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# Combining fuzzy logic and eigenvector centrality measure in social network analysis



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### HIGHLIGHTS

- A combined algorithm of fuzzy inference system and eigenvector centrality is proposed.
- Social interactions are measured by different factors with different weights.
- The influencing factors in a social network are used to weight the friendship strength using the fuzzy logic.
- The most influential person is calculated using eigenvector centrality after feeding it with the fuzzy logic results.
- The method is applied on large data sets such as Facebook, Epinions, and Slashdot-zoo website.

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#### ABSTRACT

The rapid growth of social networks use has made a great platform to present different services, increasing beneficiary of services and business profit. Therefore considering different levels of member activities in these networks, finding highly active members who can have the influence on the choice and the role of other members of the community is one the most important and challenging issues in recent years. These nodes that usually have a high number of relations with a lot of quality interactions are called influential nodes. There are various types of methods and measures presented to find these nodes. Among all the measures, centrality is the one that identifies various types of influential nodes in a network. Here we define four different factors which affect the strength of a relationship. A fuzzy inference system calculates the strength of each relation, and using this matrix eigenvector measure calculates the most influential node. Applying our suggested method resulted in choosing a more realistic central node with consideration of the strength of all friendships.

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#### 1. Introduction

Social network members interact and communicate in a self-organizing globally coherent pattern that appears in and makes up the system [1,2]. These patterns become more apparent as network size increases. A global network analysis of highly related networks is not feasible due to variety and quantity of information that causes them to be uninformative. The structure of relationships between social entities can also be examined by social network analysis techniques. Large things such as persons, groups, organizations, websites or scholarly publications can be encountered as entities of a social network. Considering the growth of social networks applications of new types and methods of marketing are introduced. In Ref. [3]

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the importance of content marketing in social networks such as Facebook is described. It mentions that popular contents shared among the members can guide to intelligent advertisement and marketing for specific goods which can result in better sales and business profit.

Different fields of study like sociology utilize the obtained information from graphs by applying centrality measures. These sets of information include the relative importance of nodes and edges. Eigenvector centrality is an example of such method that uses the eigenvectors of the adjacency matrix of the network and determines most frequently visited nodes. Some of the formally established measures of centrality are betweenness, eigenvector, degree, closeness, and Katz centrality [4]. Generally, the purpose or objective of the analysis determines the type of chosen centrality measure [5,2]. There are different studies in the literature applying these methods analyzing social networks.

In some networks node explicitly states the group memberships. In Ref. [6] 230 of such real-world networks were studied. Based on the study, a methodology is presented to compare and evaluate the difference of structural definitions of network communities that correspond to ground-truth communities. Afterward, their sensitivity, robustness, and performance in identifying the ground-truth are examined. In the subject of the semantic web and the level of trust of each source, properties which merge such trusts, and a class of functions are discussed and defined in Ref. [7]. In this work, the functions are applied to data from Epinions and BibServ [8].

The stability of betweenness centrality (BC) is evaluated in Ref. [9]. A metric is used to measure the importance of the vertices in the network and introduces a group testing algorithm. The results of this study show how the ranking of the vertices changes as the networks are perturbed and the algorithm can correctly identify the high valued BC vertices of stable networks. A parametric fuzzy closeness measure which allows relaxation of the condition of all other nodes is presented in Ref. [10]. This measure is defined for unweighted networks and evaluations on real and exemplary networks indicate that new information is provided by the fuzzy measure for closeness centrality in networks that are not provided by the classical measures. This measure is more robust to observation errors in the network. An approximation of betweenness centrality is studied and defined in Ref. [11]. The purpose is to build a predictive model of social networks. The methodology describes a bounded distance approximation of betweenness centrality designed for implementation within a parallel architecture. In Ref. [12], a network flow topology based on multi-dimensional variation is presented. The dimensions are trajectories that traffic may follow and the method of spread. Measures of centrality are matched to the kinds of flows that they are appropriate for. One of the most recent works on real social networks is illustrated in Ref. [13]. The authors consider signed variants of global network characteristics such as the clustering coefficient, node-level characteristics, and link-level characteristics on a technology news website called Slashdot. The relations between members are described as friends and foes based on the positive or negative endorsement. Eigenvectors of adjacency matrices are misapplied to asymmetric networks in which some positions are unchosen. An alternative measure of centrality is suggested in Ref. [14] for these networks that equal an eigenvector. A new formulation for node-centrality is presented in Ref. [15]. The results of this study show that some property satisfaction of eigenvector centrality measure depends on normalization. The bootstrap sampling procedures are used in Ref. [16] in order to determine how sampling affects the stability of different network centrality measures. Some of the well-known measures such as high-degree, betweenness, closeness, eigenvector, PageRank are presented and evaluated with ten data sets in Ref. [17]. According to the results, new centrality measure called DegreeDistance is presented. This measure chooses high-degree seeds in an appropriate distance from each other. Results indicate that some measures are more stable than others and that stability is also a function of network and study properties. Real eigenvector and eigenvalue of a real matrix are discussed in Ref. [18]. In this work, Tian extended the real eigenvector of a real matrix to fuzzy eigenvectors. In this work, we combine the fuzzy logic and eigenvector degree centrality measures and present a model which results in many realistic inferences. The rest of the paper organized as follows; Section 2 describes and compares different centrality measures. We illustrate the new model in Section 3. Section 4 talks the experimental results and finally Section 5 draws a conclusion.

