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New method to minimize the preventive maintenance cost of series–parallel systems

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Abstract

General preventive maintenance model for input components of a system, which improves the reliability to 'as good as new,' was used to optimize the maintenance cost. The cost function of a maintenance policy was minimized under given availability constraint. An algorithm for first inspection vector of times was described and used on selected system example. A special ratio-criterion, based on the time dependent Birnbaum importance factor, was used to generate the ordered sequence of first inspection times. Basic system availability calculations of the paper were done by using simulation approach with parallel simulation algorithm for availability analysis. These calculations, based on direct Monte Carlo technique, were applied within the programming tool Matlab. A genetic algorithm optimization technique was used and briefly described to create the Matlab's algorithm to solve the problem of finding the best maintenance policy with a given restriction. Adjacent problem, which we called 'reliability assurance,' was also theoretically solved, concerning the increase of the cost when asymptotic availability value conforms to a given availability constraint.

Keywords: Preventive maintenance; Cost; Availability; Optimization; Reliability; Monte Carlo

1. Introduction

The evolution of system reliability depends on its structure as well as on the evolution of the reliability of its elements. The latter is a function of the element age on a system's operating life. Element ageing is strongly affected by maintenance activities performed on the system. Preventive maintenance (PM) consists of actions that improve the condition of system elements before they fail. PM actions such as the replacement of an element by a new one, cleaning, adjustment, etc. either return the element to its initial condition and the element becomes 'as good as new' or reduce the age of the element. In some cases, the PM activity does not affect the state of the element but ensures that the element is in operating condition. In this case the element remains 'as bad as old.'

Optimizing the policy of preliminary planned PM actions is the subject of much research activities. In the past, the economic aspects of preventive and corrective maintenance have been extensively studied for monitored components in which failures are immediately detected and subsequently repaired. Far less attention has been paid to the economics of systems in which failures are dormant and detected only by periodic testing or inspections. Such systems are especially common in industrial safety and protection systems. For these kind of systems, both the availability evaluation models and the cost factors assessment differ considerably from those of monitored components [1].

This paper develops availability and cost models for systems with periodically inspected and maintained components subjected to some maintenance strategy.

The aim of our research is to optimize, for each component of a system, the maintenance policy minimizing the cost function, with respect to the availability constraint such as $A(t) \geq A_0$, for all t, $0 < t \leq T_M$, and a given mission time T_M.

A genetic algorithm (GA) is used as an optimization technique. GA is used to solve the above-mentioned

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problem, i.e. to find the best maintenance policy using a simulation approach to assess the availability of the studied system. The solution comprises both the availability and the cost evaluation.

Properties of the applied simulation program were intensively studied in Ref. [2]. The Matlab program was also successfully used in Ref. [3] for the reliability and availability optimization based on design of a Distribution Area System under Maintenance. New improvements of the simulation program focused on enhancing of computational efficiency were implemented into the program recently, including, e.g. a parallel computing algorithm.

A similar optimization problem applied on series–parallel multi-state system was studied in Ref. [4] taking into account imperfect component PM actions. This model uses universal z-transform for reliability calculations (universal moment generating function) but the duration of the PM activity is neglected. In Ref. [4], the optimization procedure is also based on a heuristic GA. We propose in this paper to study the example from Ref. [4] and others to prove the efficiency of our model.

This introduction is followed by seven sections, which present successively the PM model for general series–parallel systems, the problem formulation, the availability calculation based on simulation technique and analytic solution of the adjacent problem, the cost optimization technique (GA), the results and illustrative data, the result comments and a conclusion.

Notations.

WRV worst reliability value
N total number of components
$\mathbf{T}_0 = (T_0(1), T_0(2), \ldots, T_0(N))$ first inspection time vector
$\mathbf{T}_0^{\mathbf{ord}} = (T_0^{(1)}, T_0^{(2)}, \ldots, T_0^{(N)})$ ordered first inspection time vector; $T_0^{(1)} \le T_0^{(2)} \le \cdots \le T_0^{(N)}$
$\mathbf{T}_\mathrm{P} = (T_\mathrm{P}(1), T_\mathrm{P}(2), \ldots, T_\mathrm{P}(N))$ solution vector of system component inspection periods
T_M mission time
$C(e(i,k))$ cost of one inspection of ith component in the kth parallel subsystem
$A(t)$ system availability at the time t
A_0 availability constraint—lower limit

2. Preventive maintenance model for general series–parallel systems

2.1. Maintenance model for basic components

In the paper we will assume that the PM actions improve the reliability of basic component to as good as new. It means that the component's age is restored to zero. The model is demonstrated in Fig. 1, where T_F is random time to failure. Each T_F is demarcated by two conversely oriented arrows, identically with inspection periods T_P.

