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Simultaneous placement of DG and capacitor in distribution network

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ABSTRACT

In this study, the planning problem concerned with simultaneous placement of distribution generation (DG) and capacitor bank in the distribution network is investigated from a local distribution company's (DISCO's) viewpoint based on minimum total cost over the planning period and applying genetic algorithm (GA). The cost terms of the objective function include risk cost of load not supplied of the customers, cost of energy loss of the system, investment costs for purchasing the DGs and capacitor banks, and maintenance costs of the installed DGs and capacitor banks. Herein, the customers' load types including residential, commercial, and industrial are modeled in the planning problem and load model-based power flow is applied instead of the conventional power flow. In addition, the failure rate of the distribution feeder is modeled respect with magnitude of the current flowing through the feeder applying several mathematical functions such as linear, power, exponential, and logarithmic. Moreover, in order to achieve realistic results, economic factors such as inflation and interest rates and technical factors including yearly load growth, hourly variations of the load, and daily variations of the load are taken into consideration in the planning problem.

1. Introduction

Analysis of power systems' failure statistics indicates that distribution system has the lowest reliability, although forced outage in distribution system has a restricted effect on end user customers compared with outages in transmission and generation systems [1]. In addition, significant percentage of power and energy loss is related to distribution network due to radial structure of distribution network and high ratio of current to voltage in distribution network.

Distributed generation (DG) placement and shunt capacitor allocation as the effective measures are taken to decrease power and energy loss of distribution system and improve its reliability. During the last decades, optimal DG and capacitor placement problems based on different purposes and applying various optimization techniques have been topic of several research works [2–20]. Summary of the literature review regarding the DG placement, capacitor placement, and simultaneous placement of DG and capacitor problems has been presented in Table 1. The investigated studies have been categorized and characterized based on the optimization techniques applied to solve the problem, type of the objective function such as single or multi-objective function, considering or ignoring hourly and daily variations of the load demand, considering or disregarding model of the customers' load types including residential, commercial, and industrial, and modeling or neglecting the feeder's failure rate alteration due to DG and capacitor placement in distribution network. As can be seen in Table 1, in all the studied DG placement problems in the literature [2–8], feeder's failure rate has not been modeled, and also time varying load modeling and customers' load types modeling have not been done simultaneously. Regarding the capacitor placement problems [9–13], just the study presented in [13] has considered all the modelings in the problem. However, the above mentioned modelings have not been considered in the papers with the aim of simultaneous placement of DG and capacitor [14–18,20] and in [19] just variability of the load demand has been taken into consideration.

In this study, simultaneous DG and capacitor placement planning problem in electrical distribution network with the aim of minimum total cost of the system over the planning period is investigated from the local distribution company's (DISCO's) viewpoint applying genetic algorithm (GA) as the optimization technique. The cost terms of the objective function include energy loss cost, risk cost of energy not supplied of the system, investment costs for purchasing DGs and capacitor banks, and the installed DGs and capacitor banks maintenance costs. Herein, the system security constraints including apparent power limit of the lines and voltage profile limits of the system buses are taken into consideration in the problem over the given planning period. In the planning

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Nomenclature

Indices and sets:

$b \in S_b$ index and set of branches
$d \in S_d$ index and set of days
$i,j \in S_i$ indices and set of buses
$x \in S_x$ index and set of customers' load types
$t \in S_t$ index and set of hours
$l \in S_l$ index and set of load levels
pp index of planning period
$y \in S_y$ index and set of years

System parameters and variables:

α,β exponents of different load models
$\mathrm{Cost}^{\mathrm{Cap}}$ value of cost for purchasing a capacitor bank
$\mathrm{Cost}^{\mathrm{DG}}$ value of cost for purchasing a DG
$\mathrm{Cost}^{M\text{-}\mathrm{Cap}}$ value of yearly maintenance cost of a capacitor bank
$\mathrm{Cost}^{M\text{-}\mathrm{DG}}$ value of yearly maintenance cost of a DG
$\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Invest\text{-}Cap}}$ investment for purchasing capacitor banks over the planning period
$\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Invest\text{-}DG}}$ investment for purchasing DGs over the planning period
$PWV(Cost_{pp}^{Loss})$ present worth value of the system energy loss cost over the planning period
$\mathrm{PWV}(\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Risk}})$ present worth value of the system risk cost over the planning period
$\mathrm{PWV}(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-}\mathrm{Cap}})$ present worth value of maintenance cost of the installed capacitor banks over the planning period
$\mathrm{PWV}(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-}\mathrm{MG}})$ present worth value of maintenance cost of the installed DGs over the planning period
FFRM Feeder's failure rate model
IFR,ITR inflation rate and interest rate
$|I_{y,d,t,b}|$ magnitude of current in yth year, on dth day, and at tth hour flowing through bth branch
$|I_{0,b}|$ magnitude of current flowing through branch b before DG and capacitor placement
$\mathrm{Loss}_{\mathrm{pp}}$ value of system active power loss over the planning period
τ_l duration of the lth load level
$\mathrm{LNS}_{y,l,i,x}^{\mathrm{FLS}}$ load not supplied of customers with load type x, in yth year, in lth load level, and at ith bus during fault locating and switching
$\mathrm{LNS}_{y,l,i,x}^{\mathrm{FR}}$ load not supplied of customers with load type x, in yth year, in lth load level, and at ith bus during fault repairing
T_{FLS} duration of fault locating and switching
T_{FR} duration of fault repairing
$N_{l,i}^{\mathrm{cap}}$ number of installed capacitor banks at bus i in lth load level
$N_{l,i}^{\mathrm{DG}}$ number of installed DGs at bus i in lth load level
$\mathrm{OF}_{\mathrm{PP}}$ objective function of the problem over the planning period
π_l^E price of electricity in lth load level
$\pi_{l,x}^{\mathrm{ENS}}$ cost of energy not supplied of customers with load type x in the lth load level
P_{0i},P_i value of nominal active power demand at operating point and current active power demand at bus i
Q_{0i},Q_i value of nominal reactive power demand at operating point and current reactive power demand at bus i
R_b resistance of branch b
$\mathrm{Risk}_{\mathrm{pp}}$ value of system risk over the planning period
$|\mathrm{MVA}_b|$ magnitude of apparent power flowing through branch b
V_{0i},V_i value of nominal voltage at operating point and current voltage at bus i
$|V_i|$ magnitude of voltage at bus i
γ_b length of branch b
Y_{ij} admittance of line between buses i and j
φ_{ij} value of phase angle of Y_{ij} in polar coordinate system
$\lambda_{0,b}$ failure rate of branch b before DG and capacitor placement
$\lambda_{f,b}$ failure rate of branch b after fully removing apparent power from the branch
$\lambda_b^{\mathrm{Lin}}, \lambda_b^{\mathrm{Pow}}, \lambda_b^{\mathrm{Exp}}, \lambda_b^{\mathrm{Log}}$ failure rate of branch b related to linear, power, exponential, and logarithmic models
$\lambda_b^{\mathrm{FFRM}}$ failure rate of branch b considering FFRM
δ_i value of phase angle of voltage at bus i in polar coordinate system

