



Robust dynamic transmission and renewable generation expansion planning: Walking towards sustainable systems



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ABSTRACT

Recent breakthroughs in Dynamic Transmission Network Expansion Planning (DTNEP) have demonstrated that the use of robust optimization, while maintaining the full temporal dynamic complexity of the problem, renders the capacity expansion planning problem considering uncertainties computationally tractable for real systems. In this paper an adaptive robust formulation is proposed that considers, simultaneously: (i) a year-by-year integrated representation of uncertainties and investment decisions, (ii) the capacity expansion lines have and (iii) the construction and/or dismantling of renewable and conventional generation facilities as well. The Dynamic Transmission Network and Renewable Generation Expansion Planning (DTNRGEP) problem is formulated as a three-level adaptive robust optimization problem. The first level minimizes the investment costs for the transmission network and generation expansion planning, the second level maximizes the costs of operating the system with respect to uncertain parameters, while the third level minimizes those operational costs with respect to operational decisions. The method is tested on two cases: (i) an illustrative example based on the Garver IEEE system and (ii) a case study using the IEEE 118-bus system. Numerical results from these examples demonstrate that the proposed model enables optimal decisions to be made in order to reach a sustainable power system, while overcoming problem size limitations and computational intractability for realistic cases.

1. Introduction

1.1. Motivation

The new objective of the Kyoto Protocol for reducing Greenhouse Gases (GHG) encourages the development of renewable energy sources within electric systems [1]. The main reason for this is to combat the upward trend in worldwide average temperatures and climate change, and thus, it is expected that vast amounts of new generation facilities, especially renewable ones, will be built in the medium-term future.

Transmission network and renewable generation expansion planning analyze the issue of how to expand or reinforce an existing power transmission network, incorporate new renewable generation facilities and dismantle the old ones in order to adequately service system loads over a given time horizon while decreasing GHG emissions. This problem is challenging for several reasons [2]:

1. Transmission and generation investment decisions have a long-standing impact on the power system as a whole.
2. Transmission and generation investments, especially new

generation sources, must be integrated appropriately into the existing system.

3. Consumption and renewable energy generation uncertainties, such as with wind and solar power plants, make resolution of the problem complicated. Note that wind power is the renewable technology that has most developed in the last decade, while the next renewable technology, in constant evolution, is photovoltaic power. The introduction of these types of renewable sources in the generation mix increases uncertainty about the feasibility of generation.
4. The expansion planning problem is by nature a multi-stage one that entails planning a horizon over several years. Keeping the dynamic complexity of the problem mostly results in computationally intractable problems.
5. Transmission expansion planning (TEP) and generation expansion planning (GEP) have usually been addressed independently, i.e. transmission planning is determined by considering that it is not possible to build new generation facilities and vice versa. However, transmission and generation expansion plans are clearly interrelated and treating them separately provides suboptimal solutions. The reason why these problems have been treated independently is that

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Nomenclature	
<i>Indices and sets</i>	
\mathcal{D}	set of demand indices
g	index for groups of generators built per phases
\mathcal{G}	set of indices of all generation units installed at the beginning of time horizon considered which cannot be removed from the system
\mathcal{G}^+	set of all prospective and independent new possible generators
\mathcal{G}_g^+	set of all prospective new generators which can be installed at different phases associated with group g
\mathcal{G}^-	set of all generators to be uninstalled or dismantled during the study period
i	index related to generators
j	index associated with loads
k	index referring to lines
l	counter index for each iteration
\mathcal{L}	set of all existing transmission lines
\mathcal{L}^+	set of prospective transmission lines
n	index related to buses
\mathcal{N}	set of networks buses
$n(i)$	bus index for the i -th generating unit
$n(j)$	bus index for the j -th demand
t	index related to time period
\mathcal{T}	set of indices of years
$\mathcal{U}^{(t)}$	set of indices of the uncertain variables for time period t
Ψ_n^D	set of indices of demand for bus n
Ψ_n^G	set of indices of generating units for bus n
<i>Constants</i>	
b_k	line k susceptance (S)
c_i^G	generator i operational cost (€/MW h)
c_i^{GI}	generator i investment cost (€)
c_k^{LI}	line k investment cost (€)
c_j^S	consumer j load-shedding cost (€/MW h)
$e_j^{(t)}$	percentage of load shed by the j -th demand for year t
f_k^{\max}	line k capacity (MW)
$h_{\mu,j}^{(t)}$	nominal value evolution factor for demand j and period t
$h_{\sigma,j}^{(t)}$	dispersion value evolution factor for demand j and period t
I	discount rate
N_y	number of study periods
$o(k)$	line k sending-end bus
$r(k)$	line k receiving-end bus
$t_i^{G^-}$	time period when generator $i \in \mathcal{G}^-$ is uninstalled or dismantled
Π_G	generation expansion investment budget (€)
Π_L	transmission expansion investment budget (€)
σ	annual weighting factor (h)
<i>Primal variables</i>	
$c_o^{(t)}$	operating cost associated with given values of upper- and middle-level variables for year t (€)
$c_{op}^{(t)}$	operating cost related to given values of upper-level variables for year t (€)
$c_{op,\nu}^{(t)}$	it corresponds to $c_{op}^{(t)}$ at iteration ν
$f_k^{(t)}$	line k power flow for year t (MW)
$f_{k,\nu}^{(t)}$	line k power flow for year t (MW) at iteration ν
$g_i^{(t)}$	power production of generating unit i for year t (MW)
$g_{i,\nu}^{(t)}$	power production of generating unit i for year t (MW) at iteration ν
$p_j^{(t)}$	power consumption of demand j for year t (MW)
$p_{j,\nu}^{(t)}$	power consumption of demand j for year t (MW) at iteration ν
$r_j^{(t)}$	load shed of demand j for year t (MW)
$r_{j,\nu}^{(t)}$	load shed of demand j for year t (MW) at iteration ν
$\mathbf{u}^{(t)}$	vector of random or uncertain parameters ($u_i^{G(t)}, u_j^{D(t)}$) for year t , including maximum generation capacities and loads (MW)
$\mathbf{u}_\nu^{(t)}$	vector of random or uncertain parameters ($u_{i,\nu}^{G(t)}, u_{j,\nu}^{D(t)}$) for year t at iteration ν
$x_k^{(t)}$	binary variable representing new line k construction at the beginning of year t
$\tilde{x}_k^{(t)}$	line k status (<i>existing vs no existing</i>) at the beginning of year t
$\tilde{x}_{k,\nu}^{(t)}$	line k status at the beginning of year t and iteration ν
$y_i^{(t)}$	binary variable representing new generator i construction at the beginning of year t
$\tilde{y}_i^{(t)}$	generator i status (<i>existing vs no existing</i>) at the beginning of year t
$\tilde{y}_{i,\nu}^{(t)}$	generator i status at the beginning of year t and iteration ν
$\theta_n^{(t)}$	bus n voltage angle for year t (radians)
$\theta_{n,\nu}^{(t)}$	bus n voltage angle for year t (radians) at iteration ν

TEP pertains to a welfare-focused agent (ISO), while GEP relates to profit-focused producers.

1.2. Literature review

Transmission and generation expansion planning have been extensively studied areas from the time power systems began to operate [3]. In recent years, these problems have been widely researched and analyzed from different viewpoints, such as: solution method, reliability, electricity market, uncertainty, environmental impact, the modeling approach, from the time horizon viewpoint, time frames, among others [4]. State-of-the-art transmission planning is introduced in [5], where different contributions are classified by the solution method, treatment of the planning horizon, by considering the electrical sector restructuring, and the tools for developing planning models. A review of generation expansion planning techniques in the face of growing uncertainty is presented in [6]. In this document, the literature review is split into several categories: (a) the modeling approach, (b) algorithms, (c) time frames, (d) time scope, and (e) others,

which makes it easier to discuss the contribution this paper has made.

