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A model for supply chain design considering the cost of quality

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ABSTRACT

Recent studies have shown that the cost of quality (COQ) is of more strategic and economic importance than previously conceived. Whereas previous works have applied COQ as an internal performance measure within companies, the purpose of this paper is to present a model for supply chain design that computes the COQ as a global performance measure for the entire supply chain. In addition, rather than assume an exogenously given COQ curve, our model computes COQ in terms of internal operational decisions such as the error rate at inspection and fraction defective at manufacturing. The model can be used to design a logistic route that achieves a minimum total cost while maintaining an overall quality level and to evaluate the impact of investment in quality to increase overall profits. The behaviour of the model is illustrated with numerical examples that show how the COQ function changes depending on various parameters.

1. Introduction

Cost of quality (COQ) or quality cost is based on tangible costs, which are recognised by the accounting system as expenses because they are based on actual transactions used in the production process. These costs constitute the widely used traditional prevention–appraisal–failure (PAF) model proposed by Feigenbaum [1]. The concept of cost of quality (COQ) has been applied successfully in manufacturing companies and service businesses [2]. Organizations that have instituted a system of quality cost measures have also experienced dramatic positive results. However, COQ has focused on in-house quality costs for an individual firm but not for an entire supply chain [3]. COQ represents a powerful measurement system that translates the implications of poor quality, activities of a quality program, and quality improvement efforts into a monetary language for managers. Moreover, COQ is a language that every stakeholder can understand which is important because it affects operating costs, profitability, and consumer need [3]. Thus, it is crucial to extend COQ as an external measure and integrate these costs into supply chain modelling.

Srivastava [3], who gives the first step towards estimating COQ in a supply chain, measures COQ in monetary terms at selected third-party contract manufacturing sites of a pharmaceutical company located in India. Another work focused on integrating COQ in the supply chain is the work by Ramudhin et al. [4]. This seminal study presents a model that integrates COQ into the modelling of a supply chain network for a single product, three-echelon system (suppliers, manufacturing plants, and customer groups). The model seeks to minimise the overall operational and quality costs. Ramudhin et al. [4] find that adding a known COQ function only for suppliers into the objective function gives a difference of approximately 16% in costs, and changes the solution. When COQ is not included, the final optimal network will choose key suppliers who have the lowest operational costs, but there is no information in regard to the quality non-conformance cost. Therefore, the selected supplier may run at a high quality non-conformance cost, and receive the same preference that a supplier which operates at

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a lower quality non-conformance cost because both suppliers have the same production cost. Thus, choices made solely on production cost could sacrifice quality and lead to additional quality non-conformance costs or corrective action costs in the next stages of the supply chain. More recently, Alzaman et al. [5] establish a mathematical model that incorporates a known COQ quadratic function incorporating a defect ratio at all supply chain nodes; COQ function is based on Juran's original model as assumed in their previous work. They consider binary variables for selected supply chain nodes and an n level bill of material. This model is applied to a case study from the aerospace industry.

In previous studies, the total quality cost function based on percentage of defective units is assumed to be given. This paper deals with the development of a model to compute the quality cost for a whole supply chain based on internal decisions within the supply chain, such as percent defective at the manufacturing plant and error rate at inspection. No previous work has addressed how the COQ curves are obtained by taking internal and operational decisions within the supply chain.

Ittner [6] states that 'few, if any, firms use experimentation to identify the shapes of their quality cost curves' and when a quality cost system is first established, it serves to observe ex post the choices made by management related to quality resources. It reflects the manager's perception of an adequate quality cost model at that time. Moreover, Campanella [7] states that some managers think that investment in quality programs will always give the company a positive impact on profit and that ignoring quality is really expensive. Other managers believe it is uneconomical working at zero defects. The real problem begins when managers from different areas, supposedly working together, operate with conflicting perspectives on quality. Usually, once quality costs are obtained, these are used to find specific improvement projects; however, this is not an easy task and it is not clear what action should be taken nor what impact it may have on the quality cost model. Since the traditional cost model represents the hypothesised shape of quality costs and relationships, it can only serve to evaluate the distribution of quality cost categories with respect to total quality costs, sales, and profit.

A planning model is needed to estimate the impact of the total cost of quality on profit when establishing new logistic routes; thus, a proactive approach is preferable rather than a reactive one (observing ex post quality data). The proposed model aims to serve as a decision making tool for engineering managers by helping with the design and quality planning of logistic routes for manufacturing plants in the design phase (i.e., not yet operating).

The remainder of this paper is structured as follows: first, a background on COQ models and operational definitions is presented. Next, a model for computing the COQ of a supply chain is provided, followed by an analysis of numerical examples. Finally, a summary of key insights and concluding remarks, as well as recommendations for future work, are presented.

2. Theory about COQ

This section presents a survey of published literature on various COQ models. According to research carried out by Sandoval-Chávez and Beruvides [8] six primary theories were found: (1) Juran's model, (2) Lesser's classification, (3) PAF model, (4) the economics of quality, (5) business management and the COQ, and (6) Juran's revised model. Schiffauerova and Thomson [9] classified COQ models into four groups of generic models: (1) PAF or Crosby's model, (2) opportunity cost models, (3) process cost models, and (4) ABC approach. Furthermore, Banasik [10] categorized the COQ models into: (1) Juran's model, (2) Lesser's contribution, (3) PAF model, (4) Harrington PQC, (5) Godfrey–Pasewak accounting COQ model, (6) Carr's service model, (7) Juran's revised COQ model, (8) Beruvides and Sandoval-Chávez opportunity cost model, and (9) Beruvides–Chiu capital budgeting model.

Modern COQ models and theory have been developed from the works of Juran, Feigenbaum, Crosby, and Freeman [3]. The COQ models that serve as the theoretical foundation for this study are the PAF classification as well as the revised Juran's model. A short background on the COQ models on which the proposed model is based is presented in the following paragraphs.

Joseph Juran's [11] analogy of 'Gold in the Mine' is defined as the 'total of avoidable costs of quality'. According to Juran et al. [12], this concept suggests that costs resulting from defects were a gold mine in which lucrative digging could be done. Juran [11] also categorises cost of quality elements as tangible and intangible.

Soon after, Feigenbaum [1] develops the prevention–appraisal–failure (PAF) classification. The PAF classification can add orderliness and uniformity to the ensuing reports. The PAF classification offers specific advantages, such as its universal acceptance, identification of different kinds of expenditures, and provision of criteria to help in deciding whether costs are quality related. The last-mentioned advantage may be the reason why neither Feigenbaum, nor the ASQC, defines the term *quality costs*. Matters are quality-related if they meet the criteria set by each category [13]. The premise behind Feigenbaum's classification [1] is the following:

> The reason for the favourable cost result of total quality control is that it cuts the two major cost segments of quality (which might be called failure and appraisal costs) by means of much smaller increases in the third and smallest segment (prevention costs) [p. 99].

The definitions for each of the categories of the PAF classification are stated by Campanella [7] as follows:

- Prevention costs are 'the costs of all activities specifically designed to prevent poor quality in products and services.'
- Appraisal costs are 'the costs associated with measuring, evaluating, or auditing products or services to assure conformance to quality standards and performance requirements.'

– Internal failure costs are the costs resulting from products or services not conforming to requirements [which] occur prior to delivery or shipment to the customer.'
– External failure costs are 'the costs resulting from products or services not conforming to requirements [which] occur after delivery or shipment of the product, and during or after furnishing of a service to the customer.'

According to Porter and Rayner [14] the basic assumptions of the PAF model are that (1) investment in appraisal will reduce failure costs and that (2) further investment in prevention activities will also reduce failure costs. The PAF classification allows practitioners to identify quality-related costs and express each category in terms of percentages of the total cost.

Later, Juran et al. [15] merge Feigenbaum's PAF concept with Juran's original concepts, the result is what is known as the traditional COQ trade-off between prevention, appraisal, and failure costs. Studies during the 1980s show that the traditional COQ trade-off model of Juran is not completely valid. Scheneiderman [16] affirms that the minimum quality cost could lie at zero defects if the incremental cost of approaching a quality level of 100% is less than the incremental return from the improvement. Scheneiderman [16] asserts that a proper way to view quality cost optimisation is on the basis of incremental economics. In response, Juran and Gryna [17] revise the traditional COQ trade-off between prevention, appraisal, and failure costs and eliminate the asymptotic behaviour of the cost of appraisal plus prevention. They thus assert that 100% quality conformance might be reached under finite prevention and appraisal cost under the conditions of the twentieth century where a growth of manufacturing and automated inspection technologies occurred. Juran and Gryna [17] limit the application of the new model to certain instances: (1) industries demanding high safety or liability concerns, (2) highly automated companies, (3) companies selling to wealthy customers, and (4) companies struggling to optimise client's expense. Moreover, Juran and Gryna [17] state that 'while perfection is a goal for the long run, it does not follow that perfection is the most economical goal for the short run, or for every situation' and that the minimal point would move close to zero defects if opportunity costs are included in the failure costs. Juran and Gryna's ideas have been challenged by Schneiderman [16], Plunkett and Dale [18], Freiesleben [19], among others. These authors argue that there is no economic level of quality, that spending on prevention can always be justified if the time horizon extends far enough into the future and that the cost-minimal quality level is equal to quality perfection.

