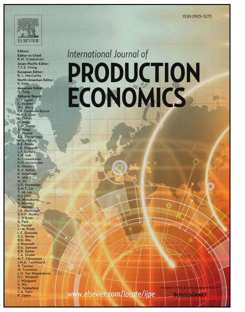

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PII: S0925-5273(17)30166-4

DOI: 10.1016/j.ijpe.2017.05.017

Reference: PROECO 6727

To appear in: *International Journal of Production Economics*

Received Date: 10 October 2016

Revised Date: 1 April 2017

Accepted Date: 29 May 2017

A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate

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Abstract

This paper investigates an alternative way to react to demand uncertainty in an integrated inventory model, namely the variation of the production rate that enables the manufacturer to reduce lead times and the corresponding demand uncertainty. To investigate the impact of variable production rates on the supply chain, this paper considers a single-vendor single-manufacturer integrated inventory model where the vendor ships finished products in multiples of full truckloads to the manufacturer. The objective of the model is to coordinate both production and distribution of the product in such a way that the total costs of the supply chain are minimized. A solution procedure is suggested, and the behaviour of the model is analysed in numerical examples. Our results illustrate that the total supply chain cost is reduced when the manufacturer's production rate is included as a decision variable in the model. These savings can generally benefit both the vendor and the manufacturer. However, in situations where coordinated decision making is initially not beneficial to the vendor, the supply chain members can benefit from a revenue sharing contract that supports the sharing of the total savings.

Keywords: *Integrated inventory model, variable production rate, stochastic demand, full truckload shipments*

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ABSTRACT

This paper investigates an alternative way to react to demand uncertainty in an integrated inventory model, namely the variation of the production rate that enables the manufacturer to reduce lead times and the corresponding demand uncertainty. To investigate the impact of variable production rates on the supply chain, this paper considers a single-vendor single-manufacturer integrated inventory model where the vendor ships finished products in multiples of full truckloads to the manufacturer. The objective of the model is to coordinate both production and distribution of the product in such a way that the total costs of the supply chain are minimized. A solution procedure is suggested, and the behaviour of the model is analysed in numerical examples. Obviously, the total supply chain cost is reduced when the manufacturer's production rate is included as a decision variable in the model. These savings can generally benefit both the vendor and the manufacturer. However, in situations where coordinated decision making is initially not beneficial to the vendor, the supply chain members can benefit from a revenue sharing contract that supports the sharing of the total savings. The model proposed in the paper at hand supports both the determination of an optimal production rate as well as the distribution of coordination benefits among the supply chain members.

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1.0 INTRODUCTION

Supply Chain Management (SCM) describes the management of materials, information and financial flows along the entire supply chain, extending over suppliers, manufacturers, distributors and customers (see Evans, 1995). The ultimate goal of SCM is to alleviate uncertainties and risks in the supply chain and to facilitate a smooth flow of materials, error-free production and an on-time delivery of products to the supply chain's customers.

To support managers in coordinating supply chains, researchers have developed so-called integrated inventory models in the past that, in their most basic form, aim on finding order and production quantities that minimize the total costs of the supply chain, instead of minimizing the costs of individual supply chain members. Starting with the work of Goyal (1976), a research stream has emerged over recent years that focuses on the coordination of operational decisions in supply chains, with a recent review of integrated inventory models being the one of Glock (2012).

The paper at hand studies a two-echelon single-vendor single-manufacturer supply chain where a vendor produces an intermediate product that is shipped in multiples of full truckloads to a manufacturer. The manufacturer transforms the intermediate product into a final product subject to stochastic end customer demand. While the vendor's production rate is fixed, the manufacturer has the opportunity to vary its production rate, which may result in a faster or slower completion of the lot size, depending on how the production rate is varied. By speeding up the production process, the manufacturer may reduce its own delivery lead time, which helps to shorten the period during which the manufacturer is at risk to run out of stock. This, in turn, reduces safety stocks and may offset the additional costs associated with varying the production rate.

The scenario studied in this paper is motivated by a case we observed in practice. A vendor supplies polymers (raw material) to a manufacturer in multiples of full truckloads. The manufacturer faces random demand by its customers, and hence keeps inventory of raw material in its warehouse. The manufacturer can increase/decrease its production rate via accelerating/decelerating the production process, which mainly includes blending and filling processes. This paper contributes to the literature by providing an integrated stochastic inventory model with a variable production rate at the manufacturer, which, to our knowledge, has not yet been addressed in the literature. In addition, the paper considers a full truckload shipment constraint that frequently governs logistics processes in practice. The paper finally proposes a solution technique for the developed model and illustrates the impact of the variable production rate and the full truckload constraint on the integrated inventory model via numerical examples.

The remainder of the paper is structured as follows: The next section summarizes the related literature, and Section 3 describes the problem studied in this paper formally and proposes a mathematical model. An efficient solution technique is presented in Section 4 along with numerical illustrations of the proposed model. Section 5 presents future research opportunities and concludes the paper.

2.0 LITERATURE REVIEW

The supply chain management literature spans a plethora of topics, ranging from daily operations scheduling and control to strategic decision problems such as facility location planning. This section discusses two streams of research that are of special relevance to the work at hand, namely I) works that study variable production rates in inventory models and II) works that investigate the coordination of multiple echelons in supply chains. In discussing the second stream of research, the focus will be on single-vendor single-buyer integrated inventory models.

2.1 Inventory models with variable production rates

Determining optimal production rates for a manufacturing system has started to attract the attention of researchers many years ago. Khouja (1994) was among the first to extend the basic Economic Production Quantity (EPQ) model to consider production volume flexibility by assuming that the production rate can be varied prior to the start of a production run. The model suggested that in volume-flexible manufacturing systems, the optimal production rate is smaller than the production rate that minimizes the unit production cost. Khouja and Mehrez (1994) extended the work of Khouja (1994) by assuming that a change in the production rate does not only affect the unit production cost, but also the quality of the product. The result of the paper indicate that for cases where an increase in the production rate causes a sharp decline in product quality, the optimal production rate is smaller than the production rate that minimizes the unit production cost. For situations where product quality does not depend on the production rate, the optimal production rate might be larger than the rate that minimizes the unit production cost. Khouja (1999) extended Khouja’s (1994) model by assuming that the production process may shift out of control with a probability that depends on the production rate. The author showed that incorporating product quality into the EPQ model with a variable production rate leads to a shorter cycle time and a smaller optimal lot size. Eiamkanchanalai and Banerjee (1999) developed a model that determines both the optimal production cycle length and production rate for a single item. In contrast to earlier works, the authors added a desirability term to the objective function (that could express a desire for unused capacity, for example), and showed that the optimal production rate can be larger or smaller than the production rate that minimizes the unit production cost.

Giri et al. (2005) introduced a variable production rate EPQ model in which the stress level of the machine varies with the production rate (i.e., a higher production rate implies a higher stress level and thus a higher failure rate). The unit production cost was expressed as a function of the production rate, and an EPQ model was developed under general failure and repair time distributions. This model was later extended to consider stochastic demand (Ayed et. al. 2012), inspection sampling (Bousalah et al. 2013), and stochastic repair time (Singh and Prasher, 2014).

Larsen (2005) introduced an EPQ model where the production cycle is composed of multiple runs at different production rates; the production rates and their corresponding runtimes were treated as decision variables in this model. The author showed that the production rates should adopt values between the demand rate and the production rate that minimizes the unit production cost, and that it should be increased over the production cycle.

Glock (2010, 2011) studied the effect of variable production rates on a two-stage and a multi-stage EPQ model with either equal- or unequal-sized batch shipments. The author investigated how production rates should be set to minimize the total costs of the system.