The main difficulty of transmission and generation expansion planning problems is taking decisions with the great amount of uncertainty associated with different factors [7]. Moreover, the integration of renewable energy into the generation mix increases uncertainty on the generation side [8]. The stochastic programming techniques enable an optimal decision to be found in problems involving uncertainty data [9]. In order to incorporate (i) demand, (ii) the equivalent availability factor of the generating plants and (iii) the transmission capacity factor of the transmission lines as random events, stochastic programming and probabilistic constraints are used in [10] in a new model for generation and transmission expansion. Uncertainty associated with intentional attacks on the transmission network has been put forward in [11] by using a stochastic programming problem with recourse, while [12] utilizes the Monte Carlo simulation and scenario reduction technique to create scenarios that simulate random characteristics of system components and load growth. Chance-constrained optimization is a type of stochastic programming which handles the stochasticity of the problem by specifying a confidence level at

which the stochastic constraints are required to hold [13]. In [14] a chance-constrained programming method was set out to solve the transmission network expansion problem bearing in mind the uncertainties of both the load and the wind farm output. The impact of integrating wind power into an existing power system is researched in [15], where the calculation of the optimal wind power capacity is also formulated as a chance-constrained programming problem. However, stochastic programming formulations result in computationally intractable problems in real-size networks. In contrast, recent breakthroughs have proved that computational tractability for realistic systems is possible by using Adaptive Robust Optimization (ARO) frameworks [16,17]. One application of ARO to transmission expansion planning is reported in [18] using a Benders decomposition scheme to solve the ARO problem, while [19] applies a column-and-constraint generation method solely based on primal cuts. Although it is true that these robust methods are more efficient than their stochastic counterparts, it is also correct that solution times for mixed-integer linear programming problems increase exponentially with respect to the size of the problem. For this reason, [20] addressed that problem by taking different features from existing algorithms. An additional advantage robust optimization offers is that it is the recommended approach for considering long-term uncertainties [2].

The number of algorithms that can be applied to address the generation and transmission expansion planning problem can be classified mainly for optimization (including decomposition techniques [21]), heuristic [22], meta-heuristic [23], genetic algorithms [24], etc. Firstly an optimization problem is proposed in [25] with a two-stage min-max model for co-optimizing the expansion of the transmission system and uncertain generation capacity with high security standards. Secondly, [26] presents a new two-phase bounding and decomposition approach to compute optimal and near-optimal solutions for large-scale investment problems using mixed integer linear programming (MILP). The decomposition phase improves Benders algorithm by accelerating the convergence of the bounds and in the lower bound an auxiliary cut is included in the Benders master problem. A heuristic algorithm for multi-stage transmission planning, considering security constraints by means of a genetic algorithm is also described in [27], based on a tree searching heuristic algorithm (TSHA) combined with a genetic algorithm (GA), whereas [28] shows a constructive heuristic algorithm for solving long-term transmission expansion planning using a DC model. An efficient metaheuristic technique based on artificial bee swarm optimization is used to configure the hybrid system put forward in [29].

Even though transmission and generation expansion planning is by nature a multi-stage problem, the complexity of this dynamic nature has meant that in most studies about transmission and generation expansion planning in the technical literature there are different simplifying assumptions. The time frame for most research studies is one year because planning and investment costs are considered annually [18,30–32]. However, in order to ensure tractability while keeping the model accurate enough; in many studies a time frame of several years, which are treated both separately and sequentially, is considered [33–35]. The use of an integrated year-by-year representation of investment decisions (dynamic approach) has been considered to be highly complex and a computationally intractable problem. For this reason, most research related to this topic focus on very small test cases or use heuristic methods [36–38]. However, in [39] the ARO formulation proposed by [20] is expanded to a dynamic approach to the transmission network expansion planning problem and computational tractability is now possible for realistic cases.

Three kinds of time periods are normally assessed such as (i) one year, (ii) 5 or 10 years and (iii) 20 to 25 years. A single 1-year period is the most frequent approximation when what is presented is a methodological novelty [18–20,40]. The challenge of long term assessment [39,41] is elevated consumption of CPU resources, where [41] considers that transmission lines have a longer life and more predictable behavior with respect to energy storage.

The transition of a power system into a sustainable system, which is the purpose of this paper, has also been addressed in various papers previously. The variability effect associated with renewable-based electricity generation on power systems dynamics is assessed in [42] with a set-theoretic method, while [43] uses a Differential Evolution algorithm to study the impact the increasing penetration of wind power technology has. Finally, a framework for the generation expansion planning problem with regard to energy efficiency solutions is developed in [44].

According to the literature review and to the best of our knowledge, there is no available method in the current literature which simultaneously deals with TEP and GEP with uncertainty, which takes an integrated multi-stage, multi-year or multi-period approach, and which is capable of dealing with realistic cases. The proposal presented in this paper intends to fill this niche.

1.3. Aims and contributions

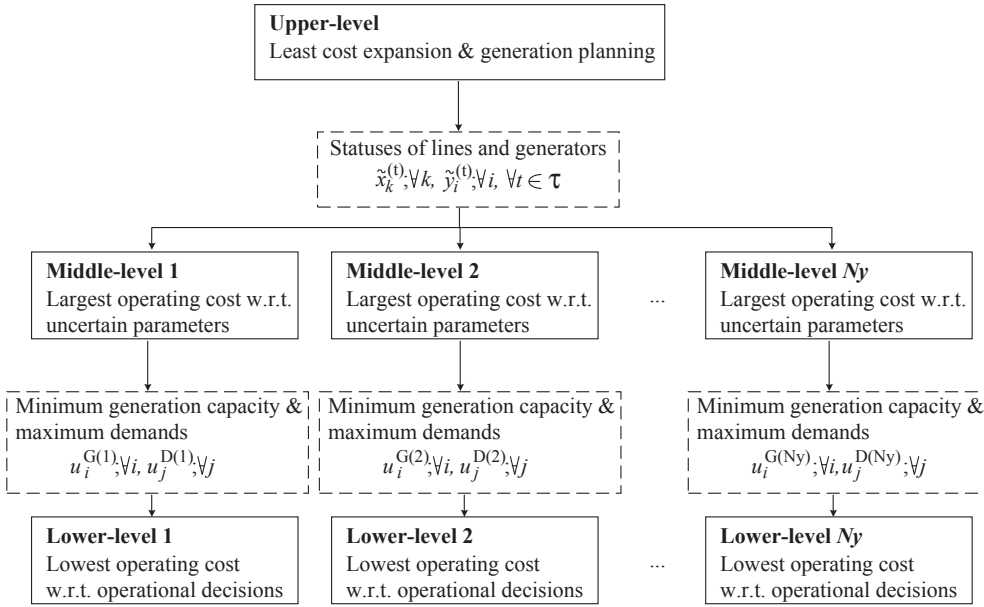
The purpose of this paper is threefold:

1. To extend the ARO formulation proposed by [39] for the dynamic approach dealing simultaneously with TEP and GEP with uncertainty.
2. To provide a highly flexible model with respect to generation capacity expansion planning possibilities:
 - (a) Conventional generation facilities (set \mathcal{G}^-) which have reached the end of their lifetimes during the time period considered can be dismantled or decommissioned.
 - (b) A set of all prospective and independent new possible generators with uncertainty in their production capacity can be considered.
 - (c) The construction of renewable generation facilities in different and sequential phases (sets \mathcal{G}_g^+ for different groups g) can be considered.
 - (d) It enables conventional facilities without uncertainties to be included.
3. To show that computational tractability for an integrated year-by-year representation of investment decisions in line capacities and generation is possible for realistic cases, ensuring the achievement of a global optimal solution.

In summary, as a mayor contribution of this paper, we address a yet unresolved and challenging problem which is of utmost practical interest since it circumvents simplifying assumptions typically adopted in the models available in the literature.

Regarding the selection of the robust approach to represent the uncertainties in generation capacities and demand values, these expansion plans are generally made for long-term planning horizons, thus it is important that future demand, decommissioning of old generating units and the increasing penetration of renewable generating units are represented appropriately. Unlike scenario based methods and chance-constrained programming, ARO neither requires accurate probabilistic information nor relies on a discrete set of uncertainty realizations that need a tradeoff between tractability and accuracy that may be hard to attain. Rather, uncertainty is modeled by decision variables within an uncertainty set, which thereby comprises an infinite number of uncertainty realizations. Hence, the size of the robust counterpart does not depend on the space dimensions of uncertainty realizations belonging to the uncertainty set, which is beneficial for implementation purposes. The uncertainty set can be built using intervals defined by lower and upper bounds for uncertain parameters. Such information may be easier to acquire than probability distributions. Moreover, the robust solution protects against all realizations of uncertainty within the uncertainty set. A worst-case setting such as this is a particularly desirable feature in planning problems. Additionally, ARO provides a flexible modeling framework to control with ease how conservative the robust solution is by means of pre-specifying user-defined uncertainty budgets or

Fig. A.1. Trilevel structure of the robust DTNRGEP problem.



conservativeness parameters that modify the uncertainty set. All these features make ARO the best option for tractability, dealing simultaneously with multi-year capacity transmission and generation expansion planning problems for realistic cases.

