Journal of Macroeconomics 46 (2015) 264–277

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Accounting for real exchange rate changes at long time horizons[☆]

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ARTICLE INFO

Article history:
Received 9 December 2014
Accepted 22 September 2015
Available online 8 October 2015

JEL Classification:
F3
F4

Keywords:
Purchasing power parity
Traded and nontradable prices
Real exchange rates
Mean squared error ratio

ABSTRACT

Engel (1999) introduced real exchange rate accounting to determine the importance of nontradables for real exchange rate movements. We extend his approach in two directions. First, we identify a potential bias in the mean squared error (MSE) measure used in previous work. Second, using the corrected MSE measure we provide new empirical evidence that nontradables explain real exchange rate movements but only at really long horizons – over decades not years.

1. Introduction

Experience with floating exchange rates since the early 1970s shows that real exchange rates are volatile and that deviations from Purchasing Power Parity (PPP) are large and persistent.[3] What explains the deviations from PPP? The consensus in the literature is that tradable prices explain short run deviations from PPP. What about deviations over longer horizons? The traditional models of open economy macroeconomics and the growth literature arising from Balassa (1964) and Samuelson (1964) assume that purchasing power parity holds for traded goods over the long run. This ensures that long run movements in real exchange rates are caused by changes in the relative price of nontradables.[4]

☆ We are grateful to two anonymous referees and the editor, William Lastrapes, for very helpful suggestions that have greatly improved the paper.

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3 Cassel (1916, 1918, 1922) is usually given the credit for the modern formulation of purchasing power parity. See Taylor and Taylor (2004) and Rogoff (1996) for literature surveys.

4 Balassa (1964) and Samuelson (1964) argued that differences in rates of productivity growth in the traded and nontraded sectors drive differences in the relative price of nontradables and hence overall price levels. The subsequent literature provides many additional mechanisms. Bhagwati (1984), for example, emphasized differences in factor proportions. Asea and Corden (1994) summarize the early literature. More recent work studies the causes of price level differences using models with trade costs, heterogeneous firms and imperfect competition, see Bergin, Glick and Taylor (2006) and Ghironi and Melitz (2005).

http://dx.doi.org/10.1016/j.jmacro.2015.09.009

Engel (1999) transformed the empirical debate on the importance of nontradables for real exchange rate movements. Using CPI data from Canada, France, Germany, Italy, Japan, and the United States, he found no support for traditional theories as traded goods prices explained real exchange rate movements at all horizons. Subsequent work using Engel's framework have largely confirmed his findings. These results have proved highly influential leading some to suggest that the tradable/nontradable distinction has little relevance for open economy macroeconomics.

This paper extends real exchange rate accounting in two directions. Our first contribution is methodological. We show, analytically and numerically, that the mean squared error (MSE) measure used by Engel (1999) is biased. This potential bias does not appear to have attracted previous attention. We also show that the bias is positive and large for relative tradable price levels derived from CPI data. This, in turn, suggests that real exchange rate accounting as previously applied will understate the importance of nontradables. Finally, we use an alternative MSE measure that does not suffer from bias.

Our second contribution is to provide new empirical evidence on the relative importance of nontradables over longer time horizons. Engel's (1999) CPI data spans 1973 to 1995 – a little over two decades. As he recognized, two decades are not sufficient to determine the role of nontradables over the very long run. We re-examine the importance of nontradables for the floating period by extending Engel's (1999) price indices to the present. This doubles the span of his data. We also correct for the bias in his MSE measures. Using the longer span data and a corrected MSE measure, we obtain results similar to Engel (1999) that the relative price of tradables dominates the short and medium run. Indeed, they account for 95% of real exchange rate changes over a 10 year horizon for all countries with the exception of Canada. For Canada, the relative price of tradables accounts for 85%. However, we find that the importance of tradables falls at very long horizons – from 30–40 years. At a horizon of 35 years, tradables explain just 24% of real exchange rate changes for Canada and 47% for Germany. They explain 60% for France and 83% for Italy. The results that nontradables matter in the very long run also hold when we use price indices derived from GDP deflators. Our findings contrast with Engel (1999) who found no role for nontradables at long horizons with the exception of Canada. The differences arise from the longer span of our data and the corrected MSE measure. We conclude that, while Engel (1999) is correct over the short and medium run, there is an important role for nontradables over the very long run.

