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Procedia Technology 22 (2016) 227 – 234

Procedia Technology

9th International Conference Interdisciplinarity in Engineering, INTER-ENG 2015, 8-9 October 2015, Tirgu-Mures, Romania

A Study on a Two-Way Post-Tensioned Concrete Waffle Slab

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Abstract

Post-tensioned construction has for a long period of time occupied a significant position, especially in the construction of bridges, storage tanks, but also in buildings. In this paper are presented the aspects of a square shaped waffle slab calculation, supported punctually and having a two-way post tensioning reinforcement disposed parabolically. It is described the waffle slab system, its characteristics, preliminary design of composing elements, technological aspects regarding the manufacturing of precast panels, details regarding used materials, the reinforcement layout and the calculation of prestressing force.

Peer-review under responsibility of the “Petru Maior” University of Tirgu Mures, Faculty of Engineering

Keywords: Prestressed concrete; waffle slab; two-way post-tensioned concrete slab; prestressing tendons; prestressing layout.

1. Introduction

Post-tensioned construction has for a long period of time occupied a significant position, especially in the construction of bridges, storage tanks, but also in buildings. The most important advantages of post-tensioning are the following: compared to reinforced concrete, a considerable saving in concrete and steel because, due to the behaviour of the entire concrete cross-section more slender designs are possible; good crack behaviour and better protection of the steel against corrosion; high fatigue strength. Slabs which are supported along four edges by beams or walls are named two-way spanning slabs since the applied loads are effectively transferred in two directions to the supported edges.

Two-way slabs and plates are those panels in which the dimensional ratio of length to width is less than 2. They may be either solid uniform slabs or, for longer spans, waffle slabs of a shape. In waffle slab construction, the slab is

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Peer-review under responsibility of the “Petru Maior” University of Tirgu Mures, Faculty of Engineering
doi:10.1016/j.protcy.2016.01.072

often made solid near the supports to increase the shear and the bending moment capacity. Square two-way spanning slabs tend to be the most economical shape since each supporting beam or wall carries the same proportion of the total load from the slab and this leads to the minimum required slab and beam depth [1],[2].

2. Analyzed structure

2.1. Description of the waffle slab. Preliminary design

Slab behaviour under the action of prestressing forces and the way the slab works during exploitation will be monitored through an experimental research of stresses and deformations on a 2,8x2,8 m element.

Considering the manufacturing process, there are two types of slabs: cast in place or precast. Considering the way the slab is supported, prefabricated slabs can be: supported punctually - on columns continuously, simply supported on the perimeter - on beams.

Waffle slab geometrical characteristics and general aspects.

The type of slab presented in this paper is a coffered precast one. The plain view of this slab has a square shape, having the length of 2800mm, as presented in Fig.1.

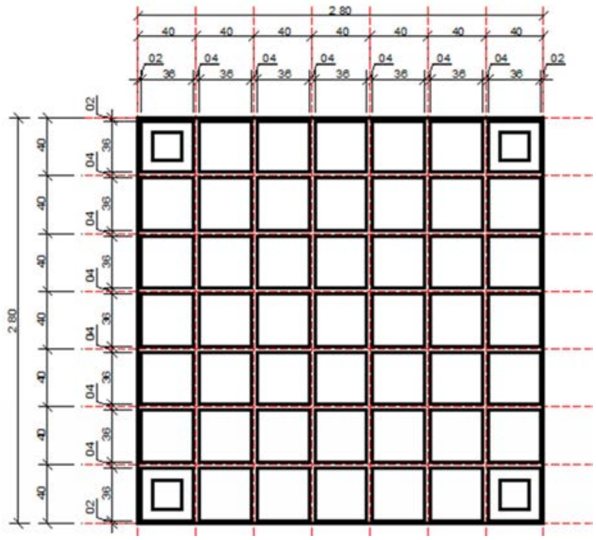

Fig.1. Precast slab layout.