Finally AlDurgam and Duffiaa (2013) provided a new application of the Partially Observed Markov Decision Process by modelling a machine with multiple machine and quality states, where in each time period, the decision maker determines the optimal production rate and the optimal maintenance action to maximize the Overall System's Effectiveness (OSE). OSE was defined as the product of availability, process rate, and quality rate. The model captured the impact of the production rate on the machine failure and scrap rates. In addition, the impact of the maintenance rate on the time the machine is down due to maintenance and enhanced availability were investigated.

2.2 Coordination of single-vendor single-buyer integrated inventory systems

Integrated inventory models (which are also frequently referred to as Joint Economic Lot Size or JELS models) have enjoyed an increased popularity in recent years. This section presents some integrated inventory models that are of special relevance to the work at hand, namely JELS models with different lot-sizing strategies and JELS models with stochastic demand and/or stochastic lead time. For a comprehensive review of the JELS literature, the reader is referred to Glock (2012).

The first single-vendor single-buyer integrated inventory model was proposed by Goyal (1976), who illustrated the economic advantage of joint lot-sizing in a simple two-stage supply chain. Banerjee (1986) extended Goyal's model and relaxed the assumption of an infinite production rate. The author implemented a so-called lot-for-lot policy for coordinating the production and consumption cycles of the vendor and the buyer. Lu (1995) extended the work of Banerjee (1986) to account for equal-sized batches that the vendor ships to the buyer. Goyal (1995) extended the works of Banerjee (1986) and Lu (1995) by assuming that subsequent batch shipments increase in size according to a geometric series, which led to another reduction in total system cost. Hill (1997) generalized this model by assuming that subsequent batches first increase in size according to a geometric series, and that batch sizes then remain constant. The optimal batch shipment policy, which also consists of a combination of unequal- and equal-sized shipments, was later proposed by Hill (1999). Hoque and Goyal (2000) studied the case of a transport facility with limited capacity and showed that the optimal policy in this case also consists of unequal-sized batches increasing by a fixed factor, followed by equal-sized batch shipments. Other authors who studied the determination of batch sizes in an integrated inventory model are Huang (2004) and Wee and Widyadana (2013), among others.

The JELS models discussed so far all assumed that demand is deterministic. Sharafali and Co (2004) presented one of the first JELS models with stochastic demand. Ben-Daya and Hariga (2004) assumed a normally distributed and lot-size-dependent lead time and derived an optimal solution for the model. This paper was extended by Glock (2009), who took account of unequal-sized batch shipments. Quyang et al. (2004) proposed another extension of this model by assuming stochastic demand with shortages allowed during lead time. In addition, the authors assumed that the lead time can be shortened at an additional cost. Jha and Shanker (2009) proposed a JELS model with controllable lead time and a service level constraint. The service level constraint guarantees that a certain level of demand is satisfied in each cycle. Glock (2012) considered a single-vendor single-buyer JELS model with stochastic demand and variable lead time. In this model, lead time can be shortened by reducing the lot size, by increasing the production rate, or by crashing a constant delay time. The author investigated how the three lead time reduction methods should be combined to minimize the total costs of the system.

The works discussed above have shown that moving from a scenario where one of the supply chain members dominates the supply chain to a scenario where a centrally coordinated solution is obtained for the supply chain improves the cost position of the supply chain as a whole. A coordinated solution may, however, place individual members of the supply chain at a cost disadvantage. To induce all members of a supply chain to participate in a coordinated solution, the supply chain may use coordination mechanisms that distribute the cooperation gain among the parties involved. The literature discusses a plethora of coordination mechanisms that may be used in a supply chain, including information sharing mechanisms (e.g., Li, 2002; Kelle and Akbulut, 2005), the design of special contracts (e.g., Panda, et. Al., 2015; Modak et. al., 2016), risk sharing mechanisms (e.g., Hou, et. Al., 2010; Linh and Hong, 2009), or strategic alliances such as vendor managed inventory or consignment stock (e.g., Zavanella and Zanoni, 2009; Ben-Daya et. al., 2013). For a comprehensive review of the supply chain coordination literature, the reader is referred to Kanda and Deshmukh (2008) and Sarmah et al. (2006).

2.3 Synthesis of both research streams

Our review of the literature showed that variable production rates have thus far only very infrequently been studied in the context of an integrated inventory model. In addition, we found that shipment constraints that very frequently apply in practice have not been considered in JELS models with stochastic demand so far. The work closest related to the paper at hand is the one of Glock (2012), who did, however, neither consider shipment constraints nor raw material purchases in his model. This paper therefore contributes to the literature by proposing a JELS model with stochastic demand and a shipment constraint where the production rate at the manufacturer can be varied. A detailed description of the proposed model is provided in Section 3.

3.0 THE MODEL

3.1 Problem description

This section develops a mathematical model for a single-vendor single-manufacturer supply chain with stochastic demand and a shipment constraint. Figure 1 illustrates how inventory develops at the supply chain parties over time. In the scenario considered here, the vendor produces a raw material at a constant rate (P_V) and sends full truckload batch shipments of size q to the manufacturer (Figure 1 – Part A). The manufacturer, Figure 1 – Part B, initiates production immediately upon arrival of the first shipment at its premises (i.e., after a lead time of length $t_i + \Delta$, where t_i represents the production lead time of the vendor needed to produce q units and Δ is the transportation lead time) by depleting the raw material received at a rate that is proportional to its production rate (αP). In each cycle, the inventory of raw material accumulates in the warehouse of the manufacturer to a maximum level, I_{max}. The manufacturer faces random end customer demand for the final product and uses a continuous review (Q, R) inventory control system (Figure 1 – Part C), Note that when the finished items inventory of the manufacturer drops to the reorder level R, the vendor reinitiates its production process. This pattern is assumed to repeat in every cycle. The objective of the model proposed in the following will be to determine the production rate, the number of full truckload shipments, the re-order point, and the production lot size that minimizes the total costs of the supply chain.

