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J. K. Whittle<sup>a</sup>, M. S. Williams<sup>a</sup>, T. L. Karavasilis<sup>b</sup> & A. Blakeborough<sup>a</sup>

<sup>a</sup> Department of Engineering Science, University of Oxford, Oxford, United Kingdom

<sup>b</sup> School of Engineering, University of Warwick, Coventry, United Kingdom

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# A Comparison of Viscous Damper Placement Methods for Improving Seismic Building Design

J. K. WHITTLE<sup>1</sup>, M. S. WILLIAMS<sup>1</sup>, T. L. KARAVASILIS<sup>2</sup>,  
and A. BLAKEBOROUGH<sup>1</sup>

<sup>1</sup>Department of Engineering Science, University of Oxford, Oxford,  
United Kingdom

<sup>2</sup>School of Engineering, University of Warwick, Coventry, United Kingdom

*This article compares the effectiveness of five viscous damper placement techniques, two standard and three advanced, for reducing seismic performance objectives, including peak interstory drifts, absolute accelerations, and residual drifts. The techniques are evaluated statistically for two steel moment-resisting frames under varying seismic hazard levels, employing linear viscous dampers and nonlinear time history analyses. Usability of the methods is also assessed. All the placement methods meet the desired drift limit but advanced techniques achieve additional improvement in drift reduction and distribution. Performance differences between the advanced techniques are minor, making usability a significant selection factor amongst the methods.*

**Keywords** Viscous Dampers; Damping Distribution; Damper Placement; Seismic Design; Moment-Resisting Steel Frame

## 1. Introduction

Supplemental damping is becoming an increasingly tested and reliable seismic design strategy, and with it has come the evolution of building guidelines to include supplementally damped structures. The placement of dampers is a critical design concern, as the distribution of damping may greatly affect a building's dynamic response and the necessary damping cost [Soong and Dargush, 1997]. However, building codes and guidelines do not prescribe a particular method for optimally placing dampers. While the 2003 *NEHRP Provisions* [BSSC, 2004] offers a methodology for determining a total damping value corresponding to a desired effective damping ratio, it does not address an optimal distribution of dampers.

A large variety and quantity of damper placement methods have been proposed. Some of the earlier research efforts include Constantinou and Tadjbakhsh [1983], Ashour and Hanson [1987], Gürgöze and Müller [1992], and Hahn and Sathiyaveeswaran [1992]. A novel heuristic placement method was the adaptation of the controllability index (previously used to determine optimal actuator locations for active structural control; Cheng and Pantelides, 1988) to sequentially place dampers where their effects are maximised [Zhang and Soong, 1992]. Considered an advancement because of its practicality, it was verified for a shear-frame model [Shukla and Datta, 1999] and a three-dimensional model [Wu *et al.*, 1997].

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Address correspondence to J. K. Whittle, Department of Engineering Science, University of Oxford, Parks Road, Oxford, OX1 3PJ, UK; E-mail: jessica.whittle@eng.ox.ac.uk

An evolution of the sequential method was the Simplified Sequential Search Algorithm (SSSA) [Lopez-Garcia, 2001], which sought to further simplify the method for passive devices by decreasing the computational-effort of optimal location indices and simulated ground motions. For linear structures with linear viscous dampers, the method was as efficient as more complex placement methods, such as Takewaki [1997] and Gluck *et al.* [1996], in terms of interstory drifts [Lopez-Garcia, 2001]. Lopez-Garcia and Soong [2002] demonstrated the SSSA's efficiency for a recommended number of procedural steps (damper sizes) based on building height. Limitations of the study include the use of few ground motions, small unrealistic effective damping ratios (less than 10% with dampers) for comparing SSSA to other methods, and the use of example structures and damper placement distributions from previous researchers, implying that the placement methods compared to SSSA were not followed in full and usability cannot be adequately compared. The method's dependency on specific ground motions (particularly sensitive to ground motion characteristics for small damping ratios; Lopez-Garcia and Soong, 2002) and proven effectiveness limited to linear structures are two limitations of the technique.

Many analytical optimal placement methods have been proposed, including methods based on the principles of active control theory [Gluck *et al.*, 1996] and gradient-based search methods, including Takewaki [1997, 2000], Singh and Moreshi [2001], and Lavan and Levy [2006]. In particular, the Optimum for Minimum Transfer Functions [Takewaki, 1997] damper placement technique (abbreviated in this article as the "Takewaki" method) is a gradient-based optimization method with the objective of minimising the sum of the interstory drifts of the transfer function, evaluated at the structure's undamped fundamental frequency. The method has since been developed further for more-complex structures, multiple performance objectives, and optimal sensitivity design to optimize total damping and distribution [Takewaki, 2009]. Since the damper placement schemes are based on the dynamic behavior of the structure alone, the Takewaki method claims independence from ground motions. Takewaki [1997] showed the efficiency of the method for two shear buildings and assumed stationary ground motions. Limitations of the technique include the objective of minimizing the sum of a performance indicator as opposed to the peak value, which is a more appropriate damage indicator, and the exclusion of design objectives in the method. The lack of verification of the 1997 method for realistic building designs and ground motion scenarios, the method's status as an early benchmark method for optimal damper placement, and its claimed independence from ground motion characteristics warrants further investigation.

Another notable analytical placement method is the Fully-stressed Analysis/Redesign procedure (abbreviated in this article as the "Lavan A/R" method), which uses engineering knowledge and a simple numerical approach for damper placement [Levy and Lavan, 2006]. Based on the principle of fully stressed design of truss members, the Lavan A/R method uses a recurrence relationship to maximize ("fully-stress") the dampers' influence on the building performance parameter (i.e., drift allowance) and minimize the total adding damping necessary [Levy and Lavan, 2006]. A slight alteration of the original Fully Stressed Analysis/Redesign procedure may be used to constrain the total damping [Lavan and Levy, 2009]. The method has been verified by formal gradient-based optimization and applied to shear-frames, industrial frames [Levy and Lavan, 2006], and 3D irregular frames [Lavan and Levy, 2006]. Levy and Lavan [2009] showed the Lavan A/R method to be more effective than active control approaches such as Gluck *et al.* [1996] in terms of interstory drifts for multiple structures and ground motions. However, to the authors' knowledge, the Lavan A/R method has not been compared to any other available advanced damper placement techniques, evaluated in terms of additional performance objectives, nor employed by other researchers from the ground-up to assess usability.

