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# Optimizing ABC inventory grouping decisions

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ABSTRACT

Inventory managers often group inventory items into classes to manage and control them more efficiently. The well-known ABC inventory classification approach categorizes inventory items into A, B and C classes according to their sales and usage volume. In this paper, we present an optimization model to enhance the quality of inventory grouping. Our model simultaneously optimizes the number of inventory groups, their corresponding service levels and assignment of SKUs to groups, under limited inventory spending budget. Our methodology provides inventory and purchasing managers with a decision-support tool to optimally exploit the tradeoff among service level, inventory cost and net profit. The model and solution are applied for an inventory classification project of a real-life company, and outperform the traditional ABC method. Computational experiments are performed to obtain managerial insights on optimal inventory grouping decisions.



## 1. Introduction

A manufacturer often keeps inventory of various raw materials and components to meet production needs. A repair shop needs to ensure availability of different parts for replacement and maintenance work. A retailer usually holds certain amount of various merchandize to satisfy market demand. A hospital must keep sufficient medical supplies of all kinds for its clinical and operational needs. In the above inventory systems, the number of stock keeping units (SKUs) may be so large that it is often not practical to control them individually (Ernst and Cohen, 1990).

One way to manage a large number of SKUs is to aggregate them into different groups, and set common inventory control policies for each group (Chakravarty, 1981). Grouping provides management with more effective means for specifying, monitoring and controlling inventory performance. From the operational perspective, grouping may achieve more efficient inventory management by reducing the overhead of managing each inventory group. Inventory policies also align better with item groups than each individual item. For instance, inventory groups with different service levels often reflect a company's order fulfillment strategy and customer relationship policies, e.g., the service level agreement (SLA). Service levels have a direct impact on the company's revenue and profit.

A well-known implementation of the inventory grouping idea is the ABC classification method widely used in industry. It was first developed by GE in the 1950s (cf. Flores and Whybark, 1986; Guvenir and Erel, 1998). In a typical ABC approach, one classifies inventory items according to their transaction volume or value. A small number of items may account for a large share of volume; an intermediate category may have a moderate percentage of volume; and a large number of items may occupy a low proportion of volume. These categories are labeled A, B and C. Taking insights from Pareto (1971), it is often found that a small percentage of the inventory items contribute to the majority of a company's sales and revenue. This has led to the *80–20 rule*. That is, the top 20% of items are given the A classification, the next 30% of items the B classification and the bottom 50% the C classification (Flores and Whybark, 1986). Alternatively, Juran (1954) claims that A-items are the highest 5% of the items in dollar value, C-items are the bottom 75% and B items are the middle 20%.

Practitioners often employ the ABC classification scheme in a three-step approach to control inventory. First, SKUs are grouped into categories according to their sales volume. Second, inventory policies, e.g. the target service levels, are determined for each group. A common wisdom to determine the service level is that one should concentrate on the A category to enhance managerial effectiveness. As a rule-of-thumb, the A-class items get the highest service level settings and C-class the lowest (Armstrong, 1985). Finally, inventory managers, in collaboration with sales management and finance, need to make sure that the inventory control policy is feasible within the available inventory and management budget.

The above ABC inventory grouping and control approach has several disadvantages. (a) According to Teunter et al. (2010), there is no clear guideline in the literature to determine the service level for each group. (b) Since the grouping decision is made

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independent from and before the service level decision, their interactions have not been exploited, thus neither of the two decisions can be optimal. (c) Because the available budget was not considered until the last step, there is no guarantee that the grouping and/or service level decisions made in the first two steps are feasible. Thus one often needs to iteratively revise the grouping and/or service level decisions until feasibility is reached. This can be a tedious process for a large number of SKUs, and may lead to sub-optimal solutions. These deficiencies have motivated us to develop a new optimization approach to enhance the existing ABC inventory grouping and control decisions.

Our model and solution will help inventory and operations managers to simultaneously optimize: (i) the number of classification groups for the SKUs; (ii) optimal assignment of each SKU to a group; (iii) target service level for each group; and (iv) optimal allocation of available inventory budget to groups of SKUs. These decisions are made to maximize the total net profit, subject to explicit inventory budget constraints. We have implemented our methodology for an industrial products' distributor using real-life inventory data.

The remainder of this paper is organized as follows. Section 2 reviews the related research literature and highlights contribution of our work. Section 3 formally describes the addressed optimization problem and presents a mixed-integer linear programming (MILP) formulation to model it. In Section 4, we provide a case study of our approach on a real-world inventory grouping application. A comprehensive computational experiment is conducted to further examine the behavior and performance of our model when problem parameters vary. The computational results and managerial insights are presented in Section 5. Finally, Section 6 draws conclusion and discusses future research directions.

## 2. Related literature

Optimizing inventory classification and grouping decisions have been intensively studied in the research literature of inventory and operations management. The existing research can roughly be classified into two lines of works: one considering only the inventory clustering/classification issues, and the other addressing both inventory grouping and control.

While the classical ABC analysis makes grouping decisions based solely on a volume/cost metric (cf. Pareto, 1971), a vast line of research generalizes it into a multi-criteria clustering framework. For instance, Flores and Whybark (1986, 1987) developed a multi-criteria ABC analysis approach by considering other classification criteria such as obsolescence, lead times, substitutability, reparability, criticality and commonality. They employ a qualitative approach using the concept of joint criteria matrix. Partovi and Burton (1993) proposed a systematic approach to quantify the priority of inventory items through the analytic hierarchy process (AHP, Saaty, 1980). An artificial neural network (ANN) approach was developed by Partovi and Anandarajan (2002) to learn the optimal weights of different criteria. They show that ANN outperforms an alternative statistical approach based on the multiple discriminate analysis (MDA). Bhattacharya et al. (2007) proposed a method, called TOPSIS, to account for various conflicting criteria having incommensurable measures.

Other researchers approach inventory grouping as an optimization problem. Notably, linear programming approach, based on the *data envelopment analysis* (DEA), has been developed by Ramanathan (2006) and Ng (2007), and recently improved by Hadi-Vencheh (2010) and Chen (2011). The advantage of DEA based approach is that it is able to alleviate the impact of subjectivity on the criteria weights. Chen et al. (2008) proposed a case-based distance model to find optimal classification thresholds using quadratic programming. Hadi-Vencheh and Mohamadghasemi (2011) developed a combined AHP-DEA methodology to account for ambiguity of decision-maker's judgments. For large instances, various metaheuristic methods have been developed including genetic algorithm (Guvenir and Erel, 1998) and particle swarm optimization (Tsai and Yeh, 2008) among others.

All the aforementioned works address a pure inventory grouping/clustering problem without explicitly considering inventory policy and performance. Although researchers have found ways to implicitly incorporate inventory control measures in the multi-criteria framework, their grouping solutions do not address the question whether the *three* (A–B–C) group classification scheme is optimal, neither do they consider the interactions between inventory grouping and control decisions.