## 2. Methods

Network analysis takes advantage of different types of methods. These methods assess different aspects of the network and try to clarify some properties about entities in the network. Centrality methods are indicators of the most important vertices within a graph. Some of the interesting applications include identifying the most influential person or favorite content in a social network, a spread of disease, and new marketing opportunities. Centrality concepts were first developed in social network analysis, and the defined terms are reflecting the sociological origin of such networks in most a proper way [14]. Here we introduce different types of centralizers. The degree centrality is defined as the number of links incident upon a node. The degree can be interpreted in terms of the opportunity of the node to catch the flowing interactions in the network. The degree centrality of a vertex v for graph G is defined as below, where V and E indicate vertices and edges, respectively.

$$C_D(v) = \deg(v).$$

Closeness centrality is another method used in connected graphs. The natural distance between all pairs of nodes is metric used in this method. This metric is based on the length of the shortest paths. The closeness of a node is defined by Bavelas

as the reciprocal of the farness which is defined as the sum of node's distances from all other nodes.

$$C(x) = \frac{1}{\sum_{y} d(y, x)}.$$

Therefore, the more central is a node, the lower is its total distance from all other nodes [19].

$$H(x) = \sum_{y \neq x} \frac{1}{d(y, x)}.$$

Betweenness centrality measure quantifies the number of occurrence of the node as a bridge along the shortest path between two other nodes. The betweenness of a vertex v in a graph G := (V, E) is computed as follows [20]:

$$C_B(v) = \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

where  $\sigma_{st}$  is the number of shortest paths from node *s* to *t* (also known as information pathways), and  $\sigma_{st}(v_i)$  is the number of shortest paths from *s* to *t* that pass through  $v_i$ . Eigenvector centrality is a measure of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node than equal connections to low-scoring nodes. Google utilizes a variant of the eigenvector centrality as its PageRank method. Another related centrality measure is Katz centrality [4] which uses the adjacency matrix to find eigenvector centrality. Let  $A = (a_{v,t})$  be the adjacency matrix or a given graph G := (V, E). If vertex *v* is linked to vertex *t*,  $a_{v,t} = 1$  and otherwise  $a_{v,t} = 0$ . The centrality score of vertex *v* is defined as:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in G} a_{v,t} x_t$$

here M(v) indicates set of v's neighbors and  $\lambda$  is a constant. In vector notation the eigenvector equation can be rewritten as below:

 $Ax = \lambda x.$ 

Bonacich [14] suggested that the eigenvector of the largest eigenvalue of an adjacency matrix could make a good network centrality measure. Unlike degree, the eigenvector weights are according to their centralities. Eigenvector centrality can also be seen as a weighted sum of not only direct connections but indirect ones of any length [16]. Eigenvectors measure has numbers of advantages over conventional graph theory-based measures of centrality: it can be used with valued or signed graphs; it can be used for negatively connected exchange networks; it allows for variations in the degree. When applied to standard binary-valued graphs these measures are especially sensitive to situations in which a high degree node is connected to many low degree nodes or a low degree node is connected to a few high degree nodes. The measure is, therefore, distinctively appropriate when centrality is ultimately driven by differences in degree. Tian [18] tries to find a relationship between real eigenspace and fuzzy eigenspace. He presents and proves a sufficient condition to existence of solution vector of fuzzy linear system  $A\tilde{x} = \tilde{y}$  where A is a crisp square matrix,  $\tilde{x}$  and  $\tilde{y}$  are fuzzy vectors. Here we feed a Fuzzy inference system a fuzzy matrix in which the elements represent a level of interactions between each member in the community and the resulted matrix that is used in fed to eigenvector algorithm is a real matrix with crisp values. As it can be inferred from different centrality methods, they generally consider the connectivity of nodes in the network but do not give so much information about the quality of the relations. In order to include the quality of relations in these methods, we need another inference system. Here we combine fuzzy inference system with eigenvector centrality and prestige method to calculate the central node in the network. Fuzzy inference system with the power of descriptive variables can be so useful in order to describe the quality of the relations. We run the algorithm on a sample community of Facebook to evaluate the correctness of the results.