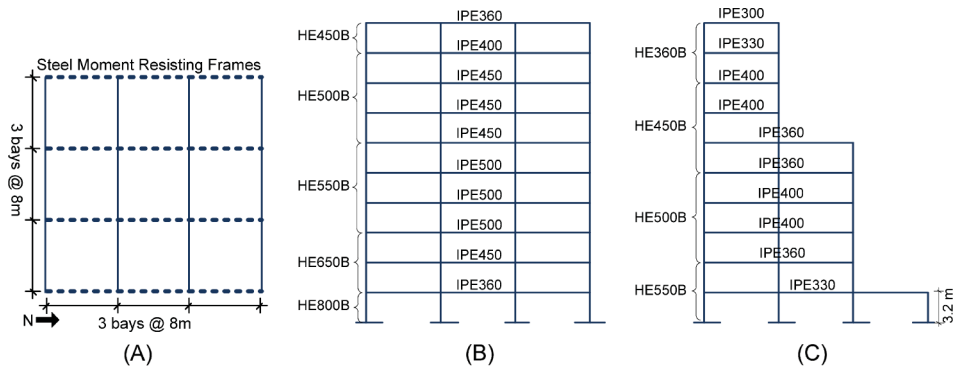
Computationally-intensive evolutionary methods have notably included genetic algorithms, such as Singh and Moreschi [2002] and Apostolakis and Dargush [2010]. Recent methods consider multi-objective optimization [Lavan and Dargush, 2009] and structural softening incorporated with a strategic damper placement [Cimellaro and Retamales, 2007]. Takewaki [2009] presented a more comprehensive list of contributions to the field of damper placement and concludes that despite the large quantity of information, structural engineers lack tools necessary for placing dampers optimally in a structure. Comparisons of practical, existing placement methods may provide insight into the effectiveness of certain methods and their usability for practicing engineers, but few comparisons exist for realistic design scenarios.

The purpose of this article is to present a thorough comparison of three advanced methods and two standard methods for realistic design scenarios and performance levels. The standard damper placement methods selected are Uniform damping and Stiffness Proportional damping methods, and the advanced damper placement methods are the Simplified Sequential Search Algorithm (SSSA) [Lopez-Garcia, 2001], the Optimal Damper Placement for Minimum Transfer Functions (Takewaki) [Takewaki, 1997], and the Fully Stressed Analysis/Redesign method (Lavan A/R) [Levy and Lavan, 2006]. The three advanced techniques were selected because they cover a range of methodologies and avoid the pitfalls of computationally-intensive methods. The comparison is made in terms reductions in peak interstory drifts, absolute accelerations, and residual drifts. The performance of the placement techniques are evaluated statistically for two steel moment resisting frames under varying seismic hazards levels. In addition, the usability and time efficiency of each damper placement method is assessed.

## 2. Building Designs

### 2.1. Bare Frame

Two steel MRF buildings, one regular and another irregular in elevation, were designed according to the Eurocode (EC3 [BS EN 1993-1-1, 2005], EC8 [BS EN 1998-1, 2004]). Both ten-story buildings had floor heights of 3.2 m and the same first floor plan (Fig. 1a), with a lateral force resisting system of MRFs in the north-south direction and braced frames in the east-west direction. The single MRF in the north-south direction was designed, neglecting plan irregularities of the irregular building. Typical gravity loads ( $4 \text{ kN/m}^2$  dead



**FIGURE 1** Building designs: (a) plan; (b) regular building profile; (c) irregular building profile.

load and  $2 \text{ kN/m}^2$  live load) and an assumed 5% inherent damping were selected. The MRFs were designed using response spectrum analysis and 0.3g PGA and Eurocode soil B site conditions. A high behavior factor (or strength reduction factor) of 6.5 was selected for the regular building, and a reduced behavior factor of 5.2 for the irregular building, to account for vertical irregularities.

Seismic performance levels selected include the frequently occurring earthquake (FOE), which is 40% of the design basis earthquake (DBE), the DBE (10% probability of exceedance in 50 years), and the maximum considered earthquake (MCE) (2% probability of exceedance in 50 years), which is 150% of the DBE [Somerville *et al.*, 1997]. A serviceability limit of 1% peak interstory drift under the FOE was selected. This achieves the Immediate Occupancy performance based design level for the FOE and Life Safety for the DBE [FEMA, 2000]. The final building properties are presented in Fig. 1b,c and Table 1, where the peak drift is based on the response spectrum analysis.

## 2.2. Added Damping

A strategic amount of added damping in the form of linear viscous dampers was calculated to achieve a linear elastic building performance under the DBE, causing no permanent damage to structural members, and thereby increasing the building performance from a Life Safety level to an Immediate Occupancy level under the DBE.

Using the equal displacement approximation and the 2003 NEHRP provisions [BSSC, 2004], a total amount of viscous damping was calculated, such that the buildings with dampers would achieve near 1% peak interstory drift under the DBE. The 2003 NEHRP provisions ([BSSC, 2004], Table 15.6-1) presents the relationship between damping B-values and total effective damping ratios for damped systems, where total damping is the sum of the effective damping from viscous dampers and inherent structural damping. The damping B-values were selected to achieve a realistic compromise between quantity of total damping and desired response (1.10% for the regular building and 1.18% for the irregular building, Table 2). The effective damping ratio from viscous dampers was translated into a total viscous damping coefficient  $C_t$ , which corresponds to damper sizes. This was calculated using the strain energy method [BSSC, 2004], whereby the effective viscous damping ratio ( $\xi_{\text{dampers}}$ ) is equivalent to the ratio of the energy dissipated in one cycle of the viscous damping system and the maximum strain energy dissipated by the structural system at its fundamental period.

The design properties of the buildings with added dampers are presented in Table 2. A damper typology of a chevron-braced frame with two horizontal dampers, one on either side of the brace apex, was selected as a common installation position.

## 2.3. Optimization Problem

The general optimization problem is to minimise the seismic response of the building by strategically placing the viscous dampers. The total damping is constrained to the same

**TABLE 1** Building properties

Building	FOE peak interstory drift (%)	DBE peak interstory drift (%)	1 <sup>st</sup> Natural period (s)	2 <sup>nd</sup> Natural period (s)	3 <sup>rd</sup> Natural period (s)
Regular	0.88%	2.20%	2.05	0.70	0.38
Irregular	0.99%	2.47%	2.31	0.93	0.47

TABLE 2 Building properties with added dampers

		Estimated peak interstory drifts (%)				Total damping	Damping from viscous dampers	
		<i>DBE</i>		<i>MCE</i>			$\xi$	$\xi$
Building	B-value	Bare frame	Frame with dampers	Bare Frame	Frame with dampers	Total (%)		
Regular	2.0	2.20%	1.10%	3.30%	1.65%	37%	32%	81200
Irregular	2.1	2.47%	1.18%	3.71%	1.77%	40%	35%	33700

value, allowing a fair comparison of the damper distribution schemes for each placement method.

2.4. Standard Placement Methods

The Uniform damping and Stiffness Proportional damping methods are simple means of distributing the total damping throughout the building. Uniform damping is an obvious and intuitive approach, in which the total added damping  $C_t$  is simply divided equally between the  $n$  floors, where  $C_i$  is the total damping at each floor:

$$C_i = \frac{C_t}{n}. \tag{1}$$

This approach has been used in many buildings, e.g., uniform distribution of viscoelastic dampers within the late World Trade Centre and the Santa Clara County Building in San Jose, California [Soong and Dargush, 1997]. However, Singh and Moreschi [2002] suggested it may not be the most effective approach.

