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On the Continuity Mathematics Curriculum between Primary and Secondary School

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Abstract

In this paper we intend to present a review of the National Curriculum continuity in mathematics, in terms of content, from the primary to the secondary. The issues on which we have committed are related to „outputs" learning mathematics fourth class and „inputs" appropriate grade. Because a complete analysis of mathematics curricular content can not be included in an article like this, we summarize only the learning content „fraction". As far we know, continuity issues relating to both the content of their purchases to specific skills training to students, standards and performance descriptors etc., relative to a subject or a curriculum area, in recent formulation, not be found treated in the literature, even in the didactics of discipline.

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Keywords: continuity, curriculum, mathematics, mathematics education.

1. Introduction

In the framework of the curriculum theory, the concept of „continuity of the curriculum" has till now at least three aspects which can be analyzed, if we do not consider its other aspects, like the curriculum continuity for the students' transition from the primary school to the secondary school. All these three aspects concern the continuity of written curricula, the operational curricula and the learning experience and each of them are extensively approached and debated from the point of view of the curriculum theory. It follows that when we thought to design a

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curriculum, we must relate this activity to issues of curriculum scope, sequence, integration, and continuity. These four issues, all relate to how to select and organize the essence of a curriculum, what should be it content (things children understand and information children acquire), learning experiences (out of which children make their own meanings and that stimulates their own unique growth), and skills (specific competencies that children acquire) or values (moral and ethical stances and perspectives on our world) (Shiro, 2007). On the other hand, the curriculum design involves a form into which curriculum is cast or organized, what comes, in generally, to organize it through designs such as disciplines, fields, units, etc., and its guidance orientations made about children, knowledge, education, and the needs of society (Manley-Delacruz, 1990). All these are the base of Tyler's innovational ideas, who introduced concepts such as syllabus, curricula and the data sources (data sources) from which we must proceed when we project a curriculum, and that is: the society, the pupil, the subject matter (society, student and subject matter) (Tyler, 1950). Eisner and Vallace have developed the curriculum concept using the technological concept in which the measure for the curriculum is given by behavioural objectives, time on task, sequential learning, positive reinforcement, direct instruction, achievement testing, mastery in skills and content, and teacher accountability (Eisner & Vallace 1970). Furthermore, linked to the technological processes, Eisner and Vallance identify another four new concepts for the curriculum, i.e. cognitive processes, self actualization, social reconstruction, and academic rationalism, which turn on that they propose a kind of technological process, which is desired without outcomes and less on a focus on the substance of curriculum. The recent researches in the curriculum matter reveal the fact that the curriculum measure should contain other observable measurements for the educational process, which is governed on a curriculum base. For example, the continuity in achieving specific competences in a field, i.e. mathematics, should be ensured all the way through the scholar trajectory and not only by cycle.

It seems that there are no recent empirical studies of curriculum continuity in mathematics (Coad & Jones, 1999), although, some empirical studies of curriculum continuity in other disciplines of schooling were already done (Capel, Zwozdiak-Myers & Lawrence, 2003).

We will analyze from this perspective few examples of mathematics curriculum continuity from our country.

2. On the Continuity of the National Mathematics Curriculum

In the Romanian education system, it is considered that the National Curriculum represents the coherent ensemble of the educational plans and the scholarship programs from the pre-universitary education, stipulated by the National Law of Education (the Law 1/2011). So, in the pre-universitary education applies the National Curriculum elaborated in order to meet the specific personal development needs and the needs of the labour market and the needs of each community, based on the subsidiary principle. The new National Curriculum determines a few fundamental changes (Nicolescu & Petrescu, 2013):

- placing education – as process – in the centre of school activities;
- orientating teaching to developing capacities and skills, through developing competencies needed for problem solving and using participating strategies in the didactic activity;
- making the teaching offer more flexible;
- adapting contents of studying to the quotidian reality, with the preoccupations, interests and skills of the pupil;
- selecting and organizing of the objectives and contents by the „not a lot, but good" principle; it is important not only what we learn, but also how well, when, why is learnt and how is it going to be used later on;
- the possibility to design individualized educational programs, motivational for the pupils, orientated to innovation and personal accomplishment;
- responsabilization of all educational agents involved in designing, monitoring and evaluation of the curriculum.