The problem to find the optimal vector $\mathbf{T}_\mathrm{P}$ is closely connected with another problem, i.e. to find the optimal first inspection time vector $\mathbf{T}_0$. Of course, it makes no sense to carry out inspections in the beginning of the life of a system, when both the system and its basic components are very reliable. Consequently, the preliminary calculations must be realized to find the optimal T_0 for each of basic components. The optimal vector $\mathbf{T}_0$ must be constructed so that it takes into account both cost and reliability view.

2.2. General series–parallel structure

Optimal PM plan is found for a general series–parallel structure that is shown in Fig. 2.

2.3. Cost model

Cost of the above-mentioned PM policy of a given system is simply given by summarizing each of the PM inspections done on the components that are under maintenance

$$C_{\mathrm{PM}} = \sum_{k=1}^{K} \sum_{i=1}^{E_k} \sum_{j=1}^{n_{e(i,k)}} C_j(e(i,k)).$$

$n_{e(i,k)}$ represents the total number of inspections of the ith component in the kth parallel subsystem in the course of mission time;
$C_j(e(i,k))$ is the cost of the jth inspection of the ith component in kth parallel subsystem;
E_k is the number of components in given kth parallel subsystem;
K is the number of parallel subsystems;

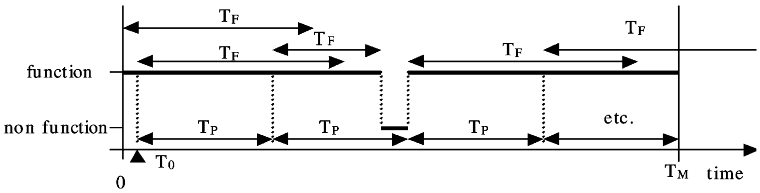

Fig. 1. PM model for periodically tested elements.

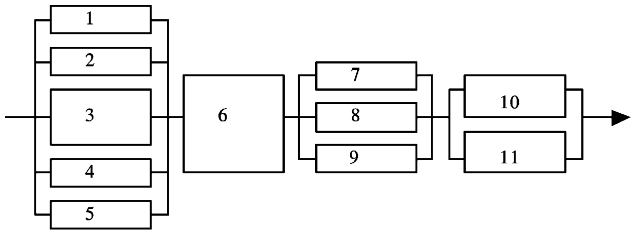

Fig. 2. General series–parallel structure.

$N = \sum_{k=1}^{K} \sum_{i=1}^{E_k} e(i,k)$ is the total number of components.

In most cases, the cost of inspection of a basic component is constant in the course of mission time, i.e.

$$C_{\mathrm{PM}}(e(i,k)) = \sum_{j=1}^{n_{e(i,k)}} C_j(e(i,k)) = n_{e(i,k)} \times C(e(i,k),$$

where $C(e(i,k))$ is the cost of one inspection of the ith component in kth parallel subsystem.

$$n_{e(i,k)} = 1 + \left\lfloor \frac{T_{\mathrm{M}}(e(i,k)) - T_0(e(i,k))}{T_{\mathrm{P}}(e(i,k))} \right\rfloor$$

is the integer part of the fraction, and T_{M}, T_0, T_{P} are, respectively, mission time, first inspection time and inspection period of a given component.

3. Problem formulation

Basic assumptions of this paper are as follows:

1. A system consisting of subsystems connected in series is considered, Fig. 2. Each subsystem contains different components connected in parallel. Each component is characterized by its failure rate function $h_j(t)$, and PM cost of one inspection; $C(e(i,k))$ is cost of one inspection of the ith component worked in kth parallel subsystem.
2. Testing actions or inspections are carried out periodically for jth basic component with the period of $T_{\mathrm{P}}(j)$. Inspections are ideal, which means that the component is renewed-model as good as new is assumed. The inspection of the jth component begins at the time $T_0(j)$.
3. The time in which a component is not available due to PM activity is negligible if compared to the time elapsed between consecutive activities.

The aim of the research is to optimize, for each component of a system, the maintenance policy minimizing the cost function C_{PM} and respecting the availability constraint $A(t) \geq A_0$, for all t, $0 < t \leq T_{\mathrm{M}}$, and a given mission time T_{M}. Consequently, we have to find optimal, cost minimizing vectors $\mathbf{T}_{\mathrm{P}} = (T_{\mathrm{P}}(1), T_{\mathrm{P}}(2), \ldots, T_{\mathrm{P}}(N))$, and $\mathbf{T}_0 = (T_0(1), T_0(2), \ldots, T_0(N))$, under given availability constraint.

