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Qiang LuoDongyun YiWenqiang Yang

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Qiang Luo, Dongyun Yi and Wenqiang Yang
Department of Mathematics and Systems Science, College of Science, National University of Defense Technology, Changsha, China

Abstract

Purpose – The purpose of this paper is to answer the question that what the best shape of fuzzy sets is in fuzzy systems for function approximation which is essential in many applications of fuzzy systems.

Design/methodology/approach – The uniform approximation rates indicate the approximating capabilities of fuzzy systems for function approximation. By Fourier analysis, the uniform approximation rates are estimated for the fuzzy systems with various shapes of if-part fuzzy sets in the case of single-input and single-output. Based on the approximation rates, the relationships between the approximating capabilities and the shapes of fuzzy sets are developed and compared.

Findings – The since functions as the input membership functions in fuzzy systems are proved to have the almost best approximation property in a particular class of membership functions.

Research limitations/implications – From the viewpoint of function approximation, the input membership functions are not necessarily positive in fuzzy systems.

Practical implications – For engineers, the sinc-shaped membership function is a good choice to improve their fuzzy systems in real applications.

Originality/value – The uniform approximation rates of fuzzy systems for function approximation are estimated. Mathematically, the relationships between the approximating capabilities and the shapes of fuzzy sets are analyzed for fuzzy systems.

Keywords Fuzzy control, Cybernetics, Systems theory, Approximation theory

Paper type Research paper

1. Introduction

Fuzzy systems are widely used today (Huang *et al.*, 2006; Chen, 2005; Cheng and Li, 2004; Zeng and Singh, 1996a), and their theoretical foundation attracted many scholars' attention. The relationship between the fuzzy sets (i.e. the membership functions) and the approximation rates of the fuzzy systems in function approximation is one of the important research areas (Zeng and Singh, 1996a; Mitaim and Kosko, 2001). The earliest work about this relationship may be published by Zeng and Singh (1996a). They discussed the pseudo trapezoid shape membership functions. In 2001, Mitaim and Kosko (2001) discussed what the best shape for fuzzy sets in function approximation is. By exploring a wide range of candidate if-part sets and deriving supervised learning laws that tune them, they found that no set shape emerges as the best shape. In Mitaim and Kosko (2001), they also showed that the sinc function $\sin(x)/x$

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often converges fastest and with greatest accuracy among their candidates, but we still do not know the theoretical reason for its performance in the fuzzy systems.

Hence, in this paper, we assume that the training data are sampled by some distribution function. And the approximation rates of the fuzzy systems could be estimated by employing the theories which are similar to the theories used in the probabilistic neural networks (PNN) (Specht, 1990), and the general regression neural networks (GRNN) (Specht, 1991). Most PNN and GRNN only use the Gaussian function as the kernel, but there are many choices of the fuzzy sets in the fuzzy systems. So we considered many types of the shape of the fuzzy sets in this paper. Based on the approximation rates, the relationship between the various shapes of the input membership functions and the uniform approximation rates can be studied.

In the following of this paper, we suppose that the analytical form of a continuous function $f(x)$ defined on the closed interval $U \subset \mathbf{R}$ is unknown but the input-output behavior of $f(x)$ for any $x \in U$ is known, so the function $f(x)$ is similar to a black-box (Wang, 1997). Without loss of generality, we also specify that $f: U = [-1, 1] \rightarrow V \subset \mathbf{R}$, where V is a bounded subset of $\mathbf{R}$. And the input-output data of f on U are $(x_i, y_i)_{i=1}^n$, where $(x_i)_{i=1}^n$ is considered as the random sample of size n from the absolute continuous distribution function $H(x)$ with the density function $h(x)$. Although the training sample $(x_i)_{i=1}^n$ could be the random sample from any distribution function, in this paper $h(x)$ is determined by the denominator of the fuzzy system and is only required to be bounded away from 0 in U by some $\varepsilon_0 < 0$, i.e. $h(x) > \varepsilon_0$ for any $x \in U$. Based on the training data, the fuzzy system $F_n(x)$ approximates the function $f(x)$. Theoretically, the size of the random sample can be as large as we want. We will estimate the uniform approximation rate for the fuzzy system, where this rate is defined by $\|F_n(x) - f(x)\|_\infty = \sup_{x \in U}|F_n(x) - f(x)|$. Based on the approximation rates, the fuzzy systems with various shapes of fuzzy sets are analyzed and compared. Specially, the sinc functions as input membership functions of fuzzy systems are proved to have the almost best approximation property in a class of input membership functions.