All these fundamental changes have been transposed, among other, in the new curriculum for the primary cycle through writing the education program based on specific competences that the pupils must acquire without précising the mathematic contents that the didactical project should be based on. So, that remains to be decided by the teacher, considering the specific of its class or pupils. The changes mentioned above are both good and bad, because they introduce new variables in the teaching-learning-evaluation processes, like the expertise of the primary cycle teacher in choosing the content of the mathematics, especially for realising the last three strategies. Therefore, the importance of the teacher accountability in measuring the curriculum grows.

In this paper we intend to present a review of the National Curriculum continuity in mathematics, in terms of content, from the primary to the secondary. The issues on which we have committed are related to „outputs" learning mathematics fourth class and „inputs" appropriate grade. Because a complete analysis of mathematics curricular content can not be included in an article like this, we summarize only the learning content „fraction". In curriculum theory, the concept of continuity in its transition from primary to secondary education, it is defined only in terms of transfer of methods and establishing explicit connections to the curriculum. As a particular example, we can mention the discipline of mathematics, in which we find from primary school up to twelfth class, regardless of the profile, real, humanistic, professional, vocational, etc. Moreover, the literature in mathematics didactics school is divided into cycles, i.e. for primary mathematics teaching, mathematics teaching (algebra, analysis, geometry, etc.) for middle school or high school. Each of these staff is focused more on aspects of teaching, learning and assessment of mathematics specified in corresponding national curriculum school cycle. Speaking of math curriculum continuity through the terms of cyclical repetition characteristics of mathematics content across it or just through the design of teaching and learning in mathematics from the initial state to a class of students is very less and not is not consistent with the academic progress of students in mathematics. Since the 2000s, studies and analyzes on the reform of the education system in Romania, insert the phrase curricular continuity and change, on the consistent implementation of the National Curriculum, this approach assumes that an organic whole with interdependent parts. Thus, the focus is on the interrelationships between disciplines and subject areas, and between subjects - horizontally (at the same level of schooling) and vertical (curricular cycles) in the implementation of the official curriculum. So continuity curriculum is seen only in terms of the curriculum, including ensuring consistency across its components and its links to the aims of the education system.

This is necessary because it has been noticed that for a large number of pupils, who attend the secondary school, making the connections between the previous and ongoing knowledge is not a component of their mathematics learning strategy. The mathematics teachers should explain more the connexions that could be established between the mathematics knowledge from different levels of conceptualization. But, in order to do so it is absolutely necessary for the teacher to know mathematics curriculum, at least for the "neighbouring" years for that cycle and at least from the learning content point of view (Attard, 2010). The method used to make the connection between the fraction concept and the rational number concept, when we go from the primary cycle to the secondary one, underlines the discontinuity of the mathematics curriculum.

For example, in (Fadiño Pinilla, 2007) is given a thorough analysis of mathematics education issues regarding teaching and learning the fraction. In fact, the fractions whilst not a part of academic Knowledge, are nevertheless an issue of Mathematics Education as an object of knowledge, a knowledge that we could call "scholastic". One of the major ideas from (Fadiño Pinilla, 2007) is that a simple "definition" of the fraction is not enough for the pupil to learn and understand the concept. As a matter of fact, in (Fadiño Pinilla, 2007), there are identified at least twelve different interpretations of the term fraction, which can be approached at the primary cycle level. Some of them are not even reminded or considered in our national mathematics curriculum, and others, even though mentioned, are not treated in relation with the fraction notion. A fraction is an important part of work on probability, which corresponds to its original definition like the frequencies of the realization of an event over the possible cases. But it is forgotten this particular characteristic of the fraction when, in the fourth class, is teaching likelihood estimate and justify an event. More, in scores fractions have a quite different explanation and seem to follow a different arithmetic.