4. Availability calculation based on simulation technique and analytic solution of the adjacent problem

4.1. Availability calculation based on simulation technique

Basic availability calculations of the paper were done by using simulation technique. In fact, the technique is employed when analytical techniques have failed to provide a satisfactory mathematical model or defy solution of the problem in closed form or the solution becomes unwieldy. The principle behind the simulation technique is relatively simple and easy to apply. However, the common real time simulation techniques are slow and take a lot of time to provide accurate results. Nevertheless, this technique is the only practical method of carrying out reliability studies, particularly when system is maintained and arbitrary failure and repair distributions are used or some special repair or maintenance strategy is prescribed.

Availability assessment method applied in the paper is based on the simulation program firstly used and tested in Ref. [2]. Many improvements of the program have been done recently, as e.g. parallel version. The parallel version of the simulation algorithm brings many improvements of the basic direct simulation technique, resulting in higher computational efficiency. Oriented Acyclic Graph, composed from nodes and edges, was used as a system representation. The parallel simulation technique is based on construction of a special Course Of Life (COL) sequence of transformed transition times subjected to a part of Acyclic Graph. Consequently, the corresponding part of Acyclic Graph could be effectively evaluated from reliability point of view. Large variety of maintenance strategy applied on basic components is allowed using the algorithm. Flexibility and good computational properties of the program made us decide to use the program within the analysis.

4.2. Analytic calculation of availability to solve adjacent problem

4.2.1. Basic concepts from the reliability theory

Consider structures capable of two states of performance, either complete success in accomplishing an assigned function or complete failure to function. Similarly, the components from which the structures are constructed are assumed capable of only the same two states of performance. The performance of the structure is represented by an indicator $\boldsymbol{\varphi}$, which is given the value **1** when the system functions and **0** when the system fails. The performance of each of the n components in the structure is similarly represented by an indicator x_i, which takes the value **1** if the ith component is functioning and **0** if ith component is failed $(i = 1, 2, \ldots, N)$.

It is assumed that the performance of a structure depends deterministically on the performance of the components which is characterized by the function φ of $\mathbf{x} = (x_1, x_2, \dots, x_N)$; $\varphi(\mathbf{x})$ is called the *structure function* of the structure [5].

For structures in which each component if functioning contributes to the functioning of the structure, certain hypotheses appear intuitively acceptable:

(i) $\varphi(\mathbf{1}) = 1$, where $\mathbf{1} = (1, 1, \dots, 1)$;
(ii) $\varphi(\mathbf{0}) = 0$, where $\mathbf{0} = (0, 0, \dots, 0)$;
(iii) $\varphi(\mathbf{x}) \geq \varphi(\mathbf{y})$ whenever $x_i \geq y_i$, $\forall i = 1, 2, \dots, N$.

Hypothesis (i) states that if all the components function, the structure functions. Hypothesis (ii) states that if all the components fail, the structure fails. Finally, hypothesis (iii) states that functioning components do not interfere with the functioning of the structure. Structures satisfying (i), (ii), and (iii) are called *coherent*. Sometimes the *monotonic* term is also used since such structures are characterized by a monotonic structure function which is equal to 0 at $\mathbf{0}$ and 1 at $\mathbf{1}$.

Assuming a probability distribution for the performance of the components, we obtain the *availability of the ith component* as follows

$$p_i = P[X_i = 1] = E[X_i],$$

where X_i is the binary random variable designating the state of component i (this expression is the reliability with the same X_i but conditioned to the component has not failed).

The *availability* (*reliability for non-repairable systems*) of the structure is

$$h = P[\varphi(\mathbf{X}) = 1] = E[\varphi(\mathbf{X})].$$

The structure function now becomes a binary random variable. When components perform independently, we may write $h = h(\mathbf{p})$, where $\mathbf{p} = (p_1, p_2, \dots, p_N)$, $h(\mathbf{p})$ is called *reliability function* of the structure.

4.2.2. Application to the adjacent problem to be solved

Let us suppose the system from Fig. 2, which is no doubt of coherent structure. The components of the system have exponentially distributed failures and are periodically inspected. The inspections of jth component are carried out periodically at the times given by the period $T_\mathrm{P}(j)$. They are ideal, which means that each inspected component is renewed. Worst case from reliability point of view is the moment when the inspection times of all components will meet in one time point. Let us call the point as a *worst point*, which assigns the system a *worst reliability value* (WRV).

