Facility Location and Supply Chain Optimization for a Biorefinery

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ABSTRACT: This paper presents a systematic approach for the optimal production planning and facility placement of a biorefinery. A structural representation is first developed to include sources of biomass feedstock, distributed preprocessing hubs, and centralized processing facilities to produce desired products and byproducts. An optimization formulation is developed to determine the optimal supply chain, size, operational strategies, and location of the biorefinery and preprocessing hub facilities. The model considers simultaneously the optimal selection of different configurations considering the specific location configuration (centralized and/or distributed), selection of biomass, and processing facilities to determine the one with the maximum overall net profit. The objective function considers the overall sales and the costs for the feedstocks, transportation costs, capital costs for the facilities, and the operational costs for the facilities. The model also considers nonlinear economy-of-scale behavior of the capital-cost functions that are reformulated using disjunctive models to yield convex relationships to guarantee a global optimal solution. The proposed model was applied to two case studies.

1. INTRODUCTION

1.1. Background. Recently, there has been a growing interest in the development of sustainable sources of energy. In early 2010, a U.S. presidential address included several initiatives geared toward encouraging the use of renewable liquid fuels “to enhance American energy independence while building a foundation for a new clean energy economy.” 3 Steps include a long-term renewable fuels mandate of 36 billion gallons by 2022 established by Congress, 1 and to comply with the rule, some renewable fuels must achieve greenhouse gas (GHG) emission reductions compared to the petroleum derived gasoline or diesel they displace. Additionally, the Biomass Crop Assistance Program proposed by the U.S. Department of Agriculture 2 will provide grants, loans, and other financial support to encourage the use of biomass for biofuels and other bioderived chemical products. The goal of this program is to speed up the commercialization of developing biomass to bioproduction technologies, and to assist with the collection, harvesting, storage, and transportation costs of eligible biomass. 2 Similar initiatives have also been launched around the world with the objective of developing sustainable sources of energy and reducing GHG impact. These initiatives call for advances in the supply of cost-effective biomass, the development of efficient technological pathways, and the establishment of supply chains for the evolving biorefining industry.

1.2. Literature Review. Much research has already focused on the areas of agricultural yield improvement (e.g., Aguirrezebal et al., 3 Zhao et al., 4 Cossani et al., 5 and Anastasi et al. 6 ). Also, systematic approaches have been proposed for the screening and selection of technological pathways for biorefineries. Ng et al. 7 proposed a hierarchical procedure for the synthesis and screening of potentials for integrated biorefineries. Different researchers have studied individual biorefining pathways including thermal processes (e.g., Goyal et al., 8 ) and biodiesel production (e.g., Mohan and El-Halwagi, 9 Myint and El-Halwagi, 10 Pokoo-Akins et al., 11 and Qin et al. 12 ). Sammons et al. 13 incorporated economic perspective to analyze an integrated biorefinery and develop a systematic framework that evaluates environmental and economic measures for product allocation problems. Tan et al. 14 developed an extended input—output model using fuzzy linear programming to determine the optimal capacities of distinct process units given a predefined product mix and environmental (carbon, land, and water footprint) goals. Elms and El-Halwagi 15 introduced an optimization routine for feedstock selection and scheduling for biorefineries and included the impact of greenhouse gas policies on the biorefinery design. Pokoo-Akins et al. 16 included safety metrics along with process and economic metrics to guide the design and screening of biorefineries.

With respect to the supply chain optimization for biorefineries, van Dyken et al. 17 developed a mixed integer linear programming (MILP) model for the optimization of biomass supply chains by considering the effect on biomass quality of each step in transport, storage, and processing where the primary biomass quality observed was the moisture and energy content. Dansereau et al. 18 developed a margin-centric approach to the optimization of the forest-biorefinery supply chains. Gigler et al. 19 proposed a dynamic modeling approach toward optimization of agricultural or biomass supply chains, where the appearance and biomass quality were the two key parameters optimized; the appearance states are affected by handling and biomass quality is affected by processing, storage, and transportation. Freppaz 20 developed an objective function considering the sales of energy produced, plant construction maintenance costs, biomass transportation and harvesting, and energy distribution costs for decision support in determining the optimal amount of woody biomass to be used for energy instead of other competing uses. Sokhansanj and Fenton 21 presented a
dynamic integrated framework that conducts a biomass supply analysis and logistics model of collection, storage, and transport operations for supplying corn stover to a biorefinery, highlighting seasonal weather conditions. Graham et al.22 used a system to quantitatively model the geographic variation of suppliers and feed costs with environmental considerations to account for geographic differences in factors that affect the supply of biomass to biorefinery facilities. The paper by Allen et al.23 discusses part of a project into the logistic planning, management, and cost of supplying biomass fuels to a biomass-fired power station in the United Kingdom. Iakovou et al.24 presented a critical review of the state of the art for the design and management of waste biomass supply chains. Cucek et al.25,26 developed a mixed integer nonlinear programming model for the synthesis of regional renewable energy supply chains.

In the context of optimization of the location of integrated biorefineries, location science is a field addressed by operations research in which the optimal location of a new facility is determined with respect to cost, profit, distance, service time, market coverage, or some other desired attribute. Since several criteria are evaluated in order to find the optimal location, the problem is often a multiple-criteria decision-making problem. A list of common objectives when solving a location problem include minimizing the total setup cost, minimizing the longest distance from the existing facilities, minimizing fixed costs, minimizing total annual operating costs, maximizing service, minimizing average time/distance traveled, minimizing maximum time/distance traveled, minimizing the number of located facilities, and maximizing responsiveness.27

ReVelle et al.28 classified typical location problems into four broad categories (i.e., analytical, continuous, networks, and discrete models). The analytical models assume that all demands are distributed uniformly throughout a service area, the cost of locating a facility is fixed and constant throughout the service, and the transportation cost per unit per distance is a fixed value. The continuous models allow facilities to be located anywhere within the service area with demands occurring at discrete points within the area; demands are weighted on a coordinate system and distances between demands are linear. The objective of these models is to minimize the overall demand weighted distance. Network models place the location problem on a series of links and nodes with demand occurring at each node. Solutions typically involve developing special structures that yield polynomial time algorithms. Discrete models assume a discrete set of demands and number of potential locations.