A second line of research in ABC analysis explicitly addresses and exploits the relationship between inventory classification and control decisions. Early works focus on minimizing total inventory costs, i.e. the inventory holding cost plus ordering cost. They also make strong assumptions to simplify a realistic inventory control system. For instance, Crouch and Oglesby (1978) classified SKUs into a given number of groups, while minimizing the total inventory cost. Their model assumes that the inventory holding cost is the same for all the items, which rarely holds in the practical setting. Chakravarty (1981) considered a more general problem setting and showed that the optimal grouping can be obtained by ordering the items according to the product of demand rate and holding cost rate (or PDHC). The use of PDHC significantly enhances the efficiency of their dynamic programming algorithm. Aggarwal (1983) further proposed closed-form expressions to obtain optimal grouping boundaries under the assumption that the cumulative distribution of inventory value can be characterized by a Pareto function. These works share the following commonalities. Firstly, they all assume that a group has either the same order cycle or the same order quantity. This assumption sets up a generic inventory control policy for a group, which reduces the burden of managing each SKU individually. However, the implication of this assumption is that items within the same group may have different service levels, which leads to a different probability of fulfilling customer demand. Secondly, they all assume unlimited spending on inventory cost, but do not address optimal allocation of inventory budget or the tradeoff between inventory cost and service level.

Ernst and Cohen (1990) proposed a two-stage approach based on a blend of statistical clustering procedures and optimization methods. Their procedure starts with solving a clustering problem to maximize the degree of dissimilarity among inventory classes, which is computed as a statistical measure as a function of inventory item attributes and clustering decision. Once the clusters/classes are determined, the second optimization problem seeks to minimize the number of groups by assigning SKUs to selected groups, subject to generic inventory control policy for each group and various operational performance constraints, e.g. cost, lead time, inventory turnover ratio, etc. Ernst and Cohen's approach provides a more general way for inventory grouping and control, but does not directly optimize inventory performance measures. Its two-stage nature may also lead to sub-optimal grouping decisions.

The work of Korevaar et al. (2007) optimizes the inventory budget using a nonlinear optimization model. Their decision variables include whether or not to stock an SKU, the safety stock level and reorder points of SKUs to achieve an optimal budget that achieves a specified service level target. Their model is solved by a simulated annealing metaheuristic.

Teunter et al. (2010) recently developed an optimization model to simultaneously optimize inventory classification and control decisions. Rather than using the service level as performance measure, they proposed an alternative metric, known as *fill rate*, i.e. the fraction of demands that are satisfied directly from stock on hand, to be the classification criterion. A nonlinear optimization model is solved to minimize the total inventory costs, as the sum of three components: cycle stock cost, safety stock cost and shortage (backlog) cost. While Teunter et al.'s approach focuses on cost minimization, our model emphasizes finding the optimal tradeoffs among revenue, service level, inventory stock and management cost to maximize profit.

Table 1 summarizes our review of the literature in terms of key modeling features: objective function, performance criteria (single or multiple), model formulation (linear or nonlinear), whether considering inventory budget constraint or not, whether optimizing number of inventory groups, and whether considering the overhead management cost for inventory groups. It is evident that our work contributes the existing research literature by providing a combination of new modeling features. Our integrated decision-support tool may assist inventory and purchasing managers to make inventory grouping and service level decisions subject to the available inventory budget. Our solution simultaneously optimizes the tradeoffs among profit, inventory investment and customer satisfaction (via service level), and optimally allocates a company's available budget for inventory spending.

## 3. Optimization model

We start with a formal description of the addressed optimization problem, then present a mixed-integer linear programming (MILP) model formulation and discuss its properties.

### 3.1. Problem description

Consider $N$ inventory items or SKUs. Each SKU $i = 1, \ldots, N$ has an average monthly demand of $d_i$ with a standard deviation of $\sigma_i$. We assume that the demand of each SKU follows a normal distribution $\mathcal{N}(d_i, \sigma_i)$. The lead time of SKU $i$ is known to be $l_i$ months. Each SKU $i$ has a gross profit $\pi_i$, which is its selling price minus purchasing cost. To simplify inventory management process and reduce overhead cost, the inventory manager's task is to classify the $N$ inventory items into groups, then to set up a generic inventory policy, i.e. service level, for each group. The inventory holding cost for SKU $i$ is $c_i$ per unit. Clearly, setting a 99.99% service level for all the SKUs achieves the highest revenue, but is not practically feasible because doing so also incurs a significant amount of inventory cost. The company has an inventory stocking budget of $B$ available for the planning horizon. The inventory manager must optimally allocate $B$ to the SKUs to maximize the total net profit.

Let $j = 1, 2, \ldots, M$ be $M$ candidate groups, each of which is associated with a service level $\alpha_j \in [0, 1)$. When demand is normally distributed, it is well-known that the inventory level of SKU $i$ to achieve $\alpha_j$ (in group $j$) can be computed in a standard way as the sum of mean demand plus safety stock (cf. Ballou 2004)

$$d_i l_i + z_j \sigma_i \sqrt{l_i}, \tag{1}$$

where $z_j$ is the $z$-value associated with $\alpha_j$ in the standard normal distribution.

Note that in general, the inventory level in (1) can be negative when (i) $z_j$ is negative ($\alpha_j$ is less than 50%), (ii) $\sigma_i$ is large (large variation in demand), or (iii) lead time $l_i$ is long.

From the management perspective, there is a cost $\omega_j$ for maintaining and managing an inventory group $j$. Purchasing departments often assign buyers to an inventory class to coordinate the ordering of these items. This management cost may also include additional purchasing and administrative costs incurred for each group. The benefit of the classical ABC method may be largely due to its simplicity in implementation and low administrative cost of maintaining *only three* inventory groups. Our model is able to improve and generalize the classical ABC method by optimizing the tradeoff between granularity of service level (number of groups) and management cost incurred.

Our current approach assumes that $\omega_j$ is constant, i.e. the same amount of management effort is needed for each additional group. In practice, it is possible that $\omega_j$ is a *decreasing* function of the number of groups through the economies of scale. Taking the staffing cost needed for managing inventory groups for example, it is possible for the same team to manage multiple inventory groups, so that the staffing cost per group may be diminishing with respect to the number groups. Thus our assumption of a constant $\omega_j$ is somewhat conservative for the benefit of our approach. The other possibility is that the effort of managing an inventory group might be an *increasing* function of the number of SKUs and/or inventory volumes in a group. However, to model such increasing relationship requires more data, and the resulting model will become nonlinear which requires a different set of solution methods. Such investigation goes beyond the scope of this paper.