GA parameters:

P^{Mutation} mutation probability of genes
N_{ch} size of population
a_{ch} binary variable as the indicator for selection of the chromosome for the new population
r_{ch} random number in range of (0,1)
$P_{\mathrm{ch}}^{\mathrm{Selection}}$ value of selection probability of the chromosome
f_{ch} value of fitness of the chromosome
S_f set of chromosomes fitness

problem, inflation and interest rates as the economic factors and yearly load growth and hourly and daily varying load as the technical aspects are considered. In addition, the customers' load types such as residential, commercial, and industrial are modeled and load model-based power flow is applied instead of the traditional power flow. Furthermore, feeder's failure rate respect with magnitude of the current flowing through the feeder is modeled applying linear and several nonlinear mathematical functions such as exponential, power and logarithmic functions.

In this study, in order to closely study the technical advantages of DG and capacitor placement in distribution network, operation cost of the installed DGs, the wholesale market price, and the retail market price are supposed to be equal. Therefore, the economic benefits achieved due to existence of price difference between the wholesale and retail markets, and also because of utilization of the DGs instead of purchasing electricity from the wholesale market are ignored.

The remainder of the paper is organized as follows. In Sections 2–4, feeder's failure rate, customer's load type, and time varying load are modeled, respectively. The simultaneous capacitor and DG placement planning problem is formulated in Section 5. The proposed optimization technique for solving the planning problem is presented in Section 6. Section 7 is concerned with the numerical studies and results analysis, and finally Section 8 concludes the paper.

2. Modeling feeder's failure rate

DG and capacitor placement in distribution network locally supplies part of active and reactive power demands of the customers. Therefore, magnitude of the apparent power flowing through the feeder is reduced and as a result, magnitude of the current is decreased. Based on this, the active and reactive power losses are decreased and consequently temperature of the feeder is reduced. Since high temperature has destructive effect on the feeder such as

Table 1
Summary of the literature review regarding DG placement problem, capacitor placement problem, and simultaneous placement of DG and capacitor problem.

Ref. no.	Capacitor, DG, or simultaneous placement	Optimization technique	Type of objective function	Time varying load modeling	Customers' load types modeling	Feeder's failure rate modeling	Published year
[2]	DG	Analytical	Single	N	N	N	2004
[3]	DG	DP[a]	Multi	Y	N	N	2011
[4]	DG	Analytical	Single	N	N	N	2006
[5]	DG	GA[b]/PSO[c]	Multi	N	N	N	2012
[6]	DG	Analytical	Multi	N	Y	N	2007
[7]	DG	HS[d]	Multi	N	N	N	2007
[8]	DG	GA/HS	Single	N	N	N	2011
[9]	Capacitor	GA	Single	N	N	N	2004
[10]	Capacitor	HCA[e]	Single	N	N	N	2008
[11]	Capacitor	PSO	Single	N	N	N	2010
[12]	Capacitor	SA[f]	Multi	N	Y	N	2012
[13]	Capacitor	SA	Multi	Y	Y	Y	2015
[14]	Simultaneously	ABC[g]	Single	N	N	N	2011
[15]	Simultaneously	Analytical	Single	N	N	N	2011
[16]	Simultaneously	ICA[h]/GA	Multi	N	N	N	2014
[17]	Simultaneously	PSO	Multi	N	N	N	2015
[18]	Simultaneously	PSO	Multi	N	N	N	2014
[19]	Simultaneously	GA/PSO/CSO[i]	Multi	Y	N	N	2015
[20]	Simultaneously	Analytical	Multi	N	N	N	2013

[a] Dynamic programming.
[b] Genetic algorithm.
[c] Particle swarm optimization.
[d] Heuristic search.
[e] Heuristic constructive algorithm.
[f] Simulated annealing.
[g] Artificial bee colony.
[h] Imperialist competitive algorithm.
[i] Cat swarm optimization.

causing overhead lines to sag and insulation problems in underground cables [21,22], after DG and capacitor placement, the feeder's failure rate is decreased and reliability level of the distribution network is increased. In other words, the positive impact of DG and capacitor placement on the system reliability can be demonstrated and investigated due to reduction of the feeders' failure rate.

Herein, it is assumed that branch b with $|I_{0,b}|$ as the magnitude of the initial current has initial failure rate about $\lambda_{0,b}$ and after fully compensating the active and reactive power demands of the two load points related to the branch, the flowing current through the branch is completely removed $(|I_b| = 0)$ and the failure rate of the branch is changed into $\lambda_{f,b}$. In other words, (1) holds.

$$\begin{cases} |I_b| = |I_b| \Rightarrow \lambda_b = \lambda_{0,b} \\ |I_b| = 0 \Rightarrow \lambda_b = \lambda_{f,b} \end{cases}, \quad \forall b \in S_b, S_b = \{1, \ldots, N_b\} \tag{1}$$

In this study, four mathematical functions including linear, power, exponential, and logarithmic functions are applied to formulate the feeder's failure rate models (FFRMs). These FFRMs with $|I_b|$ as the variable and A and B as the unknown parameters are presented in (2)–(5) in Table 2. In every model, the parameters A and B are determined by examining (1) in the model and solving it. The linear and nonlinear FFRMs with $|I_b|$ as the variable and $|I_{0,b}|$, $\lambda_{0,b}$, and $\lambda_{f,b}$ as the constant parameters are presented in (6)–(9) in Table 2.

In this study, value of $\lambda_{f,b}$ is considered about zero. In other words, if the branch does not convey any power, its failure rate will be zero. The curves of the branch's failure rate respect with magnitude of the current flowing through the branch considering different FFRMs are illustrated in Fig. 1. As can be seen, in all the models, if magnitude of the current flowing through the branch is zero, value of the branch's failure rate is zero, and also value of the branch's failure rate is equal to its initial value if magnitude of the current flowing through the branch is not changed. However, value of the branch's failure rate is different in every model if magnitude of the current flowing through the branch is changed. Based on this, the most and the least changes in value of the branch's failure rate are occurred in the exponential model and logarithmic model, respectively. Moreover, the constant model as the conventional model has no sensitivity respect to changes in value of the current flowing through the branch.