2. Real exchange rate accounting

This section outlines real exchange rate accounting. The approach aims to determine the contribution of relative traded and non-traded price levels to real exchange rate movements.

Let us assume that the overall price index is a geometrically weighted average of tradable and nontradable prices:

$$p_t = (1-\alpha)p_t^T + \alpha p_t^N \tag{1}$$

where p_t is the log of the overall price index, the super-scripts T and N refer to tradables and nontradables respectively and α is the share of nontradables in the price index of the base economy. For convenience, we take the US as the base economy.

Engel (1999) suggests that we decompose the real exchange rate as follows:

$$q_t = x_t + y_t \tag{2}$$

where

$$\begin{aligned} q_t &= s_t + p_t^* - p_t \\ x_t &= s_t + p_t^{T*} - p_t^T \\ y_t &= \beta(p_t^{N*} - p_t^{T*}) - \alpha(p_t^N - p_t^T). \end{aligned}$$

The asterisks represent the foreign country, β denotes the nontradable share in the foreign price index and s_t denotes the log of the nominal exchange rate. Eq. (2) divides the log of the real exchange rate (q_t) into two parts. The first term is the relative price of tradables in terms of the US denoted by x_t. The second term, y_t, is the cross-country relative nontradable-tradable price ratio weighted by the expenditure shares of nontradables.

There is a wide agreement that price levels respond slowly to nominal exchange rate changes over short time horizons. From Eq. (2), note that changes in the nominal exchange rate will change the x_t term but not the y_t term when prices are fixed in the domestic currency. Thus, tradables will explain most of real exchange rate movements over short horizons for floating exchange rates.

What explains real exchange rate changes at longer horizons? Engel (1999) proposed the following mean squared error (MSE) ratio to determine the importance of nontradables at different time horizons.[5]

$$B1(k) = \frac{\mathrm{MSE}(x_t - x_{t-k})}{\mathrm{MSE}(x_t - x_{t-k}) + \mathrm{MSE}(y_t - y_{t-k})} \tag{3}$$

Note that $B1(k)$ measures the importance of tradables for real exchange rate movements at a time horizon, k. If the ratio decreases as k increases, we may say that the importance of the relative price of tradables for real exchange rate changes falls over time.[6] Recall that the traditional approach assumes that relative purchasing power parity holds for tradables over longer time

[5] Eq. (3) assumes that there is a zero correlation between x and y which holds for our data in this paper. When the correlation between x and y is positive, $B1$ will understate the relative importance of tradables. With a negative correlation, it will overstate the importance of tradables.

[6] If x_t follows a stationary process and y_t is nonstationary this is sufficient for $B1(k)$ to decrease as k increases.

horizons. This means that the x_t term should tend towards zero and changes in the cross-county relative price of nontradables will explain real exchange rate movements.

The MSE ratio in (3) provides a simple and intuitive way to capture the importance of tradables/nontradables for real exchange rate changes over different time horizons. The approach does not require a specification of a data generating process thus freeing it from difficulties such as temporal aggregation and nonlinear dynamic adjustment that have bedeviled formal tests.

We face two difficulties in the application of real exchange rate accounting. The first issue is over what time horizon would we expect to see nontradables dominate real exchange rate movements - 1 year? 5 years? 20 years? The literature is silent on this question. Second, there is a problem with how real exchange rate accounting is applied in practice. As we show in the next section, the MSE measure used by Engel (1999) is flawed and produces biased empirical results.

3. The MSE estimator: bias and correction

To illustrate the bias in the MSE measure used by Engel (1999), let us consider an iid variable Z_i, $i = 1,\ldots,$ N with mean μ_z and variance σ_z^2. By definition, the variance of this variable equals the second moment minus the square of the mean (μ_z):

$$\sigma_z^2 = E(Z-\mu_z)^2 = E(Z^2) - {\mu_z}^2 \tag{4}$$

If μ_z is known, it follows that $\tilde{\sigma}_z^2 = \frac{1}{N}\sum_{i=1}^{N}(Z_i-\mu_z)^2$ is an unbiased estimator for σ_z^2 such that $E(\tilde{\sigma}_z^2) = \sigma_z^2$. We can also write $E(\tilde{\sigma}_z^2) = E(\frac{1}{N}\sum_{i=1}^{N}(Z_i-\mu_z)^2) = E(\frac{1}{N}\sum_{i=1}^{N}Z_i^2) - \mu_z^2$.