2. Fuzzy systems and their input membership functions

In this section, we introduce the mathematical formula of the fuzzy systems in the single-input and single-output (SISO) case. With n rules, the fuzzy system which comprises four principal components: singleton fuzzifier, product inference engine, center-average defuzzifier (see Zeng and Singh (1996b) for more details), and its rule base stores n rules:

$$R_i: \text{IF } x \text{ is } A_i, \text{THEN } y = y_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $x \in U$ is the input variable, A_i is the if-part set, and y_i is the point in V at which the then-part set $B_i(y)$ achieves its maximum value (when B_i is a singleton fuzzy set, $B_i(y) = 1$). Let $\mu_i: \mathbf{R} \rightarrow [0, 1]$ be the fuzzy membership function corresponding to the if-part set A_i. In the SISO case the fuzzy system can be expressed as follows:

$$F_n(x) = \sum_{i=1}^{n} \left[\frac{\mu_i(x)}{\sum_{j=1}^{n} \mu_j(x)} \right] y_i. \tag{2}$$

To construct the fuzzy system from the random sample $(x_i, y_i)_{i=1}^n$, we first define the ε_0- completeness for the if-part sets A_i $(i = 1, \ldots, n)$.

Definition 1. Fuzzy sets A_i $(i = 1, \ldots, n)$ as a partition on U are said to be ε_0-complete, if $\exists \varepsilon_0 > 0$, such that:

$$\frac{1}{n}\sum_{k=1}^{n} \mu_i(x) \geq \varepsilon_0(\forall x \in U).$$

In practical applications, any $x \in U$ is a possible input and should have a corresponding output. As in Zeng and Singh (1996b), we assume that the if-part partition must be an ε_0-complete partition on U. This means that at least one of the fuzzy IF-THEN rules should be fired for every $x \in U$. So when the partition is ε_0-complete, the denominator of equation (2) is bounded away from zero for all $x \in U$. Hence, the fuzzy systems are well defined.

The input membership functions $\mu_i(\circ)(i, \ldots, n)$ can have many types of shapes, and each shape affects how well a fuzzy system of IF-THEN rules approximates a target function (Mitaim and Kosko, 2001). Many classes of membership functions have been proposed in literatures (Hassine *et al.*, 2003), including the triangular functions, the normal peak functions, the pseudo trapezoidal functions, the β functions, and so on. In this paper, the input membership functions of the fuzzy systems are generated by the translations and scale transformations of one appropriately fixed scalar set function. Some related notations, concepts and properties that will be useful in this discussion are introduced below.

Definition 2. Let σ be a positive parameter tending to zero. The class of functions $\{\mu(\circ;\sigma), \sigma > 0\}$ is the kernel on the real line, if $\forall \sigma > 0$, $\mu(\circ;\sigma) \in NL_1$. The $\mu(\circ;\sigma)$ is symmetric or positive, if $\forall \sigma > 0$, $\mu(x;\sigma) = (-x;\sigma)$ or $\mu(x;\sigma) \leq 0$.

The kernel $\mu(x;\sigma)$ defines the singular integral as:

$$I_\mu(f;x;\sigma) = \int_{\mathbf{R}} f(u)\mu(x-u;\sigma)du.$$

Definition 3. The kernel $\{\mu(\circ;\sigma), \sigma > 0\}$ is approximation identity kernel, if $\exists M > 0$ and $\sigma > 0$, such that:

$$\|\mu(\circ;\sigma)\|_1 \leq M, \lim_{\sigma\to 0}\int_{\delta\leq|u|} |\mu(\circ;\sigma)| = 0.$$

Definition 4. The $\{\sigma_n\}$ is regularizing scale factor, if $\forall n \in \mathbf{N}$, such that $\sigma_n > 0$ and the series:

$$\sum_{n=1}^{\infty} \exp(-\eta n \sigma_n)$$

converge for every $\eta > 0$.

Definition 5. Let μ be a given function defined on U. The translations and scale transformations of μ are defined as $(1/\sigma)\mu(x-\alpha)/\sigma$ where $x \in U$, α is the translation factor and σ is the scale factor.

Lemma 1. Let $\mu \in L_1$ and:

$$\int_R \mu(x)dx \neq 0,$$

then the kernel of the form:

$$\left\{\frac{1}{\sigma}\mu\left(\frac{x}{\sigma}\right), \quad \sigma > 0\right\}$$

confirms an approximation identity kernel.