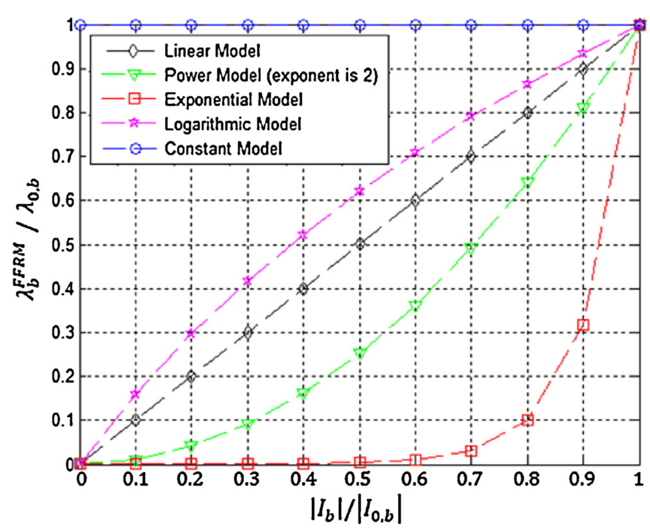

Fig. 1. Curves of feeder's failure rate (p.u.) respect with magnitude of the current (p.u.) flowing through the feeder considering different FFRMs.

3. Modeling customer's load type in load model based power flow

In a conventional power flow problem, at every PQ-bus, value of active and reactive power demands are definite and constant, value of voltage magnitude and phase angle are unknown, and (10) and

Table 2
Different FFRMs as the function of variable $|I_b|$ and with constant parameters $\left|I_{0,b}\right|$, $\lambda_{0,b}$, and $\lambda_{f,b}$.

FFRM	FFRM with unknown parameters A and B	FFRM with constant parameters $\left|I_{0,b}\right|$, $\lambda_{0,b}$, and $\lambda_{f,b}$
Linear model	$\lambda_b^{\mathrm{Lin}} = A \times \lvert I_b \rvert + B$ (2)	$\lambda_b^{\mathrm{Lin}} = \dfrac{\lambda_{0,b} - \lambda_{f,b}}{\lvert I_{0,b} \rvert} \times \lvert I_b \rvert + \lambda_{f,b}, \forall b \in S_b$ (6)
Power model	$\lambda_b^{\mathrm{Pow}} = (A \times \lvert I_b \rvert + B)^n$ (3)	$\lambda_b^{\mathrm{Pow}} = \left(\dfrac{(\lambda_{0,b})^{1/n} - (\lambda_{f,b})^{1/n}}{\lvert I_{0,b} \rvert} \times \lvert I_b \rvert + (\lambda_{f,b})^{1/n} \right)^n, \forall b \in S_b$ (7)
Exponential model	$\lambda_b^{\mathrm{Exp}} = e^{A \times \lvert I_b \rvert + B}$ (4)	$\lambda_b^{\mathrm{Exp}} = e^{\left(\left(1/\lvert I_{0,b} \rvert\right) \times \ln(\lambda_{0,b}/\lambda_{f,b}) \right) \times \lvert I_b \rvert + \ln(\lambda_{f,b})}, \forall b \in S_b$ (8)
Logarithmic model	$\lambda_b^{\mathrm{Log}} = \ln(A \times \lvert I_b \rvert + B)$ (5)	$\lambda_b^{\mathrm{Log}} = \ln \left(\dfrac{e^{\lambda_{0,b}} - e^{\lambda_{fb}}}{\lvert I_{0,b} \rvert} \times \lvert I_b \rvert + e^{\lambda_{f,b}} \right), \forall b \in S_b$ (9)

Table 3
Exponent of active and reactive power demands of different customers' load types [23].

Customers' load type	α	β
Industrial (Ind.)	0.18	6.00
Residential (Res.)	0.92	4.04
Commercial (Com.)	1.51	3.40

(11) hold.

$$P_{0i} = \sum_{j \in S_i} \left|V_i\right| \left|V_j\right| \left|Y_{ij}\right| \cos\left(\varphi_{ij} - \delta_i + \delta_j\right), \forall i \in S_i,$$
$$S_i = \left\{1, \ldots, N_i\right\} \quad (11)$$

$$Q_{0i} = \sum_{j \in S_i} \left|V_i\right| \left|V_j\right| \left|Y_{ij}\right| \sin\left(\varphi_{ij} - \delta_i + \delta_j\right), \forall i \in S_i \quad (12)$$

However, in a real electric power system, values of variable parameters of the system components are not equal to their nominal amounts and the system does not work at nominal operating point. For instance, in a real electrical distribution network, voltage profile at each bus is not equal to its nominal value due to power loss in feeders. Values of active and reactive power demands of load points have a dependency on their operating voltage profile [23]. Therefore, values of active and reactive power demands of different load types would not be constant and definite and they could be changed. Consequently, in the load model based power flow, in addition to values of voltage magnitude and phase angle, values of active and reactive power demands are variable and unknown at every load point. Behavior of different customers' load types can be mathematically modeled by symbolizing the alterations in their active and reactive power demands due to changes in their voltage profile. The voltage dependent load models are expressed mathematically in the following equations:

$$P_i = P_{0i} \left| \frac{V_i}{V_{0i}} \right|^{\alpha}, \forall i \in S_i \quad (12)$$

$$Q_i = Q_{0i} \left| \frac{V_i}{V_{0i}} \right|^{\beta}, \forall i \in S_i \quad (13)$$

where P_{0i}, Q_{0i}, V_{0i} are the nominal active power demand, nominal reactive power demand, and nominal voltage at bus i, respectively. Values of α and β as the exponents of active and reactive power demands for industrial, residential, and commercial load types are given in Table 3 [23].

4. Modeling time varying load

In this study, in order to have practical analysis, variations of the load is taken into consideration in the planning problem and the energy loss and risk calculations are done for every hour of a day. However, DG and capacitor placement problem is studied in three conditions including light, medium, and peak load levels and at every load level, the average load is applied.