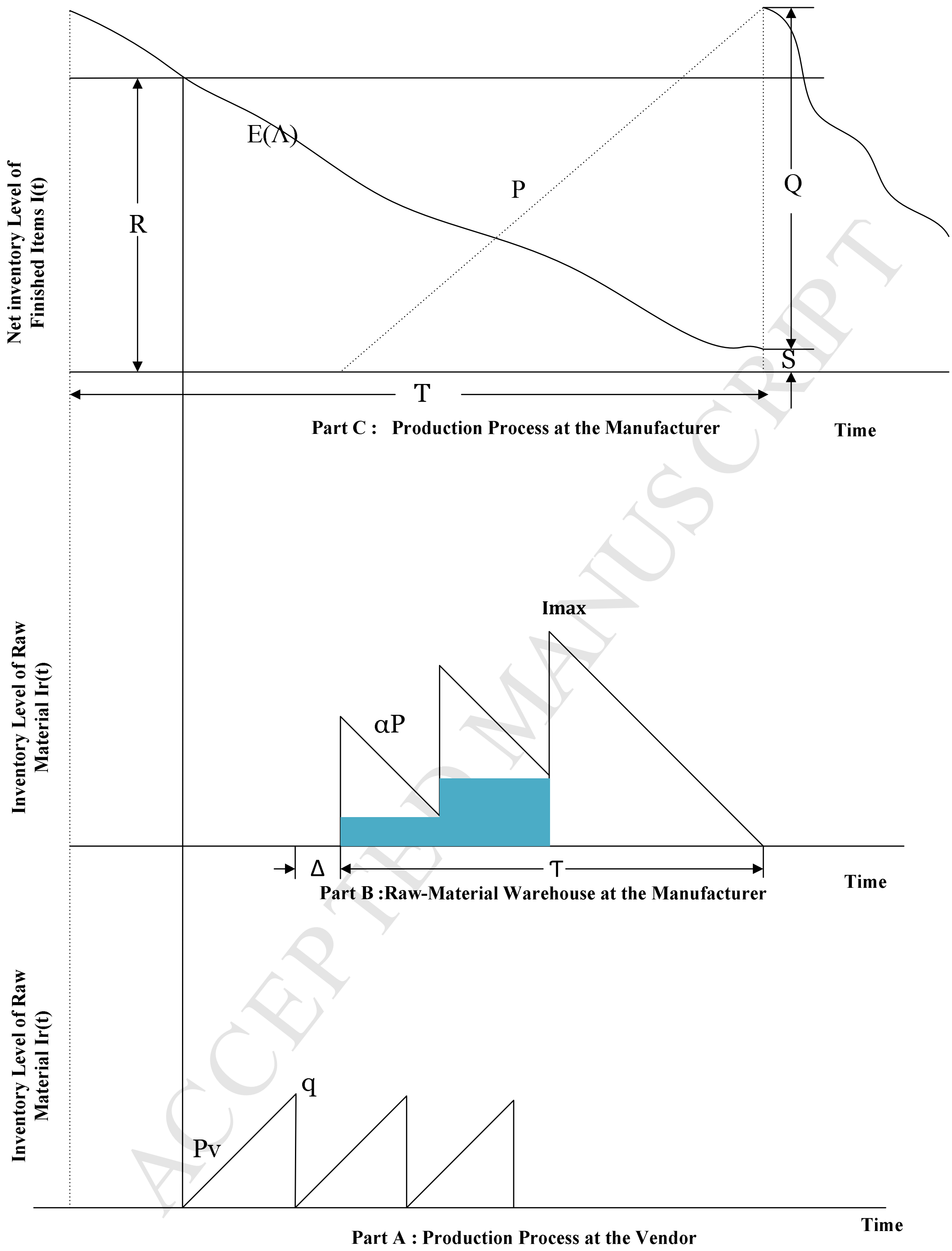

Figure 1: Inventory profile of the supply chain

In developing the proposed model, the following assumptions will be made:

- The production rate of the vendor, P_v, is known and fixed.
- Shipments are made in full truckloads of size q, and the time to produce a full truckload is t units of time.
- The capacity of the raw material warehouse of the manufacturer is limited.
- To avoid shortages at the vendor, the production rate of raw materials at the vendor is larger than the maximum inventory depletion rate of the manufacturer, i.e. $P_v > \max(d)$.
- The rate at which the raw material is depleted from the manufacturer's warehouse is directly proportional to the production rate of the manufacturer, i.e. $d = \alpha P$.
- All shortages are backordered.
- There is never more than a single production run outstanding, and the average rate of demand is constant over an infinite horizon (see, for the same assumption, Darwish et al. 2013).
- The expected number of backorders incurred per unit of time is independent of the expected number of production runs per year, provided that the stochastic process generating end customer demand is time-homogeneous.
- The demand pattern is random and modelled using a normal probability distribution.
- The production rate of the manufacturer has to be determined prior to the start of the production run. Such a production system is referred to as a "rigid system" in the literature, and it is representative for situations where a machine setup during production is technically impossible or involves prohibitively high cost. Rigid production systems have been studied by Buzacott and Ozkarahan (1980), Silver (1990), Saka and Babu (1993), Goyal (1994), Silver (1995), and Viswanathan (1995).
- The unit production cost is assumed to follow the function proposed by Khouja (1994), i.e. $f(p) = C + \frac{g}{P} + bP^{\beta}$. Here, C is the unit acquisition cost of raw material, $\frac{g}{P}$ represents the per unit cost component that is reduced as the production rate increases (e.g., labour

cost), and bP^{β} is the unit cost component that increases in the production rate (e.g., tools and rework costs).

The notations used in this paper are divided into three sets in the following: input parameters, auxiliary variables, and decision variables. Auxiliary variables are variables solely needed for calculating the decision variables.

Input parameters

A_M	:	Setup cost of the manufacturer per cycle
A_r	:	Transportation cost of raw material per truck
A_S	:	Setup cost of the vendor per cycle
b	:	non-negative parameter of the unit production cost formula as in Khouja (1994)
B	:	non-negative parameter of the unit production cost formula as in Khouja (1994)
C	:	Unit acquisition cost of raw material for the manufacturer
C_p	:	Total production cost of the manufacturer per unit of time
$E[\Lambda]$	:	Expected value of the end customer demand per unit of time
$E[Y]$	:	Expected value of the lead time demand, $E[Y]=E[\Lambda][t+\tau+\Delta]$
$f(y)dy$	:	Probability that the lead time is between y and y + dy
g	:	non-negative parameter of the unit production cost formula as in Khouja (1994)
h_v	:	Inventory holding cost of the vendor per unit of the finished item per unit of time
h_r	:	Inventory holding cost of raw material per unit per unit of time
h_m	:	Inventory holding cost of finished goods for the manufacturer per unit per unit of time
I_{max}	:	Capacity of the raw material warehouse of the manufacturer
P_V	:	Production rate in units per unit of time of the vendor
q	:	Capacity of a truck
S	:	A random variable representing safety stock
t	:	The time needed by the vendor to produce a full truckload shipment of size q
τ	:	Production lead time of the manufacturer
X	:	Unit production cost of the vendor
Y	:	A random variable representing lead time demand $[Y = \Lambda(\tau + t + \Delta)]$
z	:	Ordering or administrative cost per cycle
αP	:	Raw material consumption rate of the manufacturer in units per unit of time, α is a conversion factor from raw materials to the final product
Λ	:	A random variable representing demand per unit of time
Δ	:	Constant lead time for loading, transporting and unloading a full truck
$\emptyset$	:	Standard normal probability density function
π	:	Fixed penalty cost incurred by the manufacturer per unit short
σ_A	:	Standard deviation of the demand per unit of time
σ_Y	:	Standard deviation of lead time demand $\sigma_A\sqrt{\tau + t + \Delta}$

auxiliary variables

AC_m : Total acquisition cost of raw material for the manufacturer per unit of time
AP_v : Production cost of the vendor per unit of time
DPC : Direct production cost of the manufacturer per unit of time
$H.M_{fp}$: Total inventory holding cost of finished goods of the manufacturer per unit of time
$M.HC_r$: Raw material inventory holding cost per unit of time of the manufacturer
$M.E_c$: Ordering and transportation costs for raw material per unit of time of the manufacturer
$M.S_C$: Setup cost per unit of time of the manufacturer
S_c : Expected shortage cost per unit of time of the manufacturer
s : Expected safety stock ($s = E[S]$)
T : Inventory cycle length
$T.Cs$: Expected average total cost per unit of time of the integrated model
$T.C_V$: Long-run average cost per unit of time of the vendor
$T.C_r$: Long-run average total cost of raw material at the manufacturer's warehouse per unit of time
$T.C_M$: Long-run average manufacturing cost per unit of time
$V.S_C$: Long-run average setup cost of the vendor
$V.H_c$: Long-run average inventory holding cost of the vendor

Model decision variables

n : Number of full truckload shipments from the vendor to the manufacturer per cycle
P : Production rate of the manufacturer in units per unit of time
Q : Production lot size of the manufacturer per cycle
R : Raw material reorder point

3.2 Model Formulation

The model proposed in this paper investigates how demand uncertainty at the manufacturer influences the manufacturer's production rate, the number of full truckloads shipped from the vendor to the manufacturer, the manufacturer's reorder level, and the manufacturer's optimal production quantity. The different components of the total system cost function are developed step-by-step in the following.