Nardi et al.29 developed an optimization methodology to minimize the transportation cost of a supply chain network for grains in Argentina utilizing several feedstock origins, multiple transportation methods, and various destinations using geographical information systems (GIS) software to map resource availability, destination location, and capacity that were already in place. Parker et al.30 developed a model for biorefinery location using GIS to account for biomass availability and optimizing the total industry-wide profits considering facility location and transportation costs. This model was used to develop a reasonable biofuel supply curve for the western United States, and it considered three modes of transportation (i.e., truck, rail, and barge); however, distributed preprocessing of biomass to reduce transportation costs was not considered in this work. At present the use of preprocessing hubs to reduce transport costs to biorefineries has not been thoroughly investigated.

Discrete hub location problems are one subset of location science of interest that offer significant improvements (see Figure 1). Hubs are defined as facilities that serve as transshipment or switching points for transportation networks with multiple origins and destinations. Hub to hub transportation costs are lower as they may take advantage of economies of scale. The objective of the hub location problem is to minimize the transportation cost of a unit from its point of origin to its final destination. There are several classifications of hub problems, but the uncapped hub location problem is of particular interest. In the uncapacitated problem the number of hubs is unspecified but each hub has a predetermined fixed cost. Campbell31 outlined the different classes of discrete hub location problems and proposed integer programming techniques specific to each.

The use of hubs is of interest to the biofuels industry for their potential to reduce transportation costs and also for potential sites to preprocess biomass to a more valuable dense feedstock. Distributed preprocessing of biomass to an upgraded or denser form on site or at a fixed preprocessing facility in some cases may provide cost benefits due to improved handling and reduced transportation costs.32 Because large economies of scale are frequently required when processing biomass, a large area is required to supply required feed biomass and transportation costs can play a significant role in the viability of the plant. The problem here is to decide when distributed or centralized processing of biomass facilities is the optimal scheme for a plant. Notwithstanding the important research developed and published in the literature to date, there are opportunities for novel contributions in the area. This work is aimed at introducing the following novel contributions:

- A new superstructure is proposed for the optimal location of a distributed feedstock treatment system for the supply chain that allows simultaneous selection of either centralized or distributed configurations.
- A new mathematical model based on the superstructure is proposed for the maximization of the total profit across the entire value chain accounting for transportation costs, siting selection for the central processing facility, and siting selection for potential distributed preprocessing facilities.
- A new method is developed for determining the optimal use and placement of preprocessing hubs for feedstock densification. While considerable effort has been directed at the issue of biorefinery supply chain configuration as shown in the literature survey, no empirical method in this regard has been developed until now.

Figure 1. Traditional location problem including hubs and centralized facilities.
The model considers adequately the capital costs for the facilities considering the exponential cost functions, and it is based on a disjunctive programming model that is reformulated as a mixed integer linear programming problem to yield the global optimal solution.

2. PROBLEM STATEMENT

The problem addressed in this work is defined as follows:

Given are a set of sources for the biomass feedstocks, a set of locations available to construct preprocessing hub facilities, and a set of locations to install the central processing and distribution facilities. There are limits for the supply of the feedstock and for the demand for the products. The problem is aimed at determining the optimal configuration for the processing and distribution systems to yield the solution with the maximum total net profit considering the sales for the products and the cost for the raw materials, the transportation costs, and the operating and capital costs for the facilities.

3. MODEL FORMULATION

3.1. Description for the Model Proposed. Prior to the model formulation, the main indexes are defined. $i$ corresponds to the agricultural areas where the feed is produced, $j$ is an index to indicate the possible locations to install the hub preprocessing facilities, $k$ indicates the locations able to install the centralized processing facilities, and $l$ represents an index for the products and subproducts; finally, $n$, $m$, and $q$ are indexes for the disjunctions to determine the capital costs for the hub and central facilities.

The model formulation is based on the superstructure shown in Figure 2. It is worth noting here that the location process (see Figure 1) in this case is modeled as a source/interception/sink mass-integration representation. The sources can send the feedstocks to the preprocessing hubs and/or to the centralized biorefineries. After the preprocessing hubs process the feedstocks, they produce an intermediate product that can be sent to the central facilities for further processing and subproducts that may be sold at that location. The final product is sent to consumers from the central facilities. The existence of the facilities (hubs and central facilities) is an optimization variable that must be determined.

3.2. Mathematical Formulation. The model must determine the network configuration and the optimal flow rates to yield the process with the maximum profit. Then, the model formulation is stated as follows.

3.2.1. Maximum Feedstock Available. The total feedstock purchased from producer $i$ ($F_i$) must be less than the total feedstock available from that producer ($F_{i\text{max}}$), leading to the following constraint.

$$F_i \leq F_{i\text{max}}, \quad \forall i \in I$$

3.2.2. Supply Feedstock Balances. Feedstock purchased from each producer $i$ ($F_i$) may be routed to the nearest preprocessing hub $j$ if this is selected via $f_{ij}$ or it may bypass the hub and ship directly to the preprocessing section of the centralized facility $k$ via $h_{ik}$. This yields the next constraint.

$$F_i = \sum_j f_{ij} + \sum_k h_{ik}, \quad \forall i \in I$$

The amount of material entering the hubs and central facility from each producer must be equal to the purchased amount.