It is obvious that in the fifth class the fraction must be transformed into a rational number, a passage that is by no means without problems. In the primary cycle, on one hand, teacher does not insist on the aspect that the fraction, indifferent how it is defined, is a number that does not belong to the whole numbers class (category). So, the most important characteristic of the fraction, that is a different number from a whole one, is not even reminded in the primary cycle. At the end of the fourth year, the pupils will not understand the fraction as a number, but more as a:

a) whole number schemes (the use of counting for the identification of individual units as a means of division);

b) partitioning schemes (halving, dealing, folding or splitting); measuring schemes (fractional units seen as chunks or measures); equivalencing schemes (the reconfiguration of a certain unit in terms of other ones); relational schemes (relational understanding between parts and wholes) (Brousseau 1980), (Brousseau 1981), (Malara, 2001).

In that configuration, one can not talk about approaching writing a fraction in decimal form, especially because there are allocated only twelve hours for teaching and learning the fractions in the fourth grade. That will make it difficult for the teacher and will increase the learning efforts for the pupils from the fifth grade, when, simultaneous will be introduced the decimal writing of the positive rational numbers, the order relation and the arithmetic operations for that class.

In connection with this, for teaching mathematics at the secondary level are provided a lot of problems in terms of teaching and learning of rational numbers. We should note that at the secondary school level, teaching mathematics is, at least in our country, fewer tangents with pedagogy and psychology. Moreover, student, saw the subject in terms of pedagogy, is no longer devote research studies in this direction. Psycho-pedagogical state of the student in the middle school is considered as of an almost young adult. In other words, for he can get them in fifth grade teaching positive rational number almost simultaneously with the notions that: divisibility relationship on the set of natural numbers, the greatest common divisor, least common multiple, prime numbers, numbers reducible, writing decimal rational numbers etc.

There are neither estimated time necessary teaching learning concepts nor the effort of learning and acquisition of knowledge to specific skills training, etc. Instead, the student in primary, throughout the five years, it is held "captive" in the "universe of natural numbers" in which he only learn to perform the arithmetic operations: addition, subtraction, multiplication and division. Only at the end of fourth grade student meets for very little time the concept of the fraction written as n/m, although he knows and understands very well what is the price of $10,20$ *lei* for an object that he wants to buy it in a store, or what means $1,65\ m$ for the measurement of the own height measurement, etc.

We try to say that the primary mathematics curriculum does not contain the exterior transposition of the knowledge block concerning the decimal written of the rational numbers. So, it is not introduce the concepts of the whole part $[x]$ and the fractional part $\{x\}$ of the rational numbers. In this didactical context we can not approaching the equivalence relation on the fractions set. More, we can teach the order relation on the same set. It is true that in the fourth class we try to order only the fractions with the same denominator and thus the natural order on the positive integer set is moved between the numerators of them, but is not enough to ensure continuity of learning the concept of number rational positive for the students in the fifth grade. Also, the variation of the fraction amount in the function of the variations of the denominator or numerator it is out of the question.

As a resume, for the students at the finish of the primary school remain some uncertainties like the following:

- „The fraction n/m is or not a number?",
- „What kind of number is n/m ?",
- „Why $1/2 > 1/4$ (with out using the semiotic reasons which building for the student the constraints of his mathematical thought)?", etc.

The discussions and researches on the mathematics curriculum to ensure its continuity at the transition between primary and secondary, for example in terms of fraction and decimal writing, are of utmost importance and its consequences concerns mathematics education worldwide (Strefland, 1991). Here, we notice only that, at least for the national math curriculum design and for the guiding ideas in the researches of mathematics education, not meeting yet a concern about these of those involved in the governance of our educational system (Pareja-Heredia, 2008).

3. On the Continuity of the Sequences: Mathematics, Mathematics Curriculum, Mathematics Education

Trying to define, in terms of education sciences, the concept of continuity of the curriculum design throughout the entire school course, which means primary, secondary and high school, in order to achieve a specific didactic purpose is obviously extremely difficult. But we can, at least initially, to try to start from a concrete reality existing in today's Romanian math education.