1. <u>Setup cost per unit of time at the vendor</u>

The vendor incurs a setup cost A_S for each production cycle. The long-run average setup cost per unit of time for the vendor is given as $S_C = \frac{A_s}{T}$, which can be approximated as (see Darwish et al. (2013) and Ben-Daya and Hariga (2004)):

$$\frac{A_S E[\Lambda]}{Q} \quad (1)$$

2. Inventory holding cost per unit of time at the vendor

The inventory kept at the vendor per cycle equals the area under the inventory time plots in part A of Figure 1. Dividing the area by the expected cycle time and multiplying it with the unit inventory holding cost leads to the inventory holding costs per unit of time:

$$V.H_C = \frac{E[\Lambda]d^2 Q}{2nP^2 P_v} h_v = \frac{E[\Lambda]\alpha^2 Q}{2nP_v} h_v \quad (2)$$

3. Production cost per unit of time at the vendor

The cost of producing the raw material at the vendor per unit of time equals the product of the unit production cost function multiplied with the total quantity produced in a cycle, where the cycle time is an expected value:

$$AP_v = \alpha X E[\Lambda] \quad (3)$$

4. Long-run average cost per unit of time at the vendor

The long-run average cost per unit of time at the vendor equals the sum of Eqs. (1) to (3): $T.C_V(Q,n) = V.S_C + V.H_C + AP_v$

$$T.C_V(Q,n) = \frac{A_s E[\Lambda]}{Q} + \frac{E[\Lambda]\alpha^2 Q}{2nP_v} h_v + \alpha X E[\Lambda] \quad (4)$$

5. Raw material inventory holding costs per unit of time at the manufacturer

The costs considered here are the ordering, transportation and inventory holding costs of raw materials at the manufacturer. The inventory holding costs of raw materials at the manufacturer can be calculated using the area under the inventory time plots in Part B of Figure 1. Multiplying this area with the holding cost of raw material per unit per unit of time of the manufacturer, the long-run average inventory holding cost of raw material per unit of time can be calculated as:

$$MHC_r = \frac{Q\alpha E[\Lambda]}{2nP}\left[n\left[1-\frac{\alpha P}{P_v}\right]+\frac{\alpha P}{P_v}\right]h_r \qquad (5)$$

The maximum inventory level at the raw material warehouse of the manufacturer is not allowed to exceed the capacity of the warehouse, I_{max}. This can be expressed as $nq - (n-1)\frac{q}{P_v}\alpha P \leq I_{max}$, which leads to a constraint on the maximum number of shipments per cycle that can be rewritten as an upper bound on n:

$$n \leq \frac{P_v I_{max} - \alpha q P}{q(1-\alpha P)}$$

The manufacturer's ordering and transportation cost for raw material per unit of time is

$$ME_C = \frac{nA_r}{T} + \frac{Z}{T} = \frac{nE[\Lambda]A_r}{Q} + \frac{ZE[\Lambda]}{Q}$$

$$ME_C = \frac{E[\Lambda]}{Q}[nA_r + z] \qquad (6)$$

The long run average total raw material inventory control cost per unit of time is therefore given as

$$T.C_r[Q,n,P] = \frac{E[\Lambda]}{Q}[nA_r + z] + \frac{Q\alpha E[\Lambda]}{2nP}\left[n[1-\frac{\alpha P}{P_v}]+\frac{\alpha P}{P_v}\right]h_r \qquad (7)$$

6. <u>Inventory holding cost for the finished product per unit of time at the manufacturer</u>

Part C of Figure 1 shows that the expected net inventory at the beginning of a cycle is $S+Q$, and at the end of the cycle it is S, where S represents the safety stock (Hadley and Whitin, 1963). It is important to note that these are also the average values of the on-hand inventory when the expected number of backorders can be neglected, and since the expected demand rate is constant, the expected on-hand inventory changes linearly from $S+Q$ to S. Thus, the average inventory for the manufacturer's finished product is $\frac{1}{2}[S+S+Q]$=$[S+\frac{Q}{2}]$, and the inventory holding cost of finished goods at the manufacturer's warehouse is $\left[S+\frac{Q}{2}\right]h_m$. In addition, during the manufacturer's production run illustrated in part C of Figure 1, the average inventory holding cost per unit of time for the manufacturer's product meant to be consumed in the next cycle is $\frac{1}{T}\left[\frac{1}{2}\times\tau\times Q\right]h_m$. Given that $\tau\ =\frac{Q}{P}$ and $T=\frac{Q}{E[\Lambda]}$, the average inventory holding cost for the manufacturer can be expressed as $\frac{QE[\Lambda]h_m}{2P}$. Hence, the total inventory holding cost for the manufacturer's product per unit of time is the sum of the inventory holding cost for the average number of final products on stock and for the average work-in-progress inventory held during the production run. Thus,

$$H.M_{fp}=h_m\left[\frac{Q}{2}\left[1+\frac{E[\Lambda]}{P}\right]+S\right] \tag{8}$$

Note that the computation of the safety stock S depends on the model assumption on shortages, and thus on whether shortages are satisfied (backordered) or lost. In case shortages are lost, the safety stock, which is a random variable, is unrestricted in sign and can be computed as follows:

$$S=R-Y,\qquad E[S]=R-E[Y],$$

$$\text{since } E[Y]=E[\Lambda][\tau+\Delta+t] \text{ then } E[S]=R-E[\Lambda][\tau+t+\Delta]$$

$$t=\frac{q}{P_V}=\frac{\alpha Q}{nP_v}.$$

Substituting the expression for $E[S]$ in Eq. (8), the total inventory holding cost for the manufacturer's product per unit of time is

$$h_m[\frac{Q}{2}\left[1-\frac{E[\Lambda]}{P}\right]+R-E[\Lambda]\left[\frac{\alpha Q}{nP_v}+\Delta\right]] \quad (9)$$

7. Shortage cost

Shortage cost is the cost associated with stockouts at the manufacturer. Shortage cost occurs when the demand during lead time exceeds the reorder level (R). The shortage quantity is a random variable, and it is calculated as follows:

$$\text{Shortage} = \begin{cases} Y-R & Y>R \\ 0 & Y \leq R \end{cases}$$

The expected shortage cost per unit of time is thus given as

$$S_C = \frac{\pi E[\Lambda]}{Q}\int_R^{\infty}[Y-R]f[y]dy \quad (10)$$

8. Direct production cost at the manufacturer

Direct production cost are the cost of producing the manufacturer's product. Similarly to Khouja (1994), and without loss of generality, this paper assumes that the unit production cost is a function of the production rate.

The direct production cost per unit of time can thus be approximated as

$$DPC = E[\Lambda]\left(\frac{g}{P}+bP^B\right) \quad (11)$$

9. Total raw material acquisition cost at the manufacturer

The manufacturer's total raw material acquisition cost is the total cost of purchasing n full truckloads of raw material in one cycle. This cost is computed as follows:

$$AC_m = \alpha CE[\Lambda] \quad (12)$$

10. Total production cost per unit of time at the manufacturer

The total production cost per unit of time at the manufacturer is the sum of Eqs. (11) and (12):

$$C_p = DPC + AC_m = E[\Lambda]\left(\frac{g}{P} + bP^B\right) + \alpha CE[\Lambda] \quad (13)$$

11. Setup cost for finished products at the manufacturer

This cost is calculated as

$$MS_C = \frac{A_m E[\Lambda]}{Q} \quad (14)$$

The total cost incurred by the manufacturer per unit of time is now given as:

$$T.C_M[Q,n,P,R] = \frac{A_m E[\Lambda]}{Q} + h_m\left[\frac{Q}{2}\left[1 - \frac{E[\Lambda]}{P}\right] + R - E[\Lambda][\frac{\alpha Q}{nP_v} + \Delta]\right] + \frac{\pi E[\Lambda]}{Q}\int_R^\infty [Y - R]f[y]dy + \alpha c_2 E[\Lambda] + E[\Lambda]\left(\frac{g}{P} + bP^B\right)]$$