#### 3. Algorithm

As we mentioned before, previous methods do not consider the quality of relations in the network analysis. Different types of activities are thought important in social networks. In Facebook, for example, people post life events or ideas, like and comment on posts, tag each other in photos, and mention some people in a post. These activities are four effective factors in the quality of relations. Considering the architecture of the Twitter network, [21] measured user influence using three measures: indegree, retweets, and mentions. They clarified that popular users who have high indegree are not necessarily influential in terms of spawning retweets or mentions and influence is not gained spontaneously or accidentally [5]. Then the people who have much stronger influence in a social community, have more activities in terms of mentioning, tagging, reviewing others timeliness, commenting on life events, and sharing opinions. It can be concluded that the more people mention or tag each other in their posts, the closer they are. On the other hand, the relation strength between two people cannot be described as a crisp value and it is usually a comparative fuzzy description. As a result, the centrality methods do not reflect the *real* importance of each node in the network.

		2								
1	0.59	6.36	0.59	6.96	5.03	0.59	0.59	7.81	8.72	8.93 8.20
2	6.38	0.59	6.36	7.92	0.59	1.33	0.59	0.59	6.38	8.20
3	0.59	1.33	0.59	0.59	0.59	0.59	0.73	0.75	0.59	0.59
4	7.81	6.39	0.59	0.59	6.37	6.38	6.44	8.16	6.96	7.54
5	6.37	0.59	0.59	6.95	0.59	0.59	5.03	0.59	0.59	0.59
6	0.59	8.85	0.59	8.72	0.59	0.59	8.63	8.90	0.59	0.59
7	0.59	0.59	8.63	8.90	8.93	8.93	0.59	8.27	8.86	8.94
8	8.63	0.59	8.90	8.93	0.59	8.93	8.27	0.59	0.59	8.94
9	8.63	8.90	0.59	8.93	0.59	0.59	8.93	0.59	0.59	8.27
10	8.63	8.90	0.59	8.93	0.59	0.59	8.93	8.27	8.86	0.59

Fig. 1. Modified Adjacency Matrix (MAM) for 10 node network.

Eigenvector centrality uses an adjacency matrix in which 1 in each element indicates friendship between corresponding persons and 0 indicates no friendship. This matrix does not give any information about the strength of the relations. The degree centrality measure ranks nodes with more connections higher in terms of centrality. In directed graphs, we can either use the in-degree, the out-degree, or the combination of the degree centrality value. The interactions in a community are bidirectional, therefore, the corresponding graph of the network is a directed graph. The presented algorithm (which from now on we call it the Model) combines eigenvector centrality method and out-degree centrality value with fuzzy inference system and generates many meaningful results for the most influential person in the community.

Here we take Facebook as the network and try to find the most influential person in the community. Referring to Refs. [5,22], We limit the interactions between friends to four types of activities: liking posts, commenting on posts, tagging someone in a post, and mentioning someone in a post. These four interactions are considered as four influencing factors in a relation. Therefore, each element of adjacency matrix contains four values, one for each attribute. Each value is normalized to the scale of 100 posts for each person. For example when attribute values for *i*th node and *j*th node are [90, 70, 45, 30], they indicate that 90% of comments of *j*th person are liked and 70% are commented on by *i*th person, *j*th person has tagged and mentioned *j*th person in 45%, 30% of his comments, respectively. For large data sets such as Facebook, Twitter, Instagram using available APIs and tweaking them, enables us to extract not only the related nodes but also types and a number of interactions such as tagging and commenting in Facebook or retweeting and mentioning in Twitter.