One of the changes introduced by the new reform of Romanian education for primary school is materialized by introducing an additional hours of mathematics at the second grade, in which it will be studied fractions, multiplication table, natural sciences and physics concepts. One of the arguments that led to this change is to train students to perform mathematical calculations in a setting in which the pleasure of working with numbers is combined with the joy of knowing the real world. Moreover, it wants to give up the system of "doping" only with the knowledge in the teaching-learning process, and thus it switch to the sagging of information for settling, processing, reflection on what the student learned.

But we believe that the fair and scientific argumentations, both in terms of the mathematics and the education sciences for a real reform and an efficiency of the entire national education system must start from a framework of principles of education process seen as a subsystem of society. In short, we could say that we must first establish the principle of interaction between mathematics (as a fundamental pillar of knowledge), curriculum (as a fundamental pillar of instruction) and teaching mathematics (as a fundamental pillar of the governance processes of knowledge through learning). In other words, first we should conceptualize the continuity of the sequence Mathematics-curriculum of mathematics-mathematics didactics, and secondly we should specify the continuity of this sequence throughout the entire school, starting from kindergarten up to university.

Although necessary, in the current context of society's expectations, the changes made to the curriculum for primary school since 2014-2015 have already led to divergence of opinions dividing researchers in the education science, in the academically medium, in the parents' opinions, etc., because they were not justified in terms of principle and / or laws of pedagogy, didactics, theory instruction, etc. In our opinion, these changes were introduced at an intuitive and, quite, empirical level by a political management which means big hurry.

At the start of the above sequence for which we propose to try to define its continuity, there is Mathematics, seen it as a science of nature. To the questions like "What is Mathematics?", "Why is fundamental the role of Mathematics in knowledge?",..., we will not give the scientific argumentations because we consider that Mathematics has not need for a „defence attorney". The Mathematics defended and will defend itself, by itself. But, it is very interesting and wonderful how some students respond to such questions:

- „Math is the entire world simplified on a piece of paper ... Math is ingeniousness morphed into a tiny simple formula so we can harness its fantastic powers." (From a 13-year-old)
- „Math is the universal language of the world." (From a 12-year-old)
- „No one can live without math; it means different things to different people. But to me it means love, liberty, learning ... I could keep going but to sum it up, math is my life." (From an 11-year-old)

Although these responses may be considered only at the intuitive or common sense level of the some dedicated students to learn Mathematics, these reflect still that we can speak about the correctly perception of the primary mathematics by the students: that Mathematics is "an universal language of the world" and "It has a fantastic Powers for the knowledge". From this idea we should start when we want to design mathematics curriculum for to teach math to the students which need to learn it and to apply it and use it in line with their expectations for living in society.

We can say that one of the purposes of teaching mathematics up to the finish of the high school is that the student will can compute with real numbers, $a, b \in R$. Specifically, the student will be able to recognize, in a mathematical context, numerical calculations for addition or multiplication of two real numbers, which means that he can make getting the result as accurate or he will have to resort to approximations to obtain finally about results. More precisely, we expect for shore that the student can perform the calculus for the addition $a+b=x$, when $a, b \in Z$, but for the addition $\pi + e = x$ we will expect that the student will ask us what is the rank of approximation which is imposed for the final result of the addition or he will tell us that for such calculus he needs a computer. When the student performs the calculus he speaks in the mathematical language to us (the teachers) or to himself for to know the real world and this to „hear" him? At this rhetorical question it must answer in the first place who design the curriculum, in the second place „who teach math", and in the third place the society in which we suppose the student will act as a participative in citizen. But, will be the student face with such calculus in his future live, after he will graduate the school? Yes, we belief that the probability is close to unity, no matter what profession or job he will embrace.

In the other words, it is very hard to evaluate the skills which have to acquire the student at the mathematics within a fragmented curriculum by the school cycles, even if we follow the principle of periodic recurrence of knowledge, because we broken the natural fluency of students learning mathematics. For example, the core sequence of inclusions of number sets $N \subset Z \subset Q \subset R \subset C$ is scattered, dissipated in bits along the mathematics curriculum for only goal that the student have to learn mathematics, but without he will make the connections between the vary structures (algebraic, geometric, etc.) attached with these number sets.