Rewriting we have $E(\frac{1}{N}\sum_{i=1}^{N}Z_i^2) - \mu_z^2 = \sigma_z^2$ or

$$E\left(\frac{1}{N}\sum_{i=1}^{N}Z_i^2\right) = \sigma_z^2 + \mu_z^2. \tag{5}$$

Eq. (5) shows that $\frac{1}{N}\sum_{i=1}^{N}Z_i^2$ is an unbiased estimator for the sum of the variance and the square of the mean. It is the correct measure for the mean squared error.

We give Engel's MSE measure in Eq. (6). As we shall demonstrate, the measure is biased.

$$\mathrm{MSE}^{\mathrm{Engel}} = \tilde{\sigma}_z^2 + \bar{Z}^2 \tag{6}$$

where $\tilde{\sigma}_z^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Z_i-\bar{Z})^2$ and $\bar{Z}$ is the sample mean. Engel replaces σ_z^2 and μ_z^2 in (5) by $\tilde{\sigma}_z^2$ and $\bar{Z}^2$, respectively. The sample variance is an unbiased estimator for σ_z^2, and $\bar{Z}$ is an unbiased estimator for μ_z. However, there is a problem with his approach because $\bar{Z}^2$ is not an unbiased estimator for μ_z^2. As a result, the Engel (1999) MSE measure is biased.

Does the MSE bias matter in practice? As we shall see, the bias is potentially serious. To see why, let us consider a numerical example used by Engel (1999). Let Z_t be a series that is characterized as a random walk with drift as follows:

$$z_{t+1} = \mu + z_t + \varepsilon_t, \quad t = 1,\ldots,T,$$

where ε_t is independently and identically distributed with mean 0 and variance σ^2. Let $d_t(k) = z_t - z_{t-k}$. Then its mean and variance are $E(d_t(k)) = k\mu$ and $E(d_t(k) - E(d_t(k)))^2 = k\sigma^2$, respectively.

Eq. (7) provides our MSE measure for this case.

$$\hat{\mathrm{MSE}} = \frac{1}{N-k}\sum_{t=k+1}^{T}{d_t}^2(k) \tag{7}$$

Engel, however, used the estimator given by (8).

$$\mathrm{MSE}^{\mathrm{Engel}} = \hat{\sigma}^2(k) + (k\bar{d}(1))^2 \tag{8}$$

where $\hat{\sigma}^2(k) = T_k \cdot \sum_{t=k+1}^{T}\left(d_t(k) - k\bar{d}(1)\right)^2$ is the sample variance and where $T_k = \frac{T-1}{(T-k-1)(T-k)}$ is the small sample bias adjustment term proposed by Cochran (1988). It is straightforward to show that $\hat{\sigma}^2(k)$ is an unbiased estimator for σ^2. The problem occurs with the second term $(k\bar{d}(1))^2$ as it is a biased estimator for $(k\mu)^2$.

In Appendix A, we show the difference between the expected values of the MSE measures in (7) and (8) as follows:

$$E(\mathrm{MSE}^{\mathrm{Engel}}(d_{t-k})) - E(\hat{\mathrm{MSE}}(d_{t-k})) = \frac{1}{(T-1)}k^2\sigma^2. \tag{9}$$

From (9), we see that Engel's MSE measure is not equivalent to the correct MSE unless T goes to infinity for a given k. In other words, his measure is biased. Worryingly, the difference between the two measures increases as k approaches the sample size of T. This is important as it suggests the bias increases at long time horizons.