12. Total cost of the supply chain

The total cost of the supply chain is the sum of the vendor's and the manufacturer's total cost:

$$T.C_s[Q,n,P,R] = T.C_V(Q,P) + T.C_r[Q,n,P] + T.C_M[Q,n,P,R] = \frac{E[\Lambda]}{Q}[A_s + [nA_r + z] + A_m] + \frac{E[\Lambda]\alpha^2 Q}{2nP_v}h_v + \alpha E[\Lambda][C + X] + \frac{Q\alpha E[\Lambda]}{2nP}\left[n[1 - \frac{\alpha P}{P_v}] + \frac{\alpha P}{P_v}\right]h_r + h_m\left[\frac{Q}{2}\left[1 - \frac{E[\Lambda]}{P}\right] + R - E[\Lambda][\frac{\alpha Q}{nP_v} + \Delta]\right] + \frac{\pi E[\Lambda]}{Q}\int_R^\infty [Y - R]f[y]dy + E[\Lambda]\left(\frac{g}{P} + bP^B\right) \quad (15)$$

The following constraints have to be satisfied:

$$\frac{nq}{\alpha Q} = 1$$

$$n \le \frac{P_v I_{max} - \alpha qP}{q(1 - \alpha P)}$$

$$n \in integers$$

4.0 SOLUTION METHOD AND NUMERICAL ILLUSTRATIONS

To minimize the objective function (15), it is necessary to determine optimal values for the production quantity, Q, the re-order level, R, the production rate, P, and the number of full truckload shipments, n. First, assuming that the manufacturer has a limited production capacity, the manufacturer faces a finite range of possible production rates. We initiate our solution technique by performing a line search over the feasible range of values for P, and then for each value of P, we perform another line search on $n \in \{1, \frac{P_v I_{max} - \alpha q P}{q(1-\alpha P)}\}$. Given the fixed values of P and n, Q is obtained from the equality constraint on (15), $Q = \frac{nq}{\alpha}$. Finally, for these fixed values of P, n, and Q, notice that the objective function (15) is convex in R. Thus, R can be found by taking the first partial derivative of (15) with respect to R, while the other variables are held constant. Setting this derivative equal to zero gives $R = F^{-1}(1 - \frac{h_m q}{\pi \mathrm{E}[\Lambda]})$. Figure 2 provides a pseudocode of our suggested algorithm that determines the global optimal solutions in case they exist.

$T.C_s{}^* = big\ M$

$For\ P = P_{min} : step\ size : P_{max}$

$For\ n = 1 : step\ size\ of\ 1 : \frac{P_v I_{max} - \alpha q P}{q(1-\alpha P)}$

$$Q = \frac{nq}{\alpha}$$

$$R = F^{-1}(1 - \frac{h_m q}{\pi \mathrm{E}[\Lambda]})$$

$Compute\ T.C_s[Q, n, P, R]$

$if\ T.C_s \le T.C_s{}^*$

$T.C_s{}^* = T.C_s[Q, n, P, R]$

$Q^* = Q, n^* = n, P^* = P, R^* = R$

$End\ if$

$End\ for$

$End\ for$

$Q^*, n^*, P^*, R^* = argmin\ \ T.C_s{}^*$

$T.C_s{}^* = T.C_s[Q^*, n^*, P^*, R^*]$

Figure 2: Pseudo code of the solution algorithm for the proposed model

Remark: since Y is normally distributed with mean $E[Y] = E[\Lambda][\tau + \Delta + t]$ and standard deviation $\sigma = \sigma_A\sqrt{\tau + t + \Delta}$, noticing that $\tau = Q/P$, which equals $nq/\alpha p$, the necessary optimality conditions (obtained by substituting for $Q = nq/\alpha$, relaxing n and equating the gradient vector of $T.C_s$ to zero) will involve two integral equations (the first derivatives w.r.t n and P) which will be very complicated to solve in a closed form.

Numerical Examples

To study the impact of treating the manufacturer's production rate as an additional decision variable in the JELS model proposed in Eq. (15), we define two cases, namely:

I) The *partially integrated case* where the manufacturer is willing to collaborate with the vendor without deviating from the production rate that yields the minimum unit production cost. In this case, the manufacturer first determines the production rate that minimizes the unit production cost (11), then, given this fixed value, say P^*, the optimal values of $n, Q,$ and R are determined such that the total supply chain cost, $\mathrm{T.C_s}[\mathrm{Q}, \mathrm{n}, P^*, \mathrm{R}]$ in (15), is minimized (we refer to this minimum as $T.C_s$ − partially integrated in the following).

II) The *fully integrated case* where the manufacturer is willing to deviate from the production rate that minimizes the unit production cost (11). Here, the optimal values of Q , $n, P,$ and R are jointly determined such that total supply chain cost, $\mathrm{T.C_s}[\mathrm{Q}, \mathrm{n}, P, \mathrm{R}]$ in (15), is minimized (we refer to this minimum as $T.C_s$ − fully integrated in the following).

It is straightforward to show that $T.C_S$ of the fully integrated case is always lower than or equal to that in the partially integrated case. We present some examples to compare both cases, the partially and the fully integrated cases, and evaluate the savings for the supply chain and individually for the vendor and the manufacturer.

For the supply chain, we determine the percentage savings of the fully integrated case as compared to the partially integrated case as follows:

$$ps = \frac{T.C_s \text{ partially integrated} - T.C_s \text{fully integrated}}{T.C_s \text{ partially integrated}} \times 100\% \tag{16}$$

Similarly, for both cases, after dividing $T.C_s$ into its components ($T.C_V$ and $T.C_{rm} = T.C_r + T.C_M$), we determine the percentage savings for the vendor and the manufacturer individually as in Eq. (16) using $T.C_V$ and $T.C_{rm}$, respectively, and we refer to these savings as psv and psm for the vendor and the manufacturer, respectively:

$$psv = \frac{T.C_V \text{ partially integrated} - T.C_V \text{fully integrated}}{T.C_V \text{ partially integrated}} \times 100\% \tag{17}$$

$$psm = \frac{T.C_{rm} \text{ partially integrated} - T.C_{rm} \text{fully integrated}}{T.C_{rm} \text{ partially integrated}} \times 100\% \tag{18}$$

While ps is always larger than or equal to zero, notice that a positive psv value implies that the fully integrated case is more beneficial to the vendor than the partially integrated case, and a negative psv value implies the opposite. The same reasoning holds for psm.

Unless stated otherwise, Table 1 introduces the basic data used in all numerical illustrations of this section.

Table 1: Data used for numerical experimentation (unless stated otherwise)

A_m	2500	$E[\Lambda]$	250	z	1000
A_s	2000	g	50	Δ	0.1
A_r	500	h_v	3	π	200
b	0.035	h_r	1	α	2
B	1	h_m	5	σ_A	40
C	3.5	X	1.5	q	400

To illustrate the behaviour of our model, we vary one parameter at a time, using some of the parameters given in Table 1. The results of our numerical experiment are presented in Tables 2 to 15.