Theorem. *Let us assume a coherent system with randomly generated periods of inspection* $T_\mathrm{P}(j)$. *Apparently, the availability of the system* $\mathbf{A}_\mathrm{S}(t)$ *satisfies the condition*: $\mathbf{A}_\mathrm{S}(t) \geq \mathrm{WRV}$. *Then the minimum value of system availability* $\mathbf{min}\,\mathbf{A}_\mathrm{S}(t)$, *converges for time going to infinity, to a value greater or equal then* WRV, *which is given as follows*

$$\lim_{t\to\infty}\left[\min_{\langle 0,t\rangle} A_\mathrm{S}(t)\right] \geq \mathrm{WRV}.$$

$\mathrm{WRV} = h(\mathbf{A})$; $\mathbf{A} = (A_1, A_2, \dots, A_N)$, *where*

$$A_j = \exp\left[-\frac{T_\mathrm{P}(j)}{\mathrm{MTTF}(j)}\right]$$

is the availability of the jth *component at the end of its inspection period* $T_\mathrm{P}(j)$, $j = 1, \dots, N$.

Proof. For component availabilities $A_i(t)$ must be valid

$$A_i(t) \geq A_i \qquad \text{for } \forall i = 1, 2, \dots, N.$$

From coherent property (iii) follows that $A_\mathrm{S}(t) \geq \mathrm{WRV}$, so that $A_\mathrm{S}(t) \geq \min_{\langle 0,t\rangle} A_\mathrm{S}(t) \geq \mathrm{WRV}$ and theorem is proved for $t \to \infty$.
If we assume rational periods $T_\mathrm{P}(j) = m_j/n_j$, $j = 1, \dots, N$, where m_j, n_j are natural, then equality in the theorem is valid. Generally, we have to prove the following statement

$$(\forall \varepsilon > 0)(\exists\, \alpha \in R)(\forall t\alpha)(|\min_{\langle 0,t\rangle} A_\mathrm{S}(t) - WRV| < \varepsilon).$$

If we find such t_0 for which $\min_{\langle 0,t_0\rangle} A_\mathrm{S}(t_0) = \mathrm{WRV}$, then the statement is valid.
From given assumptions, it is easy to find t_0 and natural multipliers k_j of periods $T_\mathrm{P}(j)$, to satisfy the equation

$$k_1 T_\mathrm{P}(1) = k_2 T_\mathrm{P}(2) = \cdots = k_N T_\mathrm{P}(N) = t_0.$$

We have $t_0 = \prod_{i=1}^{N} m_i$ and $k_j = n_j \prod_{i=1, i\neq j}^{N} m_i$, $j = 1, \dots, N$. For $\forall \varepsilon > o$ we assign $\alpha = t_0$.
Answering the adjacent question, we try to find the optimal vectors $\mathbf{T}_\mathrm{P} = (T_\mathrm{P}(1), T_\mathrm{P}(2), \dots, T_\mathrm{P}(N))$, and $\mathbf{T}_0 = (T_0(1), T_0(2), \dots, T_0(N))$, minimizing cost function C_PM, and respecting the availability constraint such as $\mathrm{WRV} \geq A_0$. Cost is minimized within given mission time T_M.
Analytical Matlab program for computing the WRV value of any coherent system was completed and successfully used within the research. Consequently, we are not limited by the series–parallel structure and analysis can be done as well as for a general system from practice. □

4.3. Finding the optimal first inspection time vector $\mathbf{T}_0$

Naturally, the problem of finding the optimal vector $\mathbf{T}_\mathrm{P}$ is closely connected with another problem, namely the finding of vector $\mathbf{T}_0$ which represents the beginning of inspections of each basic component, i.e. the vector of first inspection times. We will not carry out inspections in the beginning of the life of a component, when the component is very reliable. Consequently, the preliminary calculations must be realized to find the optimal vector $\mathbf{T}_0$. The starting point for the finding of $\mathbf{T}_0$ is based on the idea that only such intervention into the system must be made, i.e. maximally effective from both reliability and cost point of view. Measure of efficiency is more or less subjective question and in many situations in practice may be dependent on concrete reliability data files. For the research we decided to use the time dependent ratio-criterion of efficiency that is defined as follows

$$\min\{R_j(t)|j=1,\dots,N\}, \qquad R_j(t)=\frac{C(j)}{\mathrm{IF}_j^B(t)},$$

where

$C(j)$ is cost of one inspection of jth component;
$\mathrm{IF}_j^B(t)$ is Birnbaum's measure of importance of jth component at time t (e.g. definition in Ref. [5]).

Actually, the Birnbaum's importance measure provides the probability that the system is in a state in which the functioning of component j is critical to system failure. The system fails when jth component fails.

For given time point, we obtain number of the component, inspection of which is optimal, i.e. for which the ratio-criterion defined above is minimal.

The following procedure determines the vector $\mathbf{T}_0=(T_0(1),T_0(2),\dots,T_0(N))$.