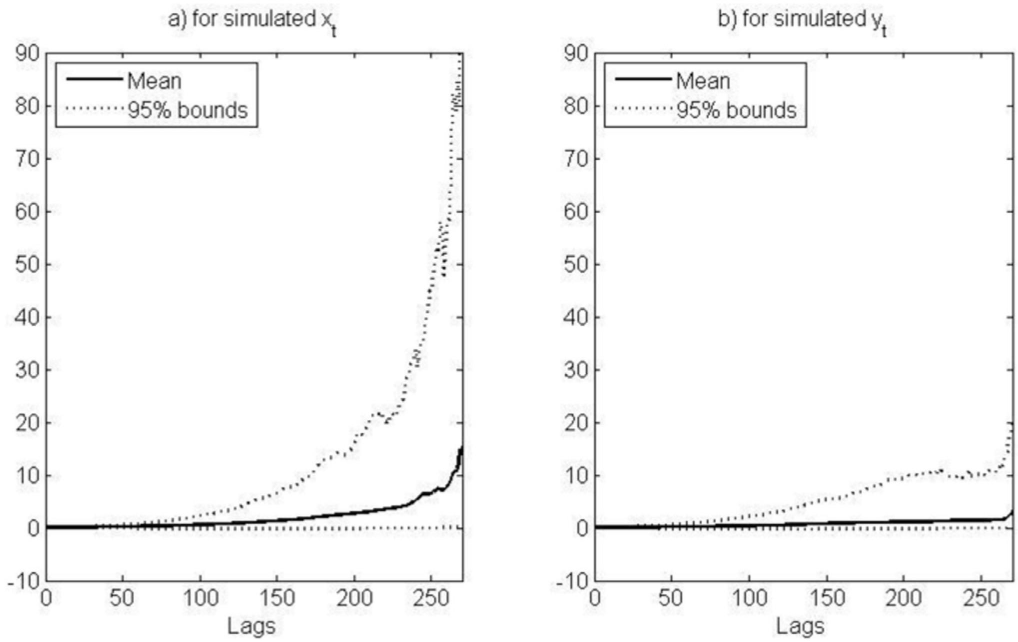

Fig. 1. The Relative bias of MSE^{Engel}.
Note: Bias is measured by $(MSE^{Engel} - \hat{MSE})/\hat{MSE}$ with the simulated data from $z_t = \mu + z_{t-1} + \varepsilon_t$ where $\mu = 0$ and $\sigma = 0.0252$ for panel a and $\mu = -3.0973e{-}04$ and $\sigma = 0.0027$ for panel b.

To have a better understanding of the likely bias, we provide a numerical example to illustrate the difference between the Engel (1999) MSE measure in (8) relative to the correct measure in (7). Let the difference relative to $\hat{MSE}(d_{t-k})$ be $M_z(k)$.

$$M_z(k) = \frac{MSE^{Engel}(d_{t-k}) - \hat{MSE}(d_{t-k})}{\hat{MSE}(d_{t-k})} \tag{10}$$

We provide simulations for the x_t and y_t variables using parameters estimated from data for the floating exchange rate period taken from Engel (1999). Engel (1999) provides monthly data for Canada, France, Germany, Italy, Japan, and the United States from 1973 to 1995. For our simulations, we estimate the x_t and y_t processes for each country as a random walk with drift where the drift is μ and standard deviation is σ.[7] The simulations use the average of the estimated μ's and σ's from these economies. We set $\mu = 0$ and $\sigma = 0.0252$ for the simulation of x_t and we use $\mu = -3.0973e{-}04$ and $\sigma = 0.0027$ for the simulation of y_t.

We repeat the simulations one thousand times. We plot the mean of the differences between the Engel MSE and the corrected MSE as well as the 25th largest and smallest values for each k in Fig. 1. The left and right panels are for the simulated x_t and y_t variables respectively. Panel *a* for the simulated x_t shows the large bias and the bias increases dramatically at longer lags. The average bias is more than 10 times the correct MSE as k approaches T. Furthermore, the 95% bounds indicate that the bias can reach up to 100 times the corrected MSE.

Panel *b* provides the results for y_t. Strikingly, the bias is smaller than for the relative price of tradables. At the 95% bounds, it does not exceed 10 times the corrected MSE. This is one tenth of the relative bias for the x_t variable.

The simulation results indicates that the Engel MSE measure is biased and the bias is large at long horizons given our assumptions about the times series properties of the x_t and y_t variables. In particular, the bias is more pronounced for the x_t variable, the relative price of tradables. This, in turn, suggests that real exchange rate accounting can understate the importance of nontradables.