Fig. 1 compares the classical Juran model with the revised model. Although, not very adequate for current manufacturing processes, the old model still provides a frame of reference for quality costs and quality improvement. Burgess (1996) carries out a quality cost simulation whose results suggest that both views can be reconciled within one model. Burgess supports the classical view in certain time-constrained conditions and the modern view under an infinite time horizon. To sum up, the main difference between the two models is the fact that 100% quality of conformance can be reached at a finite cost in the revised model.

2.1. Operational definitions

Dale and Plunkett [13] state that there is no clear definition for COQ. In general, according to Dale and Plunkett [20], a definition for quality costs that is widely accepted and which is used as an operational definition for this paper is:

> The costs incurred in the design, implementation, operation and maintenance of a quality management system, the cost of resources committed to continuous improvement, the cost of the system, product and service failures, all other necessary costs, and non-value added activities required to achieve a quality product or service.

A supply chain is defined as:

> An integrated process of various business entities working together to acquire and transform raw materials and deliver value added products to customers [21].

Therefore, the supply chain is the integration of all enterprises that work together in order to ensure the final client receives a product or service at the right time, in the right quality, and at the lowest possible cost. Before introducing the model, an operational definition for COQ across a supply chain is presented.

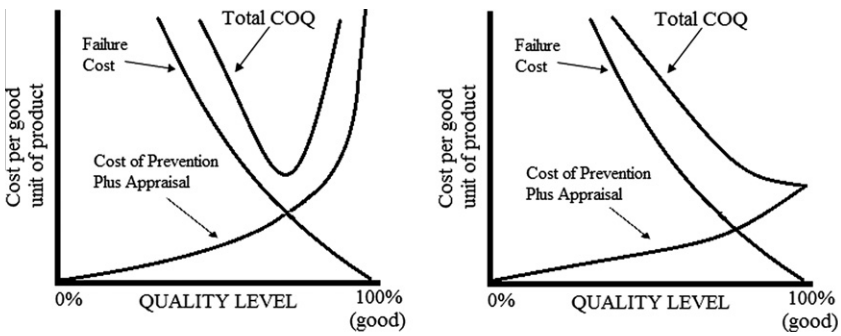

Fig. 1. Classical view on the left and the modern view on the right.

The sum of the cost incurred across a supply chain in preventing poor quality of product and/or service to the final consumer, the costs incurred to ensure and evaluate that the quality requirements are being met, and any other incurred as a result of poor quality [3].

3. Developing a quality cost model

This section presents a methodology to compute the cost incurred by various actors within the supply chain due to the cost of quality. We consider a generic consumer goods supply chain, consisting of three tiers, namely suppliers, manufacturers, and retailers. We assume that the parameter *overall quality level* (proportion of good products among all products delivered to final customers) is sufficient to represent quality. In order to simplify the problem, we assume the following: (1) this model is only appropriate in determining the required fraction defective at manufacturing and error rate at inspection to attain the minimum total quality costs for the design of new logistic routes including manufacturing plants which are not operating yet. Modifications to the model would be necessary if implemented for actual manufacturing processes which are currently working at a specific fraction defective at manufacturing and with an established inspection system. (2) This model is appropriate for manufacturing companies which follow the traditional quality cost behaviour and where external failure costs are tolerable, that is, in products where external failure is not catastrophic for customers. For instance, the aeronautical industry does not follow the traditional quality cost behaviour since this industry invests more in appraisal due to the nature of the product and does not allow external failure cost. Prior to shipment of airplanes, the customer checks the airplane by means of test flights, and if there is any nonconformity, it is fixed before shipment [22]. (3) A specific supplier among several is selected and this supplier has a fraction defective defined as products that does not fall within the quality specification limits (Y_{si}). (4) The selected supplier and retailer are external to the manufacturing plant. (5) Single sourcing is assumed. (6) Constant demand is assumed. (7) An extensive inspection system (100% inspection at the end of the manufacturing process to check component's conformance) is assumed. The implementation of quality programs might allow the company to reach higher quality levels and install a more effective and efficient monitoring system. (8) A single retailer is selected for each manufacturing plant and this retailer satisfies all customer demand. (9) All defective products are returned by customers. (10) Inspection error rate is a type II error; type II error is concerned with bad products classified as good. Type I error (good product classified as bad) is not considered.

The model computes the good and defective units at each stage as shown in Fig. 2. Although we compute the COQ for a specific selection of supplier, manufacturing plant, and retailer, the notation is presented in its general form, that is, the model is formulated to represent selection of one supplier among a set of suppliers; and likewise for plants and retailers.

Sets

- I set of suppliers
- J set of manufacturing plants
- K set of retailers

Parameters for COQ computation

- W number of components going through the manufacturing process and delivered to customers
- Ys_i fraction defective at selected supplier i
- Yr_k fraction defective at selected retailer k
- Dem customer demand
- Af fixed cost for prevention activities
- Av variable cost for prevention activities
- Bf fixed cost of inspection at the end of the manufacturing process
- Bv variable cost of inspection at the end of the manufacturing process
- Cf fixed cost for internal failure cost
- Cs loss incurred due to failure of purchased components from supplier to meet quality requirements
- Cm direct manufacturing cost per processed item that goes to rework
- Cr rework cost per item
- ϕ rework rate at the manufacturing plant j
- $\overline{C}_{EF}$ cost per defective item related to repair or replace the product
- k coefficient of the Taguchi loss function. k represents the cost of working at the specification limit divided by the width of the specification
- P_1 price per sold item paid by retailer k to manufacturing plant j
- P_2 price per 'sold as defective' item paid by retailer k to manufacturing plant j

Expressions

$GgM(yp_j) = (1 - Ys_i)W(1 - yp_j)$ represents good components with a successful manufacturing

$GbM(yp_j) = (1 - Ys_i)Wy\, p_j$ represents good components with a failed manufacture. These defective products are due to the manufacturing

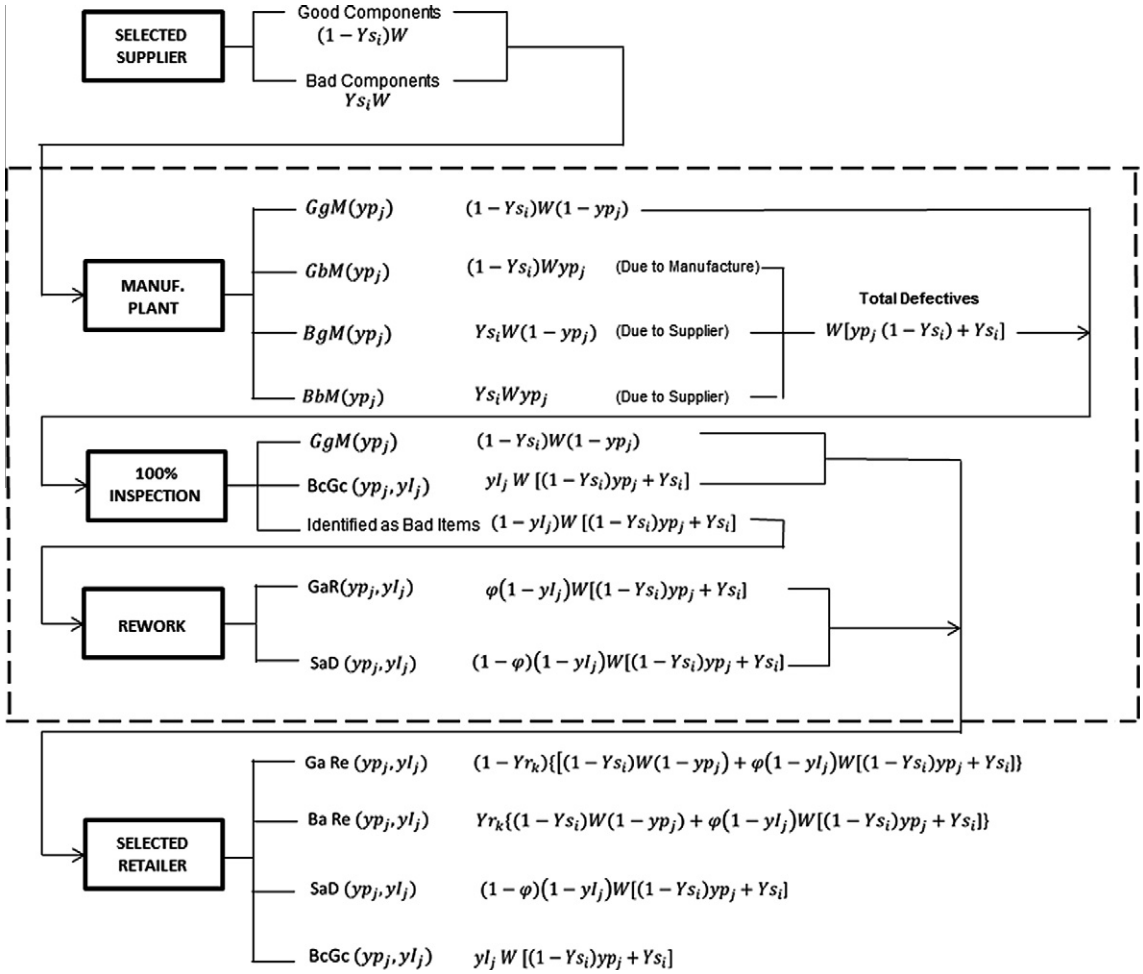

Fig. 2. Serial supply chain with a manufacturing plant and inspection after manufacture.