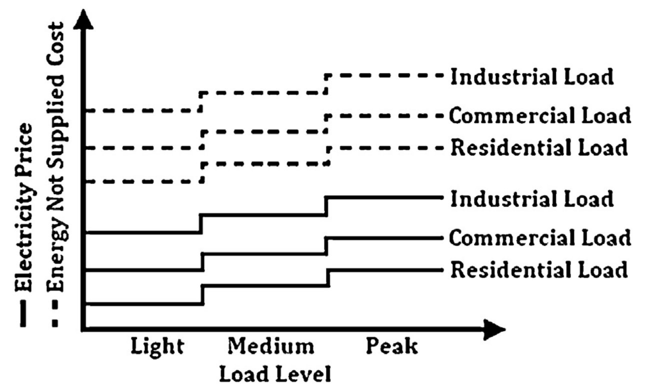

Fig. 2. Electricity price and energy not supplied cost of various types of customers at different load levels.

Also, as can be seen in Fig. 2, electricity price and energy not supplied cost of various types of the customers are modeled at light, medium, and peak load levels. Moreover, daily variations of the system load level throughout a year are taken into account. In addition, yearly load growth is considered in the problem throughout the given planning horizon.

5. Problem formulation

The objective function of the optimization problem is minimizing total cost of the system over the defined planning period considering several economic and technical factors such as inflation and interest rates, yearly load growth, hourly and daily varying load, customers' load types (residential, commercial, and industrial), different electricity prices for various types of customers at three load levels, and different energy not supplied costs for various types of customers at three load levels. Also, load model based power flow is applied instead of the conventional power flow. In addition, the feeder's failure rate and the system reliability are modelled based on different FFRMs including constant, linear, exponential, power, and logarithmic models.

As can be seen in (14), the cost terms of the objective function include the total investment for purchasing DGs, the total investment for purchasing capacitor banks, the present worth value of maintenance cost of the installed DGs, the present worth value of maintenance cost of the installed capacitor banks, the present worth value of energy loss cost, and the present worth value of risk cost over the planning horizon.

$$\mathrm{OF}_{\mathrm{pp}} = \min\left\{\mathrm{Cost}_{\mathrm{pp}}^{\text{Invest-DG}} + \mathrm{Cost}_{\mathrm{pp}}^{\text{Invest-Cap}} + \mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-DG}}\right) + \mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-Cap}}\right) + \mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Loss}}\right) + \mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Risk}}\right)\right\} \quad (14)$$

The total investment costs for purchasing DGs and capacitor banks are given in (15) and (16), respectively. As can be seen, the installed DGs and capacitor banks can be switched on or off at different load levels.

$$\mathrm{Cost}_{\mathrm{pp}}^{\text{Invest-DG}} = \mathrm{Cost}^{\mathrm{DG}}\sum_{l\in S_l}\sum_{l\in S_l} N_{l,i}^{\mathrm{DG}}, \quad S_l = \left\{1,\ldots,N_l\right\} \quad (15)$$

$$\mathrm{Cost}_{\mathrm{pp}}^{\text{Invest-Cap}} = \mathrm{Cost}^{\mathrm{Cap}}\sum_{l\in S_l}\sum_{l\in S_l} N_{l,i}^{\mathrm{Cap}} \quad (16)$$

The present worth values of maintenance costs of the installed DGs and capacitor banks over the given planning period are presented in (17) and (18), respectively [13]. As can be seen, these cost terms are related to the preserving and repairing the installed DGs and capacitor banks which are proportional to the working hours of the installed DGs and capacitor banks at different load levels.

$$\mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-DG}}\right) = \sum_{y\in S_y}\left(\mathrm{Cost}^{M\text{-DG}}\sum_{l\in S_y}\frac{\tau_l}{24}\sum_{i\in S_i} N_{l,i}^{\mathrm{DG}}\right) \times \left(\frac{1+\mathrm{IFR}}{1+\mathrm{ITR}}\right)^y \quad (17)$$

$$\mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{M\text{-Cap}}\right) = \sum_{y\in S_y}\left(\mathrm{Cost}^{M\text{-Cap}}\sum_{l\in S_y}\frac{\tau_l}{24}\sum_{i\in S_i} N_{l,i}^{\mathrm{Cap}}\right) \times \left(\frac{1+\mathrm{IFR}}{1+\mathrm{ITR}}\right)^y \quad (18)$$

In (19), value of active power loss of the system over the planning horizon is presented. Moreover, the present worth value of the system energy loss cost for the given planning period is presented in (20).

$$\mathrm{Loss}_{\mathrm{pp}} = \sum_{y\in S_y}\sum_{d\in S_d}\sum_{t\in S_t}\sum_{b\in S_b} R_b \times \left|I_{y,d,t,b}\right|^2, \quad S_y = \left\{1,\ldots,N_y\right\}, S_d = \left\{1,\ldots,N_d\right\}, S_t = \left\{1,\ldots,N_t\right\} \quad (19)$$

$$\mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Loss}}\right) = \sum_{y\in S_y}\left(\sum_{d\in S_d}\sum_{l\in S_l}\pi_l^E \times \tau_l \sum_{b\in S_b} R_b \times |I_{y,t,d,b}|^2\right) \times \left(\frac{1+\mathrm{IFR}}{1+\mathrm{ITR}}\right)^y \quad (20)$$

Value of system risk is presented in (21), which is sum of the energy not supplied of the customers during fault locating, switching, and fault repairing over the planning period [3,13]. Also, the present worth value of the system risk cost for the given planning period is presented in (22) [3,13].

$$\mathrm{Risk}_{\mathrm{pp}} = \sum_{y\in S_y}\sum_{b\in S_b}\lambda_b^{\mathrm{FFRM}} \times \gamma_b \times \left(T_{\mathrm{FLS}}\sum_{i\in S_i}\mathrm{LNS}_{y,i}^{\mathrm{FLS}} + T_{\mathrm{FR}}\sum_{i\in S_i}\mathrm{LNS}_{y,i}^{\mathrm{FR}}\right) \quad (21)$$

Furthermore, the security constraints of the system are presented in (23) and (24); where $|\mathrm{MVA}_b|$ is magnitude of the transmitted apparent power through branch b and $|V_i|$ is the magnitude of voltage at bus i.

$$\left|\mathrm{MVA}_b\right| \le \max\left|\mathrm{MVA}_b\right|, \forall b \in S_b \quad (23)$$

$$\min|V_i| \le |V_i| \le \max|V_i|, \forall i \in S_i \quad (24)$$

6. Proposed optimization technique

In this study, the problem is solved applying GA as the optimization tool. Other optimization algorithms could be used in this problem, however capability of GA for parallel optimization and its competence in complex and nonlinear environments are the main reasons for utilization of GA in this problem. As can be seen in Fig. 3, due to simultaneous installation of DGs and capacitor banks in distribution network and because of three daily load levels, every chromosome in the population is defined as a matrix with $Ni \times 6$ as its dimensions. In other words, every bus of the distribution network under study is considered as the candidate for DG and capacitor installation. Herein, inverse of value of total cost of the problem over the planning period is defined as the value of fitness of the related chromosome. In the following, different steps for applying GA in the problem are presented and described.