If we assume that at the end of the primary cycle the student shall have the skills about the additive and multiplicative structures attached to the natural number set N , we have seriously doubt that he will acquire slightly similar algebraic structures for the set of positive rational numbers Q_+ only in the fifth grade. Hence result our doubts for the construction of the rational integers Q in sixth grade, when it starts to teach the whole number set Z . Demonstration of discontinuity math curriculum, only in this direction of the building of number sets, can be developed in several ways, but we prefer to stop to these ideas.

In other words, we can say without doubt that in the current mathematics curriculum, being thought only for the primary school, it is not assured the continuity of the mathematical knowledge through all the didactical activities of teaching-learning which can be made based on it. After all, if we pay more attention to the necessary arithmetical outputs at the end of the primary school, these can be summarized at the additive and multiplicative structures on the natural numbers set, and at the summing of the fraction with the same denominator, but without acknowledging the fact that the fraction is a different number then the whole one.

From this perspective, we believe at the end of the primary cycle it should be introduced the concept that the fraction is also a number, which do not belong at the natural numbers set, and so it is rational number. In order to do so, some properties of the natural numbers, linked to the multiplication operation similar to those for the decomposition and composition of the natural numbers used in the introduction of the summing operation, should be approached.

We believe that the concept of prime number could be introduced very easily, in same way we are using the decomposition of a number in factors in the case of multiplication/division, for example $6 = 1\times 6 = 2\times 3 = 3\times 2 = 6\times 1$. So, we could create the possibility for teaching-learning the transcription of natural numbers as multiplication of powers of prime numbers. Considering that, in the fourth grade, it could be approached the relation of equivalence of fractions without introducing the decimal system, for example the fraction $1/2$ is equivalent with the fraction $2/4$ or $3/6$ etc. because we can also simplify the fractions. So, we can make several observations:

- The development of the idea of value of a fraction (that is to say a un-integer number);
- The approach of the order relation on the fraction set;
- The expansion of the arithmetic operations with fractions that do not have the same denominator.

We underline the importance of the profound study of the fraction notion, now, at the same time with the introduction of an extra hour for mathematics (or to be more precise, an hour where we study only applications of the mathematics in the real life, in the curriculum for the primary cycle, in order to understand the notions of: probability; the link between speed, time, distance; mixture or alloys; air, water composition etc.

Therefore, looking from the point of re-conceptualization of the mathematics curriculum in the primary cycle, concerning the fraction notion an important role should be allocated to the didactic of mathematics. That, must elaborate new strategies of teaching-learning of the fraction and not only, so it can ensure on one hand the connection between mathematics and that extra hour of applications of mathematics for solving real life issues, as well as the continuity of this notion with that of the positive rational number from the next cycle.

4. Conclusions

Because a complete analysis of mathematics curricular content can not be included in an article like this, we summarize only the learning content „fraction". In our short exposition we have tried to draw attention especially to researchers in curriculum theory on the multiple dimensions of the concept of continuity of curriculum and on its features in the new context of the integral education paradigm, which is foreseen to be adopted in the future. Because literacy numeracy is one of the main pillars in both mathematics educations at primary school level, and for the other school cycles, we used some aspects of it to sustain our considerations regarding the concept of continuity of the mathematics curriculum at the transition between the school cycles.

It is time together, teachers, psychologists, mathematicians, whether teachers and researchers, to sit at the same table for have the constructive debates concerning the mathematics curriculum design and an efficient implementation of it, seen along all school cycles, such that the next generation of students will receive a proper education for the knowledge based society (Bramald, Miller & Higgins, 2000), (Braund & Reiss, 2006), (Giroux, Penna &Pinar, 1981), (Manley-Delacruz, 1990), (Sriraman & English, 2010).

We consider that it should be insisted on the concept of continuity of the curriculum in general, the mathematics one in particular, in order to guaranty a coherent and stable frame for learning, so the pupil can acquire not only specific competencies linked to isolated parts of mathematics, but also general ones which can ensure the interdisciplinary and transdisciplinary. Those few aspects we have underlined represent only a small part of the complexity linked to the continuity of the mathematic curriculum. This problem remains as a constant preoccupation for us and not only for the future.

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