In our addressed inventory grouping optimization problem, the decision-maker simultaneously makes two decisions: selecting the number of inventory groups (with corresponding service levels) and

**Table 1**
Comparison of features of different inventory grouping models.

| Approaches | Obj. function | Criteria | Model formulation | Budget constraint | Optimizing # groups | Management cost |
|---|---|---|---|---|---|---|
| **Optimizing only inventory classification** | | | | | | |
| Ernst and Cohen (1990) | Minimize # of groups | Multiple | Linear | No | Yes | No |
| Guvenir and Erel (1998) | Minimize distance of expert ranking | Multiple | Nonlinear | No | No | No |
| Partovi and Anandarajan (2002) | Minimize distance of expert ranking | Multiple | Nonlinear | No | No | No |
| Ramanathan (2006) | Maximize performance score | Multiple | Linear | No | No | No |
| Bhattacharya et al. (2007) | Minimize distance from the ideal | Multiple | Linear | No | No | No |
| Ng (2007) | Maximize performance score | Multiple | Linear | No | No | No |
| Hadi-Vencheh (2010) | Maximize performance score | Multiple | Nonlinear | No | No | No |
| Chen (2011) | Maximize performance score | Multiple | Linear | No | No | No |
| Chen et al. (2008) | Minimize distance | Multiple | Linear | No | Yes | No |
| Tsai and Yeh (2008) | Maximize performance score | Multiple | Nonlinear | No | Yes | No |
| **Optimizing both inventory grouping and control** | | | | | | |
| Crouch and Oglesby (1978) | Minimize cost | Single | Nonlinear | No | No | No |
| Chakravarty (1981) | Minimize cost | Single | Linear | No | Yes | No |
| Aggarwal (1983) | Minimize cost | Single | Linear | No | Yes | No |
| Korevaar et al. (2007) | Minimize inventory budget | Single | Nonlinear | No | No | No |
| Teunter et al. (2010) | Minimize cost | Single | Nonlinear | No | No | No |
| **This paper** | **Maximize profit** | **Single/ multiple** | **Linear** | **Yes** | **Yes** | **Yes** |

assigning each SKU to an appropriate group. These decisions must be made so that the total inventory holding cost does not exceed the available inventory spending budget. The objective function is to maximize the total net profit as the difference between the total gross profit and the total inventory group management cost.

### 3.2. MILP formulation

We formulate the addressed optimization problem as a mixed-integer linear program (MILP) below.

*Parameters*

$N$: number of inventory items (SKUs)
$M$: maximum number of inventory groups
$d_i$: mean of monthly demand of SKU $i=1,\ldots,N$
$\sigma_i$: standard deviation of monthly demand of SKU $i=1,\ldots,N$
$l_i$: lead time of SKU $i=1,\ldots,N$
$\pi_i$: gross profit per unit of SKU $i=1,\ldots,N$
$c_i$: inventory holding cost per unit of SKU $i=1,\ldots,N$
$\omega_j$: fixed overhead management cost for inventory group $j=1,\ldots,M$
$B$: available budget for total inventory cost
$\alpha_j$: service level associated with group $j=1,\ldots,M$
$z_j$: $z$-value associated with the service level $\alpha_j$ of group $j=1,\ldots,M$

*Decision variables*

$y_j=1$: if inventory group $j$ is selected, and 0 o.w. for $j=1,\ldots,M$
$x_{ij}=1$: if SKU $i$ is assigned to group $j$ for $i=1,\ldots,N$ and $j=1,\ldots,M$
$v_i\geq 0$: inventory level of SKU $i=1,\ldots,N$

*Objective function*

$$\text{Maximize} \quad \sum_{i=1}^{N}\sum_{j=1}^{M}\pi_i d_i \alpha_j x_{ij}-\sum_{j=1}^{M}\omega_j y_j \qquad (2)$$

*Constraints*

$$\sum_{j=1}^{M}x_{ij}\leq 1, \quad \forall i=1,\ldots,N \qquad (3)$$

$$\sum_{i=1}^{N}x_{ij}\leq N y_j, \quad \forall j=1,\ldots,M \qquad (4)$$

$$v_i=\sum_{j=1}^{M}d_i l_i x_{ij}+\sum_{j=1}^{M}z_j\sigma_i\sqrt{l_i}x_{ij}, \quad \forall i=1,\ldots,N \qquad (5)$$

$$\sum_{i=1}^{N}c_i v_i\leq B \qquad (6)$$

$$v_i\geq 0, \quad \forall i=1,\ldots,N \qquad (7)$$

$$x_{ij}\in\{0,\ 1\}, \quad \forall i=1,\ldots,N; \quad \forall j=1,\ldots,M \qquad (8)$$

$$y_j\in\{0,\ 1\}, \quad \forall j=1,\ldots,M \qquad (9)$$

The objective function (2) maximizes the total net profit computed as the total gross profit subtracting the total overhead inventory management cost. Here the service level is treated as a *fill rate* to compute the amount of demand that can be satisfied (fulfilled) by the SKU stock. The product term $d_i\alpha_j$ is the expected average demand that can be fulfilled for SKU $i$ if $i$ is placed in the inventory group $j$ with fill rate $\alpha_j$. Then the inner summation $\Sigma_{j=1}^{M}\pi_i d_i\alpha_j x_{ij}$ over all the $M$ inventory groups computes the gross profit generated by SKU $i$ given its inventory grouping decision $x_{ij}$. Thus the entire first term $\sum_{i=1}^{N}\sum_{j=1}^{M}\pi_i d_i\alpha_j x_{ij}$ in objective function (2) is the total gross profit of all $N$ SKUs. Such concept of fill rate has also been used by other researchers (cf. Teunter et al., 2010).

Constraint (3) assigns an SKU to at most one group. Note that it is feasible not to assign SKU $i$ to any group. Proposition 1 will reveal the condition for this to happen and its implication. Constraint (4) enforces that a group must be selected in order for any SKU to be assigned to the group. In other words, if group $j$ is not selected, no SKU can be assigned to $j$. Constraint (5) computes the inventory level of SKU $i$ based on (1). Constraint (6) ensures that the total inventory holding cost does not exceed the available budget. Constraints (7) through (9) specify the domain of decision variables.

We now discuss some important properties concerning the MILP formulation. Recall that for each combination of $(i,j)$, the inventory level (1) may be negative. We need to show that this does not affect the feasibility of the system of constraints (3) through (9). For convenience, we define $\delta_{ij}$ as the inventory level of SKU $i$ if it is assigned to group $j$, i.e. $\delta_{ij}=d_i l_i+z_j\sigma_i\sqrt{l_i}$ according to (1). We state and prove Proposition 1 below.