Note that in this paper we have considered a variable cost for renewable generating units. We do not consider losses because this is the standard approach taken by the power industry, and in fact losses are not relevant considering that the robust approach takes into account the worst case, which corresponds to large demand values and low generation capacity from renewable units. In this scenario losses do not play a role in selecting which units to produce.

1.4. Paper structure

The rest of the paper is structured as follows. Section 2 describes the robust formulation of the DTNRGEP problem. The proposed decomposition method to solve the problem is described in Section 3. Section 4 provides numerical results for two examples. Finally, in Section 5 important conclusions are drawn.

2. Robust dynamic transmission network and renewable generation expansion planning formulation

TEP and GEP have been traditionally dealt with independently, however we follow the strategy given by [45] that considers the perspective of a central planner. This central planner determines the generation and transmission expansion plan that is optimal for the operation of the electric system as a whole. Thus, the central planner is responsible for carrying out optimal transmission plans and for setting incentives for private investors to build the most appropriate generation facilities for the system.

Based on the assumption above and the robust model presented in [39] for the dynamic transmission expansion planning problem under uncertainty, the formulation of the robust DTNRGEP problem can be made as a mixed-integer trilevel problem, where inner levels can also be decoupled by time period as shown in Fig. A.1: (1) the upper-level is associated with identification of the least-cost expansion and generation plan; (2) the middle level characterizes, the worst case realization of uncertainty sources for a given upper-level investment plan, for each time period considered, and (3) the lower level also provides the optimal system operation for given upper-level investment decisions and middle-level uncertainty realizations for each time period considered.

2.1. Upper-level problem

The most important variables for the upper-level correspond to binary variables $x_k^{(t)}$ which represent new line k construction at the beginning of year t , and binary variable $y_i^{(t)}$ which represents new generator i construction at the beginning of year t . The description of the remaining variables can be found in the notation section at the beginning of the manuscript. The detailed formulation of the upper-level problem is as follows:

$$\begin{aligned} \text{Minimize}_{x_k^{(t)}, \tilde{x}_k^{(t)}, y_i^{(t)}, \tilde{y}_i^{(t)}} \sum_{t \in \mathcal{T}} \frac{1}{(1+I)^{t-1}} & \left(\sum_{k \in \mathcal{L}^+} c_k^{\text{LL}} x_k^{(t)} \right. \\ & \left. + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{\text{GI}} y_i^{(t)} + \frac{c_{\text{op}}^{(t)}}{(1+I)} \right); \end{aligned} \quad (1)$$

subject to

$$\Pi_{\text{L}} \geq \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{L}^+} \frac{1}{(1+I)^{t-1}} c_k^{\text{LL}} x_k^{(t)} \quad (2)$$

$$\tilde{x}_k^{(t)} = 1; \forall k \in \mathcal{L}, \forall t \in \mathcal{T} \quad (3)$$

$$\tilde{x}_k^{(t)} = \sum_{p=1}^{p=t} x_k^{(p)}; \forall k \in \mathcal{L}^+, \forall t \in \mathcal{T} \quad (4)$$

$$\sum_{t \in \mathcal{T}} x_k^{(t)} \leq 1; \forall k \in \mathcal{L}^+ \quad (5)$$

$$x_k^{(t)} \in \{0,1\}; \forall k \in \mathcal{L}^+, \forall t \in \mathcal{T} \quad (6)$$

$$\Pi_{\text{G}} \geq \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} \frac{1}{(1+I)^{t-1}} c_i^{\text{GI}} y_i^{(t)} \quad (7)$$

$$\tilde{y}_i^{(t)} = 1; \forall i \in \mathcal{G}, \forall t \in \mathcal{T} \quad (8)$$

$$\tilde{y}_i^{(t)} = \sum_{p=1}^{p=t} y_i^{(p)}; \forall i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T}; \quad (9)$$

$$\sum_{t \in \mathcal{T}} y_i^{(t)} \leq 1; \forall i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g \quad (10)$$

$$y_i^{(t)} \in \{0,1\}; \forall i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T} \quad (11)$$

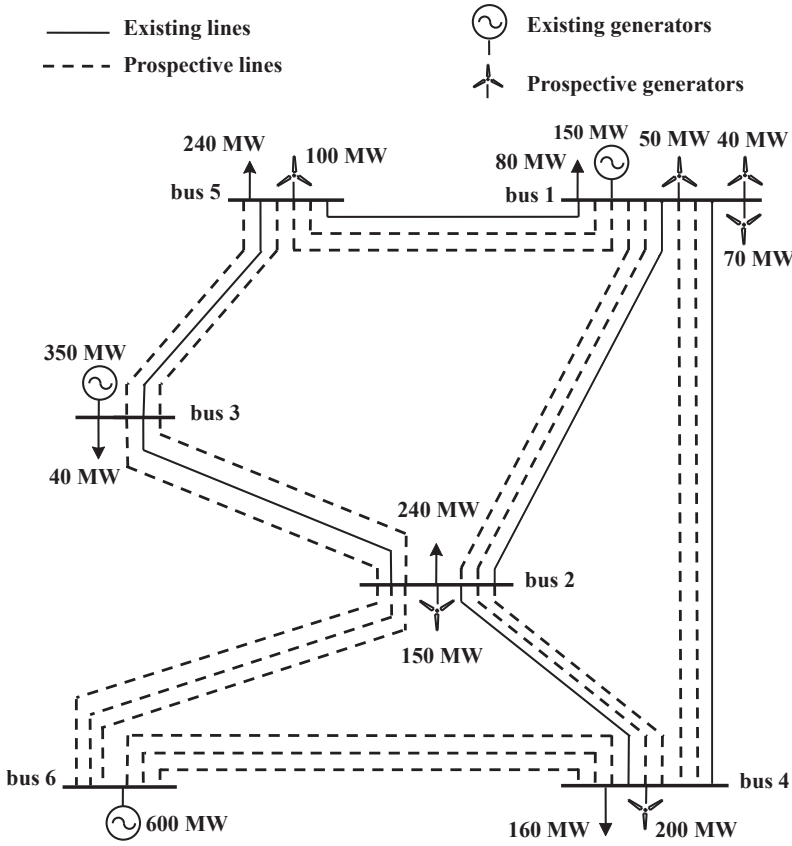


Fig. A.2. Garver's 6-bus test system.

$$\tilde{y}_i^{(t)} = 1; \forall i \in \mathcal{G}^-, \forall t = 1, \dots, t_i^{G^-} \quad (12)$$

$$\tilde{y}_i^{(t)} = 0; \forall i \in \mathcal{G}^-, \forall t = t_i^{G^-} + 1, \dots, N_y \quad (13)$$

$$y_{i+1}^{(t)} \leq \tilde{y}_i^{(t)}; \forall i \in \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T} \quad (14)$$

$$y_{i+1}^{(t)} + y_i^{(t)} \leq 1; \forall i \in \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T}; \quad (15)$$

where the objective function (1) is the total net present cost (NPC) (its derivation can be found at (A)), which comprises three terms, namely the investment costs for expansion and generation, and the worst-case operation cost which is made up of the middle and lower level problems, which will be disaggregated in the following subsections. Eqs. (2)–(6) are the constraints related to the construction of lines as presented in [39], which: (i) limit the maximum expansion investment (Eq. (2)), (ii) force the line status to 1 for all existing transmission lines at the beginning (Eq. (3)), and (iii) once the line has been constructed (Eq. (4)), (iv) ensure that no line is constructed more than once throughout the time horizon considered (Eq. (5)), and (v) establish the binary nature of line investment decisions (Eq. (6)). Constraints (7)–(15) are novel and associated with generation facilities. Constraint (7) keeps the maximum amount of generation investment within the available budget. Constraint (8) makes the generation status equal to 1 for all existing generation facilities at the beginning of the time horizon considered which cannot be dismantled (belonging to set \mathcal{G}). Constraint (9) makes the generation status equal to 1 once any generation facility belonging to set $\mathcal{G}^+ \cup \mathcal{G}_g^+$ is constructed, while constraint (10) ensures that no generation facility is constructed more than once. Restriction (11) establishes the binary nature of generation investment decisions. For generators to be dismantled during the study period ($\forall i \in \mathcal{G}^-$), constraint (12) makes the generation i status equal to 1 until the facility is dismantled, i.e. $t \leq t_i^{G^-}$, while constraint (13) makes the status equal to 0 once it is dismantled, i.e. $t > t_i^{G^-}$. Constraints (14) and (15) ensure that for each generation group to be constructed in consecutive phases,

the order of construction is sequential according to the generator set \mathcal{G}_g^+ order, thus phase $i + 1$ cannot be constructed before phase i . Note that it would be simple to consider the possibility of dismantling old lines in a similar fashion to the generation case by adapting constraints (12) and (13).