Stiffness and/or mass-proportional damping distributions are attractive because they result in a Rayleigh-type damping matrix, which does not introduce any coupling between the modes. Trombetti and Silvestri [2006] showed that a mass-proportional distribution is theoretically more effective, but can be impractical to implement. We will therefore focus on the Stiffness Proportional approach, in which the damping at story  $i$  is proportional to the lateral story stiffness  $K_i$ , where  $K_t$  is the sum of the floor stiffnesses:

$$C_i = C_t \left( \frac{K_i}{K_t} \right). \tag{2}$$

Values of  $K_i$  can be determined by applying a static lateral load distribution and then computing the ratio of interstory shear force to interstory displacement at each level.

2.5. SSSA Mode Application

The objective function of the SSSA technique is to maximise the effectiveness of the dampers, which, in turn, minimises the seismic response of the structure. However, the performance indicator being minimised is not specified. Because the behavior of a viscous damper is proportional to the velocity along the linear damper’s stroke, the optimal location is at the floor with maximum interstory velocity. The constraints are the total added viscous damping and the number of dampers. Application of the method follows a sequential



approach, as presented in Lopez-Garcia [2001]. The SSSA technique relies on time history analysis of the structure at each iteration, and creates placement schemes dependent on individual ground motion characteristics. Initial investigation revealed that a sequential damper placement procedure for each ground motion record is not efficient when considering a large ground motion suite nor is dependency on a single ground motion record realistic for design. Therefore, an adaptation of the SSSA technique was employed, termed SSSA Mode, whereby the conventional SSSA method was applied to three spectrum-compatible accelerograms and a final SSSA placement scheme was selected based on the most frequently occurring damping value at each floor within the artificial accelerograms set, subject to the constraint of the total added damping value.

Twenty dampers (i.e., 20 procedural steps) were selected for the SSSA Mode application in the 10-story buildings. A linear time history analysis with SAP2000 was run for each step of the SSSA method and for three separate artificial accelerograms to determine the final SSSA Mode configuration. The advantage of the SSSA mode adaptation is time-efficiency and a procedure that may be more easily adopted by structural engineers. Because the damper placement scheme varies only slightly for each artificial accelerogram, it can be concluded that this alteration of the SSSA technique is consistent with SSSA method's original placement objectives.

## 2.6. Takewaki Application

The objective function of the Takewaki method is to minimise the norm of the sum of the amplitudes of the interstory drifts of the transfer function, evaluated at the undamped natural frequency of the structure. The constraint is the total added viscous damping. The distribution of damping is initially assumed uniform, and the gradient-based search method is applied iteratively. Application of the Takewaki method was employed with a Matlab script, created specifically for this research based on the procedure in Takewaki [1997, 2009]. The method requires the dynamic properties of an equivalent shear model: degrees of freedom, stiffness matrix, mass matrix, and total added damping value.

The optimality index  $\gamma_j$  with respect to the floor damping coefficient is a function of the interstory drift of the transfer function  $\hat{\delta}_i$ , based on the individual floor  $i$  and total number of floors  $n$ , where  $(\cdot)_{,j+1}$  refers to partial differentiation with respect to the damping coefficient  $c_{j+1}$  [Takewaki 1997]:

$$\gamma_j = \frac{\left( \sum_{i=1}^n |\hat{\delta}_i| \right)_{,j+1}}{\left( \sum_{i=1}^n |\hat{\delta}_i| \right)_{,j}}. \quad (3)$$

The optimal index array of the current step  $\gamma_o$  includes all the optimality indices such that:

$$\gamma_o = [\gamma_1; \gamma_2; \gamma_3; \dots; \gamma_{n-1}]. \quad (4)$$

The step of the gradient search in terms of the optimality index  $\Delta\gamma$  is the difference in the final desired optimal location index  $\gamma_F$  (an array of ones) and the optimal index array  $\gamma_o$  of the current step, divided by the step size  $N_i$ :

$$\Delta\gamma = \frac{\gamma_F - \gamma_o}{N_i}. \quad (5)$$

Therefore, the larger the step size, the smaller the incremental gradient search step. The step size is selected by the user. For the regular building, a step size of 7 led to convergence, indicated by near unity values for the optimality index, for all non zero damping terms, and the objective function was minimized from 0.20 to 0.17. For the irregular building, a step size of 3 converged, and the objective function minimised from 0.23 to 0.19, with unity values of the optimality index for all non zero damping terms. In both cases, the Takewaki method redistributed the total damping with each iteration, including the removal of damping at certain floors, thus reducing the objective function (sum of the interstory drifts of the transfer function) by the final step.

## 2.7. Lavan Analysis/Redesign Application

The objective function of the Lavan A/R method is a recurrence relationship between the coefficients of damping and the floor performance indices, often selected as interstory drifts. Application of Lavan A/R followed the procedure presented in Levy and Lavan [2006] and the constrained damping recurrence relationship equation in Lavan and Levy [2009]. The Lavan A/R method was applied to the building frames using linear time history analysis and an Excel program to calculate the recurrence relationship at each step, with an assumed convergence parameter of 0.5. The Victoria, Mexico (1980) ground motion was selected as the Lavan A/R active ground motion for the regular building and the New Zealand 02 (1987) ground motion for the irregular building because they produced the largest interstory drifts in the bare frame structures. Both ground motions were selected from the Pacific Earthquake Engineering Research Center Next-Generation Attenuation Relationships strong-motion database (PEER-NGA) [PEER, 2005], and station and component details are included in Table 4. A 50% reduction of peak interstory drifts under the active ground motion was used as the performance index, more suitable than a fixed drift objective when considering a single ground motion and fixed total damping. For example, for the regular building, the peak drift of the bare frame under the active ground motion was 3.29% therefore, the allowable peak drift with dampers was 1.65%. Final damping configurations are presented in Table 3 and Fig. 2. The total added damping for the regular building is 81200 kN-s/m and for the irregular building, 33700 kN-s/m.

## 3. Dynamic Analyses

### 3.1. Modelling and Analysis Details

Nonlinear time history analysis of the buildings with added dampers was used to evaluate the damper configurations effect on building performance. The building frames were modelled in SAP2000 [CSI, 2009] using frame elements, floor diaphragm constraints, and linear link elements for the linear viscous dampers. Material and geometric nonlinearity (p-delta) were included in the nonlinear analysis, and lumped plasticity at the ends of all the frame elements was modelled with bilinear plastic hinges. Rayleigh damping was modelled with 5% damping in the 1<sup>st</sup> and 2<sup>nd</sup> modes of the buildings. Ground motion time histories were applied to the stressed state of the buildings after the nonlinear static gravity loads of the applied seismic action. Seismic masses were calculated based on the dead plus one-third live load combination.

### 3.2. Ground Motion Suite

A set of 20 ground motion records were selected from the PEER-NGA database [PEER, 2005] and scaled to model realistic seismic hazard demands on the buildings (Table 4).