**Proposition 1.** *For a pair of SKU i and group j, if $\delta_{ij}<0$, i must not be assigned to j,* i.e. $x_{ij}=0$.

**Proof.** By contradiction, suppose it is also feasible that $x_{ij}=1$. Because SKU $i$ cannot be assigned to any other group (constraint (3)), $i$ is assigned to group $j$ with $z$-value $z_j$. Thus its inventory level $v_i=\delta_{ij}<0$. Since the decision variable must be non-negative (7), there exists a contradiction. Therefore, $x_{ij}=0$ when $\delta_{ij}<0$. □

Proposition 1 implies that it is possible to keep zero inventory for an SKU $i$ if none of $\delta_{ij}$ is positive for all $j=1,\ldots,M$. This feature allows our model to determine which SKUs should be stocked and which should not. In principle, if an SKU has a low profit margin, a low average demand but a high variation and long lead time, one may opt not to stock it. This type of decision is known as *inventory (SKU) rationalization* (cf. Borin and Farris, 1990; Quelch and Kenny 1994; Byrne, 2007; Mahler and Bahulkar, 2009). Therefore, a side benefit of our model is to provide a rigorous way for optimal SKU rationalization: poor performing SKUs will be rationalized out of the inventory set and will not consume the company's inventory budget.

The MILP model is NP-hard, so that there is no polynomial algorithm to solve it to optimality. The proof of NP-hardness is established by transforming the formulation into an un-capacitated facility location problem (UFLP, cf. Drezner and Hamacher, 2004).

**Proposition 2.** *The MILP formulation (2) through (9) for inventory grouping optimization is NP-hard.*

**Proof.** It suffices to show that the MILP formulation is equivalent to a UFLP. We create a dummy group $\phi$ representing an inventory group with zero service level, i.e. $\alpha_\phi=0$, and set $\delta_{i\phi}=0$ for every SKU $i$ (by letting the safety stock to be equal to $-d_i l_i$). Then constraint (3) becomes (3'): $\Sigma_{j=1}^{M}x_{ij}=1$ for each $i=1,\ldots,N$, which means that we now assign each SKU to *exactly one* group. Since the dummy group's service level is zero ($\alpha=0$), any SKU assigned to the dummy group does not generate any profit or is there any inventory for the SKU because $\delta_{i\phi}=0$. By relaxing constraint (6), the MILP formulation (2), (3'), (4) plus (8) and (9) is equivalent to a UFLP, which is well-known to be NP-hard. Therefore, the original MILP formulation is NP-hard. □

The model formulation (2) through (9) has a single-objective function (2) to explicitly optimize inventory performance defined by the criterion of profitability. Other criteria implicitly optimized by the model include service level in (2) and (5), inventory holding cost in (5) and inventory management cost due to grouping in (2). Optimization of these implicit criteria can be achieved by the concept of *efficient frontier* through the sensitivity analysis in mathematical programming. This will be elaborated in our computational experiment in Section 5.

### 3.3. Model extension: multi-criteria optimization

In this section, we extend our single-objective optimization model to a multi-objective one that explicitly optimizes inventory performance measured by multiple criteria. Consider a set of criteria $k = 1, 2, \ldots, K$. They can be either *quantitative* such as demand volume, unit cost and lead time (cf. Flores and Whybark, 1986; Partovi and Burton, 1993); or *qualitative* such as replaceability (Guvenir and Erel, 1998) and criticality (Ramanathan, 2006). Replaceability measures the easiness of replacing an item by a different one, i.e. how easy is it to substitute an item. Criticality may reflect a company's strategic positioning of its customers.

Let $s_{ik}$ be the score of SKU $i = 1, \ldots, N$ for criterion $k = 1, 2, \ldots, K$, and $w_k$ be the weight for criterion $k$ evaluated by a decision-maker (DM) or the consensus from a group of DMs. A weight $w_k$ indicates the relative importance of criterion $k$ among all $K$ criteria, such that $\Sigma_{k=1}^{K} w_k = 1$. The weighted performance score $f_i$ of SKU $i$ can be computed as $f_i = \Sigma_{k=1}^{K} w_k s_{ik}$. We then use the well-known *weighting method* in multi-objective optimization (Cohon, 1978) to maximize the overall weighted performance score $F(\cdot)$ of all SKUs

$$\text{Maximize } F = \sum_{i=1}^{N} \sum_{j=1}^{M} f_i d_i \alpha_j x_{ij} \qquad (2')$$

Objective function (2') has as similar structure as (2) with the unit profit $\pi_i$ replaced by the weighted performance score $f_i$. The resulting multi-objective model (2') plus (3) through (9) is capable of incorporating DM's subjective judgments and opinions in optimizing inventory grouping.

## 4. A case study: optimized ABC analysis

Our research was motivated by a consulting project for an industrial products distributor of pneumatic products: air compressors, air compressor parts, valves and fittings. The company distributes products for 48 manufacturers in its sales region. The company's annual sales are approximately \$16,000,000, and 6703 SKUs were stocked in its warehouses.

During the economic expansion of the early 2000s, the company's sales grew along with the construction industry and the general economy. Inflated by strong sales, its inventory management policy had become relaxed. Going into the recession, beginning in 2008, the company found its inventory costs growing while sales decreased. Worse than these general conditions, was that the best-selling SKUs always seemed to be out-of-stock, while slow-movers continued to be purchased. An imperative decision now faced by the company's inventory manager is to classify the thousands of SKUs into reasonable groups with appropriate service levels, so that the company's limited inventory budget can be best utilized, and the effectiveness of managing these groups can be improved. The company has a planned budget of 2 million dollars, which is determined by the inventory manager to achieve approximately 6 inventory turns per year. There is an estimated fixed overhead, or management cost of 1000 dollars per inventory group. Sales and inventory data for a 12-month period is available for this project.

The company currently implements a traditional ABC approach to classify its SKUs based on their sales volume. After the A, B, and C groups and the SKUs memberships are identified, an iterative procedure is employed to set/adjust service levels for SKU groups as depicted in Fig. 1. The procedure starts with an arbitrary service level for each group based on decision-maker's experience, e.g., 95%, 87% and 80% for Class-A, Class-B and Class-C, respectively. Since this decision is made without considering the available inventory budget, the initial service levels may lead to violation of available budget. Therefore, the decision-maker needs to go back-and-forth revising the service levels until a feasible and "good" (in a heuristic sense) solution is reached.