$BgM(yp_j) = Ys_iW(1-yp_j)$ represents bad components with a successful manufacture. These defective products are due to the supplier

$BbM(yp_j) = Ys_iW\,yp_j$ represents bad components with a failed manufacture. These defective products are due to the supplier

$GaR(yp_j, yI_j) = \phi(1-yI_j)W[(1-Ys_i)yp_j + Ys_i]$ function that returns the number of good products after successful rework

$SaD(yp_j, yI_j) = (1-\phi)(1-yI_j)W[(1-Ys_i)yp_j + Ys_i]$ function that returns the number of defective products which will be sold at a reduced price

$BcGC(yp_j, yI_j) = yIjW[(1-Ys_i)yp_j + Ys_i]$ function that returns the number of bad products after entering the manufacturing process

$GaRe(yp_j, yI_j) = (1-Yr_k)\{(1-Ys_i)W(1-yp_j) + \phi(1-yI_j)W[(1-Ys_i)yp_j + Ys_i]\}$ function that returns the number of good products after the retailer that are delivered to the final customer

$BaRe(yp_j, yI_j) = Yr_k\{(1-Ys_i)W(1-yp_j) + \phi(1-yI_j)W[(1-Ys_i)yp_j + Ys_i]\}$ function that returns the number of bad products after the retailer that are delivered to the final customer

$y(yp_j, yI_j)$ overall percentage of defective products

$QL(yp_j, yI_j)$ overall quality level achieved by the supply chain

$COQ(yp_j, yI_j)$ total cost of quality given by the sum of the costs of internal and external failure, prevention, and appraisal

Decision variables

yp_j fraction defective at the manufacturing plant j

yI_j inspection error rate at the manufacturing plant j

In the following paragraphs, the computation for the quality cost categories is presented. It is worth noting that collaborative effort between the accounting and quality assurance personnel is necessary to determine the estimated quality costs to be included in the model as either fixed or variable costs.

3.1. Prevention cost

The prevention cost (C_P) is related to all activities related to the prevention of poor quality. In this model, prevention cost is linked to the production of good products after the manufacturing process as shown in Eq. (1):

$$C_P = A_f + A_v(1 - Ys_i)W(1 - yp_j), \tag{1}$$

where A_f is a fixed cost and A_v is a variable cost for prevention activities. The reasoning is that by increasing good components with successful manufacture, the overall quality level will improve and such items will not incur rework costs. It can be seen that Ys_i is considered in Eq. (1). Ys_i means that supplier selection is part of the prevention activities (other supplier related costs could be supplier reviews and purchasing prevention costs). Ys_i is a constant in the model since the company has already picked a specific supplier which meets the company's requirements.

3.2. Appraisal cost

A 100% inspection is performed at the end of the manufacturing process to verify conformance. The appraisal costs (C_A) are modelled by a fixed cost (cost of maintenance of a measurement system; that is, measurement equipment expenses, inspection labour, laboratory support, review of test, and inspection data) and a variable cost per item that is classified accurately (performance of the evaluation). Thus, the appraisal cost increases when inspection is more accurate. The appraisal cost is given by Eq. (2):

$$C_A = B_f + B_v(1 - yI_j)W, \tag{2}$$

where B_f is a fixed cost and B_v is a variable cost. W represents the number of items to be inspected (all the components going through the system are inspected).

3.3. Internal failure cost

The internal failure cost comprises the costs arising from inadequate quality discovered before the transfer of ownership [23] from manufacturing plant to retailer. After inspection, the products are classified as good products, bad products identified as bad, and bad products not identified as bad (due to error rate at inspection). The 'bad products identified as bad' enter a rework process but the source of the non-conformance could be the supplier, manufacturing or both. Campanella [7] classified the internal failure costs as: due to product/service design, purchasing, and operations (product or service). Therefore, the internal failure cost has four terms as shown in Eq. (3). The first term is a fixed cost (Cf) for corrective activities. Some of the costs that can be included are: labour for correction of non-conformance, remedial engineering, and shop down time, among others. The second term is the operations failure cost computed as the sum of the direct manufacturing cost per processed item (Cm) and a rework cost per item (Cr) times the identified good components with an unsuccessful manufacture, that is, $\phi\,(1 - yI_j)\,GbM\,(yp_j)$. We assume that components can be recovered or disassembled. The third term is the purchasing failure cost computed as the sum of: (1) losses incurred due to failure of purchased components to meet quality requirements (Cs) such as: purchased material replacement and payroll costs, (2) direct manufacture cost per processed item (Cm), and (3) rework cost per item (Cr), multiplying by the items identified as defective due to bad components $\phi\,(1 - yI_j)$ $BgM(yp_j)$ as well as the items identified as defective because of bad components and unsuccessful manufacture $\phi(1 - yI_j)$ $BbM(yp_j)$.

There are units that cannot be reworked or that management may decide not to restore. The two alternatives for those parts are to scrap them or to attempt to sell the unit 'as is'. Supposing there is a market for those items, a fourth term is added to the internal cost expression as the profit foregone by selling a defective product 'as is'. The cost is computed as the difference between P_1 profit for good product and P_2 profit for defective unit [24] times the items sold as defective or discounted products as shown in Eq. (3).

$$C_{IF} = Cf + (Cm + Cr)\phi(1 - yI_j)GbM(yp_j) + (Cs + Cm + Cr)\phi(1 - yI_j)(BgM(yp_j) + BbM(yp_j)) + (P1 - P2)SaD(yp_j, yI_j). \tag{3}$$

Finally, defective items that are not discovered by appraisal will be discovered by customers incurring in external failure costs discussed in the following section.

3.4. External failure cost

The British Standards 'BS 6143-2' [23] include opportunity cost (such as loss sales, customer claims, concessions, product liability, among others) in the external failure category which is consistent with their definition for COQ: 'cost in ensuring and assuring quality as well as loss incurred when quality is not achieved.' Thus, the opportunity costs are included in the external failure category for the model proposed in this paper.

Authors such as Juran, Lesser, Kume, Juran and Gryna, Harrington, among others, have mentioned in their cost classification the hidden costs or opportunity costs. Opportunity cost models have also been developed by Carr [2] and Sandoval-Chávez and Beruvides [25]. Sandoval-Chávez and Beruvides [25] present a model based on the PAF classification plus opportunity losses. Xerox defines the loss of opportunity as the loss of revenue from defective products, customer dissatisfaction, and complaints from customer [2]. Customer dissatisfaction has been mentioned as a relevant opportunity cost by many authors. For instance Juran [11] and Lesser [26] refer to it as loss of customer good will and Kume [27] calls it as loss of market. Juran and Gryna [28] state that sales loss can be divided into customer problems and decreased customer loyalty due to negative word of mouth.

The expression for external failure costs and opportunity costs is given by Eq. (4).

$$C_{EF} = \overline{C}_{EF}[BaRe(yp_j, yI_j) + BcGC(yp_j, yI_j)] + k(y_{rel})^2. \tag{4}$$

The total external cost (C_{EF}) is given by two terms. The first term models the cost related to customer returns which involve the action to either repair or replace the defective item. We assume that all defective products are returned by customers. The second term is based on the Taguchi loss function concept. The Taguchi loss function is part of the compendium of methods proposed by Albright and Roth [29] to measure hidden costs. Although the Taguchi loss function was first applied in manufacturing processes where the objective is to model losses in terms of the deviation between the actual value of a quality characteristic and a target value, the Taguchi loss function has also been used for non-manufacturing applications. Quigley and McNamara [30] implement it to measure quality for determining the value of a supplier's products to the firm, Kethley et al. [31] apply it to improve the selection of real estate properties by a broker, and Li [32]proposes a modified quality loss function to address service quality. Moreover, Pi and Low [33] use a Taguchi loss function for supplier evaluation and selection, defining the supplier's overall quality as one of the decision variables. The Taguchi loss function for when smaller is better has the form shown in Eq. (5). The loss constant coefficient, k, depends on the cost at the specification limits and the width of the specification [33].

$$L(y) = k(y)^2. \tag{5}$$

The quality characteristic, y, is the overall percentage defective for a given demand as shown in Eq. (6). It is worth noting that even though clients accepted such products 'as is' products sold as defective are also included for the overall percentage defective computation.

$$y = [(BaRe(yp_j, yI_j) + BcGC(yp_j, yI_j)) + SaD(yp_j, yI_j))/Dem] \times 100\%. \tag{6}$$

The retailer has a fraction defective (Yr_k) which means that retailer can turn good products into defective items. From Eq. (6) it can be seen that for a perfect inspection and manufacturing process, the target value for the Taguchi function is $Lb = [Yr_k + Ys_i(1-\phi)(1-Yr_k)] \times 100\%$ and the upper specification limit is set at $Ub = 100$ to indicate the allowable deviation from target value. Notice that $y = 100$ is the worst case, that is, when the process has 100% of defective products. In order to compute the loss for the supply chain, a relative value of the quality characteristic (y_{rel}) is obtained by subtracting the target value or lower bound from the current overall percentage defective as shown in Eq. (7).

$$y_{rel} = y - Lb. \tag{7}$$

To sum up, the total external failure cost (C_{EF}) is given by Eq. (4) and it comprises the cost of processing customer returns, and losses due to defective items. The losses incurred due to defective products are: loss of sales, complaints from customers, warranty claims, loss of market and purchaser goodwill, concessions, and product liability, among others.