First we generate the original adjacency matrix, *A*, and replace the 1s with their corresponding so-called four-valued elements. In the next stage, this matrix which we call Fuzzy adjacency matrix (FAM) is fed to the FIS. The FIS has four inputs which are the so-called attributes and one output which is the crisp value of showing the relationship quality between two specific nodes. Each element of the matrix is read by FIS and an adjacency matrix is generated in which its elements are crisp values for each friendship. We call this matrix fuzzy adjacency matrix (MAM). Each crisp value is between 0 and 10 in which 10 indicates the strongest friendship. Then MAM is fed to the eigenvector, and finally, the central node of the network is calculated. (See Fig. 2.) Pseudo-code of the algorithm is illustrated in Algorithm 1.

#### 4. Experimental results

The model was tested on one exemplary network, one community [23] and three real social networks [22,24,25,13]. The exemplary network consists of 10 people, the community is the Zachary karate club social network which contains 34 people. Facebook data has been anonymized by replacing the Facebook internal ids for each user with a new value and contains 4039 nodes and 88234 edges [25]. The other real networks are Slashdot-zoo network from Slashdot technology news website and Epinions [8]. We applied both the original eigenvector algorithm and the Model on both networks and compared the results in each situation. The adjacency matrix and fuzzy adjacency matrix for the exemplary network are illustrated in Figs. 3 and 1, respectively. The central nodes resulted after applying each algorithm on each network are illustrated in Table 1.

As it is shown, person number 4 is considered the most important person in the first network after applying the original eigenvector which is correct based on the number of relations but person number 10 is the result of the Model. Considering the MAM of this network, it is clear that person number 10 has much stronger relations, if not as many relations as person number 4, therefore, he is a more social person in the community and has more influence in general. This inference is true for the second network. Since the frequency of 1 s in  $A_{34}$  and  $A_1$  is 17 and 16, respectively, applying the original eigenvector provides person number 34 as the central. But Model results in person number 1. If we take a closer look at each row we can see that frequency of higher fuzzy values among the  $MAM_1$  of MAM is more than  $MAM_{34}$ . Therefore, it can be concluded that person number 1 is more influencing in comparison with person number 34.

#### Algorithm 1 The Model

**Require:** Network edge list Relation attribute frequency for each connection **Result:** CentralNode **Generate Adjacency Matrix of the Network** 1:  $N_{row} \leftarrow size(EdgeList, 1)$ 2:  $A \leftarrow EdgeList$ 3: for each row of network i do AdjMat(A(i, 1), A(i, 2)) = 14: 5: end for **Generate Fuzzy Adjacency Matrix** 6: **for** each row of network i **do** for each row of network j do 7: FAM(i, j) = (likeFreq., commentFreq., tagFreq., mentionFreq.)8: end for 9: 10: end for **Generate Modified Adjacency Matrix** 11:  $MAM \leftarrow FIS(FAM)$ Apply Eigenvector Algorithm and calculate central node 12:  $[u, r] \leftarrow eig(MAM)$ 13:  $Max \leftarrow r(1, 1)$ 14: **for** each row of network i **do** for each row of network j do 15: 16: **if** r(i, j) > Max then 17:  $Max \leftarrow r(i, j)$  $I \leftarrow i$ 18:  $I \leftarrow j$ 19: end if 20: 21: end for 22: end for 23:  $U \leftarrow u(:, J)$ 24: Cent  $\leftarrow U(1, 1)$ 25: NodeCent  $\leftarrow 1$ 26: for each row of network i do if U(i) > Cent then 27: Cent  $\leftarrow U(i)$ 28: NodeCent  $\leftarrow$  i 29: end if 30: 31: end for **Central Node is** NodeCent

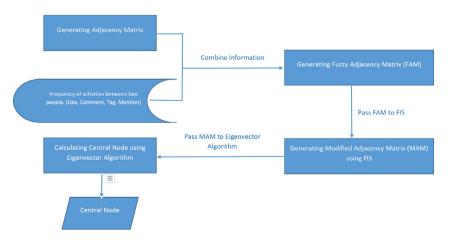


Fig. 2. Model schematic.

		1									10
	1	0	1	0	1	1	0	0	1	1	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
	2	1	0	1	1	0	1	0	0	1	1
	3	0	1	0	0	0	0	1	1	0	0
	4	1	1	0	0	1	1	1	1	1	1
٨	5	1	0	0	1	0	0	1	0	0	0
л—	6	0	1	0	1	0	0	1	1	0	0
	7	0	0	1	1	1	1	0	1	1	0 1 0 0 1
	8	1	0	1	1	0	1	1	0	0	1 1
	9	1	1	0	1	0	0	1	0	0	1
		1	1	0	1	0	0	1	1	1	0 )

Fig. 3. Adjacency matrix for 10 node network.