2.2. Middle-level problems

Given the values of the first-stage decision variables $x_k^{(t)}, y_i^{(t)}$, it is possible to establish the statuses of lines and generators $\tilde{x}_k^{(t)}, \tilde{y}_i^{(t)}$ required for the middle- and lower-level problems (Eqs. (4) and (9), respectively). Each middle-level problem identifies the worst-case uncertainty realizations yielding the largest operating cost $c_{op}^{(t)}$ in (1) for each period $t; \forall t \in \mathcal{T}$. The detailed formulation of the middle level problem for one period t is as follows:

$$c_{op}^{(t)} = \text{Maximum}_{u^{(t)} \in u^{(t)}} c_o^{(t)} \quad (16)$$

subject to

$$u_i^{G(t)} = \bar{u}_i^G - \hat{u}_i^G z_i^{G(t)}; \quad \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \quad (17)$$

$$u_j^{D(t)} = \bar{u}_j^D h_{\mu_j}^{(t)} + \hat{u}_j^D h_{\sigma_j}^{(t)} z_j^{D(t)}; \quad \forall j \in \mathcal{D} \quad (18)$$

$$\sum_{i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g} z_i^{G(t)} \leq \Gamma^G (\tilde{y}_i^{(t)}, \forall i) \quad (19)$$

$$\sum_{j \in \mathcal{D}} z_j^{D(t)} \leq \Gamma^D \quad (20)$$

$$z_i^{G(t)} \in \{0, 1\}; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \quad (21)$$

$$z_j^{D(t)} \in \{0,1\}; \forall j \in \mathcal{D} \quad (22)$$

$$z_i^{G(t)} \leq \tilde{y}_i^{(t)}; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g. \quad (23)$$

Eq. (16) represents the worst operational costs, where generation and load-shedding costs are at a maximum with respect to uncertain parameters. Constraints (17)–(22) define the polyhedral uncertainty set similarly as done in [39]. Random generation capacity $u_i^{G(t)}$ depends on binary variable $z_i^{G(t)}$, if the binary variable $z_i^{G(t)}$ is 1, maximum generation capacity is set to the nominal value $\bar{u}_i^{G(t)}$ minus the maximum deviation allowed from the nominal value $\hat{u}_i^{G(t)}$, otherwise maximum generation capacity is set to the nominal value $\bar{u}_i^{G(t)}$. The uncertain demand values $u_j^{D(t)}$, are analogous to the values shown above although in this particular case, demand nominal values and dispersion are allowed to evolve during the time horizon using parameters $h_{\mu_j}^{(t)}$ and $h_{\sigma_j}^{(t)}$. These parameters enable us to configure variation in demand values and their uncertainties over time (see reference [39] for more details). Note that constraints (17) and (18) take advantage of the fact prescribed by [19] that the worst case is when generation capacities are as low as possible and demand values are as high as possible. The level of uncertainty is controlled throughout the uncertainty budgets Γ^G and Γ^D , which sets the maximum number of generators whose maximum capacity might be different from their nominal values and the maximum load levels that might change in relation to nominal values, respectively. This is in contrast to reference [39] where the uncertainty budget for power generation was constant. In this case the uncertainty budget for each time period Γ^G is a function of the number of active generators for each period, i.e. $\tilde{y}_i^{(t)}, \forall i$. The reader is reminded that the uncertainty budget is the maximum number of generators whose maximum capacity is allowed to depart from their nominal values, if the number of generators increases, the uncertainty budget should also rise to keep a comparable level of protection against uncertainty. Note that what the appropriate selection of this function is $\Gamma^G(\tilde{y}_i^{(t)}, \forall i)$, lies beyond the scope of the paper. Finally, constraint (23) is also new in this paper and sets the binary variables related to generators to zero in case the generators are not active at time period t , thus they cannot account for the uncertainty budget in (19).

Wind farms might be modeled assuming that the corresponding uncertainty set, associated with generation capacity, must range between zero and the expected or average power production. This means that this type of generating units might be incapable of producing energy under the worst case setting, which represent the no wind case.

2.3. Lower-level problems

Finally, the middle-level variables required for the lower-level problems are maximum generation capacities $u_i^{G(t)}$ and demand values $u_j^{D(t)}$. In the lower-level problem, the operating cost $c_o^{(t)}$ for given values of upper- and middle-level variables is minimized with respect to operational decisions for each time period t . The detailed formulation for a given time period t is given as follows. Note that the dual variables associated with constraints are provided separated by a colon.

$$c_o^{(t)} = \text{Minimum}_{g_i^{(t)}, p_j^{(t)}, r_j^{(t)}, \theta_n^{(t)}, f_k^{(t)}} \left(\sigma \sum_{i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g} c_i^G g_i^{(t)} + \sigma \sum_{j \in \mathcal{D}} c_j^D r_j^{(t)} \right); \quad (24)$$

subject to

$$\sum_{i \in \Psi_n^G} g_i^{(t)} - \sum_{k|o(k)=n} f_k^{(t)} + \sum_{k|r(k)=n} f_k^{(t)} + \sum_{j \in \Psi_n^D} r_j^{(t)} = \sum_{j \in \Psi_n^D} p_j^{(t)}; \lambda_n^{(t)}; \forall n \in \mathcal{N}; \quad (25)$$

$$f_k^{(t)} = b_k \tilde{x}_k^{(t)} (\theta_{o(k)}^{(t)} - \theta_{r(k)}^{(t)}); \phi_k^{(t)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+; \quad (26)$$

$$\theta_n^{(t)} = 0; \chi_n^{(t)}; n: \text{slack} \quad (27)$$

$$f_k^{(t)} \leq f_k^{\max}; \hat{\phi}_k^{(t)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \quad (28)$$

$$f_k^{(t)} \geq -f_k^{\max}; \check{\phi}_k^{(t)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \quad (29)$$

$$\theta_n^{(t)} \leq \pi; \hat{\xi}_n^{(t)}; \forall n \in \mathcal{N} \setminus n: \text{slack} \quad (30)$$

$$\theta_n^{(t)} \geq -\pi; \check{\xi}_n^{(t)}; \forall n \in \mathcal{N} \setminus n: \text{slack} \quad (31)$$

$$g_i^{(t)} \geq 0; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \quad (32)$$

$$r_j^{(t)} \geq 0; \forall j \in \mathcal{D} \quad (33)$$

$$p_j^{(t)} = u_j^{D(t)}; \alpha_j^{D(t)}; \forall j \in \mathcal{D} \quad (34)$$

$$g_i^{(t)} \leq u_i^{G(t)} \tilde{y}_i^{(t)}; \varphi_i^{G(t)}; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \quad (35)$$

$$r_j^{(t)} \leq e_j^{(t)} u_j^{D(t)}; \varphi_j^{D(t)}; \forall j \in \mathcal{D}. \quad (36)$$

Eq. (24) represents the minimum(lower-level)-worst(middle-level) operational costs, where generation and load-shedding costs are at a maximum. The weighting factor σ is used to make investment decisions and worst-case operating costs comparable quantities. Constraints (25)–(33) represent operational constraints such as setting the power balance, line flows, reference bus, flow and voltage angle limits, etc. Check reference [39] for more details about these constraints. Restriction (34) makes the level of demand match the uncertain demand variable. Constraint (35) is novel and sets the power generation to be lower than the uncertain generation capacity variable multiplied by the binary variable $\tilde{y}_i^{(t)}$, which establishes if the corresponding generator is active for period t . If it is not, i.e. $\tilde{y}_i^{(t)} = 0$, the power generation is set to zero. Constraint (36) limits load-shedding to a percentage of the uncertain demand variable.