3.5. Deriving a quality cost function and overall quality level

The total quality cost (COQ) is computed by summing the PAF cost categories as shown in Eq. (8).

$$\mathrm{COQ}(yp_j, yI_j) = C_A + C_P + C_{IF} + C_{EF}. \tag{8}$$

The overall quality level achieved is given by Eq. (9). It is worth noting that $GaRe(yp_j,yI_j) + BaRe(yp_j,yI_j) + SaD(yp_j,yI_j) + BcGC(yp_j,yI_j)$ is equal to the customer demand (Dem) in the model presented.

$$QL = GaRe(yp_j, yI_j)/GaRe(yp_j, yI_j) + BaRe(yp_j, yI_j) + SaD(yp_j, yI_j) + BcGC(yp_j, yI_j). \tag{9}$$

4. Numerical examples

The proposed SC-COQ (supply chain-cost of quality) model can be used to find settings (inspection error rate and fraction defective at the manufacturing plant) that (1) minimise the COQ or (2) minimise the COQ subject to meeting a given quality level. For the numerical examples, the SC-COQ model is used to minimise the COQ subject to meeting a given quality level ($QL = l$) to produce the COQ curves and compare these plots with Juran's model. Several optimisations were performed depending on the range of the quality level. The range is delimited by the lower and upper bounds on QL, that is, 0 and $(1 - Yr_k)[(1 - Ys_i) + \phi\ Ys_i]$, respectively. The upper bound is attained when both yp_j and yI_j are zero in Eq. (9), that is, when there are zero defects at manufacture and no error at inspection. The lower bound is attained when both yp_j and yI_j are one in Eq. (9). MATLAB was used to generate the plots and the FMINCON function was used to minimise the total cost of quality subject to a nonlinear constraint ($QL = l$) shown in Eq. (10).

$$Yr_k[(1 - Ys_i)(1 - yp_j) + \phi(1 - yI_j)(1 - Ys_i)yp_j + \phi(1 - yI_j)Ys_i] = l. \tag{10}$$

The numerical examples were created by taking a set of base costs parameters for the COQ function so that, for reasonable quality levels (*QL*), the percentage distribution of total quality costs among various categories match the ranges reported by Ittner [6] based on a study of 49 manufacturing units belonging to 21 companies. Ittner [6] reports prevention costs ranging from 13% to 34.8% of total quality costs, appraisal costs ranging from 14.3% to 43.1%, internal failure costs ranging from 20.3% to 49.3%, and external failure costs ranging from 0.1% to 26.5%. It is worth noting that the external failure contribution to the total quality cost reported by Ittner [6] is very low. A plausible reason is that opportunity costs were unlikely to be considered or reported by the sampled manufacturing plants. Rather than limit our study to the range reported by Ittner [6] we chose to utilise a range for the external failure costs from 0% to 54%. For each numerical example, one cost parameter (e.g., fraction defective at selected supplier) is then varied within a wide range to show the impact of the cost parameter on the COQ curves (distribution of quality costs among quality cost categories), the total COQ, and the overall quality level as a function of the decision variables.

4.1. Numerical example 1

In this example, the implications that a supplier's fraction defective has on the COQ curves are explored. Here we will suppose that the company has the option to select a single supplier from among many. A set of data with a percentage distribution of total quality cost among COQ categories matching the ranges reported in Ittner [6] for a quality level between 0.3 and 0.9 was used as the base for the numerical examples.

In this numerical example 1, the supplier's fraction defective cost parameter was varied between 0.05 and 0.5. The internal decision variables (yp_j and yI_j) were optimised in order to minimise the total cost of quality while meeting a given quality level (l); the cost associated with each PAF category is shown in the y axis of Fig. 3. Z represents the total COQ, CP is the curve of the appraisal costs, CIF is the curve of the internal failure costs, CEF is the curve of the external failure costs, and CF is the total failure costs.

It can be observed from Fig. 3 that the optimal quality level which minimises the total COQ decreases when Ys_i increases. This is due to the need for a greater investment in appraisal and internal failure activities to achieve a target overall quality level since more bad components are received from the supplier. When supplier fraction defective is small, the model focuses on improving the manufacturing process through prevention activities and does not employ inspection. When Ys_i increases, the total quality costs also increase for quality levels above 0.5 as shown in Fig. 3b. Moreover, for the highest fraction defective at supplier tested (0.5), the internal failure costs increased considerably for quality levels above 0.5 The results show that it is important to consider the fraction defective when selecting suppliers. This will help to achieve a high overall quality level at minimum cost by avoiding the need to use inspection and rework to meet the quality requirements.

Fig. 4 shows overall quality level contour plots for two levels of Ys_i. A specific overall quality level can be attained by the logistic route through the use of different settings (combination of inspection error rate and fraction defective at the

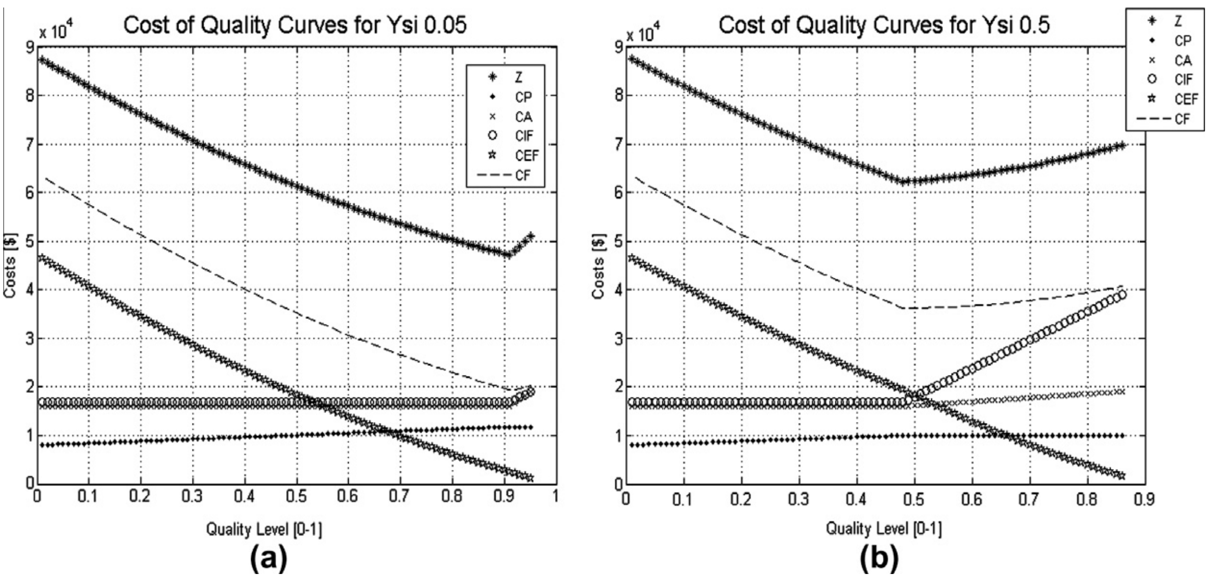

Fig. 3. Quality cost curves for 0.05 and 0.5 fraction defective at supplier.

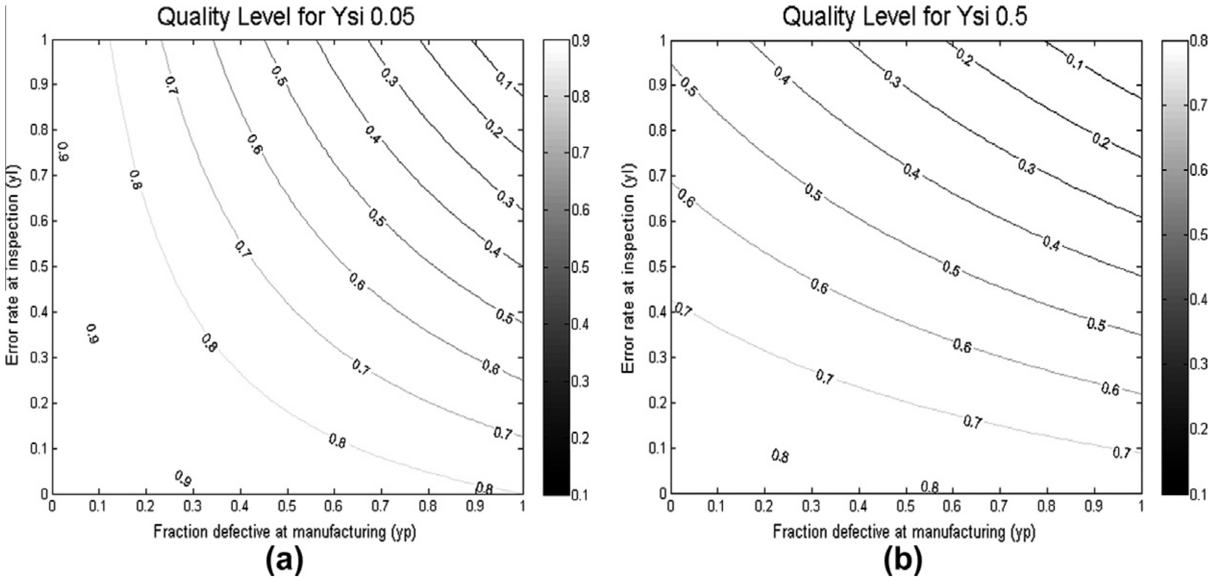

Fig. 4. Overall quality level for 0.05 and 0.5 fraction defective at supplier.

manufacturing plant); however, only one setting will result in a minimum cost of quality. The "Quality Level Graphs" plot the quality level (Eq. (9)) as a function of the internal decision variables (yp_j and yI_j). It can be seen that as the supplier percent defective increases, improving inspection is required, even at low quality levels. For instance, a 0.70 quality level can be reached at several levels of manufacturing fraction defective. One possible combination is to work with 100% defectives at manufacturing but with an error rate at inspection close to 0.10, although this situation would not be desirable from both a financial and practical viewpoint. In fact, the model will not select that option since it has a higher total quality cost than the solution with defects close to zero at manufacturing and a 0.5 error rate at inspection.