The above simulations are based on the assumption that the x_t and y_t variables from the Engel (1999) CPI data set follow a random walk with drift. Since the bias depends on the data generating process for the variables, the impact of the bias on the size of mean square error ratio of *B1* in (3) is therefore an empirical question. For example, if the magnitude of bias for the x_t and y_t variables were similar, then the effects on the MSE ratio may be small. It is also possible that the bias is larger for the y_t variable. In this situation, real exchange rate accounting will understate the importance of tradables.[8] The important point we wish to make here is that the Engel (1999) MSE measure is potentially biased and can distort our understanding of the determinants of real exchange rate movements, although the effects may vary across data sets. The next section examines how the MSE bias works in practice.

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[7] Engel's (1999) data set for the floating regime has 276 observations.

[8] We will return to this point in Appendix B.

4. Some evidence

Our empirical studies consider two data sets covering the floating exchange rate period that begins in 1973. This gives us four decades of data to examine the role of nontradables in real exchange rate movements. The first is from Engel (1999) where we extend his CPI data to January 2015. The second data set uses price data from the output side – annual price indices derived from value added deflators covering up to 2013 – to crosscheck the results.[9]

4.1. Extending Engel (1999)

As noted earlier, Engel (1999) looked at the importance of tradables/nontradables for real exchange rate changes using monthly CPI data from Canada, France, Germany, Italy, Japan, and the United States. Engel (1999) constructed his tradable price indices using the food and all goods less food sub-indices from OECD consumer price data. He built his nontradable price index from the price indices for shelter and all other services.[10] His data cover from 1973 to 1995. We extend his data to January 2015 providing us with an additional 20 years. The details of data sources and construction are in Appendix C.[11]

We begin by comparing our corrected MSE measures at different time horizons with the MSE measure used by Engel (1999) for the x_t and y_t variables defined earlier. Fig. 2 provides the results. The horizontal axis represents the lags in months. The light and bold lines are the MSE measures for x_t and y_t based on the $\mathrm{MSE}^{\mathrm{Engel}}$ and $\widehat{\mathrm{MSE}}$, respectively.

There are three points to note. First, the MSE for the relative price of tradables (x_t) greatly exceeds that for the cross-country relative price of nontradables (y_t) with the exception of Canada. The difference reflects the effects of exchange rate changes on the relative price of tradables. Second, the Engel MSE measures for x_t and y_t variables are biased upwards. The bias is much larger for x_t, the relative price of tradables, particularly at long horizons. Properly measured, the MSE for x_t, tends to fall at very long time horizons with the exception of Japan. The result is plausible as it implies that tradable prices are not permanently drifting apart. Using the Engel measure, the MSE for the x_t, variable increases at longer horizons.

Finally the MSE for the cross-country relative price of nontradables (y_t) increases over all horizons. This pattern is consistent with the notion that relative nontradable price levels are driven apart over time by the forces of economic growth. If the patterns continue to hold then nontradables will dominate real exchange rate movements eventually. Whether we find this result empirically will depend, however, on the data span. As we shall see even 40 years may not be long enough.

Fig. 3 compares the MSE ratio obtained using the Engel's measure, $\mathrm{MSE}^{\mathrm{Engel}}$ and the corrected measure, $\widehat{\mathrm{MSE}}$. Using the corrected measure, we find that changes in the relative price of tradables dominate real exchange rate movements at short and medium horizons. They account for 95% of real exchange rate movements for all countries at 120 months. They account for 85% of real exchange rate movements at 240 months for all countries except Canada (62%). This is roughly the span of data used by Engel (1999).

In sharp contrast, the importance of the relative price of tradables falls over time for all countries with the corrected measure. The decline is certainly slow. Nevertheless, by the time we reach a horizon of 420 months (35 years), the share of the relative price of tradables is down to 24% for Canada and 40% for Germany. It is 60% for France and 83% for Italy.[12] For Japan, the share of the relative price of tradables remains above 90%.

To summarize, the evidence suggests that nontradables matter for floating exchange rates but over very long horizons – 30–40 years. As mentioned with Fig. 2, the MSE of the y_t variable is increasing over time while that for x_t is decreasing or constant. If these patterns continue then y_t must eventually dominate for all countries. Fig. 3 shows that the time spans necessary for this to occur may be very long indeed.