6.1. Step 1: Obtaining primary data

Parameters for applying GA: These parameters include mutation probability of genes (P^{Mutation}) and size of the population (N_{ch}).

Parameters of the system under study: Values of all the parameters and technical data of the system including network configuration, impedance of each line, load demand at every bus, system protection components and their locations, the data concerned with the electricity price at different load levels, and energy not supplied costs of various customers at different load levels are obtained.

Initial population: The chromosomes of the population are initialized with random binary values.

6.2. Step 2: Updating the population

Applying crossover operator: Six crossover points are randomly selected for every pair chromosomes, and then crossover operator is applied on every two chromosomes of the population to reproduce two new chromosomes, as can be seen in Fig. 4.

Applying mutation operator: This operator is applied on every gene of every chromosome of the population with the definite probability P^{Mutation}.

6.3. Step 3: Selecting new population

Evaluating fitness of every chromosome: For every chromosome, the problem is run and if all the constraints are satisfied, fitness of the chromosome is measured.

Applying selection process: As can be seen in (25), new chromosomes are selected through the probabilistic fitness-based selection process, where fitter chromosomes are more likely to be chosen. Value of selection probability of every chromosome is calculated using (26) which is proportional to fitness of the chromosome.

$$\mathrm{PWV}\left(\mathrm{Cost}_{\mathrm{pp}}^{\mathrm{Risk}}\right) = \sum_{y\in S_y}\left(\sum_{l\in S_l}\frac{\tau}{24}\sum_{b\in S_b}\lambda_b^{\mathrm{FFRM}} \times \gamma_b \times \left[T_{\mathrm{FLS}}\sum_{i\in S_i}\sum_{x\in S_x}\mathrm{LNS}_{y,l,i,x}^{\mathrm{FLS}} \times \pi_{l,x}^{\mathrm{ENS}} + T_{\mathrm{FR}}\sum_{i\in S_i}\sum_{x\in S_x}\mathrm{LNS}_{y,l,i,x}^{\mathrm{FLS}} \times \pi_{l,x}^{\mathrm{ENS}}\right]\right) \times \left(\frac{1+\mathrm{IFR}}{1+\mathrm{ITR}}\right)^y \quad S_x = \left\{1,\ldots,N_x\right\} \quad (22)$$

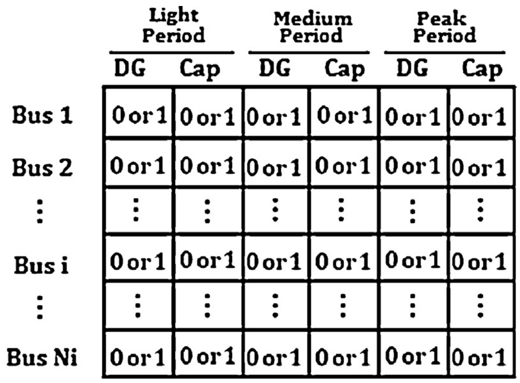

Fig. 3. Structure of the defined chromosome.

$$a_{\rm ch} = \begin{Bmatrix} 1 & P_{\rm ch}^{\rm Selection} > r_{\rm ch} \\ 0 & P_{\rm ch}^{\rm Selection} < r_{\rm ch} \end{Bmatrix} \quad (25)$$

$$P_{\rm ch}^{\rm Selection} = \frac{f_{\rm ch}}{\max(S_f)}, \quad S_f = \{f_1, \ldots, f_{\rm ch}, \ldots, f_{N_{\rm ch}}\} \quad (26)$$

6.4. Step 4: Checking termination criterion

Herein, convergence status of the optimization procedure is checked. Based on this, values of improvements in fitness of the chromosomes of the old and new populations are measured and if there are no significant improvements in them, the optimization process is finished, otherwise, the algorithm is continued form Step 2.

6.5. Step 5: Introducing the outcomes

The consequences include optimal locations of DGs and capacitor banks, minimum cost of the problem over the planning period, values of the system energy loss and system risk over the planning period, and values of the system reliability indices.

7. Numerical studies

As is illustrated in Fig. 5, the electrical distribution network under study is a 28-bus system with 28 load points and 27 branches and with different customers including industrial, commercial, and residential [13]. The system switching components installed on some branches include six switches which are normally close. Moreover, there is one normally open switching point at bus 28 for load transmission from feeder F1 onto feeder F2 while a permanent fault is occurred on feeder F1. Herein, the probability that the feeder F2 can convey the transmitted load of the feeder F1 is supposed to be about 60%. The installed circuit breaker (CB) in the beginning of the feeder F1 is automatically opened if a permanent fault occurs and it cannot be closed until the fault is removed or isolated. The CB is closed after locating the fault place and performing one of the actions below.

- Opening the related switching points for separating the defective branch.
- Repairing the defective branch if the switching action is not efficient for isolating the defective branch.

As can be seen in Table 4, the time required for locating a fault place is considered about 1 h and the time of switching action is considered zero as they can be switched online because of utilizing smart grid infrastructure. Also, repairing time and failure rate of every branch are assumed to be about 3 h and 0.3 f/km yr,