3. Solution approach

The aim of this section is to extend the decomposition method presented in [39] to solve the robust DTNRGEP problem described in Section 2. The solution procedure consists of a column-and-constraint generation algorithm [46] where the max-min middle- and lower-level problems are transformed into a single level problem, the so called subproblem, using duality. Thus, the initial three-level formulation (1)–(36) is transformed into a two-level problem: master and subproblem. The master problem and the subproblem are iteratively solved as described below and followed by an outline of the proposed iterative process.

3.1. Master problem

The master problem constitutes a relaxation of problem (1)–(36) where a set of operating constraints are iteratively added. The addition of such constraints is set up with information from the subproblem, and it enables a more robust expansion plan to be obtained at each iteration.

At iteration ν of the column-and-constraint generation algorithm, the master problem is formulated as the following mixed-integer linear program:

$$\begin{aligned} & \text{Minimize} \\ & \chi_k^{(l)} \tilde{\chi}_k^{(l)} \mathcal{Y}_k^{(l)} \mathcal{Y}_k^{(l)}, \sum_{i \in \mathcal{T}} \frac{1}{(1+I)^{l-1}} \left(\sum_{k \in \mathcal{L}^+} c_k^{\text{LI}} \chi_k^{(l)} \right. \\ & \left. g_{i,l}^{(l)} p_{j,l}^{(l)} r_{j,l}^{(l)} \theta_{n,l}^{(l)} f_{k,l}^{(l)} \right. \\ & \left. l = 1, \dots, \nu-1 \right. \\ & \left. + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{\text{GI}} \mathcal{Y}_i^{(l)} + \gamma^{(l)} \right); \end{aligned} \quad (37)$$

subject to

$$\text{Constraints (2)–(15)} \quad (38)$$

$$\gamma^{(l)} \geq \frac{\sigma}{(1+I)} \sum_{i \in \mathcal{G} \cup \mathcal{G}^+} c_i^{\text{G}} g_{i,l}^{(l)} + \frac{\sigma}{(1+I)} \sum_{j \in \mathcal{D}} c_j^{\text{S}} r_{j,l}^{(l)}; \quad (39)$$

$$\gamma^{(l)} \geq 0; \forall t \in \mathcal{T} \quad (40)$$

$$\begin{aligned} & \sum_{i \in \Psi_n^{\text{G}}} g_{i,l}^{(l)} - \sum_{k|o(k)=n} f_{k,l}^{(l)} + \sum_{k|r(k)=n} f_{k,l}^{(l)} + \sum_{j \in \Psi_n^{\text{D}}} r_{j,l}^{(l)} \\ & = \sum_{j \in \Psi_n^{\text{D}}} p_{j,l}^{(l)}; \lambda_{n,l}^{(l)}; \forall n \in \mathcal{N}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \end{aligned} \quad (41)$$

$$f_{k,l}^{(l)} = b_k \tilde{\chi}_k^{(l)} (\theta_{o(k),l}^{(l)} - \theta_{r(k),l}^{(l)}); \phi_{k,l}^{(l)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (42)$$

$$\theta_{n,l}^{(l)} = 0; \chi_{n,l}^{(l)}; n: \text{slack}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (43)$$

$$f_{k,l}^{(l)} \leq f_k^{\text{max}}; \hat{\phi}_{k,l}^{(l)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (44)$$

$$f_{k,l}^{(l)} \geq -f_k^{\text{max}}; \check{\phi}_{k,l}^{(l)}; \forall k \in \mathcal{L} \cup \mathcal{L}^+; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (45)$$

$$\theta_{n,l}^{(l)} \leq \pi; \hat{\xi}_{n,l}^{(l)}; \forall n \in \mathcal{N} \setminus n: \text{slack}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (46)$$

$$\theta_{n,l}^{(l)} \geq -\pi; \check{\xi}_{n,l}^{(l)}; \forall n \in \mathcal{N} \setminus n: \text{slack}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (47)$$

$$g_{i,l}^{(l)} \geq 0; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (48)$$

$$r_{j,l}^{(l)} \geq 0; \forall j \in \mathcal{D}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (49)$$

$$p_{j,l}^{(l)} = u_{j,l}^{\text{D}(l)}; \alpha_{j,l}^{\text{D}(l)}; \forall j \in \mathcal{D}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \quad (50)$$

$$\begin{aligned} & g_{i,l}^{(l)} \leq u_{i,l}^{\text{G}(l)} \mathcal{Y}_i^{(l)}; \varphi_{i,l}^{\text{G}(l)}; \\ & \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1 \end{aligned} \quad (51)$$

$$r_{j,l}^{(l)} \leq e_j^{\text{D}(l)} u_{j,l}^{\text{D}(l)}; \varphi_{j,l}^{\text{D}(l)}; \forall j \in \mathcal{D}; \forall t \in \mathcal{T}; l = 1, \dots, \nu-1; \quad (52)$$

where the additional decision variables $g_{i,l}^{(l)}$, $p_{j,l}^{(l)}$, $r_{j,l}^{(l)}$, $\theta_{n,l}^{(l)}$, and $f_{k,l}^{(l)}$, corresponding to $g_i^{(l)}$, $p_j^{(l)}$, $r_j^{(l)}$, $\theta_n^{(l)}$, and $f_k^{(l)}$, respectively, are associated with the demand values and generation capacities identified by the subproblem at iteration l through $u_{j,l}^{\text{D}(l)}$ and $u_{i,l}^{\text{G}(l)}$.

The objective function (37) is identical to (1) except for the last term associated with operational costs, where $\frac{c_{\text{op}}^{(l)}}{(1+I)}$ is replaced by $\gamma^{(l)}$ that relates to year on year operational costs, and it represents the pointwise maximum within all linear approximations of $\frac{c_{\text{op}}^{(l)}}{(1+I)}$. Expression (38) includes upper-level constraints. Constraints (39) are primal decomposition cuts, where the operating costs corresponding to the uncertainty realizations identified at iteration l represent lower bounds for each $\gamma^{(l)}$. The nonnegativity of each $\gamma^{(l)}$ is imposed in (40). Finally,

constraints (41)–(52) correspond to lower-level constraints (25)–(36).

3.2. Subproblems

At each ν iteration of the column-and-constraint generation algorithm, the subproblem, for each time period, determines the worst-case uncertainty realizations yielding the maximum operating cost for a given upper-level decision provided by the previous master problem. Mathematically, each subproblem is a mixed-integer linear max-min problem comprising the two lowermost optimization levels (16)–(36) parameterized in terms of the upper-level decision variables given $\tilde{\chi}_{k,\nu}^{(l)}$, $\tilde{\mathcal{Y}}_{i,\nu}^{(l)}$, which is cast as a single-level equivalent that relies on duality theory. Thus, operating costs $c_{\text{op}}^{(l)}$ for each time period $t; \forall t \in \mathcal{T}$ are obtained as follows:

$$\begin{aligned} c_{\text{op}}^{(l)} = & \text{Maximize} \\ & \mathbf{u}^{(l)} \lambda_n^{(l)} \phi_k^{(l)} \chi_n^{(l)} \hat{\phi}_k^{(l)}, \\ & \check{\phi}_k^{(l)} \check{\xi}_n^{(l)} \check{\xi}_n^{(l)} \alpha_j^{\text{D}(l)} \varphi_i^{\text{G}(l)} \varphi_j^{\text{D}(l)} \\ & \left\{ \begin{aligned} & \sum_{k \in \mathcal{L} \cup \mathcal{L}^+} \left(\hat{\phi}_k^{(l)} - \check{\phi}_k^{(l)} \right) f_k^{\text{max}} + \sum_{n \in \mathcal{N} \setminus n: \text{slack}} \pi \left(\hat{\xi}_n^{(l)} - \check{\xi}_n^{(l)} \right) \\ & + \sum_{i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g} \left(u_i^{\text{G}(l)} \tilde{\mathcal{Y}}_{i,\nu}^{(l)} \varphi_i^{\text{G}(l)} \right) \\ & + \sum_{j \in \mathcal{D}} \left(u_j^{\text{D}(l)} \alpha_j^{\text{D}(l)} + e_j^{(l)} u_j^{\text{D}(l)} \varphi_j^{\text{D}(l)} \right) \end{aligned} \right\} \end{aligned} \quad (53)$$

