

## **A New EPQ Model with Considering Preventive Maintenance, Imperfect Product, Shortage and Work in Process Inventory**

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### **Abstract**

The determination of optimal production lot size has been always noticed by the researchers and various models have been proposed in this area. Among the issues under less consideration in these models is the issue of machine failure. Obviously, the machine may be encountered during the work with random failures and stop working, this makes clear the need for applying an appropriate maintenance policy for the machinery. In this paper by the consideration of some assumptions as authorized shortage, taking into account the work in process inventory and the possibility of producing imperfect products with and without the ability to re-work, EPQ model were investigated to select preventive maintenance policy for the machinery. Also by minimization of the system total cost over a period of time, a formula for determining the optimal production lot size is presented and by the presentation of various numerical examples, its applications is shown.

Key words: Work In Process Inventory, Shortage, The Optimal Production Lot Size, Maintenance, Imperfect Production

### **1.Introduction**

Economic Production Quantity (EPQ) is a simple mathematical model in the field of inventory management in a production-inventory system which is applied in industry as one of the most popular models of Lot-sizing in the field of inventories management and control [15]. This model can be considered as the expansion of the Economic Order Quantity EOQ which has been obtained by considering the constant production rate in the economic order quantity [16]. In other words, if instead of buying goods from other suppliers, the company decides to produce them itself, the economic production quantity is often used to determine the optimal production lot size. This model has been developed in order to adapt to the company's operations by taking into account different modes as limited production and warehouse capacities, multi-level production, random demand, single-machine production, performing rework on imperfect product, work in process inventory and etc [14,21,22].

Regardless of the accepted level of this model in industrial fields, there are some unrealistic assumptions as setup and holding cost and demand rate resulting from input parameters of the models which force the researchers to study more about the practical applications of this model [12].

Hayek and Salameh have developed an EPQ model for the case where the imperfect products percentage has a uniform distribution. The main assumptions used in their model include: back orders are authorized and all

imperfect products undergo reworking in order to achieve the desired quality, and reworking time is also considered in the model [11]. Chiu was developed Hayek and Salameh model with the assumption that instead of all imperfect products, only some of them undergo reworking to achieve the desired quality and the remaining quantity will be sold in auction price. In his model with regard to back demand and a random rate for imperfect parts, wastes and reworking of reworkable imperfect parts, he minimized system costs [8]. Schwaller allocated a certain percentage of a received lot to the imperfect parts and considered an inspection operation with fixed and variable costs to detect and remove imperfect parts [19]. Jamal et al investigated the problem of economic production quantity due to the reworking process for single-production mode and obtained economic production quantity for single-production mode [13].

In many models, the researchers performed modeling with regard to back orders as a key factor. Cardenas Baron presented a mathematical model for EPQ with back orders consideration for single-step production system, an EPQ without back orders consideration and an EPQ with reworking [2,4,5]. He has also suggested a new method that can calculate the optimal product amount and back orders level with regard to linear and fixed costs [3]. Chang et al developed EPQ model with back orders by the application of a simple couple methods [6]. Sphicas has also offered the EPQ model with back orders by considering two kinds of classic back orders costs without derivatives. He has identified expensive and attractive back orders and then analyzes them [23]. Drake et al has considered production lot planning to handle a two-stage system in which the final product has been planned by the application of an EPQ model with partial back orders. In their study the components production is under control using the EPQ models without back orders. They also presented an optimal closed inventory storage strategy for an EPQ model with partial back orders in a fixed rate  $\beta$  [9]. Sharma have taken into account the partial back orders in a model of analyzing the potential factor for cost savings, with this requirement that with the purpose of moving from alternative production toward continuous production, the production rate in a classic EPQ model should reduce in order to prolong the production phase of each production- inventory cycle [20]. Chiu et al have presented a method which is used to determine the optimal time of a process in an EPQ model accompanied by the scraps, rework and random machine failures [7]. Haji et al have examined a single machine multi-product problem with reworking and determined its production values [10]. Rosenblatt and Lee have proposed an EPQ model for a system which covers imperfect products. In their model, the basic premise is that the production system manufactures 100% imperfect goods from zero position of the production to a certain time which is a random variable. At this point the system is out of control and until the end of the production period it will start to produce imperfect products with a production rate with the assumption that the distribution of elapsed time until the moment when the system is out of control is being exponential. The back orders are also not allowed in their model [17].