TABLE 3 Damping configuration schemes

Floor	Added damping (kN-s/m)									
	Regular building					Irregular building				
	Uniform	Stiffness proportional	SSSA mode	Takewaki	Lavan A/R	Uniform	Stiffness proportional	SSSA mode	Takewaki	Lavan A/R
10	8120	2760	4060	0	910	3370	673	1685	1526	392
9	8120	4290	4060	4632	3994	3370	1131	1685	2089	1561
8	8120	5100	8120	7014	6070	3370	1559	1685	2509	1724
7	8120	5530	12180	8944	9208	3370	1881	3370	2959	2967
6	8120	6330	8120	10222	9677	3370	2326	3370	3820	3519
5	8120	7010	8120	11402	10130	3370	2730	3370	4546	4528
4	8120	7380	12180	12616	13350	3370	3121	5055	5184	5342
3	8120	8150	12180	13480	15523	3370	3529	6740	5720	8255
2	8120	10190	12180	12890	12338	3370	4689	6740	5347	5411
1	8120	24450	0	0	0	3370	12061	0	0	0

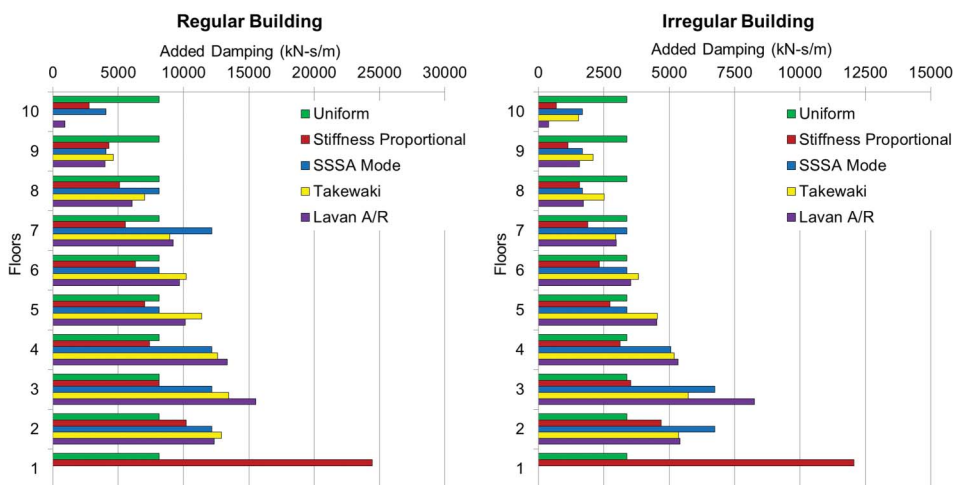


FIGURE 2 Final damping configuration schemes (color figure available online).

The absence of near-fault characteristics and Eurocode soil B classification were the primary selection criteria. The ground motions were normalized to the same hazard level (i.e., DBE or MCE), so that performance objectives at specific hazard levels could be evaluated from the ground motions. This was performed by scaling the ground motions to the same pseudo-spectral acceleration (PSA) (columns 7 and 8, Table 4) at the buildings' fundamental frequencies and 5% inherent damping (Table 5). The PSA values in Table 5 are taken from the design spectrum multiplied by the respective behavior factor  $q$ .

3.3. Accelerogram Set for SSSA Mode

The modified SSSA technique (SSSA Mode) required a set of three spectrum-compatible accelerograms. Created with the Seismic Record Processing (SRP) software [Karabalis et al., 1992], the accelerograms met EC8 requirements for artificial earthquakes [BS EN 1998-1, 2004]. SRP uses existing ground motions as a baseline template and iteratively alters these accelerograms to meet the frequency characteristics of the provided elastic target response spectrum [Karabalis et al., 2000]. Figure 3 displays the response spectra of the accelerograms, the elastic target spectrum of the buildings, and indicates the buildings' first three natural periods (Table 1).

3.4. Performance Indicators

Peak interstory drift, absolute acceleration, and residual interstory drift were selected as key performance indicators. Interstory drift indicates potential damage to structural and non structural members, while absolute floor acceleration corresponds to damage of building contents and sensitive equipment. Residual drift indicates the permanent damage to the structural members and feasibility of post-earthquake repair. Peak interstory drift is calculated as the maximum drift of adjacent floors over the time history, represented as a percentage of the total floor height, absolute floor acceleration taken as the maximum absolute value of the total floor acceleration over the time history, and residual drift measured as the final, stationary peak interstory drift.

TABLE 4 Ground motion suite

From PEER-NGA (2005)					DBE scale factor	
Ground motion	Station name	Component	Location	PGA (g)	Regular building	Irregular building
1 Imperial Valley 1979	Cerro Prieto	H-CPE237	USA	0.157	4.31	5.64
2 Loma Prieta 1989	Hollister - S & P	HSP000	USA	0.371	1.13	1.14
3 Loma Prieta 1989	Woodside	WDS000	USA	0.080	4.77	4.29
4 Manjil 1990	Abbar	ABBAR-T	Iran	0.496	0.88	0.88
5 Cape Mendocino 1992	Fortuna - Fortuna Blvd	FOR000	USA	0.116	2.66	2.17
6 Landers 1992	Desert - Hot Springs	LD-DSP000	USA	0.171	5.17	5.87
7 Northridge 1994	LA - W 15th St	W15090	USA	0.104	3.56	3.57
8 Northridge 1994	Moorpark - Fire Sta	MRP180	USA	0.292	4.02	4.19
9 Northridge 1994	N Hollywood - Cw	CWC270	USA	0.271	1.92	1.40
10 Northridge 1994	Santa Susana Ground	5108-360	USA	0.232	3.56	4.08
11 Northridge 1994	LA - Brentwood VA	0638-285	USA	0.164	4.60	3.57
12 Northridge 1994	LA - Wadsworth VA	5082-235	USA	0.303	3.09	3.09
13 Kobe 1995	Nishi-Akashi	NIS090	Japan	0.503	2.25	1.93
14 Kobe 1995	Abeno	ABN090	Japan	0.235	4.22	2.44
15 ChiChi 1999	TCU105	TCU105-E	Taiwan	0.112	2.40	2.25
16 ChiChi 1999	CHY029	CHY029-N	Taiwan	0.238	2.34	1.74
17 Hector 1999	Hector	HEC090	USA	0.337	1.90	2.06
18 Imperial Valley 1940	USGS 117 El Centro Array #9	I-ELC180	USA	0.313	2.11	1.77
19 New Zealand 02 1987	99999 Matahina Dam	A-MAT083	NZ	0.256	4.05	4.80
20 Nahinni Canada 1985	Site 3	S3270	Canada	0.148	13.93	14.61
21* Victoria Mexico 1980	UNAMUCSD6604 Cerro Prieto	CPE045	Mexico	0.621	4.10	3.44

\*Not included in the final ground motion suite; used as the active ground motion for Lavan A/R placement method.