In addition to being a tedious number-crunching process, the inventory control policies suggested by the existing approach are often sub-optimal. One solution obtained from the ABC procedure of Fig. 1 is shown in Table 2. The inventory manager raises the following issues: (i) the annual sales of A-Class ranges from \$186.23 to \$38,461.89, yet all of them are assigned to the same high service level; (ii) the difference (\$0.34) between the annual sales of the bottom A-Class SKU

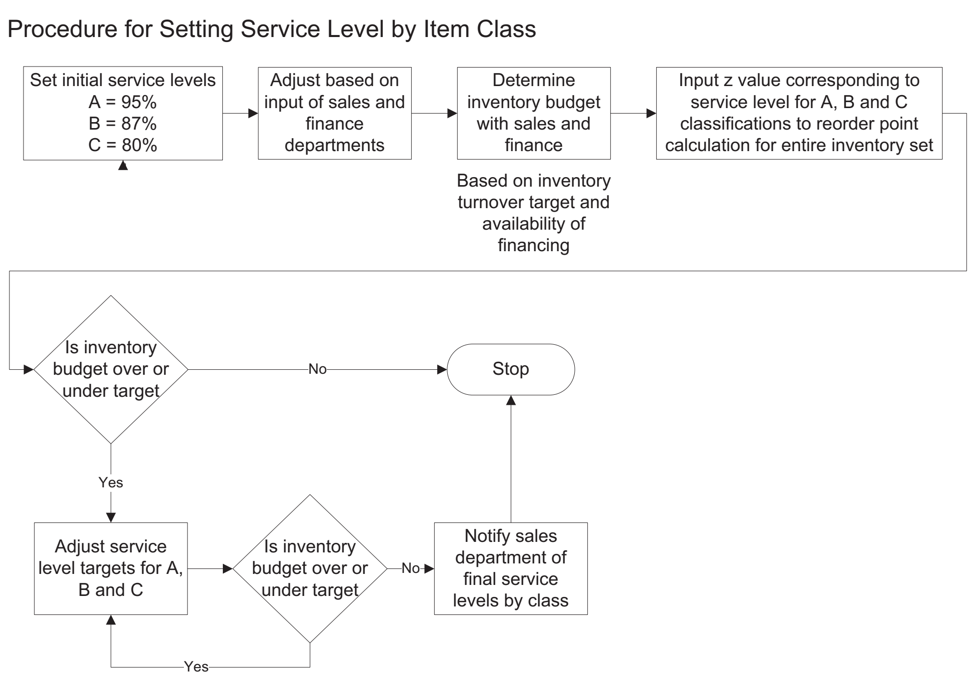


**Fig. 1.** Flow chart of an iterative ABC procedure for inventory grouping.

and top B-Class SKU, and the difference ($0.01) between the bottom B-Class SKU and top C-Class SKU, is not significant enough to justify the difference of their service levels; (iii) the solution makes no recommendation about SKU rationalization, although the manager believes that the company should keep no inventory for certain SKUs.

The inventory manager would like to answer the following questions:

- Is the three-group (A–B–C) scheme optimal? Specifically, should the company manage more groups than three, with more differentiation and granularity of service levels to achieve more profit?
- Is annual sale a reliable criterion for SKU-group assignment?
- How to optimally allocate the company's available inventory budget to SKUs?
- Should the company exclude some SKUs out of inventory? If so, which SKUs?

We implement the MILP model presented in Section 3 to answer these questions. In our implementation, a total of 108 potential inventory groups are considered including 99 groups with service levels from 1% to 99% (with an increment of 1%), plus 9 groups with service levels from 99.1% to 99.9% (with an increment of 0.1%). The purpose of considering the additional nine groups is to granulize the continuous service level space, as these higher service levels are usually assigned to more important SKUs.

Our MILP model was solved by the branch-and-cut (B&C) method in integer programming through CPLEX 12.1 on a desktop PC with Pentium 3.3 GHz CPU speed and 8 G RAM. The default CPLEX settings for B&C were used. It took CPLEX about 5 h to find optimal solution (and prove optimality). Comparing with the days or even weeks of time spent on the manual iterative procedure, this is clearly an improvement of solution efficiency for the company.

An optimal solution found by our MILP model recommends 8 inventory groups instead of 3. It generates 3.85% more profit than the ABC classification solution. The service levels, group size, inventory spending and profits of the eight groups are provided in Table 3. We compute the return of investment (ROI) in the last column as a measure to quantify the benefit of keeping the corresponding inventory group.

**Table 2**
Inventory grouping and service level solution from the ABC approach.

| SKU rank | ABC class | Service level (%) | Annual sales ($) |
|---|---|---|---|
| 1 | A | 93 | $38,461.89 |
| 1341 | A | 93 | $186.23 |
| 1342 | B | 90 | $185.89 |
| 3352 | B | 90 | $45.97 |
| 3353 | C | 80 | $45.96 |
| 6703 | C | 80 | $0.02 |

The optimal inventory grouping and service level solution differ significantly from the one found by the ABC procedure. The group with the highest 99% service level accounts for only 8.55% of the total 6703 SKUs, but has the highest gross profit. However, it is far less than the 80% of profit as suggested by the ABC scheme. Furthermore, while the ABC solution assigns the bottom 50% of SKUs to the C-Class (Flores and Whybark, 1986), our optimal solution has grouped 60% of the items into some intermediate service level groups, i.e. 98%, 96% and 93%. It is evident that our optimal solution has roughly assigned SKUs with higher ROIs higher service levels, which is intuitively plausible. This result suggests that when the company has limited inventory budget available, the ROI might be a better choice than sales alone (as in the ABC analysis) to be the classification criterion. In addition, our MILP model also serves as an SKU rationalizer by identifying 66 SKUs with zero service level and inventory.

## 5. Computational experiment

We perform additional computational experiments to examine the behavior of our MILP model when some key problem parameters vary. In particular, we would like to understand the impact of management cost per group $\omega$, and the available inventory budget $B$ on the optimal inventory grouping solutions. Our goal is to obtain managerial insights for practitioners to better characterize optimal inventory grouping and service level strategy.

### 5.1. Sensitivity analysis of problem parameters

We let management cost per group $\omega$ vary in the interval of [200, 2000] with an increment of 200, which leads to 10 values for $\omega$. The available inventory budget $B$ is changed in the interval of [1,000,000, 3,000,000] with an increment of 200,000, which gives 11 values for $B$. The MILP model is solved for a total of $10 \times 11 = 110$ combinations of $\omega$ and $B$.

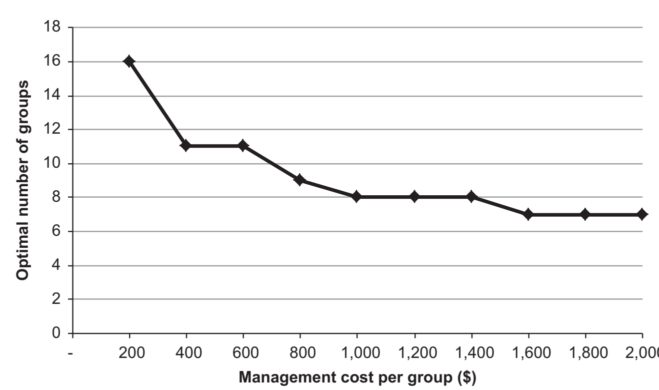


**Fig. 2.** Optimal number of groups when management cost per group increases.

**Table 3**
Optimal inventory grouping and service level solution found by the MILP model.

| Group with service level (%) | # of SKUs (%) | Inventory spending ($) | Gross profit ($) | ROI |
|---|---|---|---|---|
| 99 | 573 (8.55) | $298,657 | $1,340,921 | 4.49 |
| 98 | 1196 (17.84) | $435,320 | $1,290,743 | 2.96 |
| 96 | 1537 (22.93) | $539,320 | $1,094,234 | 2.02 |
| 93 | 1282 (19.13) | $410,762 | $649,392 | 1.58 |
| 87 | 941 (14.04) | $233,731 | $275,350 | 1.17 |
| 73 | 487 (7.27) | $71,398 | $76,216 | 1.06 |
| 39 | 621 (9.27) | $10,783 | $23,644 | 2.19 |
| 0 | 66 (0.98) | $0 | $0 | – |