Table 2: Effect of the demand standard variation on the system

	Partially integrated case							Fully integrated case							% savings		
σ_A	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
40	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	600	12369.6	1883.3	10486.3	9.3	0	10.7
60	3	598	600	490	13875.6	1883.3	11992.3	3	433	600	600	12864.7	1883.3	10981.3	7.3	0	8.43
80	3	598	600	529	14119.1	1883.3	12235.8	4	402	800	800	13306.6	1675	11631.6	5.8	11.1	4.94
100	3	598	600	567	14362.6	1883.3	12479.3	4	435	800	800	13709	1675	12034	4.6	11.1	3.57
120	4	598	800	688	14777	1675	13102	4	471	800	800	14131	1675	12456	4.4	0	4.93
140	4	598	800	726	15029	1675	13354	4	511	800	800	14578.9	1675	12903.9	3	0	3.37

Table 3: Effect of the demand standard variation on the system, $q = 300$

	Partially integrated case							Fully integrated case							% savings		
σ_A	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
40	4	598	600	425	13724	1808.3	11915.7	4	365	600	600	12359	1808.3	10550.7	9.9	0	11.5
60	4	598	600	462	13959.3	1808.3	12150.9	4	404	600	600	12808.2	1808.3	10999.9	8.2	0	9.5
80	4	598	600	498	14194.5	1808.3	12386.2	5	395	750	749	13260.6	1641.7	11618.9	6.6	9.2	6.2
100	4	598	600	535	14429.8	1808.3	12621.4	5	428	750	749	13658.7	1641.7	12017	5.3	9.2	4.8
120	4	598	600	572	14665	1808.3	12856.7	5	464	750	749	14076	1641.7	12434.3	4.0	9.2	3.3
140	5	598	750	673	15014.5	1641.7	13372.8	5	503	750	750	14514.3	1641.7	12872.6	3.3	0	3.7

Table 4: Effect of the demand standard variation on the system, $q = 200$

	Partially integrated case							Fully integrated case							% savings		
σ_A	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
40	6	598	600	397	14023.6	1733.3	12290.3	6	344	600	600	12581.9	1733.3	10848.6	10.3	0.0	11.7
60	6	598	600	432.5	14250.3	1733.3	12517	6	378	600	600	12989.4	1733.3	11256	8.8	0.0	10.1
80	6	598	600	468	14477.1	1733.3	12743.7	6	416	600	600	13422	1733.3	11688.7	7.3	0.0	8.3
100	6	598	600	503.5	14703.8	1733.3	12970.4	7	419	700	700	13818.5	1614.3	12204.2	6.0	6.9	5.9
120	6	598	600	539.1	14930.5	1733.3	13197.2	7	455	700	700	14229.6	1614.3	12615.3	4.7	6.9	4.4
140	6	598	600	574.6	15157.2	1733.3	13423.9	7	494	700	700	14661.3	1614.3	13047.1	3.3	6.9	2.8

To study the effect of a change in the standard deviation of demand on the supply chain and its members, we consider the base-case scenario parameters (Table 1). For σ_A, we consider different values ranging from 40 to 140 with a step size of 20. The results are summarized in Table 2, where it can be seen that the total supply chain savings measured by ps decrease as σ_A increases. For the fully integrated case, the system can balance an increase in uncertainty by producing faster and/or increasing the reorder point. Starting at $\sigma_A = 40$, as compared to the partially integrated case, it is beneficial for the fully integrated system to increase R to its maximum value ($R^* = Q^* = 600$), and, being consistent with Khouja's (1994) result, to deviate from the production rate of 598 units/unit of time which, in this case, minimizes the unit production cost. An increase in σ_A then induces the fully integrated system to further react to this increase in demand uncertainty by increasing the production rate as a second mechanism to protect itself against shortages. Notice that the full integration was beneficial for both the vendor and the manufacturer for σ_A-values of 80 and 100. For the remaining values of σ_A, only the manufacturer benefited directly from full integration.

Tables 3 and 4 further illustrate the impact of σ_A for different truck sizes ($q = 300$ and $q = 200$, respectively). As the truck size decreases (Tables 2-4), the supply chain performs better in terms of higher ps- and psm-values due to the increasing system flexibility provided by smaller trucks. However, the psv-values tend to decrease as q decreases; this is mainly because the manufacturer is no longer forced to produce large lots and to use high reorder points when q is large in the partially integrated case. As in Khouja (1994), the optimal production rate is smaller than the production rate that minimizes the unit production cost (P^* =598), with P^* tending to further decrease with decreasing values of q.

Table 5: Effect of the truck capacity on the system

	Partially integrated case							Fully integrated case							% savings		
q	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
400	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	599	12369.6	1883.3	10486.3	9.3	0	10.7
440	3	598	660	488	13626.5	1837.6	11788.9	3	381	660	659	12379.9	1837.6	10542.3	9.1	0	10.6
480	3	598	720	524	13664	1804.4	11859.5	3	374	720	719	12444.2	1804.4	10639.8	8.9	0	10.3
520	2	598	520	452	13696	2101.5	11594.4	3	368	780	779	12551.1	1781	10770.1	8.4	15.3	7.11
560	2	598	560	480	13661	2062.9	11598.2	3	363	840	839	12692.1	1765.2	10926.9	7.1	14.4	5.79
600	2	598	600	507	13655.2	2033.3	11621.9	2	447	600	599	12689.5	2033.3	10656.2	7.1	0	8.31

Table 5 illustrates the effect of the truck capacity on the system, for both the partially and the fully integrated cases. As can be seen, given the assumption that the vendor does not allow partially filled trucks, an increase in the truck capacity leads to lower lot size flexibility and consequently to a decreased ps-value. However, the results also indicate some mutual benefits for both the vendor and supplier that result from full integration: the vendor's psv at q=520 and q=560 is due to the manufacturer producing larger lot sizes in the fully integrated case compared to the partially integrated case.

Next, we explain the pattern observed in the Q^* and R^* values. Since we only varied the full truckload capacity, q, an increase in the truckload capacity led to a decrease in the number of shipments, n, in discrete steps as $n \epsilon \ integers$. Also, due to the equality constraint $nq/\alpha Q = 1$, for a given value of n, an increase in q entails that both Q^* and R^* increase; P^* then tends to decrease to avoid a (too) fast buildup of inventory. As an example, consider $n^* = 3$ for the fully integrated case of Table 5. We notice that if q increases from 400 to 560, Q^* also increases from 600 to 840, R^* increases from 599 to 839, and P^* decreases from 389 to 363.

Table 6: Effect of the manufacturer's holding cost on the system

	Partially integrated case							Fully integrated case							% savings		
h_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
4	4	598	800	542	13122.5	1675	11447.5	4	349	800	800	11775.9	1675	10100.9	10.3	0	12
5	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	599	12369.6	1883.3	10486.3	9.26	0	11
6	3	598	600	448	14131.6	1883.3	12248.2	3	384	600	600	12928.5	1883.3	11045.2	8.51	0	9.8
7	3	598	600	444	14626.7	1883.3	12743.4	3	380	600	599	13492.2	1883.3	11608.8	7.76	0	8.9
8	3	598	600	440	15118	1883.3	13234.6	3	376	600	599	14056.5	1883.3	12173.2	7.02	0	8
9	2	598	400	356	15474.7	2300	13174.7	3	373	600	599	14624	1883.3	12740.6	5.5	18.1	3.3

Table 7: Effect of the manufacturer's holding cost on the system, $q = 300$

	Partially integrated case							Fully integrated case							% savings		
h_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
4	5	598	750	493.8	13191.6	1641.7	11549.9	5	342	750	748.4	11777	1641.7	10135.3	10.7	0.0	12.2
5	4	598	600	424.7	13724	1808.3	11915.7	4	365	600	599.9	12359	1808.3	10550.7	9.9	0.0	11.5
6	4	598	600	420.2	14220.9	1808.3	12412.6	4	361	600	599.4	12937.2	1808.3	11128.8	9.0	0.0	10.3
7	4	598	600	416.3	14713.7	1808.3	12905.4	4	357	600	599.7	13516.1	1808.3	11707.8	8.1	0.0	9.3
8	3	598	450	351	15110	2086.1	13023.8	3	405	450	449.6	13977	2086.1	11890.9	7.5	0.0	8.7
9	3	598	450	348.3	15490.5	2086.1	13404.4	3	401	450	449.6	14394.7	2086.1	12308.6	7.1	0.0	8.2