![Figure 2. Superstructure for the model.](image-url)
3.2.3. Material Balances for the Mixers Prior to the Hubs. The total material processed by each hub is defined as \( F_j \), and this must be equal to the sum of the material from any feedstock \( i \) (\( f_{ij} \)).

\[
F_j = \sum_{i} f_{ij}, \quad \forall j \in J
\]  

(3)

3.2.4. Hub Processing Balances. The primary product at each hub \( (l = 1) \) is passed on as the feed to the centralized processing facility, so it is removed from the product slate via \( G_j \) in a manner that maintains model flexibility to sell intermediate products at hubs should this become the optimal business strategy.

\[
P_{lj}^{\text{hub}} = \alpha_{lj}^{\text{hub}} F_j - G_j, \quad \forall l = 1, \quad \forall j \in J
\]  

(4)

For products other than the main \( (l > 1) \) such as meal, heat, and power, the subproducts may be sold directly from the hub, and the material balance is stated as follows:

\[
P_{lj}^{\text{hub}} = \alpha_{lj}^{\text{hub}} F_j', \quad \forall l \geq 2, \quad \forall j \in J
\]  

(5)

Here, \( \alpha_{lj}^{\text{hub}} \) is the process yield per input for each product or subproduct \( l \) at each hub \( j \). If a hub is not designed with the ability to leverage a particular subproduct, its yield is set as zero for that facility.

3.2.5. Mass Balance for the Inlet to the Central Processing. The feed to the central processing facility remains segregated in two categories: material that has passed through the hubs and has been through the preprocessing step \( G_j \) and material transported directly to the central facility \( H_k \) that still requires the intermediate processing step before conversion to biodiesel.

\[
G_k' = \sum_{l} G_{lk}, \quad \forall k \in K
\]  

(6)

\[
H_k' = \sum_{l} h_{lk}, \quad \forall k \in K
\]  

(7)

An additional equation is required to determine the inlet to the central processing facility \( k \) (\( G_k' \)) as the sum of preprocessed biomass from the hubs (\( G_k' \)) and from the central preprocessing units (\( H_k' \)):

\[
K_k' = G_k' + H_k', \quad \forall k \in K
\]  

(8)

3.2.6. Balances for the Central Facilities Processing. The formation of products is also treated differently at the central processing facility since products from the preprocessing step may be passed on to the final processing step or sold as intermediate products at the central site. Therefore, we have the following equation for the main product \( (P_{lk}^{\text{cen}}) \), where \( \alpha_{lk}^{\text{cen}} \) is the mass conversion factor for material \( l \) at site \( k \):

\[
P_{lk}^{\text{cen}} = \alpha_{lk}^{\text{cen}} H_k', \quad \forall l = 1, \quad \forall k \in K
\]  

(9)

For the byproducts

\[
P_{lk}^{\text{cen}} = \alpha_{lk}^{\text{cen}} K_k', \quad \forall l \geq 2, \quad \forall k \in K
\]  

(10)

The demand at each distribution point is valued by a commodity price specific to each location and is limited by a maximum demand constraint.

\[
P_{lk}^{\text{cen}} \leq P_{lk}^{\text{cenMAX}}, \quad \forall l \in L, \quad \forall k \in K
\]  

(11)

\[
P_{lj}^{\text{hub}} \leq P_{lj}^{\text{hubMAX}}, \quad \forall l \in L, \quad \forall j \in J
\]  

(12)

3.2.7. Objective Function. The objective function seeks to maximize profits while accounting for product sales, feedstock cost, transportation cost, preprocessing hub location assignment, central facility location assignment, and other operating costs. The general format of the objective function is stated as follows:

\[\text{Profits} = \text{ProductSales} - \text{FeedstockCost} - \text{TransportationCost} - \text{FacilityCapitalCost} - \text{VariableOperatingCosts}\]

(13)

Each section of the objective function is explained further in the sections below.

3.2.8. Product Sales. Vegetable oil, biodiesel, meal, syngas, heat, and power may be produced in the central and hub facilities. The diverse product slate may be subdivided into intermediate products that can be further processed in the existing value chain, and final products which are terminal products that require no further processing. Subproducts are considered a class of final products whose production is not essential to the process. To account for production of a varied product slate at multiple potential locations, the following formulation is used:

\[
\text{ProductSales} = \sum_{k} \sum_{l} \sum_{i} \text{cenprod} P_{lk}^{\text{cen}} + \sum_{l} \sum_{j} \text{subprod} P_{lj}^{\text{hub}}
\]  

(14)

\( P_{lk}^{\text{cen}} \) and \( P_{lj}^{\text{hub}} \) are the amount, either mass, MMBtu, or kW of product \( l \) formed at central facility location \( k \) or at hub location \( j \) that is valued at \( C_{lk}^{\text{cenprod}} \) and \( C_{lk}^{\text{subprod}} \). It is worth noticing here that both the centralized and hub facilities are able to produce final products and subproducts.

3.2.9. Feedstock Cost. The cost of feedstock used is simply the sum of the amount of feedstock purchased from each supplier \( i \) (\( F_i \)) plus any oil, if available, purchased by the centralized biodiesel plant \( k \), \( K_k \). \( C_{i}^{\text{biomass}} \) and \( C_{ik}^{\text{oil}} \) are the prices of feedstock and fresh oil purchased, respectively. Then, the feedstock cost is calculated as follows:

\[
\text{FeedstockCost} = \sum_{l} \sum_{i} C_{i}^{\text{biomass}} F_i + \sum_{l} \sum_{k} C_{ik}^{\text{oil}} K_k
\]  

(15)