In many manufacturing systems, WIP or work in process inventory is one of the components of inventory costs which is usually not considered in the Lot-Sizing formulas [16]. Boucher is the first person who calculated the Lot-Size value with respect to WIP for group technology systems [1]. Samaddar and Hill have proposed the exact mathematical formulas fully described the variance of the changes for the state of reduced setup process being able to achieve precise circumstances in which WIP leads to the improved or not improved process [18]. we can say that almost in all performed researches, the work in process inventory has a kind of correlation with the built-in parameters such as number of the machines and their processing rate, middle buffer capacity and also the assumptions related to the failure rate and machine repair.

Rasti et al have determined the optimal production lot size with regard to work in process inventory, imperfect product and re-work [16]. This paper is in fact a generalization of their model in which preventive maintenance of the machine in order to prevent failures during the manufacturing process as well as shortage as back orders have been regarded. The preventive maintenance is based on a determined schedule in specific periods and it avoids sudden failure and unplanned repairs and increases the useful life of the device. section 2 defines the problem with useful assumptions and parameters. In Section 3, problem modeling has been presented based on the defined assumptions and parameters and numerical examples are included in section 4, and finally the results of the model and the areas of its future development have been expressed in the last section.

## 2. Defining the problem

In this paper a manufacturing process of a single product and a single-machine is under the consideration which manufactures products periodically. Raw materials needed to process are ordered in the form of lots of size  $Q$  transferred to the production unit. In each production period, cumulative products as  $Q$  which has been placed in a container near the machine, are placed under the machine. The production process is in this way: after the operations on a set of input materials, in addition to producing perfect products, a percentage of imperfect products are produced too. With the completion of a course of machining, all products are inspected to determine their perfect or imperfect ones. Perfect products are transferred to a separate container, but imperfect products are divided to two groups of reworkable and non-reworkable. Reworkable imperfect products enter the

process one more time and they are placed under the machine and because of the more precise machining they all change to perfect products and transferred to the perfect products but non-reworkable imperfect products transferred to another container to be sold at the end of the process. Here the inventory shortage are authorized as back orders in this way that in each period with regard to the imposed costs to the system, shortage up to the amount of  $b$  are authorized and then at the beginning of the next period it will be compensated.

Possibility of creating discontinuity during the production is one of the main assumptions considered in this model. Therefore the machine which performs the duty of machining on the raw parts is stopped in each period of production at a specific time by the operator based on the preventive maintenance policies and immediately placed under the preventive maintenance in an average time by the maintenance department of the organization and afterwards the process of machining on the raw parts will be again resumed.

In this process, the work in process inventory is made up of three components: raw parts, perfect products and wastes which are kept in separate containers and in order to reduce transportation amount in the system the components enter the container and exit from it together. In each period, its demands plus back orders of the previous period are satisfied from the perfect products existed in the warehouse. The goal here is to achieve an optimal level of production so that the total system cost is minimized over a period of time.

### 2.1. Parameters of the model

$Q$	Production lot size per cycle
$D$	Demand rate of Perfect products
$S$	Set up time per cycle(min)
$\lambda$	Raw material cost per unit
$\omega$	Manufacturing cost per each product
$\overline{WIP}$	Average of the work in process inventory
$T_M$	Cycle time
$E$	Set-up cost per cycle
$b$	Authorized shortage per cycle
$K$	Shortage value
$M$	Manufacturing time per unit of product
$W$	Average stock
$P_1$	Ratio of reworkable imperfect products
$P_2$	Ratio of non-reworkable imperfect products
$Q_{EFQM}$	Optimal production lot size
$\$$	Shortage rate cost
MTTR	Average time of doing Preventive Maintenance operations
$\gamma$	Maintenance rate cost
$\theta$	Inspection cost per unit
$i$	Holding cost rate (unit of money per unit of money per unit time)
$\overline{V_{WIP}}$	Average value of the total work in process inventory
$\overline{C_{WIP}}$	Work in process holding costs per unit time
$C_D$	Set up costs per unit time
$C_S$	Purchase costs per unit time
$C_H$	Inventory holding costs per unit time
$C_b$	Shortage costs per unit time
$C_R$	Maintenance costs per unit time
$C_I$	Inspection costs per unit time
$TC_{PM}$	Average total costs per unit time

### 2.2. Assumptions

In this problem for the sake of modeling, the following assumptions are considered:

- Discontinuity during the production process is allowed for preventive maintenance operation on the machine that after doing it machining of raw parts again will be resumed like the initial state.