Table 8: Effect of the manufacturer's holding cost on the system, $q = 200$

	Partially integrated case							Fully integrated case							% savings		
h_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
4	6	598	600	402	13524.7	1733.3	11791.3	7	333	700	699.1	11991.8	1614.3	10377.5	11.3	6.9	12.0
5	6	598	600	397	14023.6	1733.3	12290.3	6	344	600	600	12581.9	1733.3	10848.6	10.3	0.0	11.7
6	6	598	600	392.7	14518	1733.3	12784.6	5	363	500	499.9	13155.2	1900	11255.2	9.4	-9.6	12.0
7	5	598	500	347.1	14963.9	1900	13063.9	5	359	500	500	13641.7	1900	11741.7	8.8	0.0	10.1
8	5	598	500	344.1	15379.9	1900	13479.9	5	356	500	499.5	14132.1	1900	12232.1	8.1	0.0	9.3
9	5	598	500	341.3	15793	1900	13893	5	353	500	499.4	14622.8	1900	12722.8	7.4	0.0	8.4

Table 6 illustrates the effect of an increase in h_m on the system. An increase in h_m makes it more and more expensive to keep inventory (and therewith safety stock) in the system, which for a constant Q^*, induces the fully integrated system to decrease the values of P^* and R^*, which is in contrast to the partially integrated case. Hence, in the fully integrated case, the system benefits from production flexibility by lowering the production speed from $P^* = 598$ that was obtained in the hierarchical case. Hence, slowing down production allows reducing the build-up of inventory. The additional unit production cost that result from varying the production rate from $P^* = 598$ is offset through savings in inventory holding cost. Still, for both the partially and the fully integrated case, an increase in the manufacturer's holding cost leads to a decrease in the lot size, resulting in a lower number of shipments.

The 18.1% savings of the vendor at h_m=9 in the fully integrated case is due to the manufacturer receiving three shipments every cycle, instead of two as in the partially integrated case; this change, which led to a decrease in the manufacturer's savings, was responsible for the cost reduction on the vendor's side. Also, notice that the Q^*-values were equal for the partially and the fully integrated case for all h_m-values, except for $h_m = 9$ ($Q^* - \text{partially integrated} = 400, Q^* - \text{integrated} = 600$), which implies a higher setup cost at the manufacturer in the partially integrated case as compared to the fully integrated case; this explains the sharp decrease in psm (3% at $h_m = 9$) compared to the other psm-values of Table 6.

In terms of lot sizing, reorder levels and production rate decisions, Tables 7 and 8 show similar results than Table 6 for the partially and fully integrated cases. The main difference is that the ps- and psm-values tend to increase slightly as q decreases (more system flexibility). However, the psv-values tends to decrease to the extent of having a negative value in Table 8. Hence, in this case, the reduction of q was more beneficial to the manufacturer.

Table 9: Effect of the manufacturer's setup cost on the system

	Partially integrated case							Fully integrated case							% savings		
A_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
2500	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	599	12369.6	1883.3	10486.3	9.3	0	11
7500	4	598	800	536	15331.6	1675	13656.6	4	345	800	799	14114.2	1675	12439.2	7.9	0	8.9
12500	5	598	1000	619	16631.6	1550	15081.6	5	335	1000	967	15635.8	1550	14085.8	6	0	6.6
17500	6	598	1200	701	17730.9	1466.7	16264.2	5	335	1000	967	16885.8	1550	15335.8	4.8	-5.7	5.7
22500	6	598	1200	701	18772.5	1466.7	17305.9	6	365	1200	1038	17991.1	1466.7	16524.4	4.2	0	4.5
27500	6	598	1200	701	19814.2	1466.7	18347.5	7	392	1400	1105	18999.9	1407.1	17592.8	4.1	4.1	4.1

Table 10: Effect of the manufacturer's setup cost on the system, $q = 300$

	Partially integrated case							Fully integrated case							% savings		
A_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
2500	4	598	600	424.7	13724	1808.3	11915.7	4	365	600	599.9	12359	1808.3	10550.7	9.9	0.0	11.5
7500	6	598	900	550.5	15416	1530.6	13885.4	5	337	750	749.7	14178.2	1641.7	12536.5	8.0	-7.3	9.7
12500	6	598	900	550.5	16804.9	1530.6	15274.3	6	320	900	899.2	15685.7	1530.6	14155.1	6.7	0.0	7.3
17500	6	598	900	550.5	18193.8	1530.6	16663.2	7	343	1050	958.6	16974.9	1451.2	15523.7	6.7	5.2	6.8
22500	6	598	900	550.5	19582.7	1530.6	18052.1	8	365	1200	1012	18090.8	1391.7	16699.1	7.6	9.1	7.5
27500	6	598	900	550.5	20971.5	1530.6	19441	9	386	1350	1061	19086.6	1345.4	17741.2	9.0	12.1	8.7

Table 11: Effect of the manufacturer's setup cost on the system, $q = 200$

	Partially integrated case							Fully integrated case							% savings		
A_m	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
2500	6	598	600	397	14023.6	1733.3	12290.3	6	344	600	600	12581.9	1733.3	10848.6	10.3	0.0	11.7
7500	6	598	600	397	16107	1733.3	14373.6	7	329	700	698.9	14476.2	1614.3	12861.9	10.1	6.9	10.5
12500	6	598	600	397	18190.3	1733.3	16457	9	319	900	875.1	15992.2	1455.6	14536.6	12.1	16.0	11.7
17500	6	598	600	397	20273.6	1733.3	18540.3	11	351	1100	949.4	17291.7	1354.5	15937.1	14.7	21.9	14.0
22500	6	598	600	397	22357	1733.3	20623.6	12	365	1200	985.9	18398.7	1316.7	17082	17.7	24.0	17.2
27500	6	598	600	397	24440.3	1733.3	22707	13	379	1300	1020	19394.5	1284.6	18109.9	20.6	25.9	20.2

Table 9 illustrates that, as the setup cost of the manufacturer increases, the number of shipments and the corresponding lot size increase in both the partially and the fully integrated scenario.

In terms of percent savings, Table 9 highlights three possible scenarios, namely: I) the total supply chain savings materialize only at the manufacturer (psv=0 and psm>0), II) both parties realize savings (both, psv and psm >0), III) the vendor experiences a 5.7% loss. The 5.7% loss for the vendor due to full integration at $A_m = 17500$ is due to the manufacturer making five shipments every cycle instead of six in the partially integrated case, which imposes a loss on the vendor. In fact, since ps-fully integrated is always greater than ps-partially integrated, the fully integrated scenario, in this case, can still be made attractive to the vendor by properly sharing the cost savings that occur at the manufacturer in case the vendor does not benefit from the cooperation. The same applies to all other cases with negative psm- or psv-values.

Tables 10 and 11 repeat the experiments of Table 9, but with different truck sizes ($q = 300$ and $q = 200$, respectively). Tables 9 to 11 show that for the partially integrated case, due to the fixed production rate of 598, the system tries to offset higher setup costs by mainly increasing Q^* and R^*, which tend to take smaller values as q decreases. For the fully integrated case, Tables 9 to 11 indicate that, at given A_m, the system has maintained almost the same Q^* and R^* levels for the different truck sizes. However, as q decreases, the system achieved a better performance (i.e., for a given A_m, as q decreases, ps, psm, and psv increase), which is caused by slowing down the production rate and sending smaller trucks more frequently.

Finally, the reduction in the truck size was very beneficial to the vendor and the manufacturer reflected by the increase in both the psm- and psv-values.