3.2.10. Transportation Cost. The transportation cost is the sum of costs for transporting raw feedstock to preprocessing hubs or directly to the centralized facility and the cost of transporting oil from hubs to the centralized facility. \( F_{ij} \), \( g_{ik} \), and \( h_{ik} \) are the amount of mass moved from producer to hub, hub to central, and producer direct to central, respectively. \( C_{ij}^{\text{trans}} \) and \( C_{ik}^{\text{trans}} \) are the freight cost per ton per mile. Freight costs are a function of the mode of transportation used; trucks, rail, or barges may be used to move materials along the supply chain, each with a different cost per ton per distance. Hub to central transportation is generally less expensive as the mode of transportation is more developed. In reducing the transportation costs, the optimization routine seeks to reduce the total weighted distance among all

facilities. Then, the total transportation cost is stated as follows:

\[
\text{TransportationCost} = \sum_i \sum_j c_{ij}^{\text{trans}} f_j + \sum_j \sum_k C_{jk}^{\text{trans}} g_{jk} + \sum_j \sum_k C_{jk}^{\text{trans}} h_{jk}
\]

(16)

3.2.11. Facility Capital Costs. Next, the capital cost of locating a central facility or preprocessing hub must be considered; otherwise the model would seek to build a facility at every candidate location to reduce the transportation costs. The capital cost for each hub \( j \) preprocessing central facility \( k \) and central facility \( k \) can be calculated as follows:

\[
\text{FacilityCapitalCosts} = \frac{\sum_j \text{Cost}_{j}^{\text{hub}} + \sum_k \text{Cost}_{k}^{\text{cenPrep}} + \sum_k \text{Cost}_{k}^{\text{cen}}}{\text{lifetime}}
\]

(17)

The capital cost of a facility is assumed to be most heavily dependent upon the size of the facility. Potential locations with varying access to utilities or different needs specific to a location may also cause variability in location costs. It is worth noticing that usually the capital costs for the facilities follows a relationship of exponential capacity ratio with exponent (i.e., Costs = \( A + B(Capacity)^c \)), where \( A \) and \( B \) depend upon the type of facility and \( c \) is an exponent to account for scaling economies usually between 0.6 and 0.7).\(^{33}\) In addition, these facilities are restricted by a given maximum capacity: when this maximum capacity is overloaded an additional unit must be installed. A disjunctive formulation is used to linearize capital cost versus facility capacity curves. Figure 3 shows a schematic representation of the capital cost functions linearized. The preprocessing hub is used as an example.

The primary function of the preprocessing hub is to extract vegetable oil from the feedstock oil seeds, likely using hexane solvent extraction techniques. Modular packaged units of fixed total capacity are available to perform this task with potentially multiple units located at one site to meet capacity requirements, and an example of the disjunction used for hub capital cost is shown in section 3.2.12.

3.2.12. Capital Cost for Preprocessing Hub Facilities. To determine the capital cost for the hub facilities, the nonlinear relationships are discretized into a set of linear expressions through the following disjunction:

\[
\forall n \in N \left[ \frac{F_{jn}^\text{MIN}}{F_j} \leq y_{jn} \leq \frac{F_{jn}^\text{MAX}}{F_j}, \forall j \in J \right]
\]

The preceding disjunction states that the linear equation to determine the capital cost for the hubs depends on the capacity (as noted in Figure 3). The Boolean variable \( Y_{jn} \) is used to activate each term of the disjunction. Therefore, when a given capacity is selected (i.e., \( Y_{jn} \) is true), the corresponding capital cost equation is selected and the corresponding binary variable \( y_{jn} \) is 1. To model the preceding disjunction, the convex hull technique\(^{34,35}\) is used and the following algebraic relationships are obtained.

First, only one disjunctive term can be selected, and this is modeled as follows:

\[
\sum_n y_{jn} = 1, \forall j \in J
\]

(18)

The continuous variables are disaggregated for each segment \( n \) considered, first for the flow rates

\[
F_j = \sum_n f_{jn}, \forall j \in J
\]

(19)

and then for the cost

\[
\text{Cost}_{jn}^{\text{hub}} = \sum_n C_{jn}^{\text{hub}}, \forall j \in J
\]

(20)

Then, the constraints inside the disjunctions are stated in terms of the disaggregated variables:

\[
\frac{F_{jn}^\text{MIN}}{F_j} \leq y_{jn} \leq \frac{F_{jn}^\text{MAX}}{F_j}, \forall j \in J, \forall n \in N
\]

(21)

\[
C_{jn}^{\text{hub}} = \alpha_{jn} y_{jn} + b_{jn} f_{jn}, \forall j \in J, \forall n \in N
\]

(22)

Finally, upper and lower limits are imposed for the disaggregated variables:

\[
f_{jn}^\text{MIN} \leq f_{jn} \leq f_{jn}^\text{MAX}, \forall j \in J, \forall n \in N
\]

(23)

\[
f_{jn} \geq 0, \forall j \in J, \forall n \in N
\]

(24)

\[
C_{jn}^{\text{hub}} \geq 0, \forall j \in J, \forall n \in N
\]

(25)

To explain the previous relationships, we have the following. When a segment of the disjunctive terms is selected, then the associated Boolean variable \( Y_{jn} \) is true and the associated binary variable \( y_{jn} \) must be equal to 1. For all other cases, the Boolean and binary variables are false and 0, respectively; then, since the upper limits are given by eqs 21 and 23 for the segments not selected, the associated continuous disaggregated variables are 0, and the variables that are able to have values larger than 0 are the ones for the disjunctive term selected. For eqs 19 and 20 the continuous variables are equal to the disaggregated variables for the disjunctive term selected and the relationships are stated in terms of these disaggregated variables by relationships 21 and 22.

This same method is used to determine the capital cost of each facility. Cost curves are modified to reflect differences in location.
suitability or land costs. The total capital cost is then annualized throughout the expected lifetime of the project.