- Inspection of the products was in the form of one hundred percent and inspection time was considered to be zero.
- Parts that are under reworking become entirely perfect products.
- Parameters such as product demand rate, setup duration of the process, the percentage of imperfect parts and etc are deterministic.

### 3. Modeling

Suppose the organization had concluded according to the technical specifications of the machine and its standards that the best policy is to do preventive maintenance on it. As mentioned before, in this case before the machine encounters failure, the maintenance operation is done on it according to a periodic schedule. In this model we assume that during each period of production, the machine stopped by the operator to take necessary actions on it. We now examine the behavior of the work in process inventory position. In this case diagrams of the raw parts location, the perfect products and the wastes during a period are as figure 1. As mentioned earlier in this model the work in process inventory is consist of raw parts, perfect products and wastes. In figure 1- (1) at first the machine has been launched on time  $s$  and then the machining operations have been done on the raw parts located in a container near the machine. After a determined time passed from the zero position of the machining operation on the raw parts, the machine is stopped by the operator and maintenance operations are done immediately on it in the average time of  $MTTR$  by the maintenance personnel of the organization, and then the machining operation on the raw parts has been resumed until the inventory of these parts becomes zero. It should be mentioned that the first machining has been done on total raw parts and then on  $p_1$  re-workable raw part (i.e. totally on  $Q(1 + P_1)$  units of raw part) and therefore the inventory of the perfect product at the end of a period has reached to the value of  $Q(1 - P_2)$ .

Figures 1 - (2) and 1 - (3) are related to inventory of perfect products and wastes that have increasing rate by the beginning of machining process and accordingly reaches the values of  $Q(1 - P_2)$  and  $P_2 Q$  at the end of a period. Also in this situation diagram of inventory position for the perfect products which are stored in the warehouse in order to meet customer needs and back orders is shown in Figure 2. In Figure 2 at the beginning of each period, perfect products existed in the warehouse have been consumed with the demand rate of  $D$  in order to satisfy customer demand in the same period and back orders of the previous period and due to the authorized shortage in the model, eventually the inventory of warehouse products has reached to  $-b$  unit.

First we calculate the time of a period, time of a period (the interval between two starts of operations) in this case is consisted of the total setup time, machining time on  $Q(1 + P_1)$  units of raw units and also the average time spent on machine maintenance operations:

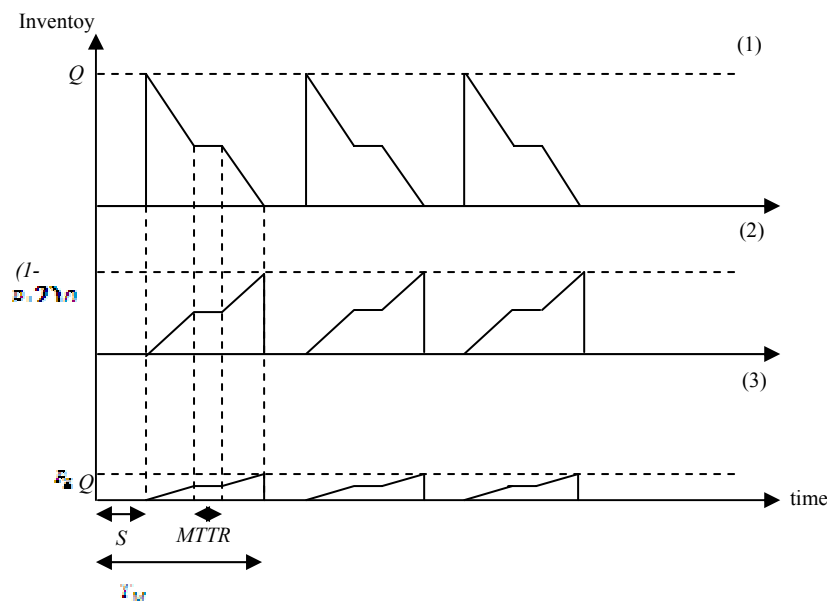


Fig. 1. Work-in-process inventory including (1) raw materials (2) good quality products and (3) wastage.