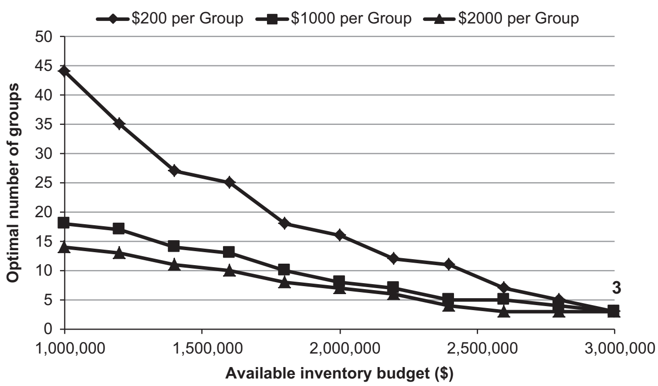


**Fig. 3.** Optimal number of groups when inventory budget varies.

As shown in Fig. 2, a higher management cost per group will discourage selecting more inventory groups. Such relationship appears to be nonlinear. We state Insight 1 below.

**Insight 1.** As the management cost per group increases, the optimal number of inventory groups will decrease at a diminishing rate.

The nonlinear relationship between the optimal number of groups and management cost per group justifies the need of an optimization model for decision-support. Without the solution offered by our MILP model, it will not be straightforward to determine the optimal number of inventory groups.

To understand the impact of available inventory budget, we plot the relationship between the optimal number of groups and the inventory budget in Fig. 3, with the management cost per group being fixed at different levels: $200, $1000 and $2000. One observes that as more inventory budget is available, there will be fewer inventory groups in an optimal solution; when inventory budget becomes tight, the optimal number of inventory groups will increase. As an extreme case, when there is plenty of capital available for inventory investment, the optimal number of groups will converge to *three* as suggested by the ABC method. One should realize that even in this case the ABC approach might still be sub-optimal because it does not optimally assign SKUs to the inventory groups (service levels).

Fig. 3 also shows that the above relationship between the optimal number of groups and the inventory budget appears to be similar for different management costs per group, but with different sensitivity. When the cost of managing an inventory group is low, the optimal number of groups is more sensitive to the inventory budget. That is, a small decrease of inventory budget may lead to a significant increase in optimal number of inventory groups. These observations are summarized by Insights 2 and 3 below.

**Insight 2.** It is optimal to select more inventory groups when the available inventory budget is tight; there is less incentive to select more inventory groups when there is plenty inventory budget available.

**Insight 3.** The relationship between the optimal number of inventory groups and available inventory budget in Insight 2 is more significant when the management cost per group is low.

Insight 2 justifies the benefit and value of our optimization model when the inventory budget is tight, because in such cases there is more incentive to better allocate the available budget, by differentiating their service levels. Insight 3 suggests that there is incentive to reduce management cost per group, possibly by using electronic data interchange (EDI), to facilitate managing multiple inventory groups.

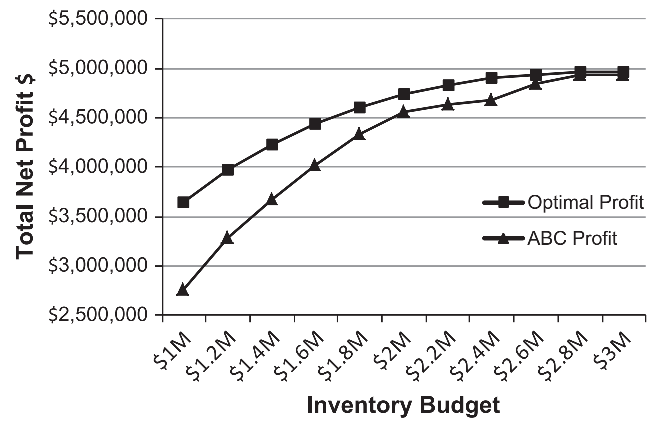


**Fig. 4.** Profit found by the MILP model and ABC method with different budgets.

We next examine how the profit found by the optimization and ABC methods change when the available inventory budget varies in Fig. 4. The net profit increases rapidly as the available budget increases from $1,000,000 to $1,500,000 initially. As the budget is increased further, the rate of change decreases, reflecting a diminishing return of the inventory investment. Insight 4 follows.

**Insight 4.** The net profit increases as the inventory budget increases, at a decreasing rate. That is, the inventory investment has a diminishing return on profitability.

Our optimization model is able to help a company answer interesting and important what-if type questions about the benefit of increasing inventory budget. For example, how much more profit can be generated if the company increases its inventory budget by 0.5 $million? Due to the nonlinear relationship between net profit and inventory budget in Insight 4, the answer depends on what the company's current budget is. For instance, if the company is currently spending 1 $million, increasing the budget to 1.5 $million will generate over $250,000 more net profit; however, if the company is already spending 2.5 $million, increasing to 3 $millions brings less than $100,000 more net profit. Without the decision-support of an optimization model it would not be intuitive to quantify the incentive of increasing inventory budget because of their non-linear relationship.

The sensitivity analysis in Fig. 4 also offers a *constraint method* in multi-objective optimization to exploit the best tradeoff between two objectives (cf. Cohon 1978). The optimal net profit curve defines an *efficient frontier* showing the best possible (maximum) total net profit for certain inventory holding budget. Since the net profit curve obtained by the ABC method always lies within the efficient frontier, our optimization model always outperforms the ABC method in solution quality. Further comparison leads to Insight 5.

**Insight 5.** The optimization model performs significantly better than the ABC method when the available inventory budget is tight. Its advantage over the ABC method diminishes when there a plenty of inventory budget available.

When the inventory budget is ample, the optimal number of groups converges to three (Insight 2), so that the quality of ABC solution is close to that of the optimal solution; when the inventory budget is tight, it becomes necessary to have more groups than three, thus the advantage of the MILP's optimal solution over the ABC solution becomes more significant.

