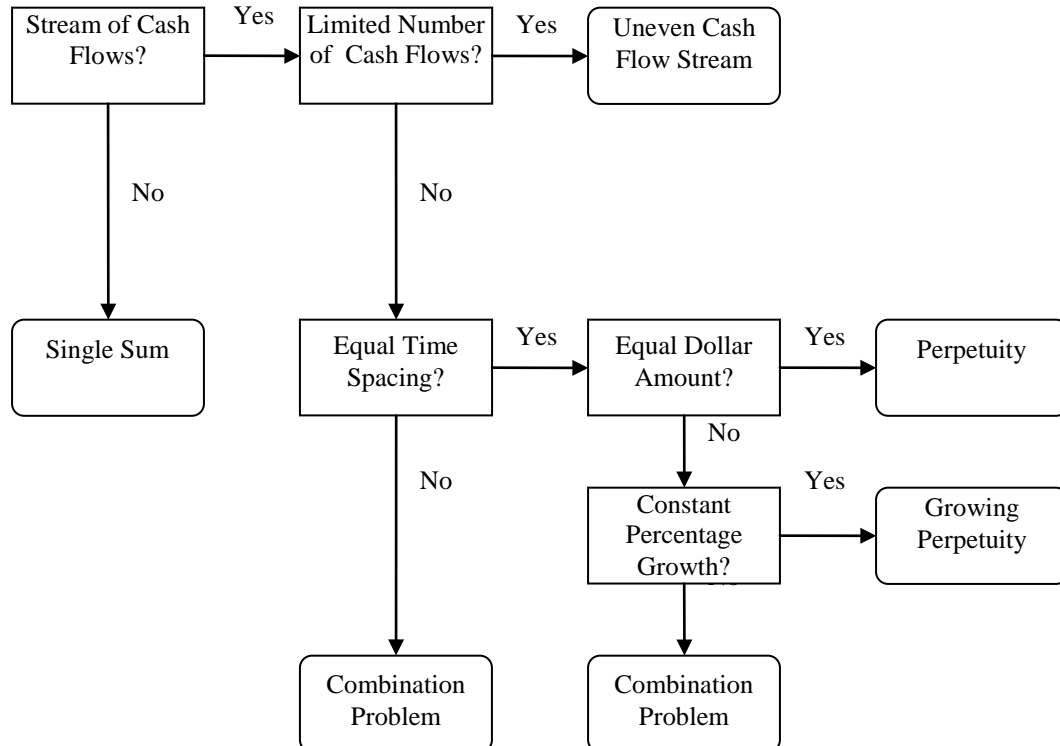

Table 2 is a modified version of Table 1. The table is different in two ways. First, it is different in the sense that the entire column associated with annuities is eliminated. A second difference is that the presence and absence indicators for the Uneven Cash Flow Stream are modified to allow for annuity type of problems. Again, this represents a substantial simplification of the Jalbert Technique.

TABLE 2: MODIFIED TABLE FOR THE CLASSIFICATION OF CASH FLOWS

	Single Sum	Perpetuity	Growing Perpetuity	Uneven Cash Flow Stream
Series of Cash Flows	No	Yes	Yes	Yes
Limited Number of Cash Flows	Yes	No	No	Yes
Each Cash Flow is for an Equal $ Amount	N/A	Yes	No	Yes or No
Equal Time Interval Between Cash Flows	N/A	Yes	Yes	Yes or No

Unfortunately, when the technique is simplified as is done here, one loss in functionality results. Many, if not all, time value of money calculators are unable to solve for the number of payments in an uneven cash flow stream. However, the calculators are able to solve for the number of payments in an annuity problem. Thus when using the modified technique students lose the capability of solving for the number of payments in an annuity problem. As such, there is a tradeoff between the Jalbert technique and the Jalbert, Jalbert and Chan technique. The Jalbert technique has slightly more functionality but is more difficult to use. The Jalbert, Jalber and Chan technique is easier to use but losses one element of functionality. As such a tradeoff must be made in selecting the best technique to use for a given class.

CONCLUDING COMMENTS

Students frequently experience difficulty identifying the appropriate time value of money (TVM) technique to apply to a problem. We suggest that this difficulty might be due in part to imprecise definitions. This paper develops a modification to a recently developed technique in order to identify the appropriate TVM technique to a problem. Similar to the technique developed in Jalbert (2002), this technique attempts to appeal to students who benefit from visual aids and systematic methodologies. The new technique developed here is very usefull and helps students to understand annuity techniques. The method developed here is a substantial simiplification of the prior technique. Specifically, students are no longer required to learn annuity techniques. Rather, annuity problems are treated as special cases of uneven cash flow stream problems. The price of this simplification is the loss of one element of functionality. That element of functionality is solving for the number of payments in an annuity problem. The Jalbert (2002) Technique is best suited for students that are comfortable with annuity techniques and wish to have full functionality. The model developed in this paper is best suited for students who have difficulty understanding annuity techniques, and are willing to sacrifice some functionality in order to avoid learning the annuity techniques. While the model developed here is a substantial simplification on the Jalbert Technique, like the Jalbert Technique, the model does not accommodate time varying interest rates. Further research might develop a model similar to that developed here that can accommodate such nonstationary interest rates.

REFERENCES

Bloom, B. (1956). Taxonomy of Educational Objectives, Handbook I: Cognitive Domain, New York: McKay.

Eddy, Albert and Gene Swanson (1996), "A Hierarchy of Skills Approach to Teaching Accounting Present Value," *Journal of Accounting Education 14(1) p. 123-131.*

Jalbert, Terrance (2002) "A New Method for Teaching the Time Value of Money," Terrance Jalbert, *Journal of the American Academy of Business, Cambridge* Vol. 2(1), September 2002 p. 72-79.