According to the coffered slab system definition, this slab has a network of ribs that form a square mesh, covered by a reinforced concrete plate. The plate has a two-way reinforcement consisting of steel meshes and the ribs will embed in concrete the prestressing tendon. Passive reinforcement is placed in the precast coffers. All the prestressed concrete ribs have the same dimensions of the cross-section.

The concrete plate that covers the ribs mesh, will have the minimum thickness considered as it follows:

$$h_f \geq \left(\frac{\ell}{30}\right) = \frac{330mm}{30} = 11\ [mm] \quad (1)$$

Where ℓ is the length of a cell side. From technological reasons the plate thickness will be greater that 40mm.

Considering that this slab is cast in laboratory conditions, where manufacturing errors (geometrical imperfections) are eliminated, and the dimensions of the slab and its component elements reproduce a real slab at the scale of 1:10, the covering plate thickness will be 50mm. The prestressed concrete ribs follow the bending moment directions, thereby no additional moments of torsion are created. For this reason, in the plane view, the ribs are positioned on two orthogonal directions, and the cross-section of each rib has the dimension of 70x120 mm.

The overall set-up and the cross-section dimensions of the prestressed concrete slab are presented in Fig. 2.

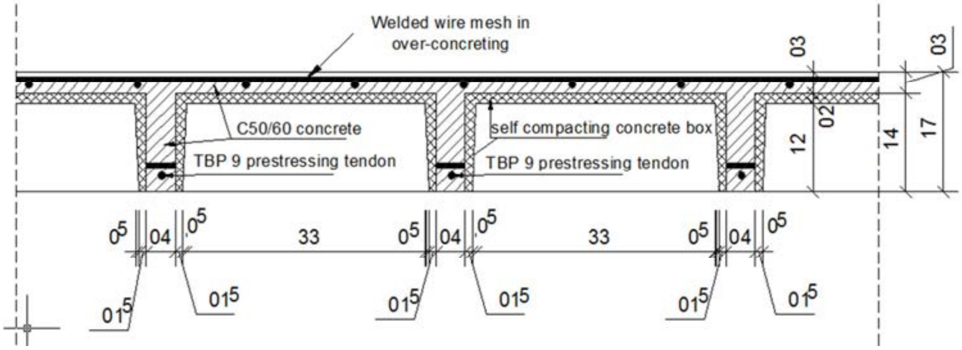

Fig. 2. Transversal section, dimensions of the elements that compose the waffle slab.

The concrete ribs will embed the prestressing cable strands. The prestressing tendon layout is a parabolic curve, so that the prestress bending moments will have an opposite sign to the bending moments of the dead loads (self-weight of the covering plate and of the ribs) and of live loads (applied using a spatial device, under the action of a hydraulic press).

2.2. Technological aspects regarding the precast panels

The panels, necessary to be used for the manufacturing of this slab, will be made of reinforced concrete and will have the role, when the slab is made, of the permanent formwork. External dimensions and their aspect are presented in Fig. 2 and 3.

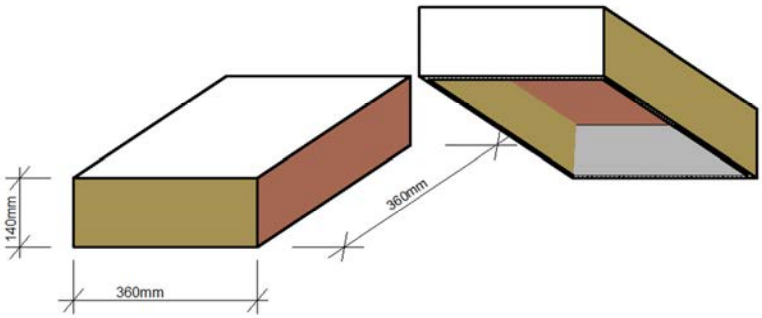

Fig.3. Reinforced concrete prefabricated panels.