The proof of Lemma 1 is omitted here for the reason of space. Using Definition 5 and Lemma 1, the scale transformations of a function $\mu \in L_1$ confirm an approximation identity kernel $\{\mu(x;\ \sigma_n),\ \sigma_n > 0\}$ with the regularizing scale factors $\{\sigma_n\}$ in a particular form:

$$\mu(x; \sigma_n) = \frac{1}{\sigma_n}\mu\left(\frac{x}{\sigma_n}\right). \tag{3}$$

If the translations $\mu(x - x_i;\ \sigma_n)$ of equation (3), which are generated by the sample $(x_i)_{i=1}^n$, construct an ε_0-complete partitions on U, then by replacing $\mu_i(x)$ with $\mu(x - x_i; \sigma_n)$ we can construct the fuzzy systems as follows:

$$F_n(x) = \sum_{i=1}^{n}\left[\frac{(1/\sigma_n)\mu((x - x_i)/\sigma_n)}{\sum_{j=1}^{n}(1/\sigma_n)\mu((x - x_j)/\sigma_n)}\right]y_i \equiv \sum_{i=1}^{n}\left[\frac{\mu_n(x - x_i)}{\sum_{j=1}^{n}\mu_n(x - x_j)}\right]y_i, \tag{4}$$

where $x \in U$ and $(y_i)_{i=1}^n$ are the corresponding outputs of $(x_i)_{i=1}^n$.

3. The uniform approximation rates of fuzzy systems

Definition 6. If α is a positive number, the αth order absolute moment of μ can be defined as follows:

$$m(\mu; \alpha) = \int_{\mathbf{R}} |u|^\alpha |\mu(u)| du.$$

Lemma 2. Let $\mu \in NL_1$ be a symmetric function, for some $0 < \alpha \leq 2$, whose αth order absolute moment exists and is finite (Butzer and Nessel, 1971). Then $\forall f \in Lip^*(C(U); \alpha)$:

$$\|I_\mu(f; \circ; \rho) - f(\circ)\|_{X(U)} = O(\rho^{-\alpha})(\rho \to \infty),$$

where the kernel $\{\mu(x;\ \rho)\}$ must be in the form $\{\rho\mu(\rho x)\}$.

Theorem 1. Let the kernel $\mu(\circ; \sigma_n)$ in equation (4) be symmetric. If for some $0 < \alpha \leq 2$ the αth order absolute moment of the kernel exists and is finite, then $\forall f \in Lip^*(X(U); \alpha)$:

$$\|F_n(x) - f(x)\|_\infty \leq \frac{C_1}{\varepsilon_0 \sigma_n}\left(\frac{\log n}{n}\right)^{1/2} + \frac{C_2}{\varepsilon_0} \cdot O(\sigma_n^\alpha) \text{ a.s.}$$

The proof of Theorem 1 is omitted here for the reason of space. Under the conditions of Theorem 1, it is clear that:

$$\|F_n(x) - f(x)\|_\infty \leq O\left(\left(\frac{\log n}{n}\right)^{\alpha/2(1+\alpha)}\right), \quad \text{a.s.}$$

when the $\sigma_n \simeq ((\log n)/n)^{1/2(1+\alpha)}$.

Corollary 1. Suppose that the conditions of Theorem 1 hold, and $h(x)$ is an uniform density function on U, choosing $\sigma_n \simeq ((\log n)/n)^{1/2(1+\alpha)}$, then $\forall f \in Lip^*(C(U); \alpha)$:

$$\|F_n(x) - f(x)\|_\infty \leq O\left(\left(\frac{\log n}{n}\right)^{\alpha/2(1+\alpha)}\right), \quad \text{a.s.}$$

4. Comparisons among several input membership functions

In this section, based on the uniform approximation rates for the fuzzy systems as function approximators, we show the theoretical relationships between the different performances of fuzzy systems and the different shapes of input membership functions, including triangle, trapezoid, Gaussian, quadratic set function, Laplace set functions, the π function, the Cauchy function, and so on. The definitions of the candidates are listed in Table I. One can easily verify that these membership functions all satisfy the following conditions:

- $0 \leq \mu(x) \leq 1, \forall x \in \mathbf{R}$;
- $\mu \in L_1$ and the norm of μ is not zero; and
- μ is symmetric.

These functions can be examples of the functions which satisfy the conditions of the input membership functions in the fuzzy systems equation (4). To show the relationships between the function approximation performance of fuzzy systems and the different shapes of input membership functions, the theorem can be used by simply validating the conditions in these theorems. Table II lists the validating results of the above candidates.