Table 12: Effect of the vendor's holding cost on the system

	Partially integrated case							Fully integrated case							% savings		
h_v	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
3	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	599	12369.6	1883.3	10486.3	9.26	0	10.7
5	3	598	600	452	13832.2	2083.3	11748.8	3	389	600	599	12569.6	2083.3	10486.3	9.13	0	10.7
7	3	598	600	452	14032.2	2283.3	11748.8	3	389	600	599	12769.6	2283.3	10486.3	9	0	10.7
9	3	598	600	452	14232.2	2483.3	11748.8	3	389	600	599	12969.6	2483.3	10486.3	8.87	0	10.7
11	3	598	600	452	14432.2	2683.3	11748.8	3	389	600	599	13169.6	2683.3	10486.3	8.75	0	10.7
13	3	598	600	452	14632.2	2883.3	11748.8	3	389	600	599	13369.6	2883.3	10486.3	8.63	0	10.7

Table 13: Effect of the vendor's holding cost on the system, $q = 300$

	Partially integrated case							Fully integrated case							% savings		
h_v	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
3	4	597.6	600	424.7	13724	1808.3	11915.7	4	365	600	599.9	12359	1808.3	10550.7	9.9	0.0	11.5
5	4	597.6	600	424.7	13874	1958.3	11915.7	4	365	600	599.9	12509	1958.3	10550.7	9.8	0.0	11.5
7	4	597.6	600	424.7	14024	2108.3	11915.7	4	365	600	599.9	12659	2108.3	10550.7	9.7	0.0	11.5
9	4	597.6	600	424.7	14174	2258.3	11915.7	4	365	600	599.9	12809	2258.3	10550.7	9.6	0.0	11.5
11	4	597.6	600	424.7	14324	2408.3	11915.7	4	365	600	599.9	12959	2408.3	10550.7	9.5	0.0	11.5
13	4	597.6	600	424.7	14474	2558.3	11915.7	4	365	600	599.9	13109	2558.3	10550.7	9.4	0.0	11.5

Table 14: Effect of the vendor's holding cost on the system, $q = 200$

	Partially integrated case							Fully integrated case							% savings		
h_v	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
3	6	597.6	600	397	14023.6	1733.3	12290.3	6	344	600	600	12581.9	1733.3	10848.6	10.3	0.0	11.7
5	6	597.6	600	397	14123.6	1833.3	12290.3	6	344	600	600	12681.9	1833.3	10848.6	10.2	0.0	11.7
7	6	597.6	600	397	14223.6	1933.3	12290.3	6	344	600	600	12781.9	1933.3	10848.6	10.1	0.0	11.7
9	6	597.6	600	397	14323.6	2033.3	12290.3	6	344	600	600	12881.9	2033.3	10848.6	10.1	0.0	11.7
11	6	597.6	600	397	14423.6	2133.3	12290.3	6	344	600	600	12981.9	2133.3	10848.6	10.0	0.0	11.7
13	6	597.6	600	397	14523.6	2233.3	12290.3	6	344	600	600	13081.9	2233.3	10848.6	9.9	0.0	11.7

Tables 12 to 14 illustrate the effect of the vendor's holding cost on the system. Since the vendor holds no significant inventory (as compared to the manufacturer), s/he produces and ships continuously (Figure 1, Part A), and because of the demand satisfaction constraint the number of shipments do not change. Changes in h_v and q resulted only in an increase in the total supply chain cost and decreased ps-values, while psm remained constant. A further observation from Tables 12 to 14 is that the smaller the truck size, the better the performance of the system in terms of ps- and psm-values.

Table 15: Effect of the vendor's setup cost on the system

	Partially integrated case							Fully integrated case							% savings		
A_s	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	n^*	P^*	Q^*	R^*	$T.C_s$	$T.C_V$	$T.C_{rm}$	ps	psv	psm
2000	3	598	600	452	13632.2	1883.3	11748.8	3	389	600	599	12369.6	1883.3	10486.3	9.3	0	10.7
7000	4	598	800	536	15331.6	3237.5	12094.1	4	345	800	799	14114.2	3237.5	10876.7	7.9	0	10.1
12000	5	598	1000	619	16631.6	4050	12581.6	5	335	1000	967	15635.8	4050	11585.8	6	0	7.92
17000	6	598	1200	701	17730.9	4591.7	13139.2	5	335	1000	967	16885.8	5300	11585.8	4.8	-15	11.8
22000	6	598	1200	701	18772.5	5633.3	13139.2	6	365	1200	1038	17991.1	5633.3	12357.7	4.2	0	5.95
27000	6	598	1200	701	19814.2	6675	13139.2	7	392	1400	1105	18999.9	5871.4	13128.5	4.1	12	0.08

Similar to the impact of the manufacturer's setup cost on the system, Table 15 illustrates that an increase in the setup cost of the vendor entails an increase in the number of shipments and the corresponding manufacturer's lot size, both in the partially and the fully integrated scenarios.

In terms of percent savings, there are four possible scenarios with respect to the combination of psv and psm, namely: I) psv=0 and psm>0, II) psv<0 and psm>0, III) psv>0 and psm>0, and IV) psv>0 and psm<0.

If the number of shipments, n^* in the fully integrated case exceeds that of the partially integrated case, then psv will be positive; in case they are equal, psv will be zero; otherwise, psv will be positive indicating that full integration is more beneficial to the vendor than partial integration. It can also be observed that as the setup cost increases, psm tends to decrease, and in some cases, psm even took on negative values; this effect is due to the increase in the lot size that the manufacturer has to produce in every cycle and the accompanying higher reorder level.

Finally, the experiments of Table 15 were replicated but with different truck sizes ($q = 300$ and $q = 200$, respectively), and all the observations made earlier on Tables 10 ($q = 300$) and 11 ($q = 200$) when compared to Table 9 ($q = 400$, baseline scenario as per Table 1), apply to our results for $q = 300$ and $q = 200$ when compared to $q = 400$ (Table 15). Hence, to keep the paper short, we decided not to report the additional tables here.

5.0 CONCLUSION

This paper considered the single vendor-single manufacturer joint economic lot size problem under stochastic demand. As compared to previous works, we developed an integrated mathematical model that investigates the impact of a variable production rate on the system. A simple, yet fast solution technique was used to solve the model and obtain globally optimal solutions. In numerical examples, we showed that the cost incurred in a supply chain system can be reduced by controlling the manufacturer's production rate. This reduction in total supply chain cost can be beneficial to the vendor and the manufacturer. Some examples showed that varying the manufacturer's production rate can benefit the manufacturer, but lead to disadvantages on the vendor's side. These disadvantages, however, can be offset by implementing a proper sharing mechanism for the total supply chain savings.

The paper at hand showed that in case of full truckload shipments, the truck capacity restricts the lot sizing decisions strongly, which negatively influences the performance of the system. First, it restricts the integration and may make it more difficult to coordinate, as the cooperation gain that can be redistributed is lower. Secondly, it may imply that the supply chain should evaluate whether shifting to less-than-truckload shipments or offering quantity discounts on large trucks is beneficial. In both cases, a variable production rate introduces additional flexibility into the supply chain, which may help to reduce total cost.

As a future work, our model could be extended by incorporating quality issues resulting from variable production rates. Prior research has shown that deviating from the design production rate of a machine may lead to lower yield rates, which would have to be taken into account when coordinating the production policies of a vendor and a manufacturer. Clearly, investigating maintenance policies or the reworking of defective items in this scenario would be promising. A second option to extend our work would be the integration of quantity discounts the vendor could offer to the manufacturer to induce the manufacturer to consider the vendor's position in making production and distribution decisions.

Acknowledgment

The authors wish to thank the anonymous reviewers for their constructive comments on an earlier version of this paper that helped to improve its quality substantially.

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