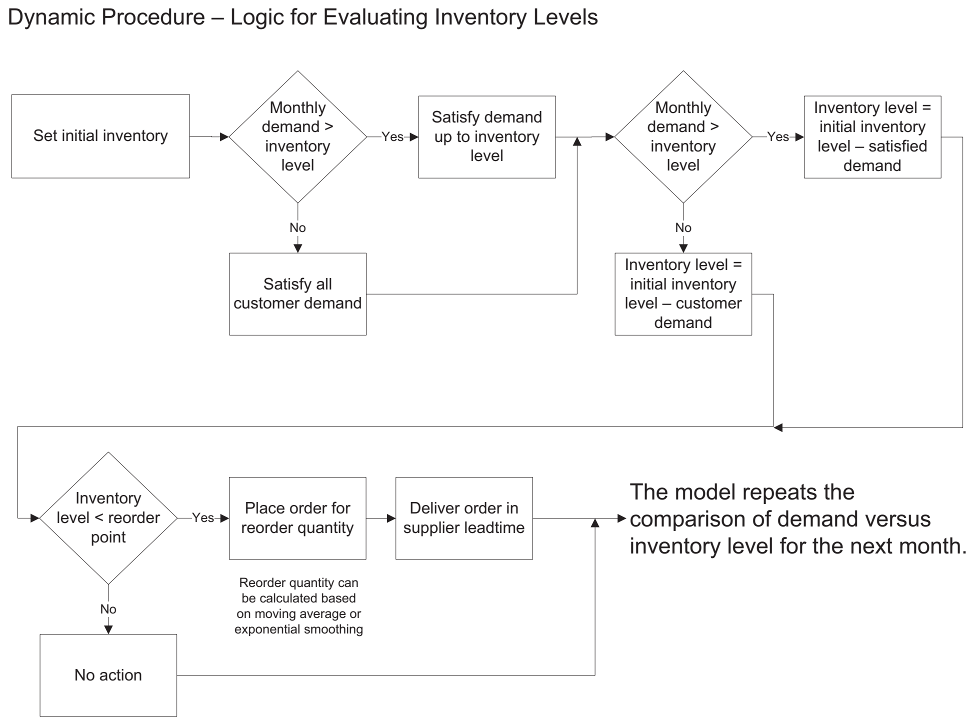


**Fig. 5.** Dynamic procedure to implement the inventory group solution.

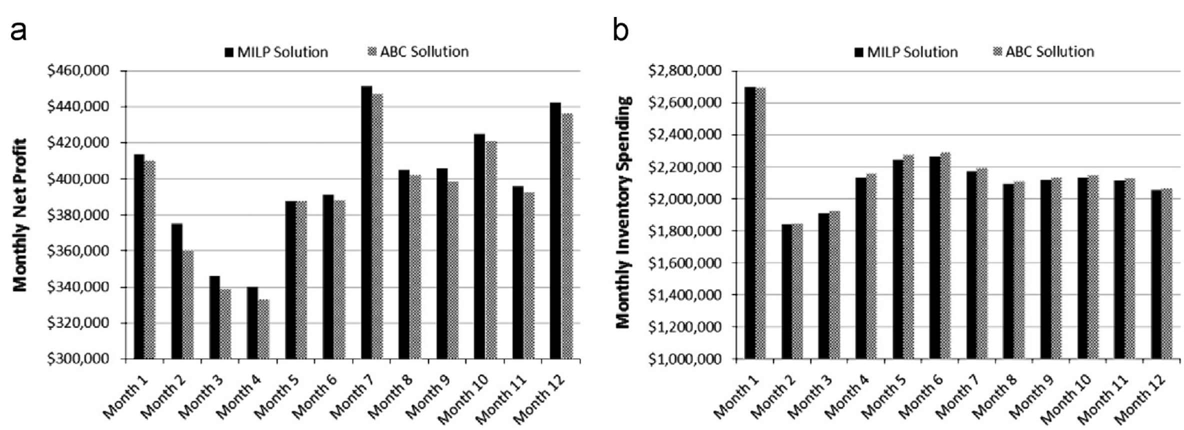


**Fig. 6.** Monthly performance of the MILP and ABC solutions. (a) Monthly net profit and (b) monthly inventory spending.

### *5.2. Dynamic implementation results*

To examine the performance of our approach over time, we implement the MILP and ABC solutions over a course of 12 months. The 6703 SKUs' 12 month actual demand data of the distributor in the case project are used for this experiment. The flow of the dynamic procedure is depicted in Fig. 5. Given the SKU grouping and service level solutions suggested by either solution (the MILP model or the ABC method), the reorder point of each SKU can be computed. The procedure starts with some initial inventory. Then the actual demand is observed to update both the satisfied demand and remaining inventory level. If the remaining inventory is below the reorder point, an order is placed with quantity equal to the average monthly demand. The lead times for all SKU orders are assumed to be 1 month. The above process repeats for every time period to evaluate the overall solution performance.

Since our MILP solution performs significantly better than the ABC solution when the budget is tight, we first examine the case where the budget is ample ($ 3million). Fig. 6 compares the profit obtained (a) and inventory cost spent (b) in each month. We state the following two hypotheses.

**Hypothesis 1.** The MILP solution generates higher profit in each month than the ABC solution does.

**Hypothesis 2.** The MILP solution spends less inventory cost in each month than the ABC solution does.

The two-sample paired $t$-test is performed to test the two hypotheses. Both are supported at confidence level greater than 99.99%. Insight 6 follows.

**Insight 6.** Our MILP solution consistently outperforms the ABC solution in inventory spending and profitability in the multi-period setting.

We next vary the available budget level to implement the dynamic procedure. Fig. 7 summarizes the advantage of MILP solution in both multi-period (dynamic) and one-period (static) settings with different budget levels. In the dynamic setting, the percentage of improvement of MILP solution over the ABC solution decreases from 4.5% to 1.3% when the available inventory budget increases from $ 1 million to $ 3 million. In the static setting, the percentage of improvement is significantly higher (over 30%) when budget is $ 1 million, but lower (0.57%) when budget is $ 3 million.

We now examine why a dynamic implementation of the MILP solution shows less advantage over the ABC solution compared with the static implementation. Note that an order to replenish is only activated when the on-hand inventory level triggers the reorder point. When the actual monthly demand is low enough and the on-hand inventory level remains higher than the reorder point, no replenishment order will be made, so that the service level determined by either the MILP or ABC solution will in fact be greater than the target. This is often the case for SKUs with low and sporadic demand. Fig. 8 shows examples of demand distributions of four SKUs. The demand in (a) appears to be symmetric and normally distributed, as assumed in our MILP model; the one in (b) is non-symmetric and biased toward lower demand; the demand in (c) and (d) is more sporadic with extremely low quantity. The existence of demand patterns such as (b), (c) and (d) deviates the assumption of normal demand distribution in the MILP model, and leads to less frequent replenishment of the corresponding items, thus mitigating the disadvantage of the ABC solution.