TABLE 5 PSAs under seismic hazard levels

Seismic hazard level	PSA at $T_1$ ( $\text{m/s}^2$ )	PSA at $T_1$ ( $\text{m/s}^2$ )
	DBE	MCE
Regular building	3.83	5.75
Irregular building	3.06	4.59

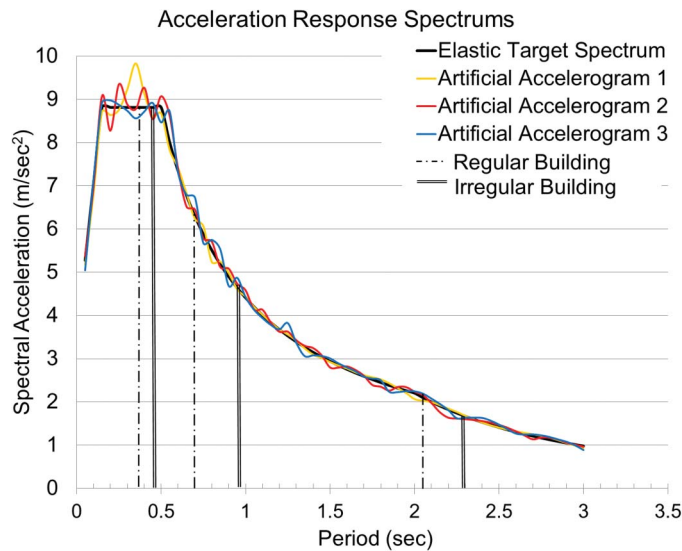


FIGURE 3 Acceleration response spectrums of the artificial accelerogram set (color figure available online).

4. Results

Modal damping due to the damper placement schemes and results of the regular and irregular buildings are presented. The performance of the damper placement schemes is compared in terms of peak interstory drifts, absolute accelerations, and residual interstory drifts. All values presented are the median results of the 20 ground motion suite to best capture the dominant trends and avoid large influence of outlying results. However, as standard deviations of the median do not exist, the scatter of the peak interstory drift results is presented in terms of the standard deviations of the mean values.

4.1. Modal Damping

The contribution of the damping distributions to modal damping is presented in Table 6 for each damper scheme and the first three building modes. The damping ratios were calculated using undamped eigenvector analysis and an energy-based equation for effective damping [Whittaker et al., 2003]. All five damper placement methods achieve greater first mode damping than the design value (Table 6, row 2), with the exception of the Stiffness Proportional method for the irregular building, which produces a lower fundamental damping ratio than desired. The advanced techniques produce similar

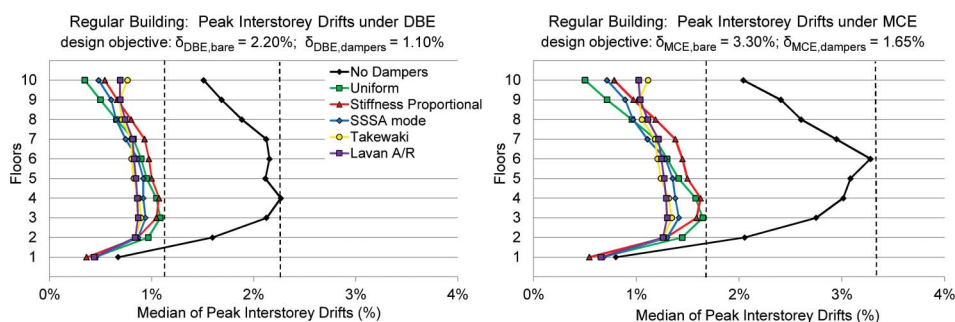
**TABLE 6** Damper schemes contribution to modal damping

Damper schemes	Modal damping ratios ( $\xi_{\text{dampers}}$ )					
	Regular building			Irregular building		
	Mode 1	Mode 2	Mode 3	Mode 1	Mode 2	Mode 3
Design with dampers	32%	—	—	35%	—	—
Uniform	39%	—	—	43%	—	—
Stiffness Proportional	33%	81%	—	32%	54%	80%
SSSA Mode	46%	—	—	48%	83%	83%
Takewaki	48%	93%	—	49%	86%	—
Lavan A/R	48%	93%	—	49%	68%	94%

fundamental damping ratios, over 30% greater than the design damping ratios. Table 6 reveals that there is less damping in modes two and three in the irregular building, and the lowest modal damping ratios occur with the Stiffness Proportional scheme. The total added damping generally overdamps mode 4 and higher for both buildings (indicated by dashes).

#### 4.2. Example 1. Regular Building

The results of the regular building's performance are presented in Figs. 4–7. Figure 4a compares the added damper placement schemes in terms of the median peak interstory drift distributions under the DBE. The drift design objective of the bare frame under the DBE ( $\delta_{\text{DBE, bare}}$ ) and the frame with dampers ( $\delta_{\text{DBE, dampers}}$ ) is noted with dashed lines in Fig. 4. All the damper schemes achieve less than 1.10% peak interstory drift, thereby meeting the DBE design objective (1.10% with added dampers, Table 2) and reducing the bare frame drifts by more than half. Both the Takewaki and Lavan A/R schemes result in peak interstory drifts best approaching a desirable, uniform drift distribution. Stiffness Proportional and Uniform produce the least uniform drift distributions, with Uniform overdamping the upper floors and Stiffness Proportional overdamping the first floor such that floors three and four are not effectively damped. Figure 4b compares the distributions under the MCE. The MCE drift distributions mirror the DBE results and display a 50% increase in the drifts of the damped frame over the DBE interstory drifts, as to be expected for a predominantly linear building



**FIGURE 4** Regular building – median of peak interstory drifts under (a) DBE and (b) MCE (color figure available online).

TABLE 7 Regular building – maximum of peak interstory drifts

	DBE ground motion suite	MCE ground motion suite
	$\delta_{\text{DBE, bare}} = 2.20\%$	$\delta_{\text{MCE, bare}} = 3.30\%$
	$\delta_{\text{DBE, dampers}} = 1.10\%$	$\delta_{\text{MCE, dampers}} = 1.65\%$
	% (mm)	% (mm)
No dampers	2.27% (72.5)	3.28% (104.9)
Uniform	1.08% (34.7)	1.64% (52.6)
Stiffness Proportional	1.07% (34.3)	1.62% (52.0)
SSSA Mode	0.94% (30.1)	1.41% (45.2)
Takewaki	0.90% (28.7)	1.34% (43.0)
Lavan A/R	0.87% (27.7)	1.30% (41.6)

response. Under the MCE, the design objective for added dampers (1.65% interstory drift) is met by all of the damper placement schemes.

Table 7 presents the maximum interstory drifts of all floors (i.e. maximum interstory drift for the Uniform scheme occurs at floor 3 from Fig. 4a). The Uniform and Stiffness Proportional damper schemes produce maximum interstory drifts within 1% of each other, while the 3 advanced techniques result in lower peak interstory drifts, with little disparity between the three schemes. Lavan A/R produces the lowest peak interstory drift, 3% lower than the Takewaki method, under both ground motion scenarios. Absolute interstory drifts in millimetres as presented in Table 7 particularly highlight the small differences between the methods.