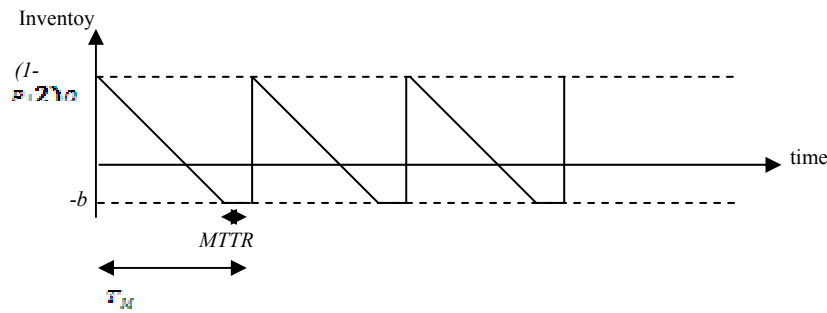


Fig. 2. Work-in-process inventory including good quality products in the warehouse.

$$T_M = s + M(1 + R_1)Q + MTTR \tag{1}$$

Total system costs over a period of time is:

$$TC_{PM} = C_D + C_S + C_{WIP} + C_M + C_b + C_R + C_I \tag{2}$$

3.1. Set up costs per unit time

In each period the machine starts manufacturing process with a fixed cost of E. therefore setup costs of a process in a period of time is:

$$C_D = \frac{E}{T_M} \tag{3}$$

3.2. Inspection costs per unit time

Inspection is only done once on a product, because by performing an inspection, it becomes clear that a product is perfect or imperfect but needs re-working or is a waste. Besides by re-working, an imperfect product changes to a perfect product. So, inspection cost of products over a period of time is obtained by the following relation.

$$C_I = \frac{pQ}{T_M} \tag{4}$$

3.3. Inventory holding costs per unit time

The average of work in process inventory over a period of time according to Figure 1 for raw parts, perfect products and wastes is calculated as below:

$$WIP = \frac{\frac{1}{2}Q(MQ(1 + R_1) + MTTR)}{T_M} + \frac{\frac{1}{2}Q(1 - R_2)(MQ(1 + R_1) + MTTR)}{T_M} + \frac{\frac{1}{2}QR_2(MQ(1 + R_1) + MTTR)}{T_M} \tag{5}$$

The average value of work in process inventory over a period of time is:

$$V_{WIP} = \frac{\frac{1}{2}Q(MQ(1 + P_1) + MTTR)}{T_M} \lambda + \frac{\frac{1}{2}Q(1 - P_2)(MQ(1 + P_1) + MTTR)}{T_M} \omega + \frac{\frac{1}{2}QP_2(MQ(1 + P_1) + MTTR)}{T_M} \omega = \frac{\frac{1}{2}(\omega + \lambda)(M(1 + P_1)Q^2 + MTTR \cdot Q)}{T_M} \dots\dots\dots (6)$$

Thus the holding cost of work in process inventory over a period of time is:

$$C_{WIP} = \frac{\frac{1}{2}i(\omega + \lambda)(M(1 + P_1)Q^2 + MTTR \cdot Q)}{T_M} \dots\dots\dots (7)$$

3.4. Inventory holding costs per unit time

According to figure 2 the inventory average of the perfect products in the warehouse is equal to:

$$W = \frac{(1 - P_2)^2 Q^2}{2D} \dots\dots\dots (8)$$

The cost of inventory holding in the warehouse over a period of time is:

$$C_H = \frac{i\omega \frac{(1 - P_2)^2 Q^2}{2D}}{T_M} \dots\dots\dots (9)$$

3.5. Shortage costs per unit time

Due to figure 2 the shortage value over a period is equal to:

$$K = \frac{b^2}{2D} + MTTR \cdot b \dots\dots\dots (10)$$

Thus the cost of dealing with the shortage over a period of time is:

$$C_b = \frac{\hat{\pi} \left( \frac{b^2}{2D} + MTTR \cdot b \right)}{T_M} \dots\dots\dots (11)$$

3.6. Purchase costs per unit time

in every period, order Q is purchased then the cost of raw parts over a period of time is obtained by the following equation:

$$C_S = \frac{\lambda Q}{T_M} \dots\dots\dots (12)$$

3.7. Maintenance costs per unit time

The cost of preventive maintenance operations over a period of time is equal to:

$$C_R = \frac{\gamma \cdot MTTR}{T_M} \dots\dots\dots (13)$$

Therefore the function of system total cost over a period of time is as follows:

$$TC_{PM} = \frac{E}{T_M} + \frac{\theta Q}{T_M} + \frac{\frac{1}{2}i(\omega + \lambda)(M(1 + P_1)Q^2 + MTTR \cdot Q)}{T_M} + \frac{i\omega \frac{(1 - P_2)^2 Q^2}{2D}}{T_M} + \frac{\pi \left( \frac{b^2}{2D} + MTTR \cdot b \right)}{T_M} + \frac{\lambda Q}{T_M} + \frac{\gamma \cdot MTTR}{T_M} \dots\dots\dots (14)$$