3.2.13. Capital Cost for Preprocessing Central Facilities. To determine the capital costs for the preprocessing central facilities, the following disjunction is used to have linear relationships in a given set of $q$ intervals:

$$
\forall q \in Q \quad \left[ X_{kq}^{\text{MIN}} H_{kq}^{\text{MIN}} \leq H_{kq} \leq H_{kq}^{\text{MAX}} \right], \quad \forall k \in K
$$

This way, when the Boolean variable $X_{kq}$ is true (i.e., the binary variable $H_{kq}$ is 1), this segment for the capital cost function is activated. The preceding disjunction is reformulated as a set of algebraic equations using the convex hull reformulations as follows.34,35

Only one segment can be selected:

$$
\sum_{q}^{Q} X_{kq} = 1, \quad \forall k \in K
$$

(26)

The continuous variables $H_{kq}^\prime$ and $\text{Cost}_{\text{cenPrep}}^k$ are disaggregated for each segment $q$ ($h_{kq}^\prime$ and $c_{kq}^\text{cenPrep}$) as follows:

$$
H_{kq} = \sum_{q}^{Q} h_{kq}, \quad \forall k \in K
$$

(27)

$$
\text{Cost}_{\text{cenPrep}}^k = \sum_{q}^{Q} c_{kq}^\text{cenPrep}, \quad \forall k \in K
$$

(28)

The constraints inside the disjunctions are stated in terms of the disaggregated variables:

$$
H_{kq}^{\text{MIN}} X_{kq} \leq h_{kq}^\prime \leq H_{kq}^{\text{MAX}} X_{kq}, \quad \forall k \in K, \quad \forall q \in Q
$$

(29)

$$
C_{kq}^\text{cenPrep} = c_{kq} X_{kq} + d_{kq} h_{kq}^\prime, \quad \forall k \in K, \quad \forall q \in Q
$$

(30)

Finally, upper and lower limits are imposed for the disaggregated variables:

$$
C_{kq}^\text{cenPrep} \leq C_{kq}^\text{cenPrepMAX} X_{kq}, \quad \forall k \in K, \quad \forall q \in Q
$$

(31)

$$
h_{kq}^\prime \geq 0, \quad \forall k \in K, \quad \forall q \in Q
$$

(32)

$$
c_{kq}^\text{cenPrep} \geq 0, \quad \forall k \in K, \quad \forall q \in Q
$$

(33)

To explain these relationships, when the Boolean variable $X_{kq}$ is true for a central preprocessing facility $k$ and a given segment $q$, then the associated binary variable $h_{kq}$ is equal to 1 (notice that only one segment $q$ must be equal to 1 by eq 26 for each location $k$) and the associated disaggregated variables can be greater than 0 (see relationships 29 and 31); for other segments where the binary variables are 0, the associated disaggregated variables are 0 (by relationships 29 and 31). Finally, the relationships inside the disjunction are stated in terms of the disaggregated variables by relationships 29 and 30, and these relationships are activated only for the case when the associated binary variables are 1; for the other cases these relationships are inactivated.

3.2.14. Capital Cost for Central Facilities. To determine the capital costs for the central facilities, the following disjunction is used in order to have linear relationships:

$$
\forall k \in M \quad \left[ Z_{km}^{\text{MIN}} K_{km} \leq K_k \leq Z_{km}^{\text{MAX}} K_{km} \right], \quad \forall k \in K
$$

In the preceding disjunction, the capital cost function for the central facilities is linearized in $m$ segments; therefore, the Boolean variable $Z_{km}$ becomes true when the segment $m$ is activated, while the associated binary variable $z_{km}$ becomes 1. Notice than only one segment $m$ is selected. The preceding disjunction is algebraically reformulated using the convex hull technique as follows.34,35

Only one segment can be selected:

$$
\sum_{m}^{M} z_{km} = 1, \quad \forall k \in K
$$

(34)

The continuous variables $K_k^\prime$ and $\text{Cost}_{\text{cen}}^k$ are disaggregated for each segment $m$ (i.e., $K_{km}^\prime$ and $c_{km}^\text{cen}$, respectively) as follows:

$$
K_k^\prime = \sum_{m}^{M} k_{km}^\prime, \quad \forall k \in K
$$

(35)

$$
\text{Cost}_{\text{cen}}^k = \sum_{m}^{M} c_{km}^\text{cen}, \quad \forall k \in K
$$

(36)
Table 2. Transportation Costs for Case Study 1 (USD/ton)

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<th>producer</th>
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<td>6.82</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Pricing Costs for Case Study 1

<table>
<thead>
<tr>
<th>parameter</th>
<th>description</th>
<th>value (USD/ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{biomass}}^i$</td>
<td>oil seed market spot price</td>
<td>varied</td>
</tr>
<tr>
<td>$C_{\text{oil}}^i$</td>
<td>vegetable oil spot price</td>
<td>not used</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^i$</td>
<td>vegetable oil contract price</td>
<td>700</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^{2k}$</td>
<td>meal contract price</td>
<td>250</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^{3k}$</td>
<td>biodiesel contract price</td>
<td>800</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^q$</td>
<td>vegetable oil contract price</td>
<td>700</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^{j2}$</td>
<td>meal contract price</td>
<td>250</td>
</tr>
<tr>
<td>$C_{\text{cen prod}}^{j3}$</td>
<td>biodiesel contract price</td>
<td>800</td>
</tr>
</tbody>
</table>

The constraints inside the disjunctions are stated in terms of the disaggregated variables:

\[
K_{km}^{\text{MIN}} z_{km} \leq k^i_{km} = K_{km}^{\text{MAX}} z_{km}, \quad \forall k \in K, \quad \forall m \in M \tag{37}
\]

\[
C_{km}^{\text{cen}} = r_{km} z_{km} + s_{km} k^i_{km}, \quad \forall k \in K, \quad \forall m \in M \tag{38}
\]

Finally, upper and lower limits are imposed for the disaggregated variables:

\[
C_{km}^{\text{cen}} \leq C_{km}^{\text{cen MAX}} z_{km}, \quad \forall k \in K, \quad \forall m \in M \tag{39}
\]

\[
k^i_{km} \geq 0, \quad \forall k \in K, \quad \forall m \in M \tag{40}
\]

\[
k^i_{km} \geq 0, \quad \forall k \in K, \quad \forall m \in M \tag{41}
\]

The explanation of the preceding disjunction is similar to the one for the preprocessing hubs.