The mean of peak interstory drifts for the regular building is displayed in Fig. 5. The standard deviations of drift under the DBE are similar amongst the damper placement methods at each floor. Defining the maximum standard deviation as the largest standard deviation of peak interstory drifts amongst all floors, each damping distribution yields a maximum standard deviation that is within 8% of the average of all five maximum standard deviations (0.24%) under the DBE. The dispersion is greatest for the bare frame (0.40% and 0.84% maximum standard deviations under the DBE and MCE, respectively).

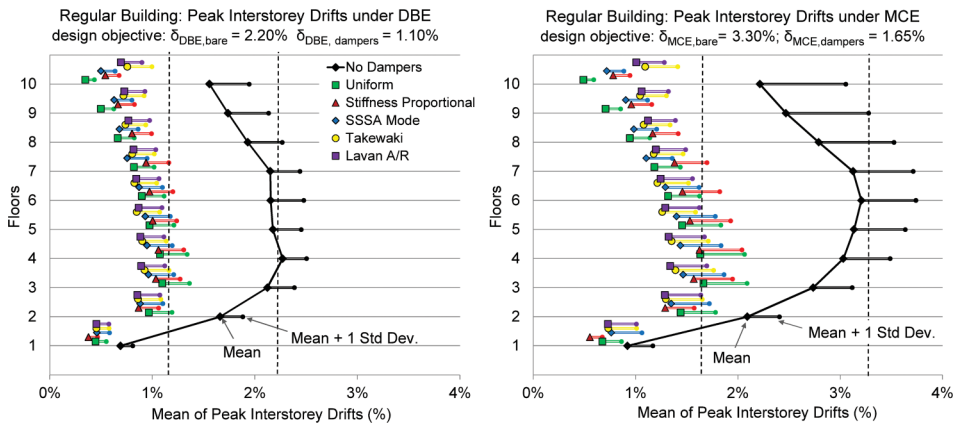


FIGURE 5 Regular building – mean and standard deviation of peak interstory drifts under (a) DBE and (b) MCE (color figure available online).

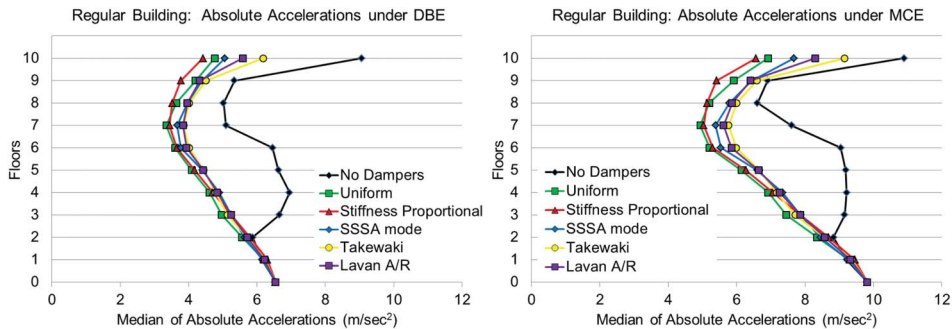


A larger dispersion of drifts occurs in the damped frame under the MCE, with an average of 0.39% maximum standard deviation, with all methods within 11% of this value. The dispersion is largest in the internal floors (2–7) corresponding to the largest peak interstory drifts.

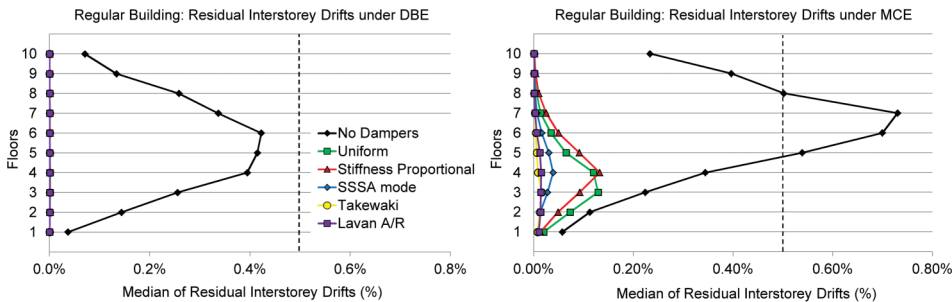
Figure 6a compares the placement techniques in terms of absolute accelerations of the regular building under the DBE. All the damper placement schemes reduce the absolute accelerations of the bare frame at all floors except the 1<sup>st</sup> floor. The maximum peak accelerations in the damped frames occur at the first floor and are within a narrow range of 6.17–6.30 m/s<sup>2</sup>. Similar distributions and narrow range of maximum peak accelerations (9.25–9.46 m/s<sup>2</sup>) are exhibited in the damped frame under the MCE (Fig. 6b). The maximum peak accelerations of the bare frame (at the roof) are reduced by an average of 30% under the DBE and 14% under the MCE with the added dampers (peak occurring at the first floor).

In terms of overall distribution, the Uniform and Stiffness Proportional schemes are the most effective at reducing accelerations at floors 5–10. For example, under the MCE at the roof, Uniform achieves a 10% reduction and Stiffness Proportional a 14% reduction from the nearest advanced method, SSSA, which may be attributed to the standard methods apportioning large damping at the base and roof of the building.

The building with added dampers experiences negligible residual interstory drifts under the DBE (confirming elastic building performance with dampers), but experiences large residual drifts in the bare frame, 0.42% at floor 6 (Fig. 7a). McCormick *et al.* [2008] recommends a permissible residual drift limit of less than 0.5%, based on realistic repair



**FIGURE 6** Regular building – median of absolute accelerations under (a) DBE and (b) MCE (color figure available online).



**FIGURE 7** Regular building – median of residual interstory drifts under (a) DBE and (b) MCE (color figure available online).

costs and human tolerance of drifts. The bare frame under the MCE (Fig. 7b) achieves peak residual drifts near 0.75% that would render the building economically unsalvageable after the earthquake. However, the addition of viscous dampers reduces the residual drifts to less than 0.15% for the standard placement methods and less than 0.05% for the advanced placement methods.

4.3. Example 2. Irregular Building

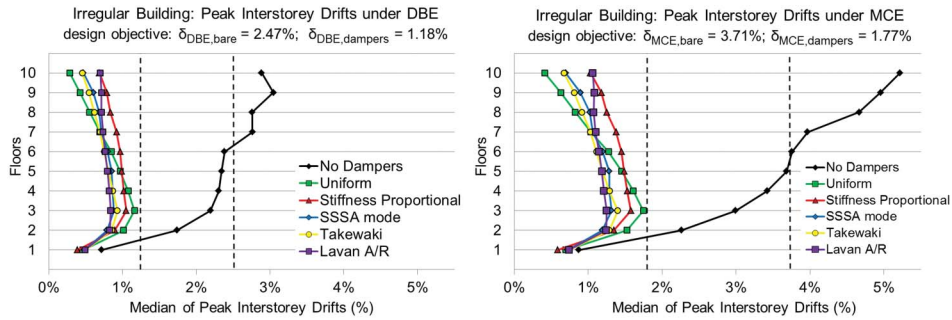
The results of the irregular building’s performance are presented in Figs. 8–11. Figure 8a compares the damper placement schemes in terms of median peak interstory drift distribution under the DBE. The upper tower (floors 7–10) of the bare frame experiences larger drifts than designed, indicating the limits of response spectrum design for vertical irregularities. All of the added damper schemes achieve less than 1.18% peak interstory drift, thus meeting the design objective and reducing maximum drifts in the bare frame by 62% in the Uniform case (least reduction, at floor 3) and effectively damping the upper tower of the frame. The Lavan A/R method produces the most uniform drift distribution and the best reduction in the maximum interstory drift (Table 8), more than a 10% improved reduction of the bare frame compared to the Uniform method, closely followed by the SSSA Mode and Takewaki schemes. Uniform produces the least uniform drift distributions.