Considering that the walls of the panels are extremely thin, their thickness ranging between 10 to 15 mm, and that in this thickness is introduced a reinforcement mesh ø3/100xø3/100mm, the remaining available distance for concrete to flow imposes the choice of aggregate size to the maximum of 4 mm. On the other hand, the same small thickness of the panel walls does not allow the inside vibration of concrete, therefore the surface vibration has been analyzed, but this too has proven to be impossible due to the fact that the element is extremely fragile and has extremely small dimensions, this technology having the risk of affecting the quality and conformity of the precast panel.

Also, it is intended to have an overall strength of the slab greater or at least equal to C50/60 concrete strength class.

In conclusion, considering the above presented facts, for the manufacturing of the prefabricated panels, self compacting concrete has been chosen, with the maximum size of the aggregate of 0-4mm, and the concrete strength class of C50/60.

2.3. Choice of materials

Concrete – class C50/60 high strength concrete.
Concrete strength at transfer, at t=3 days results from the equations (2) and (3)

$$f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm} \qquad f_{cm}(3\ days) = 0{,}66298 \cdot 58 = 38{,}45\ [MPa] \tag{2}$$

$$f_{ctm}(t) = \beta_{cc}(t) \cdot f_{cm} \qquad f_{ctm}(3\ days) = 0{,}66298 \cdot 4{,}1 = 2{,}72\ [MPa] \tag{3}$$

Also:

$$\beta_{cc}(t) = exp\left\{s\left[1 - \frac{28}{t}^{1/2}\right]\right\} \qquad \beta_{cc}(t) = exp\left\{0{,}2\left[1 - \frac{28}{3}^{1/2}\right]\right\} = 0{,}6629 \tag{4}$$

Where *s* is a coefficient considered according to the cement type and is equal to 0,2 for high strength and rapid hardening cement.

According to [3], the modulus of elasticity for concrete class C 50/60 is denoted by E_{cm} and its value is 37 [GPa].

$$E_{cm}(t) = \left[\frac{f_{cm}(t)}{f_{cm}}\right]^{0,3} \cdot E_{cm} \qquad E_{cm}(3\ days) = 32{,}71[GPa] \tag{5}$$

2.4. The prestressing tendons

For more accurate values, tests will be carried out on the prestressing tendons in order to determine experimentally their ultimate strength. According to [4], the strength of the prestressing tendon is denoted by R_{pk}, but considering the actual norms, in this paper, the notation from Eurocode 2 [5] will be used.

Theoretically, from [6] TBP9 has the characteristic strength (Rpk notation in bibliography): f_{pk}= 1760 [MPa].

Eurocode 2 [5], states that the design strength of TBP9 is computed as it follows:

$$f_{pd} = \frac{f_{p\,0,1k}}{\gamma_s} = \frac{0{,}9 \cdot 1760}{1{,}15} = 1377{,}4\ [MPa] \tag{6}$$

The geometrical characteristics of the tendons that are used.

TBP9 is composed of 7 wires, and each wire has the diameter of ø3mm, resulting a strand of 7ø3,
Where:

- the nominal diameter of the prestressing tendon TBP9 is $\emptyset_{p,nom}$=D=9,2 [mm]; the diameter of one wire from the tendon: d_p=d_c= 3 [mm]; TBP9 tendon cross section area: A_p= 49,93 [mm]; the mass per unit length of the tendon: G_p=0,388÷0,423 [kg/m]

The design of the tendon layout.

The waffle slab presented in this paper has been reinforced with TBP 9 post-tensioned tendons, vertically placed inside the slab's ribs with a parabolic layout. Analyzing the case of a simply supported on the contour slab, the layout design and the stresses that are considered are presented in Fig 4.