3.2.15. Operating Cost. The operating cost considers variable costs of operations including labor, supervision, utilities, maintenance, supplies, lab charges, royalties, catalyst, solvents, taxes, and insurance. As an approximation in this model, it is assumed that all of these charges are directly linearly dependent upon production levels. \(F_j, K_k,\) and \(H_k\) are the plant inlet feed rates (see Figure 2). \(\text{Cost}_{\text{op}}^{\text{p}}, \text{Cost}_{\text{op}}^{\text{cen}},\) and \(\text{Cost}_{\text{op}}^{\text{cen Pre}}\) are the operating cost charges in U.S. dollars (USD) per mass processed at each facility; then, the total operating cost is given by

\[
\text{OperatingCost} = \sum_j \text{Cost}_{\text{op}}^{\text{p}} F_j + \sum_k \text{Cost}_{\text{op}}^{\text{cen}} K_k + \sum_k \text{Cost}_{\text{op}}^{\text{cen Pre}} H_k
\]

Estimates for the variable operating cost of biodiesel production range from roughly $93 to $111 per ton of oil processed (e.g., van Gerpen” and Carriquiry”).

Figure 5. Capital cost function for preprocessing facilities.

3.3. Model Remarks

- The model formulation is an MILP problem; therefore, a global optimal solution is guaranteed.
- The model considers typical exponential capital cost behavior for the processing facilities.
- The superstructure considers simultaneously distributed and centralized configurations.

4. CASE STUDIES

Two cases of study are used to show the applicability of the proposed methodology.

4.1. Case Study 1. For this example problem, the case study to determine the optimal location of hubs and central facilities for the biomass processing to yield biodiesel is considered to show the applicability of the proposed methodology. Six locations with specific feedstock availabilities are considered (in Figure 4, identified by diamonds), there are two locations to install the central facilities identified in Figure 4 by triangles, and there are...
also two locations to install preprocessing hubs (identified by squares in Figure 4). Tables 1 and 2 show the distances and the unitary transportation costs between the different locations considered for this case.

Information regarding the costs of feedstock, intermediate products, and final products is shown in Table 3, and the process yields are shown in Table 4. These values were chosen to roughly correspond to an oil seed crop used to create FAME biodiesel. Much of the data used is best guess or order-of-magnitude estimates and should not be considered empirical. For more accurate requirements, experimental data or previously reported methodologies). Notice that the distributed solution presents significant savings with respect to the centralized solution for all scenarios for the feed price analyzed.

Table 9 shows the feedstock purchasing variation for case study 1. The behavior obtained is explained as follows: as the price of the feed increases, preference is given to producers closer to the selected production facilities.

4.2. Case Study 2. The distributions for the producers, hubs, and central facilities are shown in Figure 8 for this case study. This case considers the possibility of installing two central facilities and two hubs, whereas six producers are considered. Figure 8 also shows the feedstock available for each producer. The distances and the transportation costs are shown in Tables 10 and 11, respectively.

The same capital cost functions for the facilities used in the first case study are used in the second case study (including the correlation data for these capital cost functions). The proposed model consists of 219 continuous variables, 38 binary variables, and 325 constraints. To analyze the results, several simply reasonable order-of-magnitude guesses to display the functionality of the model and should not be taken as accurate estimates of facility capital costs.

Table 6. Data for Central Processing Facility Capital Cost for Disjunctive Relationships

<table>
<thead>
<tr>
<th>capacity interval (tons/year)</th>
<th>$c_{km}$</th>
<th>$d_{km}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0—15 000</td>
<td>2 000 000</td>
<td>250</td>
</tr>
<tr>
<td>15 001—75 000</td>
<td>3 750 000</td>
<td>135</td>
</tr>
<tr>
<td>75 001—150 000</td>
<td>6 750 000</td>
<td>90</td>
</tr>
<tr>
<td>150 001—225 000</td>
<td>15 750 000</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 7. Data for Central Preprocessing Unit Capital Cost

<table>
<thead>
<tr>
<th>capacity interval (tons/year)</th>
<th>$r_{kk}$</th>
<th>$s_{kk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0—40 000</td>
<td>200 000</td>
<td>20</td>
</tr>
<tr>
<td>40 001—200 000</td>
<td>800 000</td>
<td>5</td>
</tr>
<tr>
<td>200 001—240 000</td>
<td>–2 200 000</td>
<td>20</td>
</tr>
<tr>
<td>640 001—800 000</td>
<td>2 600 000</td>
<td>5</td>
</tr>
<tr>
<td>240 001—400 000</td>
<td>1 400 000</td>
<td>5</td>
</tr>
<tr>
<td>400 001—440 000</td>
<td>–4 600 000</td>
<td>20</td>
</tr>
<tr>
<td>440 001—600 000</td>
<td>2 000 000</td>
<td>5</td>
</tr>
<tr>
<td>600 001—640 000</td>
<td>–7 000 000</td>
<td>20</td>
</tr>
</tbody>
</table>
scenarios are proposed: for scenario 1 the price of feed was fixed but the transport costs were varied. For scenario 2, the transport costs were modified by varying the dollar per ton per mile rate as shown in Table 12. Transportation cost factors were the primary manipulated parameter in this case study to observe its impact on the final configuration. The result is that, as the transportation cost factor from producer to hub is lowered, more feedstock is pulled through the hub and the hub size increases. For this case study the feed cost was fixed as $305/ton, except for scenario 3 the feed prices from producers 4 and 5 were decreased to $295/ton. Decreasing the value of stranded feedstock also pulls more feedstock into the supply chain and causes hub sizes to increase to gain access to stranded feedstocks. Table 13 shows the results for the different scenarios analyzed for example 2. Even though the configuration is the same for all scenarios, the material distributions are different. In addition, the total profit for the centralized solution is always less than the distributed solution; notice that previously reported methodologies only considered central processing facilities. The utilization of hubs extends the reach of the