Step 1
Calculate dependence on availability of given system on time for mission time T_M, supposing no maintenance; $i=1$; $\mathbf{T}_0=(+\infty,+\infty,\dots,+\infty)$.
Step 2
Obtain the time point t_i in which the system availability value A_0 is reached.
Step 3
If $t_i<T_\mathrm{M}$, then t_i is ith component of ordered first inspection time vector $\mathbf{T}_0^{\mathrm{ord}}$; $T_0^{(i)}=t_i$; $\mathbf{T}_0^{\mathrm{ord}}=(T_0^{(1)},T_0^{(2)},\dots,T_0^{(N)})$.
Step 4
Determine component No. of j, using the above-mentioned ratio-criterion applied in the time t_i; $1\le j\le N$. Then $T_0(j)=T_0^{(i)}=t_i$.
Step 5
Recalculate dependence on availability of the system on time under the maintenance actions given by first inspection times of all relevant components in all time points $T_0^{(k)}=t_k$; $k=1,\dots,i$.
Step 6
$i=i+1$, $i\le N$, return to Step 2.

Using the procedure we obtain full vector $\mathbf{T}_0$. However, in some cases there is no necessary to use all components of the vector. That is just in the case when repeating inspections of one or more system components brings more effective way, under the given criterion, to satisfy given availability constraint A_0. Consequently in such cases, it is necessary to select those elements that will be maintained. Final decision about system interventions must be made in good accordance with given cost matrix.

5. Cost optimization technique

The GAs were developed by John Holland in 1967 [6–8] at the Michigan University. The implementation of the GA consists to create an initial population with given size (number of individuals). Then by a selection process similar to that of the natural selection, which is defined by an adaptation function, the second step is to select the individuals who will be crossed. These individuals are represented by a chromosome in the GA. Then a current population is created by crossing of the individuals. The passage from a current population to another is called generation. For each generation, the algorithm keeps the individual with the best criterion value. The coding and the construction of the chromosome, representing the individual in the population, is the most important step of the algorithm.

The general structure of the GA according to Davis [6] is as follows.

Step 1
Initialization of the chromosomes population.
Step 2
Evaluation of each chromosome of the population.
Step 3
Creation of new chromosomes using crossing and mutation operators.
Step 4
Evaluation of the new chromosomes.
Step 5
Removing of the not selected chromosomes.

The last step is the final stop test (one considers, e.g. the iteration count or the no improvement of the solution value on a certain iteration count, etc.). If the test is not verified, go to Step 3.

5.1. Solution coding

We adopted the *direct coding*, i.e. the chromosome gives directly information about the problem solution. Each chromosome is composed of subchromosomes. The genes of these chromosomes are the durations between two maintenance interventions for each component in the system. It is real number randomly selected in the interval [LB, UB] according to a uniform distribution (LB is the lower bound and UB the upper bound).

The initial population, composed of N individuals, is built using the previous presented algorithm (Section 4.3). At first, we calculate an inspection time vector $\mathbf{T}_0$.

5.2. Reproduction (selection method)

The reproduction process consists of selecting the population elements ready to reproduce by evaluating their force, i.e. the ready ones are the strongest. This evaluation is based on an adaptation function (G), which is the objective function in the case of maximization without constraint. In the case of problems with constraints, we have to assure the feasibility of the solution before calculating the objective function. For the minimization problem, the adaptation function used is G', $G' = \mathrm{Cst} - G$, G being the adaptation function for maximization problem. The constant Cst, is selected so that the quantity G' remains always positive. From a generation to another we sort all the individuals, in the intermediate generation, and choose the N best ones.

5.3. The crossing method

The crossing is the genetic operator that allows, starting from two individuals of a given generation, to create one or more other individuals of the following generation. We choose randomly to be crossed, with a probability of $p_c \geq 0.7$ (called crossover probability), $N/2$ couples of individuals.

Among the variety of crossover operators, we adopt those produce from each crossing two children. This operator is generally called 1X. This crossover operator consists of generating randomly a point, called *crossover point*, and combines the different parts of the parent chromosomes to construct the children ones.

5.4. The mutation method

The purpose of the mutation is to bring diversity among genes. The mutation, contrary to the crossing, should not be too often applied because good genes in the individuals might be lost. The mutation probability adopted is $P_m \leq 0.07$.

It consists in modifying a part of gene in a random way. This modification consists in permuting between two genes chosen randomly for each selected chromosome.

5.5. The population size and the generations number (stop conditions)

The size of the population was fixed at $N = 50$. It appears that, if this size is too low, there will be risk to obtain not enough varied solutions by the individual crossing. With regard to the number of generations, we started from the value of 100. Performing many control tests, we decided finally to fix the value to 2000. At the end of 2000 consecutive generations, the algorithm stops.