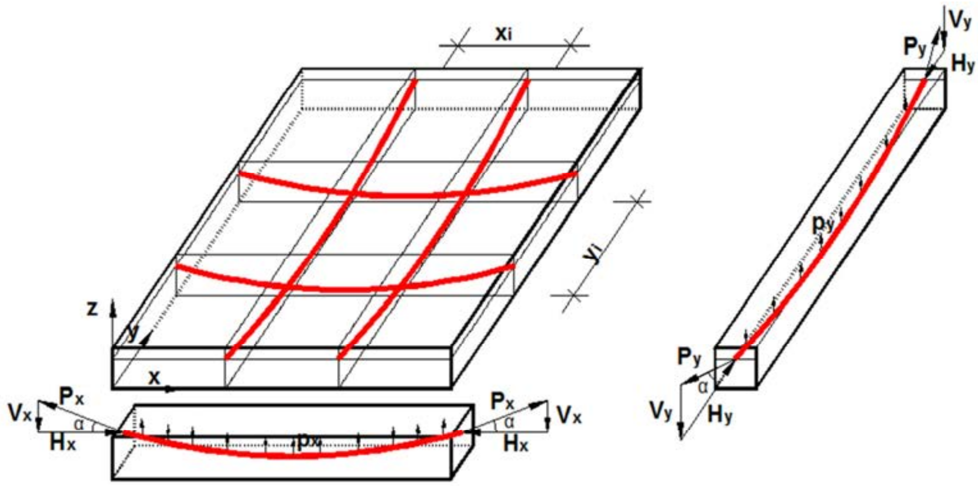

Fig. 4. Simply supported prestressed beam

The case analyzed in this paper presents a slab supported punctually in the corner panels, on columns.

Concrete cover:

According to [3] and [5] the minimum concrete cover is defined as it follows:

$$c_{min} = max\begin{Bmatrix} c_{min,b} \\ c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add} \\ 10mm \end{Bmatrix} \tag{7}$$

Where:

$c_{min,b}$ is the minimum layer for proper bonding; $c_{min,dur}$ is the minimum cover resulting from environmental conditions; $\Delta c_{dur,\gamma}$ additional safety cover, $\Delta c_{dur,st}$ the reduction of concrete cover in the case of stainless steel;

$\Delta c_{dur,add}$ the reduction of concrete cover in the case of additional protection;

Tolerances admitted by design are denoted with Δc_{tot}.

Therefore, the nominal concrete cover will be considered as it follows:

$$c_{nom} = c_{min} + \Delta c_{tot} = c_{min,dur} + \Delta c_{dur,\gamma} - \Delta c_{dur,st} - \Delta c_{dur,add} + \Delta c_{tot} \tag{8}$$

Considering that during the execution of the prestressed slab there is a quality control system having the possibility of checking the concrete cover, we have Δc_{tot}= 5 mm. We can also approximate the nominal diameter of the tendon to be 10mm, therefore we will take $c_{min,b}$= 10mm.

In equation (8), in the calculation of c_{min} has been considered the fact that the tendon is not made of stainless steel and considering the possible exposure of the element to chemically aggressive environment and also the concrete recipe may imply the use of additives. In conclusion, the value for the nominal concrete cover has resulted 30 mm.

The equivalent cross-section characteristics

The cross-section of a rib is presented in Fig. 5.