Figure 8b compares the distributions under the MCE. The bare frame produces unrealistic upper-story interstory drifts, but the damped frame meets the design objective of 1.77% peak drift for every placement configuration. The Uniform scheme very closely meets the design objective, followed secondly by the Stiffness Proportional scheme, which achieves a 10% reduction of the Uniform method peak drifts (Table 8). The advanced methods for the irregular building achieve a maximum interstory drift of all floors ranging from 1.25–1.40%, substantially smaller than the drift design objective of 1.77%. In fact, in both buildings, the advanced techniques reduce the drift beyond the design objective. For example, in the best case, the Lavan A/R method reduces the maximum interstory drift by 21% of the original design objective for the regular building under DBE and MCE and by 29% for the irregular under DBE and MCE.

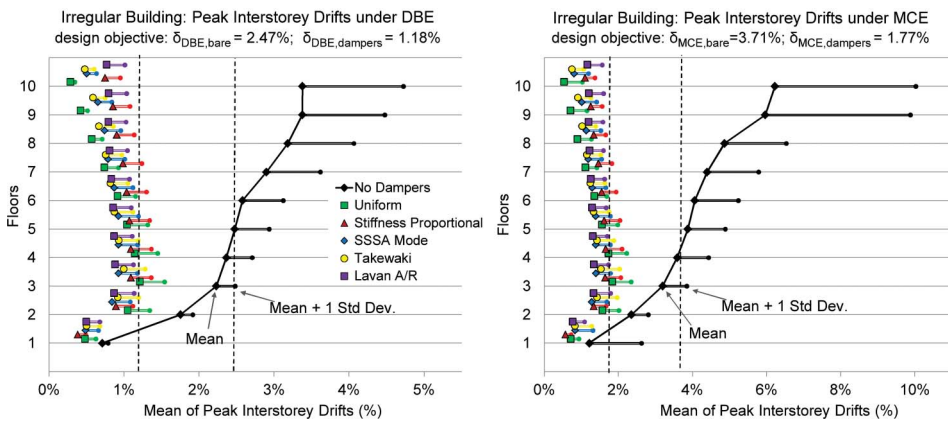
Unlike the regular building, the Takewaki method is less effective than the SSSA Mode method for reducing the maximum peak interstory drifts, by more than 5% under the DBE and MCE. However, Takewaki best minimises the sum of the interstory drifts, a reflection of its objective function of minimising the sum of interstory drifts of the transfer function.

TABLE 8 Irregular building – maximum of peak interstory drifts

	DBE ground motion suite	MCE ground motion suite
	$\delta_{\text{DBE, bare}} = 2.47\%$ $\delta_{\text{DBE, dampers}} = 1.18\%$	$\delta_{\text{MCE, bare}} = 3.71\%$ $\delta_{\text{MCE, dampers}} = 1.77\%$
	% (mm)	% (mm)
No dampers	3.05% (97.5)	5.22% (167.0)
Uniform	1.16% (37.2)	1.75% (55.9)
Stiffness Proportional	1.05% (33.7)	1.58% (50.5)
SSSA Mode	0.88% (28.2)	1.31% (42.0)
Takewaki	0.93% (29.8)	1.40% (44.8)
Lavan A/R	0.84% (26.9)	1.25% (40.0)



**FIGURE 8** Irregular building – median of peak interstory drifts under (a) DBE and (b) MCE (color figure available online).

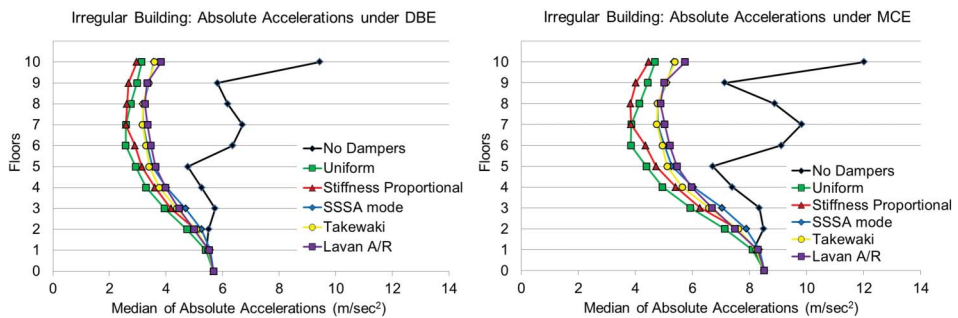


**FIGURE 9** Irregular building – mean and standard deviation of peak interstory drifts under (a) DBE and (b) MCE (color figure available online).

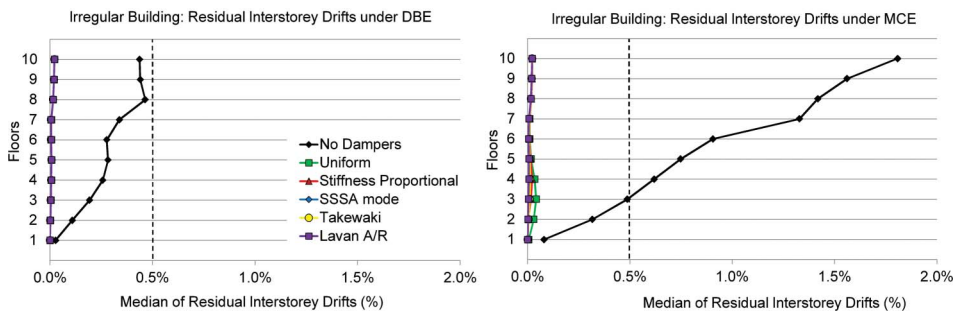
The mean of peak interstory drifts for the irregular building is displayed in Fig. 9. The mean drifts of the bare frame accentuate the dispersion and unrealistic drifts in the upper half of the frame, and it can be concluded that the bare frame has inadequate performance under both hazard levels. The dispersion of mean drifts under the DBE and MCE for each damper placement scheme is similar at each floor, apart from the very small standard deviations of the Uniform method at the upper floors. Maximum standard deviation occurs with the Uniform method, 0.33% and 0.51%, under the DBE and MCE, respectively. The dispersion of drifts with added dampers is greater under the MCE than the DBE by an average of 70% for the maximum standard deviations.