Table 1
Parameters of system components

No. of component	Probability distribution	MTTF $= 1/\lambda_0$ (years)	$C(e(i,k))$
1	Exp	12.059	4.1
2	Exp	12.059	4.1
3	Exp	12.2062	4.1
4	Exp	2.014	5.5
5	Exp	66.6667	14.2
6	Exp	191.5197	19.0
7	Exp	63.5146	6.5
8	Exp	438.5965	6.2
9	Exp	176.0426	5.4
10	Exp	13.9802	14
11	Exp	167.484	14

6. Results and illustrative data

Consider a series–parallel system consisting of four parallel subsystems connected in series, Fig. 2. The system contains 11 basic components with different reliability and PM cost data. The reliability of each component is defined by an exponential distribution with the failure rate $\lambda_0 = 1/$MTTF presented in Table 1. This table also contains the PM cost $C(e(i,k))$ of each component. The basic data are exponential modification of those Weibull data presented in Ref. [4].

6.1. Calculations for the mission time $T_M = 25$ years

6.1.1. Availability constraint $A(t) \geq A_0$; $A_0 = 0.9$

Obtained solution: the component Nos 1, 2, 4, 7, 8, 9, 10 are not maintained (Table 2). The dependence on availability of time is demonstrated in Fig. 3.

Comparisons:

- If we continue development, the first inspection time vector without GA optimization we obtain $\mathbf{T}_0 = (22, 22, 18, 24.5, 14, 9.5, \infty, 20, 24.5, 20, 12)$, the cost of which is $C_{PM} = 91.6$.

  If we use the structural importance of jth component [5], in place of Birnbaum's measure of importance at time t, trying to find the optimal first inspection time vector $\mathbf{T}_0$, we obtain $\mathbf{T}_0 = (9.5, 11, 11, 11, 21, 16, 11, 11, 11, 11.5, 14.5)$, the cost of which is $\mathbf{C}_{PM} = 98.1$. This policy satisfies the constraint $A(t) \geq 0.9$ during given mission time, without GA optimization.

Table 2
The best cost C_{PM} obtained with the first inspection times and system component inspection periods ($A(t) \geq A_0$; $A_0 = 0.9$, $T_M = 25$ years

No. of component	3	5	6	11
$\mathbf{T}_P$	8.79	12.64	10.83	10.82
$\mathbf{T}_0$	18	14	9.5	12
C_{PM}	84.3			

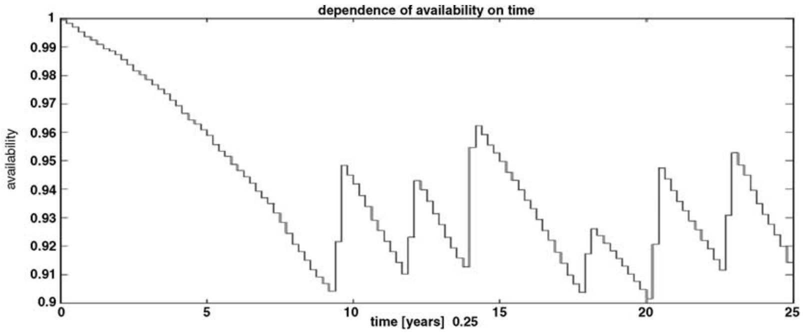

Fig. 3. Dependence on availability of time under availability constraint $A(t) \geq 0.9$.

Table 3
The best cost C_{PM} obtained with the first inspection times and system component inspection periods (WRV $\geq A_0$; $A_0 = 0.9$, $T_M = 25$ years)

No. of component	3	5	6	8	11
$\mathbf{T}_P$	2.40	5.96	9.34	2.81	4.94
$\mathbf{T}_0$	18	14	9.5	20	12
C_{PM}	133.1				

Table 4
The best cost C_{PM} obtained with the first inspection times and system component inspection periods ($A(t) \geq A_0$; $A_0 = 0.9$, $T_M = 50$ years)

No. of component	3	5	6	8	11
$\mathbf{T}_P$	9.466	8.554	10.301	12.767	10.573
$\mathbf{T}_0$	18	14	9.5	20	12
C_{PM}	238.0				

6.1.2. Availability constraint WRV $\geq A_0$; $A_0 = 0.9$

Obtained solution: the basic component Nos. 1, 2, 4, 7, 9, 10 are not maintained (Table 3).

6.2. Calculations for the long-term mission time $T_M = 50$ years

6.2.1. Availability constraint $A(t) \geq A_0$; $A_0 = 0.9$

Obtained solution: the basic component Nos. 1, 2, 4, 7, 9, 10 are not maintained (Table 4). The dependence on availability of time is demonstrated in Fig. 4.