Slashdot-zoo data set contains information about all users of Slashdot news technology website providing a signed weighted graph indicating two types of relations, friends, and foes. Edges between friends and foes are weighted +1 and -1, respectively. The data set contains 79,120 vertices and 515,397 edges. Here we selected 5724 vertices and 35256 edges. Since there is no parameter specified for endorsements, we assigned normally distributed random values between 7.8 to 9.5 to positively signed edges that provide positive endorsement and similarly for the negatively signed edges, random values between 2.5 to 5.6 are assigned. A similar approach was applied on the Facebook and Epinions data set. The selected portion of Epinions and Facebook data sets contains 3794 vertices and 16244 edges and 3252 vertices and 12132 edges, respectively. The results are shown in Table 1. Since the latter mentioned network is undirected and unweighted, we assigned normally distributed random values between 2.5 to 9.5 to the edges. The results indicate that for high sparse data sets where the number of relations of different nodes is not close like Epinions data set results are the same in both algorithms. As it is shown in Table 1 node 364 is chosen as central node in both algorithms. This node has the most connections in the selected data set which is 1081 relations while the runner-up node has only 493 relations. Obviously this gap between the frequency of relations among all the members, dictates the node number 364 to be central. As it is shown in Table 1 the central nodes in Slashdot-zoo data set are 41 and 44 after applying original eigenvector and the Model, respectively. The number of relations of node 41 is 367 and as it can be concluded from Fig. 5, weak relations are considerably high. A total number of node 44 relations is 173 but as it shown in Fig. 4, the number of weak relations is 7. Considering the nature of relations in the Slashdot website which has divided the relation into two categories of friend and foe, members who have large numbers of foes are not so trusted and their reputation as an influential member is low. On the other hand, high numbers of strong relations in comparison to the number of weak relations makes a person much trusted and influential.

### Table 1 Central nodes.

	Exemplary	Zachary	Facebook	Slashdot-zoo	Epinions
Org. Eigenvector	4	34	1	41	364
The Model	10	1	2482	44	364

#### Table 2

Pairwise correlation coefficient-zachary club.

	Like	Comment	Tag	
Comment	0.943			
Tag Mention	0.985	0.947		
Mention	0.922	0.920	0.961	

#### Table 3

Pairwise correlation coefficient-facebook network.

	Like	Comment	Tag
Comment	0.914		
Tag Mention	0.932	0.895	
Mention	0.901	0.916	0.926

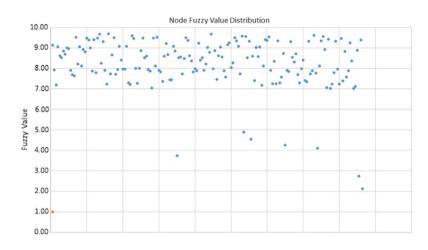
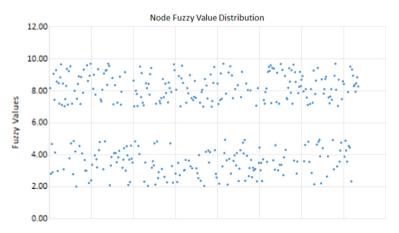


Fig. 4. Node 44 fuzzy value distribution.





The correlation coefficient of value factors for the central node in each data set was calculated and the result showed that the values of the considered factor are highly correlated and the relation between them is meaningful. Pairwise and total correlation coefficients of the factors are illustrated in Tables 2 and 3.

#### 5. Conclusion

Social network analysis is the strategy of investigating social structures through the use of network and graph theories. It characterizes networked structures in terms of nodes (individual actors or people) and the ties or edges (relationships or interactions) that connect them. There are different aspects of a social network which can be analyzed and the importance of an individual in the network is one of them. In this paper, we modified the eigenvector centrality measure and combined it with fuzzy inference system to get much more realistic results in terms of the importance of individuals of the network. Fuzzy logic took the quality of each relationship into account and the final result actually revealed the strength of the relationship. Although it has the same result for the highly sparse matrices the results are accurate and reflect true behavior of the members of the community. Future work can be focused on better defining the social parameters and altering the fuzzy system.

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