So when $0 < \alpha \leq 2$, the uniform approximation rates of fuzzy systems can be estimated by Corollary 1 as $O((\log n)/n)^{\alpha/2(1+\alpha)}) \forall f \in Lip^*(C(U); \alpha)$. From Table II, we can see that even the first order absolute moment of μ_8 or μ_9 does not exist, but for any $0 < \alpha < 1$, the αth order absolute moments of μ_8 and μ_9 exist and are finite. Compared with the αth order absolute moments of the candidates, the differences among them are the coefficients of σ_n^α. Now the uniform approximation rates for fuzzy systems with the candidates as input membership functions can also be estimated by Corollary 1. When $0 < \alpha < 1$, the uniform approximation rate is $O((\log n)/n)^{\alpha/2(1+\alpha)})$ ($\forall f \in Lip^*(C(U); \alpha)$) by properly choosing σ_n. That is to say, for the performance of the function approximation by fuzzy systems with different candidates functions as their input membership functions both the order of the absolute moment of the candidates and the order of the Lipschitz condition of the target functions are involved.

The last candidate the sinc function is very different from the others: the αth order absolute moment of μ_{10} does not exist for any $\alpha > 0$ so the above theorems cannot be applied. But as presented by Mitaim and Kosko (2001), this kind of set functions seems

Triangle set function $\mu_1(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$	*Trapezoid set function* $\mu_2(x) = \begin{cases} 2x+2, & -1 \le x \le -0.5 \\ 1, & -0.5 < x < 0.5 \\ -2x+2, & 0.5 \le x \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$
Gaussian set function $\mu_3(x) = \exp((-1/2)x^2)$	*Quadratic set function* $\mu_4(x) = \begin{cases} 1-x^2, & -1 \le x \le 1 \\ 0, & \lvert x\rvert > 1 \end{cases}$
Laplace set function $\mu_5(x) = \exp(-\lvert x\rvert)$	*π set function* $\mu_6(x) = \begin{cases} 0, & x < \frac{-3}{2} \\ \frac{8}{9}\left(x+\frac{3}{2}\right)^2, & \frac{-3}{2} \le x \le \frac{-3}{4} \\ 1-\frac{8}{9}x^2, & \frac{-3}{4} < x \le \frac{3}{4} \\ \frac{8}{9}\left(x-\frac{3}{2}\right)^2, & \frac{3}{4} < x \le \frac{3}{2} \\ 0, & \lvert x\rvert > \frac{-3}{2} \end{cases}$
H4 set function $\mu_7(x) = \begin{cases} \frac{9}{8}\left(1-\frac{5}{3}x^2\right), & \lvert x\rvert \le 1 \\ 0, & \lvert x\rvert \le 1 \end{cases}$	*Fejer set function* $\mu_8(x) = \begin{cases} \left[\frac{\sin(x/2)}{x/2}\right]^2, & x \ne 0 \\ 1, & x = 0 \end{cases}$
Cauchy set function $\mu_9(x) = 1/(1+x^2)$	*Sinc set function* $\mu_{10}(x) = \begin{cases} \frac{\sin(x)}{x}, & x \ne 0 \\ 1, & x = 0 \end{cases}$

Table I. The candidates of the input membership functions

to have some advantages in function approximation. Next, we will prove that the sinc functions as input membership functions have the almost best approximation property in a certain class of kernels.

Let Ω denote the set of kernels satisfying the following conditions:

- μ is continuous and $\mathrm{supp}(\mu) \subset U$.
- For some $0 < \alpha \le 2$, $m(\mu, \alpha)$ exists and is finite.
- μ is symmetric.
- Fourier reverse transformation ofμ exists and is denoted by $\mu^\vee$.

Choose $\mu \in \Omega$ to be the input membership function and the fuzzy system constructed as equation (4) is denoted by $F_n(x;\ \mu)$ With uniform density function $h(x)$ and regularizing factor σ_n which satisfies the conditions in Theorem 1, $\forall f \in Lip^*(C(U); \alpha)$ the uniform approximation rate can be estimated:

Table II.
The validating results of the candidates for the conditions in Theorem 1