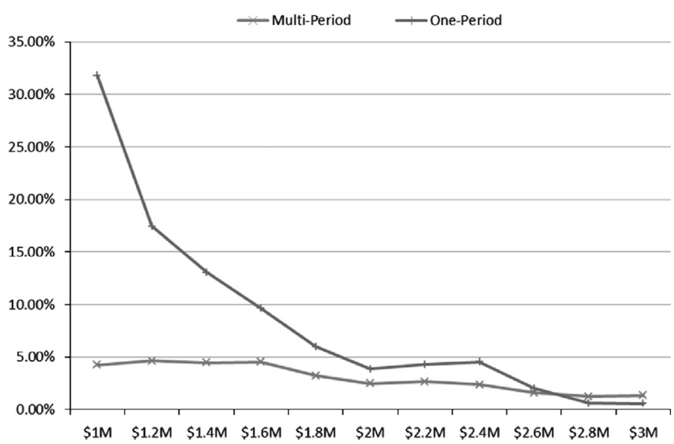


**Fig. 7.** Percentage of improvement of MILP solution over ABC solution.

## 6. Conclusion and future research

In this paper, we have developed an optimization model to simultaneously determine inventory groups, their corresponding service levels and assignment of SKUs to groups. It generalizes and enhances the well-known ABC inventory grouping approach by offering integrated, automated and optimized solutions. Our model differs from the existing optimization models in the literature with two distinctive features. First, rather than minimizing inventory cost, our model maximizes the profitability of a company. Second, our solution optimizes the tradeoff between inventory cost and profit, and optimally allocates the inventory budget to SKUs. Our approach may also serve as an SKU rationalization tool to help inventory managers decide which SKUs should better to be kept out of stock.

Our optimization model and solution are applicable to companies and organizations in various industries: manufacturing, distribution, retail and health care. We have implemented our methodology for a real-life company who distributes thousands of industrial products to business customers. Solution offered by our model has improved the company's total net profit by 3.85%, compared with past ABC solution implemented at the company. Our solution helps better manage inventories by optimally assigning service levels to SKUs and determining with SKUs should be rationalized out of stock. Moreover, the sensitivity analysis provided by our approach helps

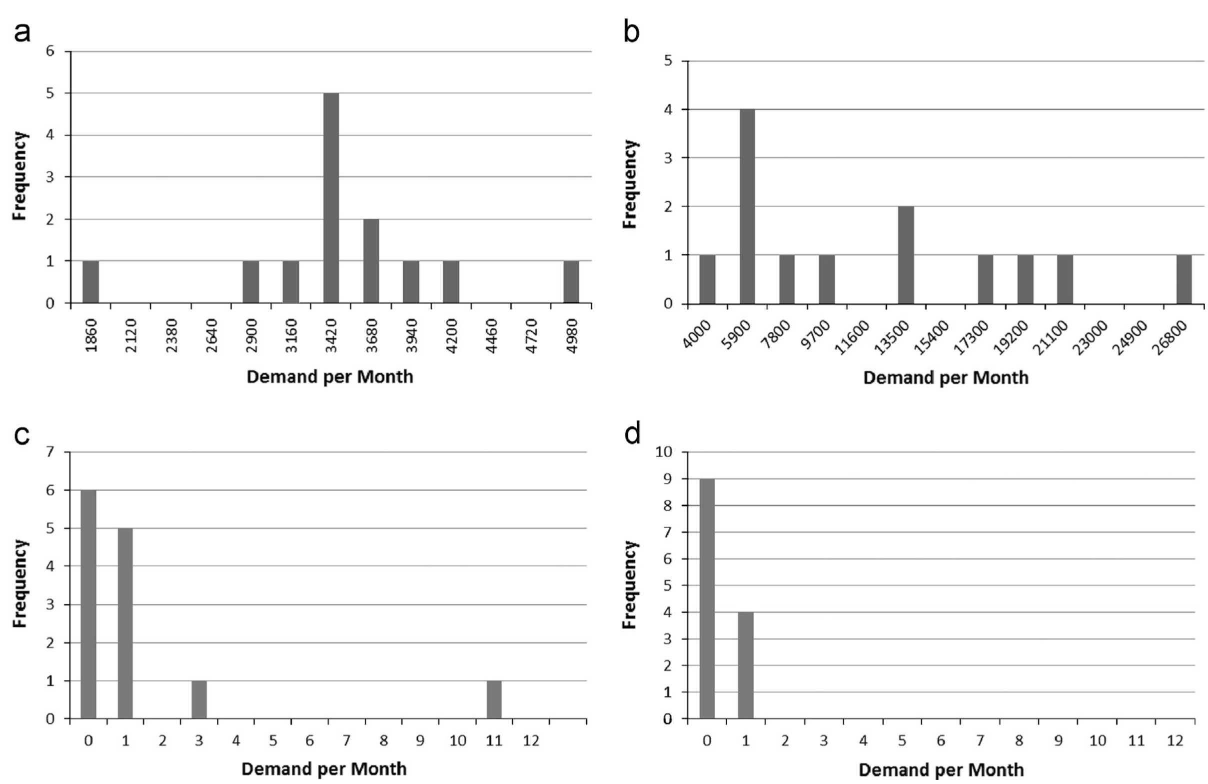


**Fig. 8.** Demand distributions of four SKUs. (a) SKU no. P/TH20106BU-250R, (b) SKU no. P/T24044NA-100, (c) SKU no. P/T89153 and (d) SKU no. P/TR910922790.

inventory managers to quantify the impact of inventory spending and inventory group management cost on the optimal inventory grouping decision and profitability.

Through a comprehensive computational experiment, we have obtained several managerial insights about optimal inventory grouping and control strategy. (i) When the management cost per group can be reduced, it is optimal to differentiate service levels for SKUs by classifying them into more granular groups. (ii) Our solution shows a diminishing return of inventory spending on the net profit, and can help a company quantify and justify the benefit of increasing inventory budget. (iii) We find that there is more incentive to increase the number of inventory groups when the available inventory budget is tight; whereas when there is plenty of budget available, it might be acceptable to aggregate SKUs into a small number of groups as in the traditional ABC approach. The capability of being able to optimally allocate limited inventory spending among SKUs is of importance in today's completive business environment.

Our work has the following limitations, which also opens the door for future study. Firstly, our current model is a one-period static model. Although it can be implemented in a rolling horizon fashion as shown in Section 5.2, it will be interesting to develop a multi-period dynamic inventory grouping model that directly optimizes the grouping decisions taking the future demand projection and trend into consideration. Secondly, the current model is based on a deterministic optimization approach, which can be improved by an integrated simulation–optimization approach to optimize inventory grouping decisions under uncertainty. In addition, due to limited availability of data, our computational study has focused on the model with single-objective. We plan to continue working with our industrial partners on inventory grouping optimization with multiple criteria. It is also our plan to consider other practical inventory management settings such as quantity discounts, perishable SKUs, and variable overhead management cost per inventory group as a function of the number of groups, the number of SKUs or volumes in each group.

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