subject to:

$$\begin{aligned} & \lambda_n^{(l)} + \varphi_i^{\text{G}(l)} \leq \frac{\sigma}{(1+I)} c_i^{\text{G}}; \\ & \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \end{aligned} \quad (54)$$

$$-\lambda_{n(j)}^{(l)} + \alpha_j^{\text{D}(l)} \leq 0; \forall j \in \mathcal{D} \quad (55)$$

$$\lambda_{n(j)}^{(l)} + \varphi_j^{\text{D}(l)} \leq \frac{\sigma}{(1+I)} c_j^{\text{S}}; \forall j \in \mathcal{D} \quad (56)$$

$$-\lambda_{o(k)}^{(l)} + \lambda_{r(k)}^{(l)} + \phi_k^{(l)} + \hat{\phi}_k^{(l)} + \check{\phi}_k^{(l)} = 0; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \quad (57)$$

$$\begin{aligned} & - \sum_{k|o(k)=n} b_k \tilde{\chi}_{k,\nu}^{(l)} \phi_k^{(l)} + \sum_{k|r(k)=n} b_k \tilde{\chi}_{k,\nu}^{(l)} \phi_k^{(l)} \\ & + \hat{\xi}_n^{(l)} + \check{\xi}_n^{(l)} = 0; \forall n \in \mathcal{N} \setminus n: \text{slack} \end{aligned} \quad (58)$$

$$\begin{aligned} & - \sum_{k|o(k)=n} b_k \tilde{\chi}_{k,\nu}^{(l)} \phi_k^{(l)} + \sum_{k|r(k)=n} b_k \tilde{\chi}_{k,\nu}^{(l)} \phi_k^{(l)} \\ & + \chi_n^{(l)} = 0; n: \text{slack} \end{aligned} \quad (59)$$

$$-\infty \leq \lambda_n^{(l)} \leq \infty; \forall n \in \mathcal{N} \quad (60)$$

$$-\infty \leq \phi_k^{(l)} \leq \infty; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \quad (61)$$

$$-\infty \leq \chi_n^{(l)} \leq \infty; n: \text{slack} \quad (62)$$

$$\hat{\phi}_k^{(l)} \leq 0; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \quad (63)$$

Table A.1

Candidate renewable generators related to Garver's 6-bus test system.

Gen.	Bus	Power capacity \bar{u}_i^{G} (MW)	O & M costs (€/MW h)	Investment cost (M€)
4	1	50	17.8	50
5	1	70	17.5	80
6	1	40	17.5	40
7	2	150	16.5	200
8	4	200	15.0	198
9	5	100	17.0	110

Table A.2
Results for Garver's 6-bus test system illustrative example.

Case study	Input $\Gamma^G, \Gamma^D, \Pi_G(\text{M€})$	Inv. cost (M€)	New lines			New generators	
			From	To	Period	Gen.	Period
a)	1, 2, 350	384.802 (36.502 + 348.300)	1	5	5	4	1
			2	3	1	5	2
			2	6	1	6	3
			2	6	8	8	8
			2	6	1	9	3
			3	5	1	–	–
			4	6	1	–	–
			4	6	1	–	–
b)	1, 2, 450	384.802 (36.502 + 348.300)	1	5	5	4	1
			2	3	1	5	2
			2	6	1	6	3
			2	6	8	8	8
			2	6	1	9	3
			3	5	1	–	–
			4	6	1	–	–
			4	6	1	–	–
c)	2, 3, 350	385.190 (36.159 + 349.031)	1	5	1	4	1
			2	6	2	5	4
			2	6	1	6	5
			2	6	1	8	8
			3	5	1	9	1
			4	6	1	–	–
			4	6	1	–	–
			4	6	1	–	–
d)	2, 3, 450	475.124 (38.000 + 437.124)	1	5	1	4	1
			2	3	2	5	2
			2	6	6	6	7
			2	6	1	7	2
			2	6	1	9	1
			3	5	1	–	–
			4	6	1	–	–
			4	6	1	–	–

$$\hat{\phi}_k^{(t)} \geq 0; \forall k \in \mathcal{L} \cup \mathcal{L}^+ \tag{64}$$

$$\hat{\xi}_n^{(t)} \leq 0; \forall n \in \mathcal{N} \setminus \text{slack} \tag{65}$$

$$\check{\xi}_n^{(t)} \geq 0; \forall n \in \mathcal{N} \setminus \text{slack} \tag{66}$$

$$-\infty \leq \alpha_j^{D(t)} \leq \infty; \forall j \in \mathcal{D} \tag{67}$$

$$\varphi_i^{G(t)} \leq 0; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g \tag{68}$$

$$\varphi_j^{D(t)} \leq 0; \forall j \in \mathcal{D} \tag{69}$$

$$\text{Constraints (17)–(22)} \tag{70}$$

$$z_i^{G(t)} \leq \hat{y}_{i,v}^{(t)}; \forall i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g. \tag{71}$$

Subproblems (53)–(71) result from substituting the third-level problem by its dual in problem (16)–(23) for each time period t . An important aspect of resolution of subproblems is the linearization of bilinear terms in the objective function (53), i.e., $\sum_{i \in \mathcal{G} \cup \mathcal{G}^+ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g} (u_i^{G(t)} \hat{y}_{i,v}^{(t)} \varphi_i^{G(t)}) + \sum_{j \in \mathcal{D}} (u_j^{D(t)} \alpha_j^{D(t)} + e_j^{(t)} u_j^{D(t)} \varphi_j^{D(t)})$. The linearization process is described in more detail in [20]. Note that variable $\hat{y}_{i,v}^{(t)}$ is considered a parameter within our subproblem. Maximization of the operating cost is subject to middle-level constraints (17)–(22), as formulated in (70) and (71). These subproblems provide the next uncertain parameter values $\mathbf{u}^{(t)}$ within the uncertainty sets to give the least desirable operational costs for each year, which will be used next in the master problem.

The resulting formulation (53)–(71) is a mixed-integer linear programming problem, which can be solved by using state-of-the-art

mixed-integer mathematical programming solvers such as CPLEX or Gurobi, thereby ensuring a global optimum is achieved.

3.3. Solution method

Once the master problem and subproblem formulations are given, the solution method consists of solving the following problems by iteration:

- **Master problem:** For given realizations of the uncertain parameters obtained from subproblems at the previous iterations, new first-stage variable $\tilde{x}_{k,v}^{(t)}, \tilde{y}_{i,v}^{(t)}$ values are calculated by means of (37)–(52). The lower bound for the optimal objective function is updated

$$z^{(lo)} = \sum_{t \in \mathcal{T}} \frac{1}{(1 + I)^{t-1}} \left(\sum_{k \in \mathcal{L}^+} c_k^{LI} x_{k,v}^{(t)} + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{GI} y_{i,v}^{(t)} + \gamma^{(t)} \right).$$

- **Subproblems, one for each year:** For given values of the first-stage decision variables $\tilde{x}_{k,v}^{(t)}, \tilde{y}_{i,v}^{(t)}$, uncertain parameters within the uncertainty set which give the least desirable operational costs (16), i.e. $\mathbf{u}_v^{(t)}$ and $c_{op,v}^{(t)}$, respectively, are calculated by solving subproblems in (53)–(71). The upper bound for the optimal objective function is updated

$$z^{(up)} = \sum_{t \in \mathcal{T}} \frac{1}{(1 + I)^{t-1}} \left(\sum_{k \in \mathcal{L}^+} c_k^{LI} x_{k,v}^{(t)} + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{GI} y_{i,v}^{(t)} + c_{op,v}^{(t)} \right)$$

The iterative scheme put forward is described step by step in the following algorithm:

Algorithm 3.1. Robust dynamic transmission network and renewable

- Input:** Selection of uncertainty budgets Γ^G and Γ^D , time periods to divide the time horizon, interest rate I , definition of the uncertainty sets for each time period, definition of the prospective lines sets \mathcal{L}^+ , prospective and independent generators \mathcal{G}^+ , prospective and independent generators to be installed at different phases \mathcal{G}_g^+ , generators to be dismantled \mathcal{G}^- and process tolerance ε . These data are selected by the decision maker.
- Step 1: Initialization.** Initialize the iteration counter to $\nu = 1$, and upper and lower bounds for the objective function $z^{(\text{up})} = \infty$ and $z^{(\text{lo})} = -\infty$.
- Step 2: Solving the master problem at iteration ν .** Solve the master problem (37)–(52). The result provides values of the decision variables $x_{k,\nu}^{(t)}$, $y_{i,\nu}^{(t)}$ and $\gamma^{(t)}$. Update the lower bound for the optimal objective function $z^{(\text{lo})} = \sum_{t \in \mathcal{T}} \frac{1}{(1+I)^{t-1}} (\sum_{k \in \mathcal{L}^+} c_k^{\text{LL}} x_{k,\nu}^{(t)} + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{\text{GL}} y_{i,\nu}^{(t)} + \gamma^{(t)})$. Note that at the first iteration the optimal solution matches the noinvestment case. Alternatively, we could start with any other vector for decision variables.
- Step 3: Solving subproblems at iteration ν for each year t .** For given values of the decision variables $x_{k,\nu}^{(t)}$, $y_{i,\nu}^{(t)}$, we calculate the least desirable operational costs within the uncertainty set $c_{\text{op},\nu}^{(t)}$, whereby we also obtain the corresponding uncertain parameters $\mathbf{u}_\nu^{(t)}$. This is achieved by solving subproblems (53)–(71). Update the upper bound for the optimal objective function $z^{(\text{up})} = \sum_{t \in \mathcal{T}} \frac{1}{(1+I)^{t-1}} (\sum_{k \in \mathcal{L}^+} c_k^{\text{LL}} x_{k,\nu}^{(t)} + \sum_{i \in \mathcal{G}^+ \cup \mathcal{G}_g^+; \forall g} c_i^{\text{GL}} y_{i,\nu}^{(t)} + c_{\text{op},\nu}^{(t)})$.
- Step 4: Convergence checking.** If $(z^{(\text{up})} - z^{(\text{lo})}) / z^{(\text{up})} \leq \varepsilon$ go to Step 5, else update the iteration counter $\nu \rightarrow \nu + 1$ and continue from Step 2.
- Step 5: Output.** The solution for a given tolerance corresponds to $x_k^{(t)*} = x_{k,\nu}^{(t)}$ and $y_i^{(t)*} = y_{i,\nu}^{(t)}$.

generation expansion planning

The advantage of this bi-level formulation given by (37)–(52) and (53)–(71) is that it has the same problem structure as that defined by [46], and therefore the proposed column-and-constraint generation method guarantees convergence to a global optimum.

4. Examples

In this section, the numerical results for an illustrative example based on the Garver system [30] and a case study using the IEEE 118-bus test system [47] are shown to analyze the joint study of transmission network and generation expansion planning.

All numerical tests have been implemented and solved using CPLEX within GAMS [48] on a Windows DELL PowerEdge R920 server with two Intel Xeon E7–4820 processors clocking at 2 GHz and 768 Gb of RAM. The tolerance for stopping in all cases is equal to $\varepsilon = 10^{-6}$.

4.1. Illustrative Example. Garver's 6-bus System

The model is initially tested on the Garver's 6-bus system depicted in Fig. A.2. This system is composed of 6 buses, 3 generators, 5 levels of inelastic demand and 6 lines. Data for generation and demand capacities, and supply and bidding prices are given in [20]. The load-shedding cost is equal to one hundred times the bidding price for each level of demand. Line data are obtained from Table I of reference [32], including construction costs.

Regarding expansion possibilities, 6 generation units and 3 transmission lines between each pair of buses could be installed. The characteristics of potential generation units, which are assumed to be wind units, are shown in Table A.1. The first three generation units belong to one group ($g = 1$) to be constructed and installed in phases (at different times) at the same bus 1, so if the model decides to include them in generation expansion planning they must be installed sequentially, i.e. $\mathcal{G}_1^+ \equiv \{4,5,6\}$. In addition, it is known that the existing generator at bus 1 is going to be dismantled at time period 8 because it will have reached the end of its useful life, i.e. $\mathcal{G}^- \equiv \{1\}$. The remaining sets associated with generation are $\mathcal{G} \equiv \{2,3\}$ and $\mathcal{G}^+ \equiv \{7,8,9\}$. Operational and investment costs for generators is presented in Table A.1. As regards the characteristics of the new possible transmission lines, they are also attained from Table I of reference [32]. The maximum available investment budget for transmission lines is 40 million euros.

The time horizon considered is 25 years and the discount rate is 10%. The weighted factor σ is equal to the number of hours in one year,

i.e. 8760, so that the load-shedding and power generation costs are related to years, which can be compared with the annualized investment cost.

Regarding the uncertainty sets, the maximum capacity of conven-

tional generators can decrease by up to 50% with respect to their nominal values, i.e. $\hat{u}_i^G = 0.5\bar{u}_i^G$; $i = 1,2,3$, while for renewable generators their maximum capacity can decrease by 100% with respect to their nominal values, i.e. $\hat{u}_i^G = \bar{u}_i^G$; $i = 4,\dots,9$. Load levels may change by a maximum of 20% with respect to their nominal values, i.e. $\hat{u}_j^D = 0.2\bar{u}_j^D$; $j = 1,\dots,5$. Finally, annual growth rates for load nominal values and dispersion are equal to 1.2%, i.e. $h_{\mu,j}^{(t)} = h_{\sigma,j}^{(t)} = 1.012^{(t-1)}$. Four case studies are analyzed, considering two different combinations of uncertainty budgets associated with generation capacities and demand values and two different generation investment budgets. It is worth stressing that the inclusion of new generators implies updating the uncertainty budget associated with generation, we use the following step function to define $\Gamma^G(\hat{y}_i^{(t)}, \forall i)$: if 1 or 2 generators are built, the generation uncertainty budget increases by one unit; if 3 or 4 generators are built, the uncertainty budget increases by two units; and finally, if 4 or 5 generators are built, the uncertainty budget increases by three units. Results about investment cost, and lines and generators built for each case study using the proposed model are given in Table A.2. Investment costs in Table A.2 are split into line and generation investments, respectively. From this table the following observations are outstanding:

- **Case a), generation expansion:**

- Building generation unit 4 at period 1, the system almost has enough generation capacity to supply demand for the worst possible scenario. There is a slight amount of load shedding, but the construction of an additional generation unit is not profitable at this time.
- Since demand progressively increases from period to period, it is required to build generation unit 5 at period 2, and generation units 6 and 9 at period 3. Note that generation units at bus 1 are built sequentially in three different phases according to constraint (14).
- In period 8, unit 1 (150 MW) is dismantled, which force the construction of generation unit 8 (200 MW).
- Given the generation units built at periods 23, 24 and 25, the system almost has enough generation capacity to supply demand for the worst possible scenario. There is a slight amount of load shedding, but the construction of an additional generation unit is not possible because of the budget limitation.

- **Case a), line capacity expansion:**

- Most of the constructed lines connect the 600 MW generation unit (bus 6) with the rest of the system.

- The rest of lines reinforce the system to allow generation to reach demand buses.
- **Case b), generation expansion:** Even though the budget has increased with respect to case a) and we could build additional generation units, the solution is exactly the same in terms of construction of generation units. The reason is that the slight amount of load shedding, corresponding to the worst possible case at periods 23, 24 and 25, does not make the investment in an additional generation unit profitable.
- **Case c), generation expansion:** This case increases uncertainty budget with respect to case a), therefore, generation capacity requirements come at earlier periods. In particular, generation unit 9 (100 MW) is built at period 1 instead of period 3. In contrast, the construction of generation units 5 and 6 can be delayed from periods 2 and 3 to periods 4 and 5, respectively.
- **Case c), line capacity expansion:** The increment of generation capacity at earlier periods forces more lines to be built at period 1 with respect to case a).
- **Case d), generation expansion:** This case increases uncertainty budget with respect to case c). This budget increment allows to build generation unit 7 at period 2 instead of generation unit 8 at period 8.