Figure 10 compares the damper placement techniques in terms of absolute accelerations of the irregular building under the DBE and MCE. All the damper placement schemes reduce the acceleration at all floors except the 1<sup>st</sup> floor and produced similar maximum absolute accelerations within a small range of 5.40–5.56 m/s<sup>2</sup>, occurring at the first floor. Similar to the regular building, both the Uniform and Stiffness Proportional schemes result in the lowest overall acceleration distribution, likely attributed to the large damping at the base for both schemes, as compared to the advanced methods (damping is predominantly on floors 2–9). Similar results occurred under the MCE.

The bare frame experiences a 0.47% maximum residual drift under the DBE, approaching the limit for economical repairs [McCormick *et al.*, 2008] (Fig. 11a). The residual drifts



**FIGURE 10** Irregular building – median of absolute accelerations under (a) DBE and (b) MCE (color figure available online).



**FIGURE 11** Irregular building – median of residual interstorey drifts under (a) DBE and (b) MCE (color figure available online).

under the MCE for the bare frame indicate unrealistic performance and likely failure under the high hazard level (Fig. 11b). However, under both the DBE and MCE, the added damper placement methods produce negligible residual interstorey drifts.

**4.4. Usability of the Placement Methods**

Finally, observations about the usability of the placement methods are presented, based on adherence to the methods’ procedures as outlined in literature. The Uniform and Stiffness Proportional methods are the simplest to apply while still achieving the desired drift limit. SSSA Mode is an easily-applied adaptation of the SSSA technique, allowing quicker application to a large number of ground motions. Despite this, SSSA mode is the most time consuming method because it requires twenty steps for three ground motions. The Takewaki technique is also time-intensive, as it requires creation and iterative application of a programming script. However, once programmed, the method is reasonably efficient, requiring only minimal inputs and operating independently from ground motions. Selection of the step-size greatly influences convergence time, but no stringent selection guidelines are provided. The Lavan A/R technique is the easiest to implement from scratch, of all three advanced techniques. Although dependent on iterative analysis with specific ground motions, this can be conducted with the same tools used for the SSSA Mode or Takewaki methods. For both examples presented, convergence occurred in less than ten iterations.

Estimates of the application time for each damper placement method are presented in Table 9. Total application time is the sum of the preparation time and analysis time for the

**TABLE 9** Approximate application time of damper placement methods

Method	Preparation time (min)	Analysis time (min)	Total time (min)	Normalized time	Assumptions
Uniform Stiffness Proportional	None 20	2 5	2 25	0.002 0.02	None Preparation time to calculate the approximate floor stiffness (variable time)
SSSA Mode	5	1320	1325	1.00	Assume (1) 20 steps and 3 ground motions (60 LTHAs), (2) 20 min per LTHA and 2 min per step to calculate maximum floor velocity and assign new damper
Takewaki	600	30	630	0.48	Preparation time to create the Matlab script and select an effective gradient search step size (variable time)
Lavan A/R	10	250	260	0.20	Assume (1) 10 steps and 1 active ground motion (10 LTHAs), (2) 20 minutes per LTHA and 5 min per step to recalculate and assign damping

method, based on the assumption of one available computer processor, an existing building model for the SSSA Mode and Lavan A/R methods, and an average of 20 min duration for a single linear time history analysis (LTHA). Times are presented normalized by the largest application time, occurring with the SSSA Mode method, and allow for a rough comparison of the time expended. The time efficiency of the standard placement methods compared to the advanced methods is obvious. The Takewaki method requires the largest preparation time yet the shortest analysis time amongst the advanced placement methods. Note that the Lavan A/R time is based on a single active ground motion. Application of the method without constrained damping would require multiple active ground motions, thereby increasing the normalized time to 0.60 for 3 ground motions and yielding less time savings than the Takewaki method. The efficiency of the SSSA Mode and Lavan A/R methods could be easily improved by using multiple computers to run the time history analyses simultaneously.

This research uses a conservative uniform damping estimation based on first-mode response to calculate the total damping. While controlling the total added damping permits

a fair comparison of the placement methods, less damping cost could be used to meet the drift performance objectives, especially for the advanced damper placement schemes, which achieved drifts much smaller than the design drift limit. Based on the modal damping ratios for the regular building, a total damping value for the advanced placement methods may be reduced by 30% as compared to uniform damping for this particular building. Note that the original recurrence relationship for the Lavan A/R method [Levy and Lavan, 2006] may be used to limit the total damping and optimally distribute damping.

Considering the comparable performance of the three advanced techniques compared to standard placement methods, it is advised to select advanced techniques based on usability. The examples presented here support the Lavan A/R method as the most suitable candidate for recommendation because of its effective limitation of drifts (as per its objective function), familiar approach using fully-stressed analysis/redesign, comparable time efficiency, and usability in application. However, the large controlled total damping used in this paper provides a level of safety for the Lavan A/R method against increases in floor drifts corresponding to variations from the active ground motion. Thorough checks of the Lavan A/R distribution under multiple active ground motions should be conducted when using a more economical reduced total damping to achieve the performance objective. These conclusions are based on idealised regular and irregular buildings with greater than 30% added damping with viscous dampers and restricted to the five methods investigated.

Because the uniform damping method is quickly and easily implemented, it is recommended that any selected advanced technique be compared against the uniform damping method for a variety of ground motion characteristics, to obtain additional confirmation of the advanced method's effectiveness. This comparison should be conducted with a few nonlinear time history analyses and would provide redundancy in the damping distribution selection. A criticism of the SSSA and Takewaki methods is the lack of performance-based design criteria within the methods, and therefore, an inability to assess the effectiveness of the damping distribution for meeting specific design criteria. However, the 2003 NEHRP Provisions [BSSC, 2004] and Whittaker *et al.* [2003] in conjunction with these damper placement techniques allows a final check that drift or modal damping design objectives have been met.

## 5. Conclusions

All of the damper placement methods achieve the desired drift objective for the regular and irregular buildings under the DBE and MCE, evaluated by the median results of 20 ground motions. This indicates that the design performance level of Immediate Occupancy has been met with the dampers, regardless of the placement scheme. In addition, all five placement schemes further reduced the absolute accelerations and residual drifts as compared to the bare frames. Peak absolute accelerations do not reveal large differences between the added damper schemes, apart from consistently smaller acceleration distributions in the upper floors for the standard placement methods. Residual drifts support the effectiveness of adding viscous damping to limit permanent deformation and suggest minor sensitivity of residual drifts to damper distributions.

Uniform and Stiffness Proportional, the simplest methods to implement, are proven to meet the design drift limits, yet do not achieve an optimal distribution of dampers in terms of best performance reduction or most uniform drift distributions. The three advanced methods show broadly comparable performance. The Lavan A/R technique achieves the best performance with the least complexity and comparable time expended to compute the damper distribution scheme. Neither the Takewaki nor SSSA Mode method is clearly more



effective considering both building results, although the Takewaki damper distribution is achieved more quickly. Overall, the performance differences between the advanced techniques should not be exaggerated, as all three produced similar placement schemes and extremely similar drift and acceleration results. Hence, usability of the method becomes an important distinguishing factor between the advanced placement methods.

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