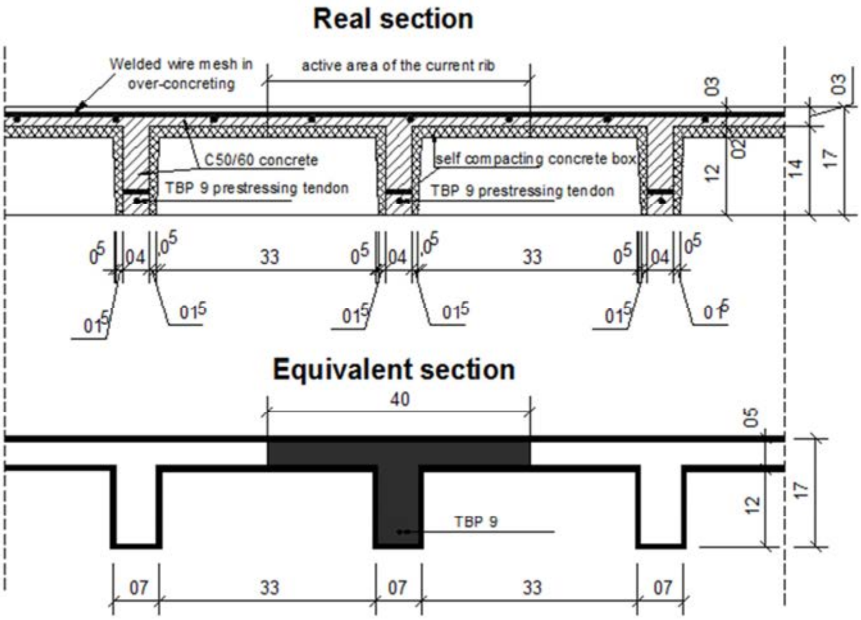

Fig. 5. Equivalent cross-section

$$h = 170\ [mm], b = 70\ [mm], h_p = 50\ [mm]$$

Concrete area $A_b = 2{,}84 \cdot 10^4\ [mm^2]$.
The equivalent cross-section centroid is denoted by $Y_g = 119{,}86\ [mm]$.
The equivalent cross-section moment of inertia $I_b = 56{,}986 \cdot 10^6\ [mm^4]$.
The sequence in which tendons are stressed

a b

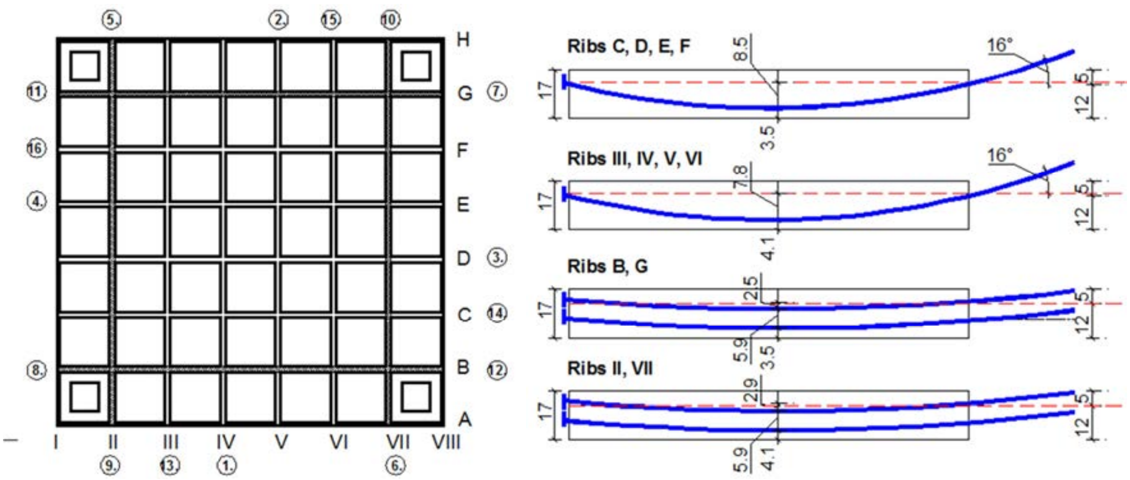

Fig. 6. (a)The sequence in which tendons are stressed; (b) The prestressing tendon layout

The way the values in the table are computed is the following.

Table 1. Geometrical characteristics of the prestressing layout.