Comparisons:

- If we continue determination the future replacement times with the same principle as the first renewals, we obtain the result: $C_{PM} = 252.9$ for the mission time $T_M = 40$ years, which results from the obtained time points $\mathbf{t} = (9.5, 12, 14, 18, 20, 22, 26, 30, 33.5, 37)$. The following components must be maintained within the time points: $(6,\ 11,\ 5,\ 3,\ 8+11,\ 6+11,\ 3+6+11,\ 3+6+11, 3+6+11, 3+6+11)$.

We see that the ratio-criterion of efficiency is only of limited use. It is of advantage to use just on local level.

6.2.2. Availability constraint WRV $\geq A_0$; $A_0 = 0.9$

Obtained solution: the basic component Nos 1, 2, 4, 7, 9, 10 are not maintained (Table 5).

6.2.3. Average availability

Fig. 5 demonstrates the average availability, when the mission time of 50 years is partitioned into four evenly long intervals per 12.5 years.

6.2.4. Availability constraint $A(t) \geq A_0$; $A_0 = 0.8$

Obtained solution: the basic component Nos 4, 7, 9, 10 are not maintained (Table 6). The dependence on availability of time is demonstrated in Fig. 6.

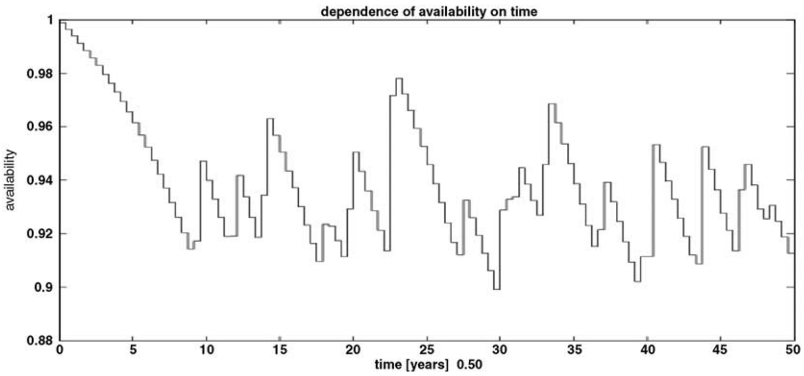

Fig. 4. Dependence on availability of time under availability constraint $A(t) > 0.9$.

Table 5
The best cost C_{PM} obtained with the first inspection times and system component inspection periods (WRV $\geq A_0$; $A_0 = 0.9$, $T_M = 50$ years)

No. of component	3	5	6	8	11
$\mathbf{T_P}$	2.67	7.35	8.62	4.33	4.75
$\mathbf{T_0}$	18	14	9.5	20	12
C_{PM}	370.6				

6.2.5. Availability constraint WRV $\geq A_0$; $A_0 = 0.8$

Obtained solution: the basic component Nos 4, 7, 9, 10 are not maintained (Table 7).

7. Result comments

We computed the results for two levels of reliability constraint, i.e. 0.9 and 0.8, and two levels of mission time, 25 and 50 years. Parallel to the calculations, we solved the adjacent problem respecting the availability constraint WRV $\geq A_0$, which in fact answers the following question: which policy is necessary to apply to prevent that the availability constraint will never be overstepped even if the worst case from reliability point of view has happened?

The policy means something like 'reliability assurance,' i.e. if we do not care for instantaneous availability $A(t)$ and even if the first inspection time vector T_0 is not respected, the policy assures that the given constraint A_0 will never be reached. Of course, the solution is in all cases the most expensive way to keep to the constraint, but in case of long-term working systems and in situations when the vector T_0 does not play critical role to reliability, the solution would be comparable with the one obtained under the constraint $A(t) \geq A_0$.

Fig. 5 represents the average availability for the mission time $T_M = 50$ years. In many technical applications we might be interested in average availability within chosen time intervals, i.e. when the instantaneous availability $A(t)$ is averaged within the intervals under obtained optimal policy. In the demonstrative example the mission time is partitioned into the evenly long intervals per 12.5 years. We can see that the average availability is highly above

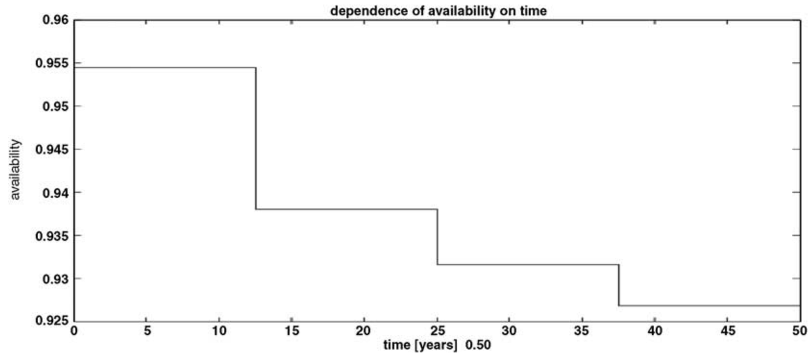

Fig. 5. Average availability for $T_M = 50$ years (partitioned per 12.5 years intervals).