$1/\sigma_n\mu(x/\sigma_n)$	$m(\mu;\alpha), \quad 0<\alpha<1$	$m(\mu;1)$	$m(\mu;2)$
μ_1	$2\sigma_n^\alpha/(2+3\alpha+\alpha^2)$	$\sigma_n/3$	$\sigma_n^2/6$
μ_2	$(2^{-\alpha}(-1+2^{2+\alpha})\alpha_n^\alpha)/(2+3\alpha+\alpha^2)$	$7\sigma_n/18$	$5\sigma_n^2/24$
μ_3	$2^{(1/2)+(\alpha/2)}\Gamma(1+\alpha/2)\sigma_n^\alpha$	$\sigma_n\sqrt{2/\pi}$	σ_n^2
μ_4	$(4\sigma_n^\alpha)/(3+4\alpha+\alpha^2)$	$3\sigma_n/8$	$\sigma_n^2/5$
μ_5	$2\Gamma(1+\alpha)\sigma_n^\alpha$	σ_n	$2\sigma_n^2$
μ_6	$(3^{1+\alpha}4^{-\alpha}(-1+2^{2+\alpha})\sigma_n^\alpha)/(6+11\alpha+6\alpha^2+\alpha^3)$	$7\sigma_n/16$	$9\sigma_n^2/32$
μ_7	$((-6+23^{((3+\alpha)/2)}5^{((-2-\alpha)/2)}+3\alpha)\sigma_n^\alpha)/(2(3+4\alpha+\alpha^2))$	$39\sigma_n/80$	$\sqrt{3/59}/25\sigma_n^2$
μ_8	$-4\sigma_n^\alpha\Gamma(-1+\alpha)\sin(\alpha\pi/2)$	–	–
μ_9	$\sigma_n^\alpha \sec\alpha(\alpha\pi/2)$	–	–
μ_{10}	–	–	–

Note: Let – denote that the integral does not exist and the γ function is $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$.

$$\|F_n(x;\mu)-f(x)\|_\infty \le \frac{C_1}{\varepsilon_0\sigma_n}\left(\frac{\log n}{n}\right)^{1/2}+\frac{C_2}{\varepsilon_0}\cdot O(\sigma_n^\alpha) \text{ a.s.} \tag{5}$$

Let:

$$I_\mu(f;x;\sigma_n)\equiv\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x-u)\frac{1}{\sigma_n}\mu\left(\frac{u}{\sigma_n}\right)du=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f^\wedge(v)e^{ixv}\mu^\vee(\sigma_n v)dv; \tag{6}$$

$$S(f;x;\sigma_n)\equiv\frac{1}{\pi}\int_{-\infty}^{\infty}f(x-u)\frac{\sin(u/\sigma_n)}{\sigma_n}\,du=\frac{1}{\sqrt{2\pi}}\int_{-\sigma_n^{-1}}^{\sigma_n^{-1}}f^\wedge(v)e^{ixv}dv. \tag{7}$$

And for a given n the best singular integral approximation error f achieved by $\mu\in\Omega$ is:

$$E_n(C(U);f)=\inf_{\mu\in\Omega}\|I_\mu(f;x;\rho)-f(x)\|_\infty. \tag{8}$$

Theorem 2. $\forall f\in Lip^*(C(U);\alpha)$, $\exists$ constants C_1 and C_2, such that:

$$\|F_n(x;\mu)-f(x)\|_\infty \le \frac{C_1}{\varepsilon_0\sigma_n}\left(\frac{\log n}{n}\right)^{1/2}+\lambda(\sigma_n)E_n(C(U);f), \text{ a.s.}$$

where:

$$\lambda(\sigma_n)=|\mathrm{IntSic}(\sigma_n)|+C_2\sigma_n+1.$$

The proof of Theorem 2 is omitted here for the reason of space. The above discussion shows that the relationships between the uniform approximation rates for fuzzy systems and the shapes of the input membership functions of fuzzy systems are too complex to tell in general which type of the shapes of input membership functions is the best type for fuzzy systems as function approximators, because the continuous condition of the target functions and the existence condition of the absolute moment of the input membership functions are both involved. But the sinc functions as input membership functions of fuzzy systems have the some advantages in a certain class of membership functions.

5. Conclusions

In this paper, the fuzzy systems are constructed with random training data through translations and scale transformations of some fixed kernels. When the partition generated by the sample $(x_i)_{i=1}^{n}$ is an ε_0-complete partition on U, we estimated the uniform approximation rates using Fourier analysis technologies. Based on these rates, the approximation accuracy of the fuzzy systems with various types of input membership functions has been analyzed and compared. Furthermore, the almost best approximation property of sinc functions as input membership functions has been obtained for a certain class of kernels.

Our results indicate that both the continuity of the target functions and the existence of the αth order absolute of the input membership functions affect the performance of fuzzy systems as function approximators. For the commonly used positive membership functions, we give the uniform approximation rates of fuzzy systems. However, from the viewpoint of function approximation, the input membership functions are not necessarily positive. For example, the sinc function is not positive and the approximation accuracy of the fuzzy systems using sinc as the input membership function is better than using most of the positive kernels. One open question here is to choose an input membership function μ whose αth order absolute moment exists to achieve the best function approximation rate by the fuzzy systems for a given target function f. We think the lower bound of the approximation error of f by the fuzzy systems can shed some light.

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Corresponding author
Qiang Luo can be contacted at: luoqiang@nudt.edu.cn

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