In conclusion, there exists a relationship between the supplier fraction defective and the quality level that achieves the lowest total quality cost. Therefore, quality costs are an aid in identifying the economic impact that the supplier fraction defective has on the supply chain and it shows the relevance of analysing supplier fraction defective to avoid making decisions based solely on price and bargaining.

4.2. Numerical example 2

The numerical example 2 shows the impact of the fraction defective at the retailer on the COQ curves, total COQ, and quality level achieved. Fig. 5 shows that as Yr_k increases, the maximum overall quality level attained by the logistic route decreases. With 0.05 fraction defective at the retailer, the optimal COQ point is at the 0.91 quality level while with 0.2 fraction defective, the optimal COQ is at the 0.76 quality level. Thus, the following rule of thumb could be applied to quickly estimate the overall quality level attained by the supply chain; the overall quality level is less than $1 - Yr_k$. This could be verified analytically from the QL formula given by Eq. (9) where $(1 - Yr_k)$ multiplies a value less than one.

Fig. 6 shows the total cost of quality as a function of the inspection error rate and the fraction defective at the manufacturing plant, that is, the evaluation of Eq. (8) for each possible combination of decision variables. Notice that the minimum quality level constraint is not considered when obtaining the total COQ. It was observed from Fig. 6 that when Yr_k increases, the total COQ increases. Additionally, the maximum quality level achieved decreases since the good products delivered to final customers depend directly on the percent defective at the retailer (see Fig. 7). Moreover, we can observe that both the minimum total COQ (Fig. 6) and the maximum quality levels (Fig. 7) coincide. Therefore, an important factor to be considered when selecting a supplier is its fraction defective in order to reduce the total quality costs and achieve a superior overall quality level in the supply chain.

4.3. Numerical example 3

In this example, the rework rate was changed. It can be seen from Fig. 8a that to achieve any given quality level when working with a rework rate of 0.2, we could choose from among numerous error rates at inspection but just a few levels of fraction defective at manufacturing. At low rework rates, it is better to invest in prevention activities to guarantee that good products reach the retailer. Investments in appraisal activities will not return significant benefits

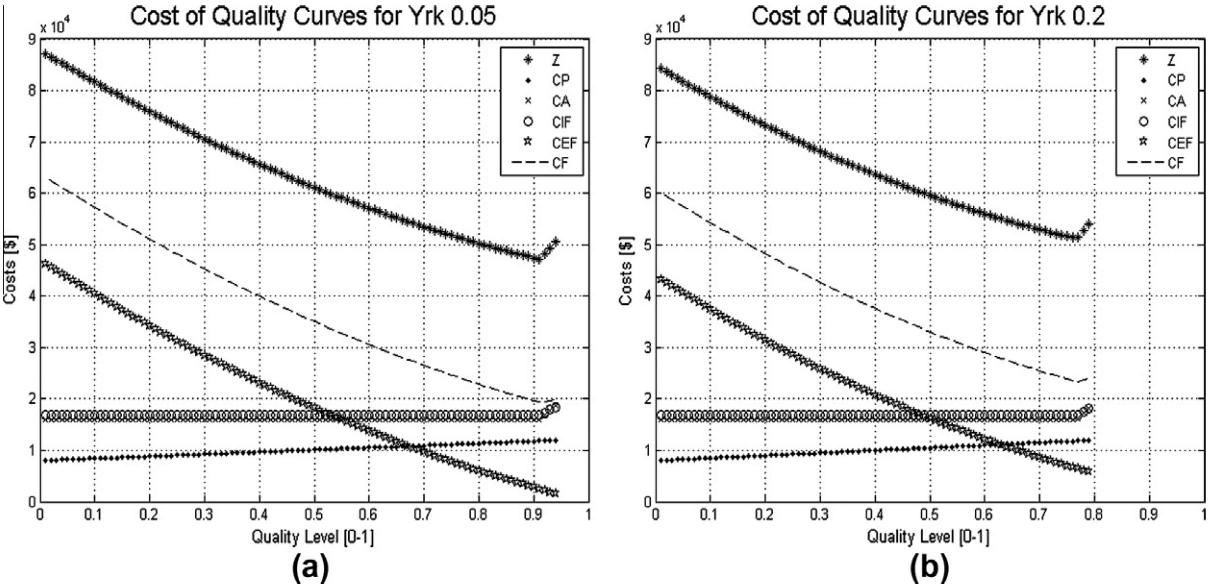

Fig. 5. Quality cost curves for 0.05 and 0.20 fraction defective at retailer.

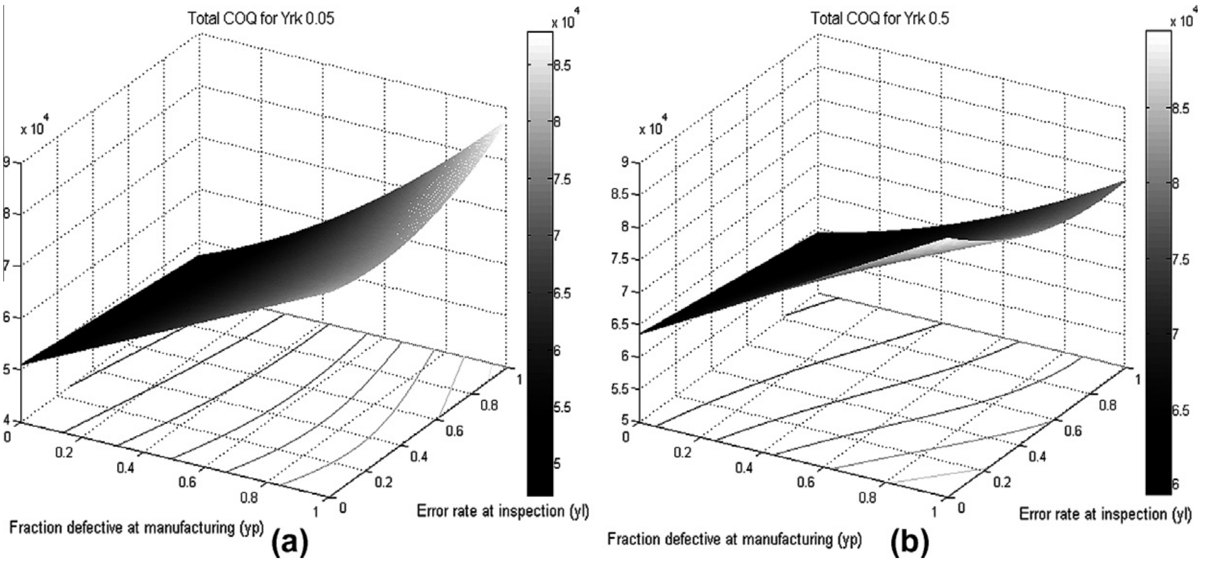

Fig. 6. Total quality costs (COQ) for 0.05 and 0.5 fraction defective at retailer.

because the output from the rework process is *GaR* (good products after rework) and *SaD* (sold as defective items). Since we are investing in inspection but there is a poor rework rate, many products will not be reworked and most of the products will be "sold as defective," incurring a foregone profit. This foregone profit is counted as part of the internal failure costs.

The total COQ is very similar for most rework rates as shown in Fig. 9 (Z curve). The minimum COQ for a given quality level is attained, for most quality levels, without using inspection and by decreasing the fraction defective at manufacturing as the minimum quality level (l) increases. Therefore, the prevention costs increases as the minimum quality level increases; this behaviour will continue until the fraction defective at manufacturing reaches a value close to zero. At this point, the inspection starts to be needed, and consequently, the internal failure costs increases. In conclusion, rework rate does not have a considerable impact on total COQ but a considerably low rework rate will result in the inspection system not returning significant benefits.

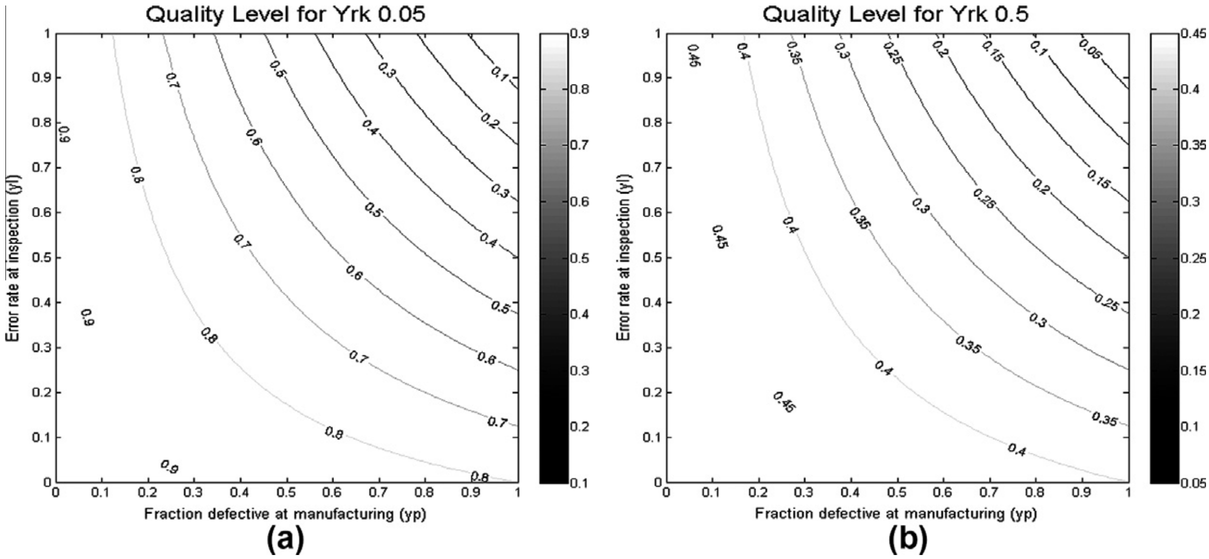

Fig. 7. Overall quality level for 0.05 and 0.5 fraction defective at retailer.