It is obvious that the above function is a curve depending on production lot size variable of Q and though it is a cost function, it should be minimized. So to obtain the optimal value of Q, it is necessary to calculate the first derivative root of the above function Q:

$$\frac{dT_{C_{PM}}(Q)}{dQ} = 0 \dots\dots\dots (15)$$

By differentiation of the above function (equation 14), we will have:

$$\frac{dT_{C_{PM}}(Q)}{dQ} = \frac{AQ^2 + BQ + C}{[M(1 + P_1)Q + S + MTTR]^2} \dots\dots\dots (16)$$

In which:

$$A = \frac{M^2(1 + P_1)^2 i (\omega + \lambda)}{2} + \frac{M(1 + P_1) i \omega (1 - P_2)^2}{2D} \dots\dots\dots (17)$$

$$B = (MTTR+S) [ i\omega [(1 - P_1)2] ^2)/D + i(\omega + \lambda)M(1 + P_1)] \dots\dots\dots (18)$$

$$C = (MTTR+S) [ i(\omega + \lambda)MTTR/2 + \lambda + \theta] - M(1+P_1)[E + \pi(b^2/2D + MTTR \cdot b) + \gamma \cdot MTTR] \dots\dots\dots (19)$$

if the quadratic equation in the numerator of the first total cost's derivative considered to be equal to zero, its positive root will be equal to the optimal value of Q:

$$AQ^2 + BQ + C = 0 \Rightarrow \dots\dots\dots (20)$$

$$Q_{EPQPM} = \frac{-B + \sqrt{\Delta}}{2A} \dots\dots\dots (21)$$

$$\Delta = B^2 - 4AC \dots\dots\dots (22)$$

3.8. Existence two distinct real roots of the debate in the first derivative of the total cost function

As we know in the quadratic equation whenever  $\Delta > 0$ , the equation has two distinct real roots, if  $\Delta = 0$  the equation has one real root and if  $\Delta < 0$  it has no real root (in fact, in this case the equation has two complex roots). So in the above equation to demonstrate two real roots, we must show  $\Delta > 0$ .

It is clear that in the above quadratic equation (equation 20) A for all values of the parameter is always positive and always for all values  $B^2 > 0$ . So if  $C < 0$ , we have:

$$C < 0, \quad A > 0 \Rightarrow 4AC < 0 \Rightarrow -4AC > 0 \Rightarrow B^2 - 4AC > 0 \Rightarrow \Delta > 0 \dots\dots\dots (23)$$

We conclude that in the above model in order to have two distinct real roots, the condition of  $C < 0$  must be enhanced, i.e.:

$$C = (MTTR+S) [ i(\omega + \lambda)MTTR/2 + \lambda + \theta ] - M(1+P_1) [ E + \pi'(b^2/2D + MTTR.b) + \gamma.MTTR ] < 0 \dots\dots\dots (24)$$

In the case of the above condition we will have  $C/A < 0$  and because of  $\Delta > 0$ , as a result according to the established theorems about relations between coefficients and roots of a quadratic equation, equation (20) has two distinct real roots that its positive root is equal to the optimal value of Q.

• It should be mentioned that in this case if  $\Delta = 0$ , regarding that for values of the parameters we always have  $A > 0$  and  $B > 0$ , so  $Q_{EPQPM} = -B/2A < 0$  that due to the negative result of the optimal value of Q, we do not consider this case.

3.9. Proved to be a convex function of the total cost

In order for the obtained value of Q which is equal to the first derivative root of the total cost function  $TC_{PM}$  to be the function minimum point, it is necessary that the cost function to be a convex function, i.e. the second derivative of the function Q must be positive. With regard to the past we have:

$$\frac{dT C_{PM}(Q)}{dQ} = \frac{AQ^2 + BQ + C}{[M(1 + P_1)Q + S + MTTR]^2} \dots\dots\dots (25)$$

By re-differentiation of the above relation and simplifying it we have:

$$\frac{d^2T C_{PM}(Q)}{dQ^2} = \frac{[2A(MTTR + S)^2 - 2M^2(1 + P_1)^2C]Q + B(MTTR + S)^2 - 2M(1 + P_1)(MTTR + S)C}{[M(1 + P_1)Q + S + MTTR]^4} \dots\dots\dots (26)$$