Table 8. Results for Case Study 1

<table>
<thead>
<tr>
<th>distributed solution</th>
<th>feed price (USD/ton)</th>
<th>objective value (USD/year)</th>
<th>hub 1 (tons/year)</th>
<th>hub 2 (tons/year)</th>
<th>central 1 (tons/year)</th>
<th>central 2 (tons/year)</th>
<th>centralized solution (USD/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>305</td>
<td>28,248,788</td>
<td>240,000</td>
<td>203,000</td>
<td>160,000</td>
<td>15,921,938</td>
<td>160,000</td>
<td>7,245,250</td>
</tr>
<tr>
<td>310</td>
<td>19,738,750</td>
<td>240,000</td>
<td>160,000</td>
<td>160,000</td>
<td>11,245,250</td>
<td>160,000</td>
<td>7,245,250</td>
</tr>
<tr>
<td>315</td>
<td>11,738,750</td>
<td>240,000</td>
<td>160,000</td>
<td>160,000</td>
<td>7,245,250</td>
<td>160,000</td>
<td>1,646,000</td>
</tr>
<tr>
<td>320</td>
<td>3,738,750</td>
<td>240,000</td>
<td>160,000</td>
<td>160,000</td>
<td>3,245,250</td>
<td>160,000</td>
<td>0</td>
</tr>
<tr>
<td>322</td>
<td>1,689,000</td>
<td>240,000</td>
<td>160,000</td>
<td>160,000</td>
<td>1,646,000</td>
<td>160,000</td>
<td>0</td>
</tr>
<tr>
<td>325</td>
<td>0</td>
<td>240,000</td>
<td>160,000</td>
<td>160,000</td>
<td>0</td>
<td>160,000</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9. Feedstock Purchase Quantities for Example 1 (tons/year)

<table>
<thead>
<tr>
<th>producer</th>
<th>305</th>
<th>310</th>
<th>315</th>
<th>320</th>
<th>322</th>
<th>325</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.50 x 10^3</td>
<td>4.50 x 10^3</td>
<td>4.50 x 10^3</td>
<td>4.50 x 10^3</td>
<td>4.50 x 10^3</td>
<td>4.50 x 10^3</td>
</tr>
<tr>
<td>2</td>
<td>5.00 x 10^3</td>
<td>5.00 x 10^3</td>
<td>5.00 x 10^3</td>
<td>5.00 x 10^3</td>
<td>5.00 x 10^3</td>
<td>5.00 x 10^3</td>
</tr>
<tr>
<td>3</td>
<td>3.25 x 10^5</td>
<td>3.25 x 10^5</td>
<td>3.25 x 10^5</td>
<td>3.25 x 10^5</td>
<td>3.25 x 10^5</td>
<td>3.25 x 10^5</td>
</tr>
<tr>
<td>4</td>
<td>1.65 x 10^5</td>
<td>1.65 x 10^5</td>
<td>1.65 x 10^5</td>
<td>1.65 x 10^5</td>
<td>1.65 x 10^5</td>
<td>1.65 x 10^5</td>
</tr>
<tr>
<td>5</td>
<td>3.00 x 10^5</td>
<td>2.25 x 10^5</td>
<td>2.25 x 10^5</td>
<td>2.25 x 10^5</td>
<td>2.25 x 10^5</td>
<td>2.25 x 10^5</td>
</tr>
<tr>
<td>6</td>
<td>1.00 x 10^5</td>
<td>1.00 x 10^5</td>
<td>1.00 x 10^5</td>
<td>1.00 x 10^5</td>
<td>1.00 x 10^5</td>
<td>1.00 x 10^5</td>
</tr>
</tbody>
</table>

Table 10. Distances for the Case Study 2 (miles)

<table>
<thead>
<tr>
<th>producers to hubs</th>
<th>producer hub 1</th>
<th>hub 2</th>
<th>producer central 1</th>
<th>central 2</th>
<th>hub central 1</th>
<th>central 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>131</td>
<td>228</td>
<td>1</td>
<td>49</td>
<td>84</td>
<td>178</td>
</tr>
<tr>
<td>2</td>
<td>52</td>
<td>410</td>
<td>2</td>
<td>230</td>
<td>106</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>46</td>
<td>321</td>
<td>3</td>
<td>142</td>
<td>56</td>
<td>311</td>
</tr>
<tr>
<td>4</td>
<td>396</td>
<td>51</td>
<td>4</td>
<td>219</td>
<td>352</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>362</td>
<td>8</td>
<td>5</td>
<td>185</td>
<td>315</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>166</td>
<td>403</td>
<td>6</td>
<td>251</td>
<td>149</td>
<td></td>
</tr>
</tbody>
</table>

Table 11. Transport Costs for Case Study 2 (USD/ton)

<table>
<thead>
<tr>
<th>producers to hubs, Cij</th>
<th>producers to centrals, Cik</th>
<th>hubs to centrals, Cjk</th>
</tr>
</thead>
<tbody>
<tr>
<td>producer hub 1</td>
<td>hub 2</td>
<td>producer central 1</td>
</tr>
<tr>
<td>1</td>
<td>13.10</td>
<td>22.76</td>
</tr>
<tr>
<td>2</td>
<td>5.25</td>
<td>40.99</td>
</tr>
<tr>
<td>3</td>
<td>4.60</td>
<td>32.14</td>
</tr>
<tr>
<td>4</td>
<td>39.59</td>
<td>5.10</td>
</tr>
<tr>
<td>5</td>
<td>36.22</td>
<td>0.80</td>
</tr>
<tr>
<td>6</td>
<td>16.60</td>
<td>40.26</td>
</tr>
</tbody>
</table>

Table 12. Scenarios for the Transport Cost Factors for Example 2 (USD/ton mile)