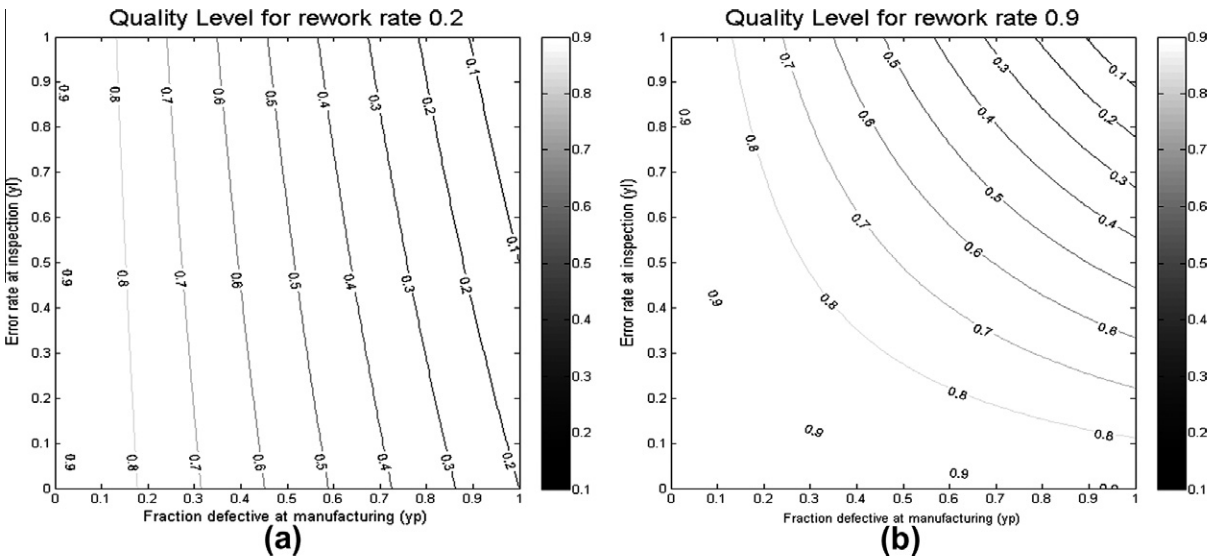

Fig. 8. Overall quality level for 0.20 and 0.90 rework rate.

4.4. Numerical example 4

In this section, we analysed the effect that the ratio of the variable prevention cost and variable appraisal cost (Av/Bv) has on the COQ curves, total COQ, and quality level achieved. First, instances generated by using the same set of costs but varying the Av/Bv ratio between 1 and 14 were run. The percentage of the prevention costs among the total quality cost is within the range reported in the literature for Av/Bv ratios less than 5.

In regard to the COQ curves, when variable prevention cost (Av) is equal to variable appraisal cost (Bv) as shown in Fig. 10a, the minimum total COQ is achieved by improving the manufacturing process through prevention activities as the required quality level increases. When the process reaches zero defects, appraisal activities are implemented to attain higher quality levels. Once more, inspection is an aid for achieving higher quality levels. Thus, investment in conformance should be more oriented to prevention rather than to appraisal activities. Moreover, the prevention curve will linearly increase with respect to the quality level; appraisal and internal failure costs remain constant until the process has been improved to zero defects and higher quality levels are required as shown in Fig. 10a. When the variable prevention cost is 14 times greater than the variable appraisal cost, a disruption in the COQ curves behaviour is observed (Fig. 10b. This

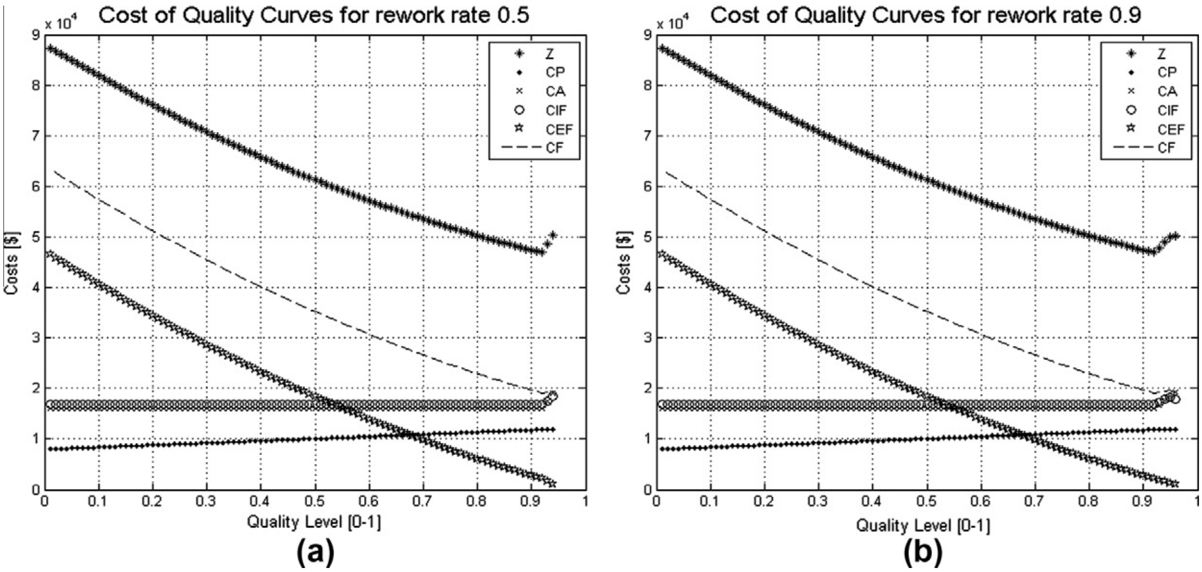

Fig. 9. Quality cost curves for 0.50 and 0.90 rework rate.

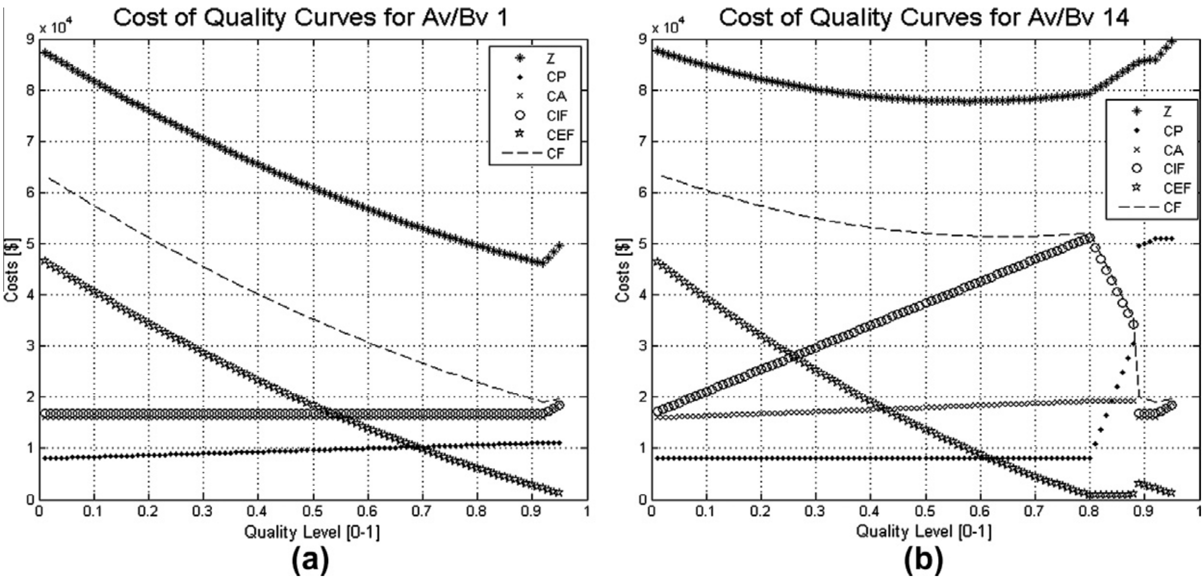

Fig. 10. Quality cost curves for *Av/Bv* ratio equal to 1 and 14.

disruption can be better understood by observing Fig. 11 where the COQ function moves its minimum from zero defects at manufacturing and error rate at inspection of 1 to a small error rate at inspection and a fraction defective close to 1 at manufacturing. Now, it would be preferable to utilise inspection rather than improving the manufacturing process to attain the required quality level at a minimum total cost.