Obviously the expression in the above denominator is always positive for all positive values of Q. We want to show that for positive values of Q, all expressions included in the numerator are positive too. With regard to the condition of  $C < 0$  (equation 24):

$$C < 0 \Rightarrow 2M^2(1 + P_1)^2C < 0 \Rightarrow -2M^2(1 + P_1)^2C > 0 \stackrel{A > 0}{\Rightarrow} 2A(MTTR + S)^2 - 2M^2(1 + P_1)^2C > 0 \dots\dots\dots (27)$$

then for all positive values of Q for the first expression of the numerator we will have:

$$[2A(MTTR + S)^2 - 2M^2(1 + P_1)^2C]Q > 0 \dots\dots\dots (28)$$

accordingly for the second and third sentences:

$$B > 0 \Rightarrow B(MTTR + S)^2 > 0 \dots\dots\dots (29)$$

$$C < 0 \Rightarrow 2M(1 + P_1)(MTTR + S)C < 0 \Rightarrow -2M(1 + P_1)(MTTR + S)C > 0 \dots\dots\dots (30)$$

So the numerator and denominator of the second derivative of total cost function for all values of Q are always positive thus  $TC_{PM}$  function is a convex function and  $Q_{EPQPM}$  point is its minimum point.

4.Numerical examples

In order to demonstrate the practical application of the model, several numerical examples are presented here. In these instances it has been tried to show the dependence of optimal Q to the changes in parameters MTTR, b,  $P_{1i}$  and  $P_2$  while keeping other parameters constant. Suppose the numerical values of other parameters of the model are as follows:

$$M = 3(\text{min}), \quad \omega = 15(\$), \quad \lambda = 10(\$), \quad D = 30(\text{unit}), \quad S = 5(\text{min}), \quad \theta = 10 \left( \frac{\$}{\text{unit}} \right), \quad \pi = 100 \quad (\$/\text{unit}), \quad \gamma = 40 \left( \frac{\$}{\text{min}} \right),$$

$$E = 300(\$) \dots\dots\dots (31)$$

The results for the optimal Q based on the changes in parameters MTTR, b,  $P_{1i}$  and  $P_2$  is shown in Table 1.



Note that in all presented numerical examples, the condition of  $C < 0$  is true. Also the behavior of  $Q_{EPQPM}$  to some parameters of Table 1 is shown in Figures 3 to 5. As obvious from Figures 3 to 5,  $Q_{EPQPM}$  has direct relation with MTTR and  $b$  parameters while having an inverse relation with  $i$ . These results can be also achieved directly with differentiation of  $Q_{EPQPM}$  from each of these parameters.

In Figure 3, with increasing time of preventive maintenance on the machine, the curve at first has an ascending trend which is sever at first and then continued by a smoother slope. This cure shows that by increasing the time of preventive maintenance on the machine, with a higher level and more accurately, the optimal production amount is increased too (as this cause the increase in machine longevity and efficiency).

In figure 4 the curve has a complete descending trend which is sever at first and becomes smoother gradually. It can be said that with respect to this curve for values less than 10%, increasing percentage of the work in process inventory holding cost has influenced the optimal production lot size severely and reduced it, therefore the holding cost of work in process inventory is one of the most critical costs calculated in this model.

Table 1. Optimal production lot size for numerical value of the parameters

NUM.	MTTR (min)	b(unit)	$\left(\frac{i}{\$ \cdot \text{min}}\right)$	$F_1$	$F_2$	C	$Q_{EPQPM}$
1	5	10	0.2	0.1	0.05	-18375	24
2	10	10	0.2	0.1	0.05	-35185	32
3	15	10	0.2	0.1	0.05	-51870	38
4	20	10	0.2	0.1	0.05	-68430	43
5	25	10	0.2	0.1	0.05	-84865	48
6	30	10	0.2	0.1	0.05	-101175	52
7	20	5	0.2	0.1	0.05	-35017.5	29
8	20	15	0.2	0.1	0.05	-102117.5	54
9	20	20	0.2	0.1	0.05	-136080	64
10	20	25	0.2	0.1	0.05	-170317.5	72
11	20	10	0.05	0.1	0.05	-69367.5	94
12	20	10	0.1	0.1	0.05	-69055	64
13	20	10	0.15	0.1	0.05	-68742.5	51
14	20	10	0.25	0.1	0.05	-68117.5	38
15	20	10	0.2	0.05	0.05	-65240	44
16	20	10	0.2	0.15	0.05	-71620	43
17	20	10	0.2	0.2	0.05	-74810	42
18	20	10	0.2	0.1	0.1	-68430	43
19	20	10	0.2	0.1	0.15	-68430	44
20	20	10	0.2	0.1	0.2	-68430	44