Table 6
The best cost C_{PM} obtained with the first inspection times and system component inspection periods ($A(t) \geq A_0$; $A_0 = 0.8$, $T_M = 50$ years)

No. of component	1	2	3	5	6	8	11
$\mathbf{T_P}$	24.344	16.507	23.136	19.773	20.504	22.107	20.508
$\mathbf{T_0}$	26.5	26.5	21	32.5	15	32.5	18
C_{PM}	106.9						

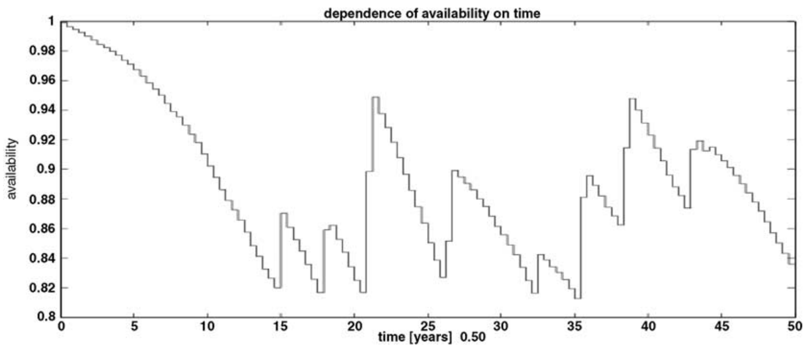

Fig. 6. Dependence on availability of time under availability constraint $A(t) > 0.8$.

Table 7
The best cost C_{PM} obtained with the first inspection times and system component inspection periods (WRV $\geq A_0$; $A_0 = 0.8$, $T_M = 50$ years)

No. of component	1	2	3	5	6	8	11
$\mathbf{T}_P$	17.86	12.18	10.14	18.84	12.74	8.84	10.97
$\mathbf{T}_0$	26.5	26.5	21	32.5	15	32.5	18
C_{PM}	154.3						

the availability constraint of $A_0 = 0.9$, particularly in the beginning phase of the mission time. In Fig. 5, we can see that even if our GA program found the four periods $T_P(3)$, $T_P(5)$, $T_P(6)$ and $T_P(11)$, in fact only the last two will be realized. First two periods are so long that in its first application exceed behind the required mission time 25 years.

Highly reliable calculations, with the constraint $A_0 = 0.9$, are characterized by big sensitivity of the obtained minimal cost results C_{PM} to small changes of the constraint value A_0. For example, if we change the value from 0.9 to 0.87, considering 50 years calculations, we obtain new solution of the vector $\mathbf{T}_P$ with cost value $C_{PM} = 203.4$ (compare with previously computed value 238), and for $A_0 = 0.85$ we obtain $C_{PM} = 185.3$.

All availability calculations are computed with the relative error of 5%, by the confidence level of 90%. The use of the newly developed GA program required the need of an automatic termination of the simulation program. Finishing on accuracy is, to our opinion, efficient way to terminate the program. Consequently, the possibility of 'finishing on accuracy' was built into the simulation program. In fact, that means that simulations are terminated just in the case, when a minimal number of successful trials is reached in the time point of worst availability value (worst during the mission time). The minimal number depends of course on given accuracy and can be obtained according to the method in Ref. [2].

8. Conclusions

This paper shows the efficiency of an optimization method to minimize the PM cost of series–parallel systems based on the time dependent Birnbaum importance factor and using Monte Carlo simulation and GAs. A theoretical approach based on the asymptotic availability value is also proposed. Starting from the results obtained for series–parallel systems, this approach can be extended to more complex systems, viz. no exponential failure rates, complex structures different than series–parallel ones, etc. according to the ability of the chosen methods (GA, simulation approach). Another extension seems possible: the improvement of the importance factor (other interesting importance factor should be studied), the study of other constraints than a minimal availability (minimal distance to the average availability), additional safety constraints, or more realist characteristics of the maintenance (imperfect maintenance, logistic delays). Also, other optimization methods would be developed and compared (simulated annealing for example) to the GA (present work or modified improved forms).

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