Moreover, appraisal and internal failure cost functions increase at a fast pace until a 0.80 quality level is reached. At this point, the error rate at inspection is zero (perfect inspection) as depicted in Fig. 10b. Consequently, improvement in manufacturing is needed to continue attaining higher quality levels. This behaviour persists up to a 0.88 quality level when the fraction defective at plant is 0.48. At a 0.89 quality level a disruption in the COQ curves plot is observed. This is due to the fact that the model reaches the minimum total quality cost at no inspection (error rate of 1) and with 3.4% of defective items at manufacturing. That is, it goes from an error rate of zero to an error rate of 1 at inspection and it reduces the fraction defective from 0.48 to around 0.04 while changing quality level (*QL*) by just 0.01. The COQ curves show that internal failure costs drop (since inspection is not used anymore) and prevention costs increase rapidly. Lastly, at a 0.92 quality level, the

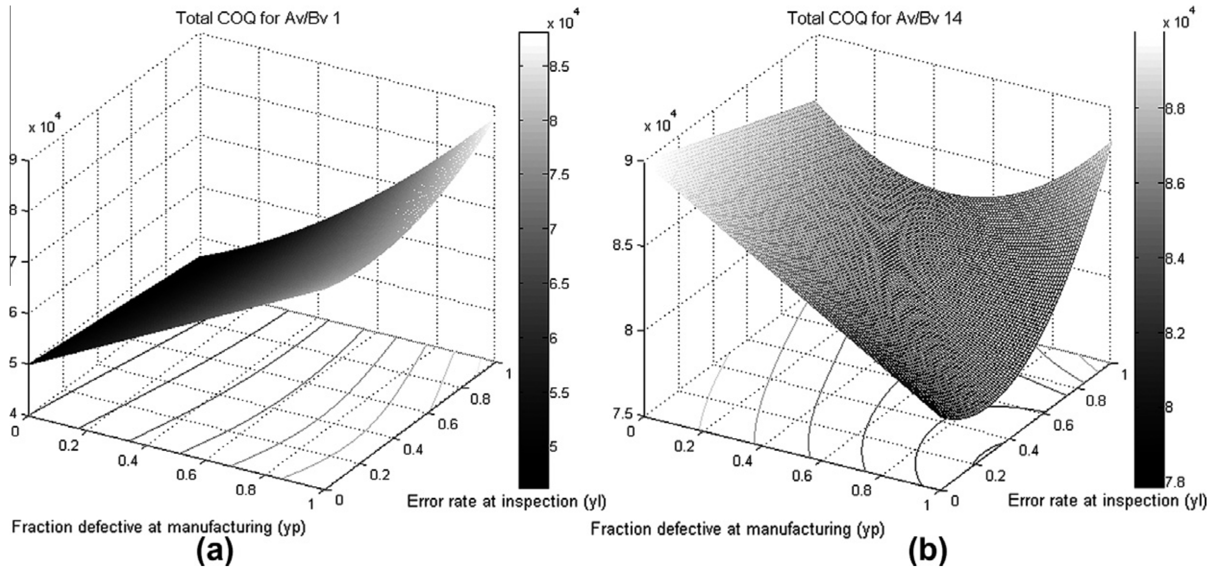

Fig. 11. Total quality costs (COQ) for *Av/Bv* ratio equal to 1 and 14.

process gets zero defects and no need for inspection. Thus, to attain a quality level above 0.92, reductions in error rate at inspection are mandatory since the manufacture process is already at zero defects.

The described behaviour does not have a great impact when designing a new logistic route with manufacturing plants that are not yet operating. Whereas the proposed model has complete freedom to change the decision variables values, this may not be the case for logistic routes with manufacturing plants that are already operating. For the latter, switching from perfect inspection to not using it at all may be a significant issue. Therefore, modifications to the model would be required to make it applicable for existing manufacturing plants. One alternative might be to include in the model a cost for switching from one configuration to a different one. Another option may be to find a suboptimal but feasible solution by looking for similar values of the decision variables that reach a similar quality level. Subsequently, a cost-benefit analysis could be conducted to evaluate the feasibility of using the suboptimal solution instead of the lowest cost solution.

4.5. Numerical example 5

Next, instances with the same set of costs but varying the variable appraisal cost (*Bv*) and the variable prevention cost (*Av*) ratio (*Bv/Av*) were solved. The ratio ranges from 1 to 50. The total quality cost is minimised by improving manufacturing through prevention activities as the required quality level increases. When process is at zero defects, then appraisal activities are implemented to attain higher quality levels. As in previous examples, inspection is an aid for achieving a higher quality level as shown in Fig. 12.

The overall quality level is not affected by the *Bv/Av* ratio. However, the total COQ function changes its minimum as shown in Fig. 13. When the ratio is equal to one, the solver finds the minimum point with close to zero defects at manufacturing and with no inspection (error rate of 1). The impact of inspection is not very clear in the plots at this value of the ratio; however, as the ratio increases, the total COQ function rotates and the minimum point becomes easier to predict. Summarizing, the results from the optimisation at each quality level show that the minimum COQ is attained, for all *Bv/Av* ratios, at very low defects at manufacturing and without the need for inspection. Moreover, the quality level with the minimum COQ, for all *Bv/Av* ratios, remains the same (0.92).

5. Discussion and managerial implications

The proposed model can generate COQ curves that resemble both the original and the revised Juran's model. The behaviour of Juran's original model is observed when supplier fraction defective is high and when the cost of implementing prevention activities to improve quality level surpasses the cost of appraisal activities. On the other hand, the behaviour of Juran's revised model is observed when working at low fraction defective at supplier and retailer, high rework rate, and when having a cost structure where the cost of prevention activities is similar or at most twelve times the cost of appraisal activities.

On the surface, the prevention and appraisal variable costs have no direct connection. However, we found that when the variable prevention cost is similar to the variable appraisal cost, the model suggests investing in prevention activities rather than in appraisal activities. This behaviour goes along with the following way of thinking: appraisal activities such as

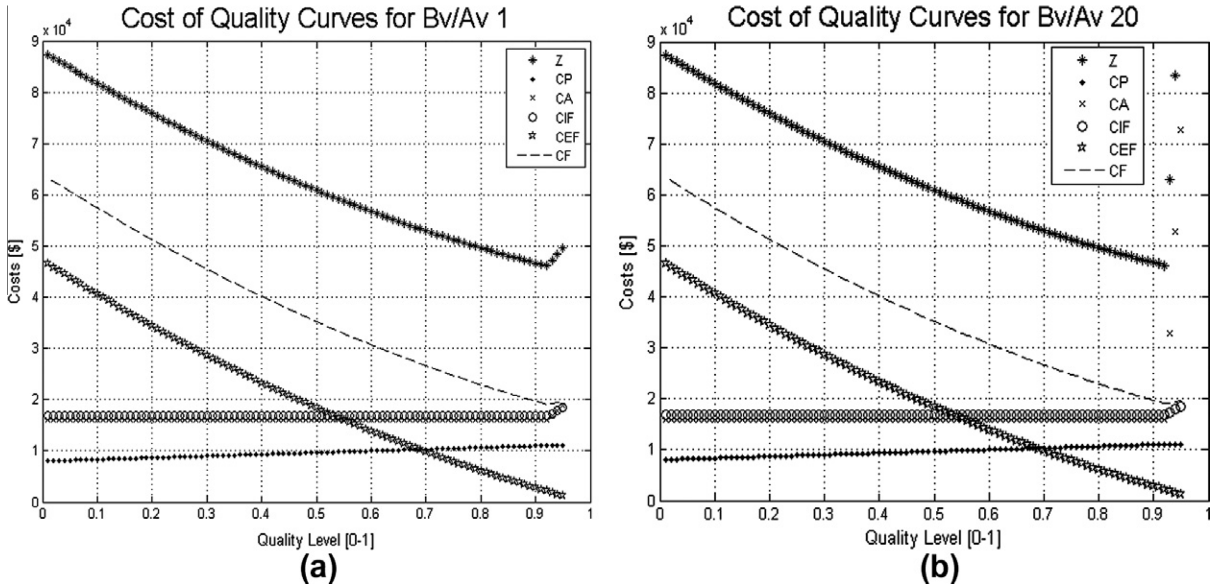

Fig. 12. Quality costs curves for *Bv/Av* ratio equal to 1 and 20.

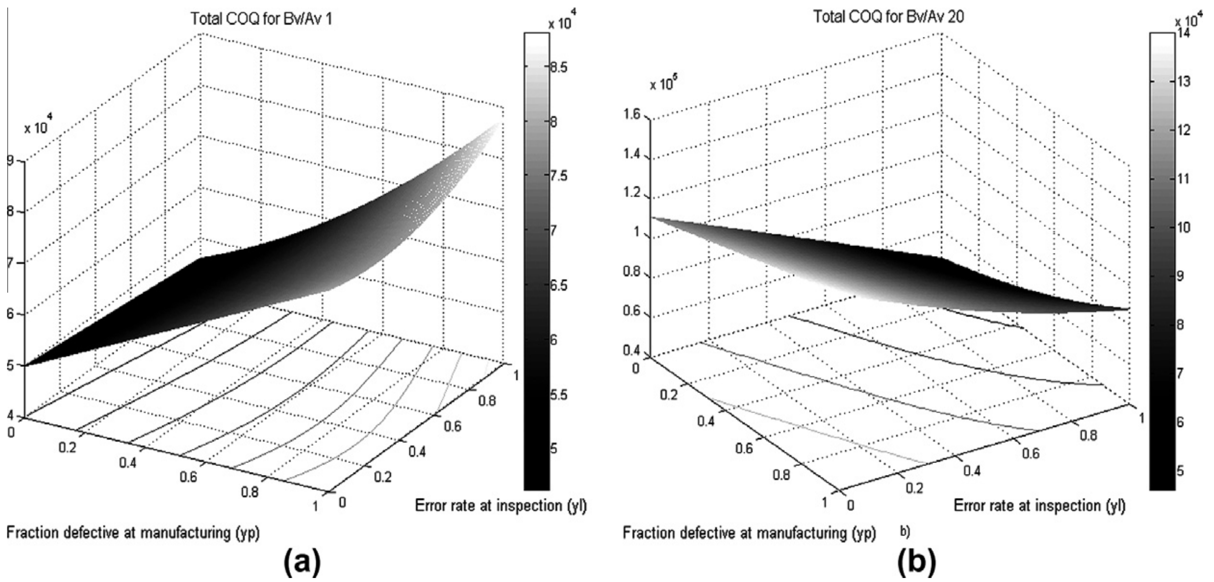

Fig. 13. Total quality costs (COQ) for *Bv/Av* ratio equal to 1 and 20.

inspection, testing, and verification are non-value-added activities [34]. Therefore, appraisal should be used only as an aid to attain higher quality levels when manufacturing is working at very low defective rate. Consequently, the proposed model utilises inspection only when prevention activities are extremely costly. In this way, inspection is not employed as a tool to cover the poor performance or capability of manufacturing operations.