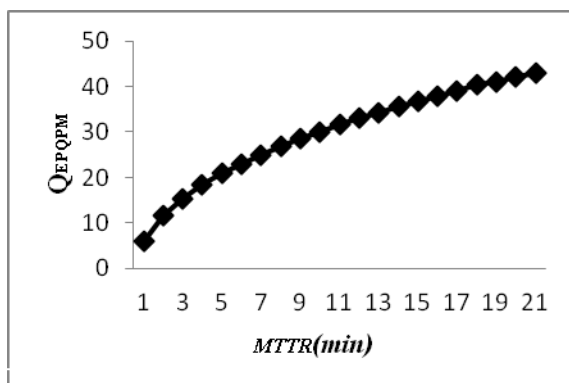


Figure 3. Behaviour chart of  $Q_{EPQPM}$  based on changes MTTR

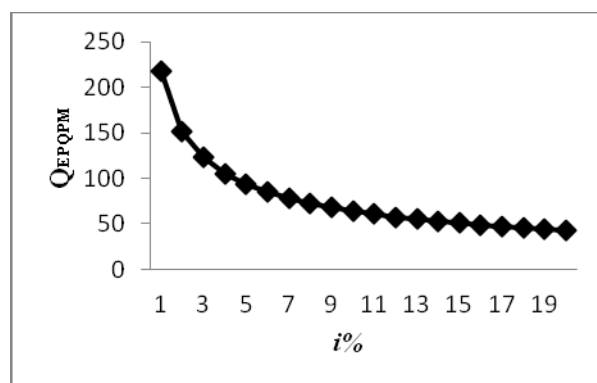


Figure 4. Behaviour chart of  $Q_{EPQPM}$  based on changes  $i$

Eventually the curve has an ascending trend in Figure 5 it means that if the model follows the backlog policy, by increasing the authorized back orders for minimizing the total cost function, we will encounter with greater amount of products. These graphs can be plotted for the other parameters of the model and found similar results.

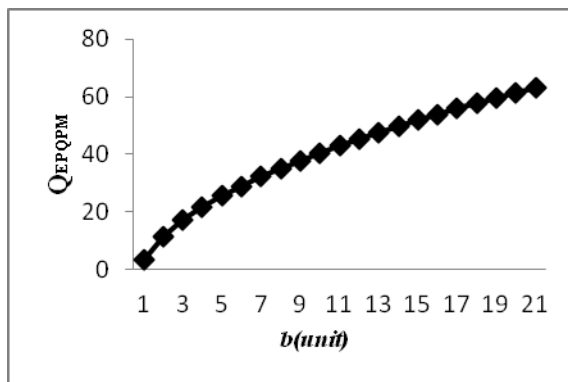


Figure 5. Behaviour chart of  $QEPQPM$  based on changes  $b$

## 5. Conclusion

In this paper the determination of optimal production lot size with regard to some factors such as shortage, the work in process inventory, manufacturing the reworkable and non-reworkable products and exercising the preventive maintenance policy on the machine was investigated and a relation to achieve optimal amount of production were presented. Temporary stopping of the machine in each production period for performing preventive maintenance on it and entering preventive maintenance time in the model as an average time is one of the most outstanding difference of this model with other proposed models and as the results show considering this factor was dramatically effective on the optimal production lot size determination. Performing preventive maintenance on the machine increases its lifetime and efficiency and avoids its sudden failure during the process therefore, it is necessary for the level of these repairs to be acceptable. In order to adapt this model with the actual conditions more, the preventive maintenance operations on machines can be considered in various states (for example once in each two production periods) or time required for its performance can be changed in some periods. It should be noted that the equipment has three types of failure rates during its lifetime that this rate is descending at the beginning of their work (due to failures caused by the assembly), almost constant during the lifetime of the machine and it is ascending during their fatigue. Thus the time required to perform preventive maintenance can be considered due to the failure rate of the machine. Area of future development in this model can be discussions on equipment reliability based on their failure rate functions and considering other types of machinery maintenance policies such as emergency maintenance (EM), corrective maintenance (CM) and ... While in this paper all parameters of the model are considered to be deterministic that for its development, the probabilistic form of some of them can be used.