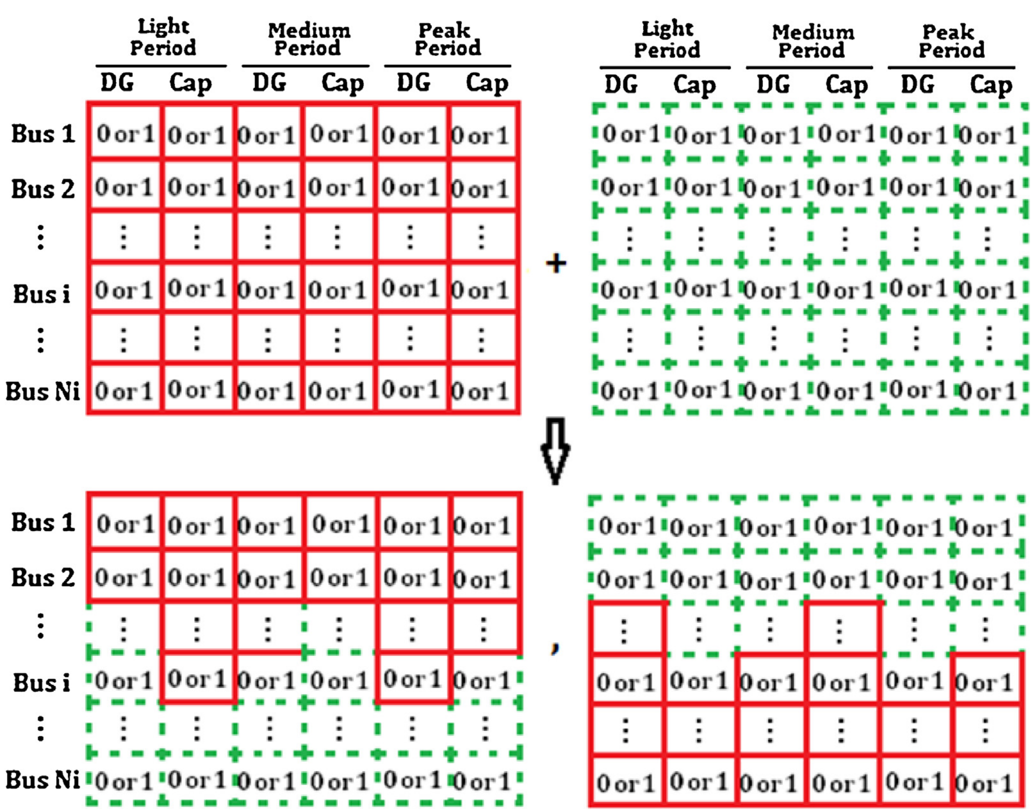

Fig. 4. Applying crossover operator on two chromosomes for reproducing new chromosomes.

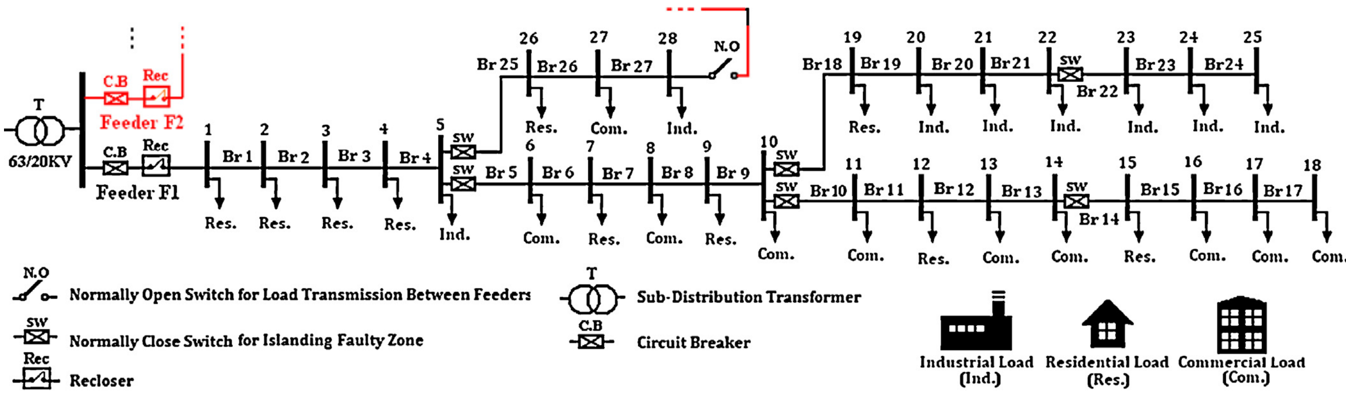

Fig. 5. The electrical distribution network under study that includes residential, industrial, and commercial customers [13].

Table 4
Data required for assessment of the system reliability [13].

Locating time of a fault place (h)	Switching time (h)	Repairing time of a defective branch (h)	Failure rate of every branch (f/km yr)	Length of every branch (km)
1	0	3	0.3	1

respectively. Herein, the length of every branch is about 1 km. The other network data including values of active and reactive demands at each bus, lines' impedances, lines' capacities, customers' load types, and number of the customers connected to each bus are presented in Table 5 [13]. The active and reactive demands are related to 21st hour of March 21st. Herein, values of minimum and maximum voltage constraints at each bus are considered about 0.95 p.u. and 1.05 p.u., respectively.

In addition, price of electricity for different types of customers and costs of energy not supplied of various customers at different load levels are presented in Table 6. Herein, the electricity prices at medium load level are related to residential, commercial, and industrial customers in Massachusetts extracted from [24]. In order to determine the electricity prices at light and peak load levels, the prices at medium load level are multiplied with 0.8 and 1.2, respectively. Moreover, energy not supplied costs of residential customers at different load levels are considered to be equal to the electricity prices introduced to them. However, due to high importance of electricity for commercial and industrial customers, energy not supplied costs of these customers are assumed to be about 5 times as the suggested electricity prices.

The hourly demand level of the system including light, medium, and peak load levels and their intervals are illustrated in Fig. 6 [13]. Also, daily demand level of system throughout one year can be seen in Fig. 7 [13].

Table 7 presents the initial data of the problem including investment cost for purchasing a DG [25], size of a DG [25], yearly maintenance cost of a DG, investment cost for purchasing a

Table 5
Data of lines and loads of the electrical distribution network under study related to 21th hour of March 1st [13].