Non-conformance costs (internal and external failure costs) have been considered to be the counterpart of conformance costs (prevention and appraisal). A common belief is that by increasing conformance costs, a company will decrease non-conformance costs. This depends on how the COQ model is constructed. If the total quality costs are associated with an internal quality level, high internal quality is associated with low appraisal cost. In the proposed model, the quality level is the overall 'external quality level' of the supply chain. Since defective items within the plant are found through inspection, achieving higher external quality levels implies more appraisal and consequently more internal failure activities. Therefore, appraisal is observed to rise as the overall quality level increases. Thus, a potential positive relationship between appraisal and internal failure costs arises. This relationship may merit further study but is beyond the scope of the present work. In regard to external failure costs, the more defects are found within the plant limits, the less external failure and opportunity costs will be incurred. Hence, the negative relationship between conformance and non-conformance costs may be due solely to external failure and opportunity costs which account for most of the non-conformance costs.

6. Conclusions and recommendations for future work

An interdependent relationship between the selected supplier and the optimal quality level as well as between the selected retailer and the optimal quality level achieved by the logistic route is observed in the numerical examples. Therefore, quality costs are an aid to identify the economic impact that supplier and retailer fraction defective have on the logistic route. Quality costs show the relevance of making good selections among available suppliers and retailers. Conformance costs, usually divided into prevention and appraisal activities, play an important role in the definition of the optimal COQ point. The proposed model seeks to utilise prevention activities and make use of appraisal activities only when necessary or when the cost of prevention activities exceed appraisal costs by a great amount. Internal failure cost plays a relevant role in the behaviour of the COQ curves and selection of optimal values for decision variables since the defective parts found through inspection are either reworked or sold 'as is'. As a result, a positive relationship between appraisal and internal failure costs emerged in the proposed model.

In conclusion, the major contribution of this research lies in developing a formal framework for computing the quality cost across a single-product three-echelon serial supply chain model and providing useful managerial insights.The proposed methodology provides an aid for engineering managers who want to translate quality into monetary terms, and moreover, into internal operational decisions such as error rate at the inspection and fraction defective at manufacturing. This will facilitate the evaluation of supplier and retailer selection and provide a better understanding of the interdependencies among defective rates at each stage of the supply chain. The results show that computing quality costs and determining the optimal COQ point when generating a new logistic route will reduce costs while maintaining the best possible quality level. Computing COQ for a supply chain is the first step in integrating it into the decision process because it allows exploring the interrelationships among business entities. Independent COQ curves based on fraction defective at each business unit are not the best way to integrate quality costs since the integration should be addressed from a systems viewpoint. We can only conclude that more research needs to be conducted and that including quality costs in supply chain modelling can provide significant benefits. The model developed provides a way to manage cost of quality across a single-product three-echelon serial supply chain. It constitutes an initial step toward more comprehensive models. Some recommendations for future work include: (1) Exploring the effect of adding an inspection before manufacturing. This may reduce the cost of processing bad components. (2) Enhance the model structure to allow the selection of different appraisal technologies. For example, automated monitoring systems and statistical quality control may be lower cost alternatives to simply intensifying the inspection. (3) Extend the model to include supplier and retailer selection, as well as plant location decisions.

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References

[1] A. Feigenbaum, Total quality control, Harvard Business Rev. 34 (6) (1956) 93–101.
[2] L. Carr, Applying cost of quality to a service business, Sloan Manage. Rev. 33 (4) (1992) 72–77.
[3] S. Srivastava, Towards estimating cost of quality in supply chains, Total Quality Manage. Business Excell. 19 (3) (2008) 193–208.
[4] A. Ramudhin, C. Alzaman, A. Bulgak, Incorporating the cost of quality in supply chain design, J. Quality Maintenance Eng. 14 (1) (2008) 71–86.
[5] C. Alzaman, A. Ramudhin, A.A. Bulgak, Heurisitc Procedures to Solve a Binay Nonlinear Supply Chain Model: A Case Study from the Aerospace Industry, in: International Conference on Computers and Industrial Engineering (CIE). Troyes, France, 2009, pp. 985–990.
[6] C.D. Ittner, Exploratory evidence on the behavior of quality costs, Oper. Res. 44 (1) (1996) 114–130.
[7] J. Campanella, Principles of Quality Costs: Principles. Implementation, and Use, 2nd ed., ASQC Quality Costs Committee, Milwaukee, WI, 1990.
[8] D.A. Sandoval-Chavez, M.G. Beruvides, A state-of-the-art matrix analysis of the cost related to quality, in: Sixth international conference on management of technology II, 1997, pp. 1253–1260.
[9] A. Schiffauerova, V. Thomson, A review of research on cost of quality models and best practices, Int. J. Quality Reliab. Manage. 23 (6–7) (2006) 647–669.
[10] M. Banasik, A study of the costs of quality in a renewable resource environment, Industrial Engineering, Texas Tech University, Lubbock, Texas, USA, 2009.
[11] Juran, Quality Control Handbook, 1st ed., McGraw-Hill, New York, 1951.
[12] J.M. Juran, F.M. Gryna, R.S. Bingham, Quality Control Handbook, 3rd ed., McGraw-Hill, New York, 1974.
[13] B. Dale, J. Plunkett, Quality Costing, 1st ed., Chapman & Hall, 1991.
[14] L. Porter, P. Rayner, Quality costing for total quality management, Int. J. Product. Econ. 27 (1) (1992) 69–81.
[15] J. Juran, F. Gryna, R. Bingham, Quality Control Handbook, McGraw-Hill, New York, 1962.
[16] A. Schneiderman, Optimum quality costs and zero defects: are they contradictory concepts?, Quality Progr 19 (11) (1986) 28–31.
[17] J.M. Juran, F.M. Gryna, Quality Planning and Analysis: From Product Development Through Use, McGraw-Hill, New York, 1993.
[18] J.J. Plunkett, B.G. Dale, Quality Costs: A Critique of Some Economic Cost of Quality Models, Int. J. Product. Res. 26 (11) (1988) 1713–1726.
[19] J. Freiesleben, On the limited value of cost of quality models, Total Quality Manage. 15 (7) (2004) 959–969.
[20] B. Dale, J. Plunkett, Quality Costing, 2nd ed., Chapman & Hall, London, 1995.
[21] B. Beamon, Supply chain design and analysis: models and methods, Int. J. Product. Econ. 55 (3) (1998) 281–294.
[22] C. Simga-Mugan, Distribution of quality costs: evidence from an aeronautical firm, Total Quality Manage. 11 (2) (2000) 227–234.
[23] BSi (British Standard) 6143-2:1990, Guide to the economics of quality. Prevention, appraisal and failure model, Global Engineering Documents, London, 1990.

[24] J.T. Godfrey, W.R. Pasewark, Controlling quality costs, Manage. Account. 69 (9) (1988) 48–51.
[25] D. Sandoval-Chávez, M. Beruvides, Using opportunity costs to determine the cost of quality: a case study in a continuous-process industry, The Eng. Econ. 43 (2) (1998) 107–124.
[26] W.H. Lesser, Cost of quality, Ind. Quality Control (1954) 11–14.
[27] H. Kume, Business management and quality cost: the Japanese view, Quality Progr. (1985) 13–18.
[28] J.M. Juran, F.M. Gryna, Quality Planning and Analysis: From Product Development Through Use, McGraw-Hill, New York, 1993.
[29] T.L. Albright, H.P. Roth, The measurement of quality costs: an alternative paradigm, Account. Horizons (1992) 15–27.
[30] C. Quigley, C. McNamara, Evaluating product quality: an application of the Taguchi quality loss concept, Int. J. Purchas. Mater. Manage. 28 (3) (1992) 19–25.
[31] R.B. Kethley, B.D. Waller, T.A. Festervand, Improving customer service in the real estate industry: a property selection model using Taguchi loss functions, Total Quality Manage. 13 (6) (2002) 739–748.
[32] M.-H.C. Li, Quality loss functions for the measurement of service quality, Int. J. Adv. Manufact. Technol. 21 (2003) 29–37.
[33] W.-N. Pi, C. Low, Supplier evaluation and selection using Taguchi loss functions, The Int. J. Adv. Manufact. Technol. 26 (1–2) (2005).
[34] P. Gupta, Reduce appraisal costs, Quality (2007) 20.