## References

- [1] Boucher, T. O. (1984). Lot sizing in group technology production systems, *International Journal of Production Research* 22, 85–93.
- [2] Cardenas Barron, L. E. (2008). Optimal manufacturing batch size with rework in a single-stage production system – a simple derivation, *Comput. Ind. Eng.* 55 (4) 758–765.
- [3] Cardenas Barron, L. E. (2010). The derivation of EOQ/EPQ inventory models with two backorders costs using analytic geometry and algebra, *Applied Mathematical Modelling*.
- [4] Cardenas Barron, L. E. (2001). The economic production quantity (EPQ) with shortage derived algebraically, *Int. J. Prod. Econ.* 70 (3) 289–292.
- [5] Cardenas Barron, L. E. (2001). The economic production quantity without backlogging derived with algebra, in: *Sixth International Conference of the Decision Sciences Institute, Chihuahua, Mexico*.
- [6] Chang, S. K. J., Chuang, J. P. C., & Chen, H. J. (2005). Short comments on technical note – the EOQ and EPQ models with shortages derived without derivatives, *Int. J. Prod. Econ.* 97 (2) 241–243.
- [7] Chiu, S. W., Wang, S. L., & Chiu, Y. S. P. (2007). Determining the optimal run time for EPQ model with scrap, rework, and stochastic breakdowns, *European Journal of Operational Research* 180, 664–676.
- [8] Chiu, Y.P. (2003). Determining the optimal lot size for the finite production model with random defective rate, the rework process, and backlogging. *Engineering Optimization* 35 (4), 427–437.
- [9] Drake, M. J., Pentico, D. W., & Toews, C. (2011). Using the EPQ for coordinated planning of a product with partial backordering and its components, *Mathematical and Computer Modelling* 53 359 375.
- [10] Haji, A., Haji, R., & Bijari, M. (2006). The newsboy problem with random defective and probabilistic initial inventory, *The 36<sup>th</sup> CIE Conference on Computers and Industrial Engineering*, 3498-3502.
- [11] Hayek, P. A., Salameh, M. K. (2001). Production lot sizing with the reworking of imperfect quality items produced, *Production Planning and Control* 12 (6), 584–590.
- [12] Jaber, M. Y., Nuwayhid, R. Y., & Rosen, M. A. (2004). Price driven economic order systems from a thermodynamic point of view, *International Journal of Production Research* 42 (24), 5167–5184.
- [13] Jamal, A. M. M., Sarker, B. R., & Mondal, S. (2004). Optimal manufacturing batch size with rework process at a single-stage production system, *Computers & Industrial Engineering* 47 77 89 89.
- [14] Nahmias, S. (1993). *Production and operations analysis, 2nd ed. Burr Ridge: Irwin*.
- [15] Osteryoung, J. S., McCarty, D. E., & Reinhart, W. J. (1986). Use of EOQ model for inventory analysis, *Production Inventory Management Journal (3rd Quarter)*, 39–45.
- [16] Rasti, M. B., Jahanbazi, M., & Bijari, M. (2011). Effects of imperfect products on lot sizing with work in process inventory, *Applied Mathematics and Computation* 217, 8328-8336.
- [17] Rosenblatt, M. J., Lee, H. L. (1986). Economic production cycles with imperfect production processes, *IEE Transactions* 18, 48–55.
- [18] Samaddar, S., Hill, C. A. (2007). Controlling adverse effect on work in process inventory while reducing machine setup time, *European Journal of Operational Research* 180, 249–261.
- [19] Schwaller, R. L. (1988). EOQ under inspection costs, *Production and Inventory Management*, 29, 22–35.
- [20] Sharma, S. (2007). Minimizing the difference between intermittent and continuous production with back orders, *Journal of Engineering Manufacture* 221 (11), 1617–1623.
- [21] Silver, E. A. (1981). Operations research in inventory management: a review and critique, *Operations Research*; 29: 628–45.
- [22] Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling, 3rd ed. New York: Wiley*.
- [23] Sphicas, G. P. (2006). EOQ and EPQ with linear and fixed backorder costs: two cases identified and models analyzed without calculus, *Int. J. Prod. Econ.* 100 (1) 59–64.