First bus	End bus	Active demand (MW)	Reactive demand (MVAr)	*R* (p.u.)	*X* (p.u.)	Line capacity (MVA)	Load type	No. of customers
1	1	0.1992	0.0905	–	–	–	Res.	21
1	2	0.1992	0.0905	0.00057	0.00029	25.5	Res.	21
2	3	0.4979	0.2263	0.00107	0.00156	24.5	Res.	55
3	4	0.4979	0.2263	0.00127	0.00116	24.5	Res.	55
4	5	0.5432	0.2354	0.00137	0.00120	23.5	Ind.	30
5	6	0.5432	0.2354	0.00111	0.00140	19.5	Com.	120
6	7	0.5432	0.2263	0.00116	0.00185	19.5	Res.	60
7	8	0.5884	0.2535	0.00143	0.00146	18.5	Com.	130
8	9	0.5884	0.2535	0.00141	0.00160	17.5	Res.	65
9	10	0.6337	0.2716	0.00150	0.00160	17.5	Com.	140
10	11	0.6790	0.2806	0.00122	0.00040	9.5	Com.	150
11	12	0.6790	0.2897	0.00133	0.00077	8.5	Res.	75
12	13	0.6790	0.2806	0.00114	0.00119	8.5	Com.	150
13	14	0.4979	0.2263	0.00137	0.00143	7.5	Com.	110
14	15	0.8600	0.4164	0.00168	0.00127	6.5	Com.	190
15	16	0.8600	0.3893	0.00164	0.00139	5.5	Res.	95
16	17	0.8600	0.3893	0.00102	0.00071	6.5	Com.	190
17	18	0.8600	0.4074	0.00102	0.00071	6.5	Com.	190
10	19	0.8600	0.4164	0.00100	0.00037	9.5	Res.	95
19	20	0.8600	0.4164	0.00136	0.00044	8.5	Ind.	47
20	21	0.8600	0.4255	0.00155	0.00097	8.5	Ind.	47
21	22	0.9053	0.4436	0.00141	0.00083	7.5	Ind.	50
22	23	0.9053	0.3621	0.00180	0.00092	6.5	Ind.	50
23	24	0.9053	0.3621	0.00159	0.00041	5.5	Ind.	50
24	25	0.9506	0.3712	0.00157	0.00036	6.5	Ind.	52
5	26	0.8600	0.3621	0.00180	0.00092	5.5	Res.	95
26	27	0.8600	0.3621	0.00159	0.00041	6.5	Com.	190
27	28	0.8600	0.3621	0.00157	0.00036	6.5	Ind.	47

Table 6
Energy prices in retail market [24] and costs of energy not supplied of various customers at different load levels.

Load level	Electricity price (Cents/kW h)	Load type	Energy not supplied cost (Cents/kW h)
Light	17.69	Res.	17.69
	13.84	Com.	69.20
	11.22	Ind.	56.12
Medium	22.12	Res.	22.12
	17.30	Com.	86.50
	14.03	Ind.	70.15
Peak	26.54	Res.	26.54
	20.76	Com.	103.80
	16.83	Ind.	84.18

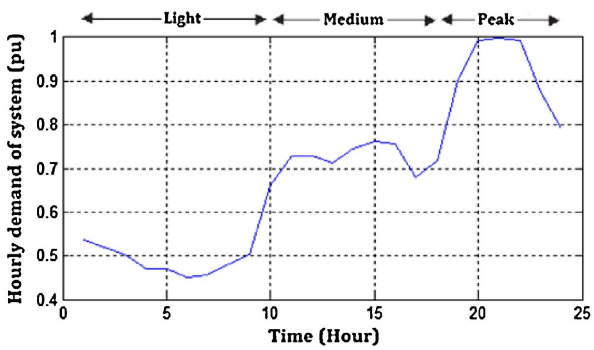

Fig. 6. Hourly demand of the system at light, medium, and peak load levels [13].

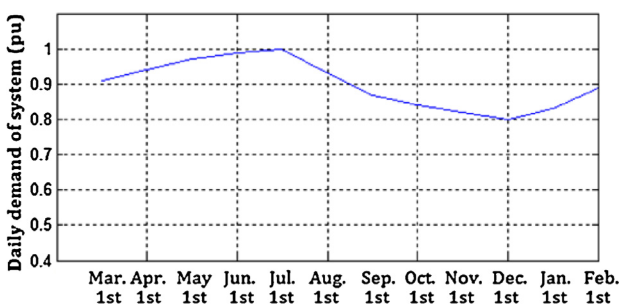

Fig. 7. Daily demand level of the system throughout one year [13].

capacitor bank [26], size of a capacitor bank [26], yearly maintenance cost of a capacitor bank, inflation and interest rates, load growth, and planning horizon.

In the problem simulations, values of the P^{Mutation} and N_{ch} as the mutation probability of the genes and size of the population are considered to be about 0.05 and 100, respectively.

7.1. Base case simulation

Herein, the considered FFRM is linear. The simulation results of the problem before and after planning are presented in Table 8. As can be seen, energy loss and risk level of the system

Table 7
Parameters and initial data of the problem.

DG investment cost ($/DG) [25]	200,000
DG maintenance cost (%/yr)	5
Size of DG (MW) [25]	1
Capacitor investment cost ($/Capacitor) [26]	4000
Capacitor maintenance cost (%/yr)	5
Size of Capacitor (MVAr) [26]	1
Inflation rate (%/yr)	10
Interest rate (%/yr)	15
Load growth (%/yr)	1
Planning period (yr)	20

Table 8
Base case simulation results before and after planning.

FFRM	Before planning	After planning
Energy loss (MW h/pp)	38,946	4945
Risk level (MW h/pp)	5380	1942
DG Investment cost ($/pp)	0	2600,000
DG Maintenance cost ($/pp)	0	1027,200
Capacitor Investment cost ($/pp)	0	32,000
Capacitor Maintenance cost ($/pp)	0	15,352
Cost of energy loss ($/pp)	4,917,682	566,550
Cost of risk ($/pp)	2,190,772	744,070
Total cost ($/pp)	7,108,454	4985,172

Table 9
Values of the system reliability indices before and after planning for the base case simulation.

FFRM	Before planning	After planning
SAIFI (int/cust pp)	162.00	65.47
SAIDI (h/cust pp)	327.39	144.57
AENS (kW h/cust pp)	2093.62	755.63

after simultaneous DG and capacitor placement are decreased about 34,001 MW h/pp and 3438 MW h/pp, respectively. Moreover, although DG and capacitor placement has extra cost about $3674,552 due to investment and maintenance costs, the total cost of the system is decreased about $2123,282 over the planning period. Fig. 8 illustrates voltage profiles of the system at light, medium, and peak load levels before and after planning. As can be seen, voltage profile of every bus at every load level is improved after optimal DG and capacitor placement.

In addition, values of the system reliability indices over the planning period are presented in Table 9. As can be seen, values of system average interruption frequency index (SAIFI) in interruption per customer over the planning period (int/cust pp), system average interruption duration index (SAIDI) in hour per customer over the planning period (h/cust pp), and average energy not supplied (AENS) index in kW h per customer over the planning period (kW h/cust pp) as the undesirable indices are decreased.

Moreover, locations of the installed DGs and capacitor banks for light, medium, and peak load levels in the base case simulation are presented in Fig. 9. As can be seen, some of the installed DGs and capacitor banks are switched on (or off) by changing the load levels throughout the day.