Regarding computational tractability, the number of iterations required are 16, 15, 10 and 16, respectively, for study cases a), b), c) and d). The maximum computing time is eight hours for case a).

4.2. IEEE 118-bus test system example

We also apply the proposed model on a bigger and more realistic case using the IEEE 118-bus test system [47] which is composed of 118 buses, 186 existing lines, 54 generators and 91 loads. We assume that the generator located at bus 4 stops working at period 8 because it reaches the end of its useful life, i.e. $\mathcal{G}^- \equiv \{4\}$. Generation capacities and demand loads can be found in [20]. The load-shedding cost is ten times

Table A.3
Candidate generators related to IEEE 118-bus test system.

Gen.	Bus	Power capacity (MW)	O & M cost (€/MW h)	Investment cost (M€)
55	1	90	15.2	120
56	4	50	17.8	50
57	4	70	17.5	80
58	4	40	17.5	40
59	6	100	17.1	110
60	10	180	15.3	184
61	14	100	17.0	110
62	14	90	15.2	120
63	18	150	16.0	145
64	20	50	17.6	50
65	20	50	17.6	50
66	20	60	15.4	55
67	21	130	16.5	135
68	22	200	15.0	198
69	27	80	15.9	90
70	38	110	16.7	123
71	39	200	15.1	200
72	50	90	17.0	118
73	51	150	16.6	153
74	62	110	16.8	103
75	75	110	16.9	147
76	80	170	16.0	164
77	88	200	14.9	198
78	93	100	17.0	110
79	94	200	15.0	198
80	96	140	16.3	158
81	101	170	15.2	180
82	114	190	15.5	191
83	116	110	16.6	112
84	118	90	17.3	102

the bidding price of each level of demand. Additionally, the same 61 existing lines given in [20] can be duplicated to build additional lines. Data for all lines are taken from [47]. The characteristics of new possible generators are shown in Table A.3 and there are two groups of generators to be installed sequentially at buses 4 and 20, i.e. $\mathcal{G}_1^+ \equiv \{56,57,58\}$ and $\mathcal{G}_2^+ \equiv \{64,65,66\}$. The investment budgets for the generators and transmission lines are 1,500 and 100 million euros, respectively. The discount rate is 10% and the time horizon is 25 years.

Due to uncertainty, conventional generators can decrease by 50% with respect to their nominal values, i.e. $\hat{u}_i^G = 0.5\bar{u}_i^G; i = 1, \dots, 54$, while for renewable generators their maximum capacity can decrease by 100% with respect to their nominal values, i.e. $\hat{u}_i^G = \bar{u}_i^G; i = 55, \dots, 84$. Demand levels may change by up to of 50% with respect to their nominal values. Annual growth rates for load nominal values and dispersion are equal to 1.2%, i.e. $h_{\mu,j}^{(t)} = h_{\sigma,j}^{(t)} = 1.012^{(t-1)}$.

Using the following uncertainty budgets $\Gamma^G = 15$ and $\Gamma^D = 20$, the DTNRGEP approach provides a total investment cost of 1599.574 million euros, 1499.666 million euros for constructing the generators shown in Table A.4 and 99.908 million euros for constructing the lines 187, 189, 191, 192, 203, 204, 205, 206, 207, 211, 223, 226, 241. Lines 189 and 204 are constructed at periods 15 and 3, respectively, while the remaining lines are constructed at the beginning of the time period considered. In terms of operational costs, a total of 1.069 billion euros are needed, of which 360.961 million euros are required for load-shedding.

Note that as in the previous example, the sequential installation of the corresponding generators is respected. Thus, three generators are built at periods 1, 2 and 3, respectively, at bus 20. Regarding computational tractability, 5 iterations are required to reach convergence in a computational time of 4 h and 28 min.

5. Conclusions

In this paper the use of robust optimization for solving the dynamic transmission and renewable generation expansion planning problem has been extended. The model put forward herein provides the initial design and the expansion plan as regards forthcoming years in terms of where and when new lines and/or generators have to be constructed. It may be assumed that the probability distributions for the random variables (uncertainty sets) change between consecutive years. The model set out herein provides an integrated approach for reaching the global optimal solution, and overcomes the size limitations and computational intractability associated with this type of problem for realistic cases.

We have solved an example with a network of 118 buses, which is a fairly large network considering the benchmark examples traditionally used in the technical literature. Results show that it has potential for application to bigger networks, especially since it is an off-line problem. Nevertheless, our computational experiments also show that the number of binary variables defining possible network and generation unit expansions is more important than the network size. In addition, the following additional conclusions are in order:

1. Generation and transmission expansion decisions are highly conditioned by uncertainties.

Table A.4
New generators for IEEE 118-bus test system.

Gen.	Bus	Period	Gen.	Bus	Period
56	4	22	73	51	1
63	18	1	74	62	1
64	20	1	76	80	1
65	20	2	77	88	1
66	20	3	79	94	1
71	39	1	82	114	1

2. Even though the cost of building new transmission lines is much lower than the cost of building new generation units. Generation expansion plans are highly conditioned by transmission expansion decisions.
3. Uncertainty budgets affect expansions plans considerably.
4. For the same uncertainty budgets, the solutions associated with different investment budgets are not incremental, hence the necessity to consider the year-by-year dynamics of the problem.

In summary, the model put forward herein constitutes a valuable tool for the difficult task of transforming conventional power systems

into sustainable systems. Inclusion of losses is a subject for further research.

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Appendix A. Objective function derivation

This appendix justifies the selection of the objective function used in this study and proves that it is coherent with expressions given in the technical literature.

Traditionally, transmission and generation expansion planning research studies only consider one target year (static approach), and planning and investment costs are calculated annually as follows:

$$R \left(\sum_{k \in L^+} c_k^{LI} x_k^{(0)} + \sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^{GI} y_i^{(0)} \right) + \sigma \left(\sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^G g_i^{(0)} + \sum_{j \in \mathcal{D}} c_j^S r_j^{(0)} \right), \quad (A.1)$$

where R is the capital recovery factor, calculated as $R = \frac{I(1+I)^{n_y}}{(1+I)^{n_y}-1}$, and n_y is the number of years considered for investment. Note that superscript “0” in (A.1) refers to the target year. Cost (A.1) is the total annualized cost, which corresponds to the annualized value of the total net present cost. However, as we have considered the dynamic version, the objection function used in this study corresponds to the total net present cost, given as follows:

$$\sum_{t \in \mathcal{T}} \frac{1}{(1+I)^{t-1}} \left(\sum_{k \in L^+} c_k^{LI} x_k^{(t)} + \sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^{GI} y_i^{(t)} + \frac{\sigma \left(\sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^G g_i^{(t)} + \sum_{j \in \mathcal{D}} c_j^S r_j^{(t)} \right)}{(1+I)} \right). \quad (A.2)$$

Both expressions must be equivalents for the static case. Assuming that (i) investment costs are made at the beginning of the study period ($t = 1$) and (ii) operating costs are constant throughout the study period, expression (A.2) simplifies as follows:

$$\left(\sum_{k \in L^+} c_k^{LI} x_k^{(1)} + \sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^{GI} y_i^{(1)} \right) + \sum_{t \in \mathcal{T}} \frac{\sigma}{(1+I)^t} \left(\sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^G g_i^{(1)} + \sum_{j \in \mathcal{D}} c_j^S r_j^{(1)} \right). \quad (A.3)$$

If we divide Eq. (A.3) by constant factor $\sum_{t \in \mathcal{T}} \frac{1}{(1+I)^t}$, it becomes:

$$R \left(\sum_{k \in L^+} c_k^{LI} x_k^{(1)} + \sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^{GI} y_i^{(1)} \right) + \sigma \left(\sum_{\substack{i \in \mathcal{G} \cup \mathcal{G}^+ \\ \cup \mathcal{G}^- \cup \mathcal{G}_g^+; \forall g}} c_i^G g_i^{(1)} + \sum_{j \in \mathcal{D}} c_j^S r_j^{(1)} \right), \quad (A.4)$$

because $\left(\sum_{t \in \mathcal{T}} \frac{1}{(1+I)^t} \right)^{-1} = R$, which implies that both expressions (A.1) and (A.2) provide the same optimal expansion plans with the static case assumption. However, only expression (A.2) allows for changes during the study period, which is why it is used in this study.

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