Rib	Parabola equation[mm] $y = ax^2 + c$	α [°]	Arc radius,r [m]	Arc length l [m]	θ[°]
C,D,E,F	$4{,}28 \cdot 10^{-5}$-84	6°84'	11,7087	2,8067	14
III, IV, V, VI	$3{,}9796 \cdot 10^{-5}$-78	6°36'	12,6031	2,8058	13
B, G	$1{,}2755 \cdot 10^{-5}$-25	2°04'	39,2115	2,8006	4
II, VII	$0{,}9694 \cdot 10^{-5}$-19	1°55'	51,5884	2,8003	30

In the expression of the parabola equation, c is given by the " $Y_G - a_p$" expression, as it is noticed from Fig. 6(b), where a_p represents the distance from the centroid of the tendon cross section to the underside of the rib.

α is the angle between the tangent to the prestressing cable layout and the axis of the rib cross section centroid, and may be computed as it follows $\text{tg}\,\alpha = \frac{\partial y}{\partial x}$ when $x = 1400$ mm.

Radius r, circular arc length l and θ, were determined using AutoCAD software, by a 1:1 scale drawing of the considered element.

2.5. Computation of the prestressing force

Maximum force applied to the prestressing tendons.

In order to determine this force, it is necessary to determine the corresponding effort to a remanent deformation of 0.1mm in the prestressed reinforcement which in the considered case is the TBP9 tendon, denoted with $f_{p0,1k}$. This value, $f_{p0,1k}$ is generally given by the manufacturer of the tendon, can be determined experimentally, or, according to [3], "*if there is insufficient data[…] $f_{p0,1k}/f_{pk} = 0{,}9$*", allegation which frequently appears in technical literature [7],[8].Therefore, we can consider:

$$f_{p0,1k} = 0{,}9 \cdot f_{pk} \qquad f_{p0,1k} = 0{,}9 \cdot 1760 = 1584\ [MPa] \tag{9}$$

Knowing that the area of the tendon is A_p=49 [mm^2], according to [8], [3], and [5], the prestressing force reinforcement can be computed:

$$P_{max} = A_p \cdot \sigma_{p,max} \tag{10}$$

$\sigma_{p,max}$ is the maximum stress in the reinforcement and is determined with the following relation:

$$\sigma_{p,max} = min\,\{k_1 \cdot f_{pk};\ k_2 \cdot f_{p0,1k}\} = \sigma_{max} = min\,\{0{,}8 \cdot 1760;\ 0{,}9 \cdot 1584\} = 1408\ [MPa] \tag{11}$$

In conclusion, the maximum force applied to the prestressing reinforcement is $P_{max} \approx 69\ [kN]$

Initial prestressing force at the time $t=t_o$

The initial prestressing force is considered to be applied to the concrete immediately after tensioning and anchorage. According to [3], it cannot be greater than

$$P_{m0}(x) = A_p \cdot \sigma_{pm0}(x) \tag{12}$$

where:

$$\sigma_{pm0}(x) = min\{k_7 \cdot f_{pk};\ k_8 \cdot f_{p0,1k}\} = min\{0{,}75 \cdot 1760;\ 0{,}85 \cdot 1584\} = 1320[MPa] \quad (13)$$

Lastly, the initial prestressing force is $P_{m0}(x) \approx 65\ [kN]$.

3. Conclusions

One way prestressed slabs have disadvantages such as: large slab thicknesses, a relatively insufficient ductilization under exceptional loads due to statical determinacy and do not increase torsional moment resistance, while two-way prestressed slabs eliminate these disadvantages and allow the compliance with other requirements such as achieving perfectly plane surface. Due to the fact that in our country the post-tensioned slabs for linear elements are mostly used, the experimental study which will follow the presented paper may be considered a useful tool for understanding the post-tensioning of surface elements. In this paper has been presented the design, calculation and layout, of prestressing reinforcement (TBP9 tendons) that are to be placed in the ribs of a waffle type slab. It is mentioned the sequence the tendons are stressed, the geometrical characteristics of their layout and is presented the computation of prestressing force in terms of maximum force applied to the TBP9 tendons and initial prestressing force at time $t=t_o$. It is to be mentioned that the angular change in tendon profile causes a transverse force on the member which 'balances' structural dead loads.

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