7.2. Studying the problem with different FFRMs

Herein, the problem is studied for every FFRM to demonstrate the possible differences and inconsistencies in values of the problem results, values of the reliability indices, and optimal locations of the DGs and capacitors. Table 10 presents the problem simulation results for different FFRMs including constant, power, exponential, and logarithmic models and compare them with the consequences

Table 10
The problem simulation results after planning considering different FFRMs.

FFRM	Lin.	Const.	Pow.	Exp.	Log.
Energy loss (MW h/pp)	4945	11,188	6040	9412	5250
Risk level (MW h/pp)	1942	5457	1138	543	2171
DG Investment cost ($/pp)	2600,000	2000,000	2400,000	2200,000	2400,000
DG Maintenance cost ($/pp)	1027,200	666,000	948,210	756,310	993,360
Capacitor Investment cost ($/pp)	32,000	32,000	32,000	32,000	32,000
Capacitor Maintenance cost ($/pp)	15,352	15,352	15,352	15,352	15,352
Cost of energy loss ($/pp)	566,550	1267,900	708,530	1078,100	610,760
Cost of risk ($/pp)	744,070	2216,400	425,830	190,490	842,870
Total cost ($/pp)	4985,172	6197,652	4529,922	4272,252	4894,342

Table 11
Values of the system reliability indices after planning considering different FFRMs.

FFRM	Lin.	Const.	Pow.	Exp.	Log.
SAIFI (int/cust pp)	65.47	162.00	42.46	25.01	71.97
SAIDI (h/cust pp)	144.57	372.39	88.10	46.10	158.94
AENS (kW h/cust pp)	755.63	2123.70	443.08	211.62	845.08

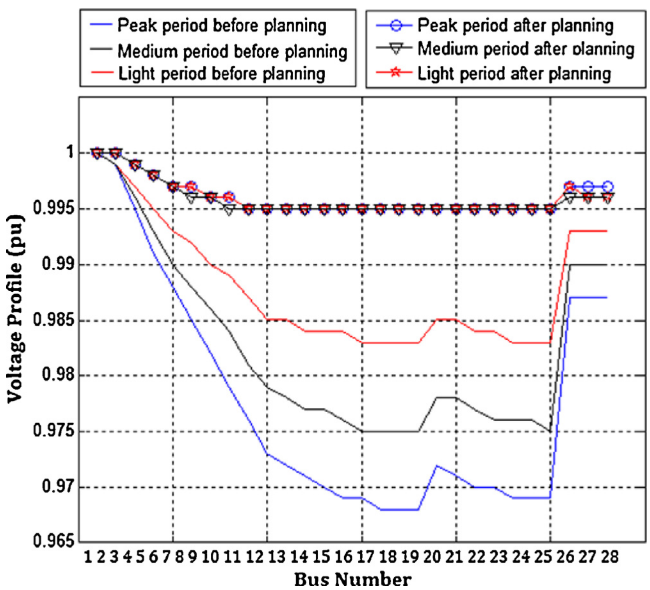

Fig. 8. Voltage profile of the system at light, medium, and peak load levels before and after simultaneous DG and capacitor placement in base case simulation.

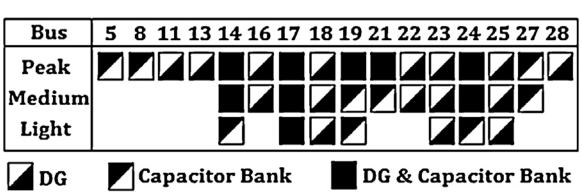

Fig. 9. Optimal locations of DGs and capacitors at light, medium, and peak load levels for base case simulation.

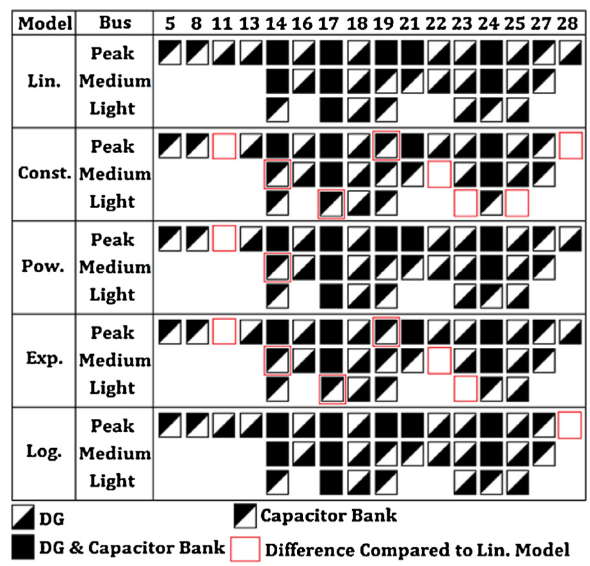

Fig. 10. Optimal locations of DGs and capacitors at light, medium, and peak load levels considering different FFRMs.

of the base case simulation (linear FFRM). As can be seen, there are considerable dissimilarities in the results. Therefore, this reality demonstrates the necessity for real modeling of the FFRM in the simultaneous DG and capacitor placement planning problem. Herein, considering exponential and constant models for the FFRM result in the least and the most total cost of the problem, respectively. Moreover, values of the system reliability indices are presented in Table 11. As can be seen, the best and the worst values of the system reliability indices are related to the exponential and constant FFRMs, respectively.

Optimal locations of the DGs and capacitor banks at light, medium, and peak load levels considering different FFRMs are illustrated in Fig. 10. Again, as can be seen, considering different FFRM affects the optimal locations of the DGs and capacitor banks at various load levels. Thus, studying simultaneous DG and capacitor placement problem without realistic modeling of the FFRM would not be efficient after installing the DGs and capacitor banks in the physical distribution network.

8. Conclusion

The planning problem of simultaneous DG and capacitor bank placement in distribution network was investigated from the local DISCO's viewpoint based on minimum total cost over the planning period considering several economic and technical factors and modeling the customers' load types and the feeder's failure rate. After optimal DG and capacitor placement, the system energy loss and risk level over the planning period were decreased noticeably, and also voltage profiles of all the system buses at different load levels were improved.

It was proven that considering different feeder's failure rate models in the simultaneous DG and capacitor placement planning problem can notably affect the simulation results. In other words, assuming incidental model for the feeder's failure rate in the simulations would not be beneficial after implementing the outcomes on the real system. Therefore, comprehensive studies must be carried out by the local DISCO to determine an appropriate model for feeder's failure rate. In addition, it is noteworthy to mention that suboptimal consequences of the problem and its misleading outcomes due to the unreal modelings can affect the results of the subsequent planning problems.

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