<table>
<thead>
<tr>
<th>scenario</th>
<th>i to j</th>
<th>j to k</th>
<th>i to k</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.07</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.02</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.02</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 13. Results for Case Study 2

<table>
<thead>
<tr>
<th>distributed solution</th>
<th>objective value (USD/year)</th>
<th>hub 1 (tons/year)</th>
<th>hub 2 (tons/year)</th>
<th>central 1 (tons/year)</th>
<th>central 2 (tons/year)</th>
<th>centralized solution (USD/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25,124,220</td>
<td>325,000</td>
<td>218,500</td>
<td>160,000</td>
<td>12,856,370</td>
<td>12,856,370</td>
</tr>
<tr>
<td>2</td>
<td>19,618,430</td>
<td>200,000</td>
<td>196,000</td>
<td>150,000</td>
<td>15,066,746</td>
<td>15,066,746</td>
</tr>
<tr>
<td>3</td>
<td>25,621,890</td>
<td>225,000</td>
<td>200,000</td>
<td>160,000</td>
<td>12,856,370</td>
<td>12,856,370</td>
</tr>
</tbody>
</table>
biorefinery to include feedstock suppliers that were previously considered unprofitable.

5. CONCLUSIONS

This paper presents a mathematical programming model for the optimal placement of distributed biorefineries. The model includes the optimal selection of biomass from different sources, and the possibility to send it to preprocessing hub facilities or send it directly to central processing and distribution facilities. The model is able to determine the amount of each source sent to each facility and the amount of products and subproducts that must be produced for each facility to determine the maximum total net profit considering the transportation costs and the operating and capital costs for the facilities. The capital costs for the facilities consider the power-law behaviors that are modeled through a set of disjunctive formulations to linearize the model as a mixed integer linear programming problem to guarantee the global optimal solution of the problem. The application of the proposed methodology shows that the distributed configurations usually represent better solutions than the centralized solutions. No numerical complications were observed in the solutions of the examples analyzed.

■ NOMENCLATURE

Decision Variables

- \( C_{\text{cenPrep}} \) = central preprocessing \( k \) capital cost
- \( C_{\text{cen}} \) = central cost in linearized interval
- \( C_{\text{hub}} \) = hub cost in linearized interval
- \( C_{\text{hubPrep}} \) = central preprocessing cost in linearized interval
- \( C_{\text{hub}} \) = hub capital cost
- \( C_{\text{cen}} \) = central \( k \) capital cost
- \( F_i \) = purchased feedstock
- \( f_{ij} \) = feedstock routed \( i \) to \( j \)
- \( f_i \) = feedstock sum into hub \( i \)
- \( f_j \) = hub feed rate in linearized interval
- \( \text{FacilityCapitalCost} \) = total facilities capital cost
- \( \text{FeedstockCost} \) = total feedstock cost
- \( G_j \) = intermediate leaving hub \( j \)
- \( G_k \) = sum of intermediate into central \( k \)
- \( H_i \) = sum of raw feed to central \( k \)
- \( H_k \) = processed feed leaving centralized preprocessing
- \( h_{ij} \) = central preprocessing feed rate in linearized interval
- \( k_i \) = make-up oil
- \( k_j \) = sum of intermediate and processed raw feed at \( k_j \)
- \( k_{ih} \) = central feed rate in linearized interval
- \( P_{ij} \) = product \( i \) leaving hub \( j \)
- \( P_{ik} \) = product \( i \) leaving central \( k \)
- \( \text{ProductSales} \) = total product sales
- \( \text{Profit} \) = total profit
- \( \text{TransportationCost} \) = total transportation cost
- \( \text{VariableOperatingCosts} \) = total variable operational costs
- \( s_{ik} \) = binary variable for interval selection
- \( X_i \) = Boolean variable for interval selection
- \( y_{ij} \) = binary variable for interval selection
- \( Y_k \) = Boolean variable for interval selection
- \( z_{km} \) = binary variable for interval selection
- \( Z_{km} \) = Boolean variable for interval selection

Parameters

- \( G_{\text{hub}} \) = hub product yields
- \( G_{\text{cen}} \) = central product yields
- \( b_{mk} \) = linearization constant
- \( c_{km} \) = central product price
- \( c_{\text{hubPrep}} \) = hub product price
- \( c_{\text{biomass}} \) = feed price
- \( c_{\text{oil}} \) = make-up oil price
- \( c_{\text{trans}} \) = producer to hub freight cost
- \( c_{\text{trans}} \) = hub to central freight cost
- \( \text{Cost}_{\text{hub}} \) = central operating cost
- \( \text{Cost}_{\text{hubMax}} \) = maximum cost of hub
- \( \text{Cost}_{\text{cenMax}} \) = maximum cost of central
- \( \text{Cost}_{\text{cenPrepMax}} \) = maximum cost of central preprocessing section
- \( \text{d}_{km} \) = linearization constant
- \( \text{FMIN} \) = maximum central product demand
- \( \text{FMAX} \) = linearized interval maximum
- \( \text{FMIN} \) = linearized interval minimum
- \( \text{RMAX} \) = linearized interval maximum
- \( \text{RMIN} \) = linearized interval minimum
- \( \text{RMAX} \) = linearized interval maximum
- \( \text{RMIN} \) = linearized interval minimum
- \( \text{RMAX} \) = linearization constant
- \( \text{RMIN} \) = linearization constant
- \( \text{SMin} \) = linearization constant
- \( \text{SKin} \) = linearization constant

Indices

- \( i \) = agricultural area
- \( j \) = possible location to install a hub
- \( k \) = possible location to install a centralized facility
- \( l \) = product or byproduct
- \( m \) = index for the disjunction of capital costs for hubs
- \( n \) = index for the disjunction of capital costs for centralized facilities
- \( q \) = index for the disjunction of capital costs for centralized facilities

■ REFERENCES