Fig. 3 shows that the bias in the Engel (1999) MSE measure works to increase the share of the relative price of tradables. It is also noticeable that the bias would not change the overall conclusions –the relative importance of tradables falls but only at very long time horizons.[13] Rather, the bias works to reduce the quantitative importance of nontradables.

4.2. GDP deflators

Until now, we have followed Engel (1999) and focused on CPI data. The CPI collects price data at the final expenditure stage. These prices contain wholesale and retail margins that are largely nontraded. The presence of nontraded margins has led some scholars to construct alternative price indices for tradables using output deflators. Since these price indices are measured at the farm or factory gate, they arguably provide a better measure of tradable price levels, see Betts and Kehoe (2006, 2008). The drawback is that monthly data are not available for such indices. It is also difficult to obtain quarterly data on a consistent basis for a large group of economies.

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9 We are indebted to an anonymous referee for the suggestion to use output deflators.

10 See Appendix A of Engel (1999) for details. We obtained his data set from: http://www.ssc.wisc.edu/~cengel/Data/JPE/RealExchangeRateData.htm.

11 As explained in Appendix C, our weights differ from Engel (1999).

12 The Canadian results may reflect the high degree of integration between Canada and the US.

13 The reason that the bias does not change the overall conclusion may be because we have a relatively long series. The bias, however, matters with shorter spans. Using the span covered by Engel (1999), from 1973 to 1995, we find that the share of tradables falls for all economies except Japan with the corrected measure. In contrast, the share of tradables does not decline using the Engel measure except for Canada. We provide these results in Appendix B.

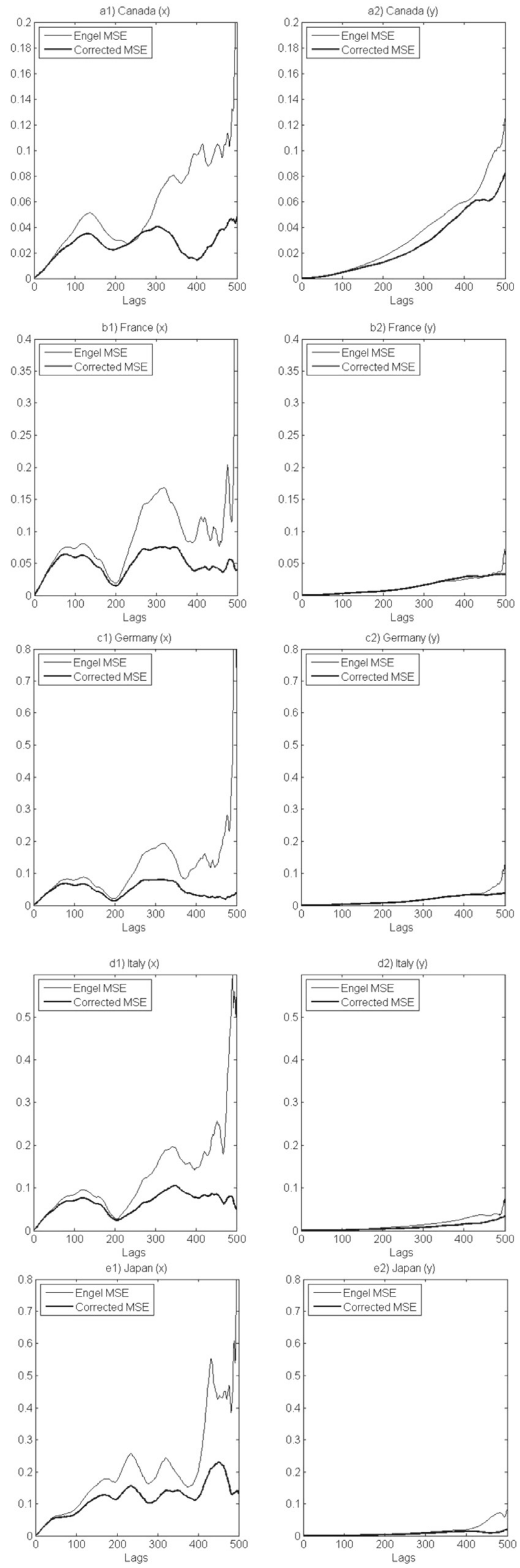

Fig. 2. Comparing the MSE measures.

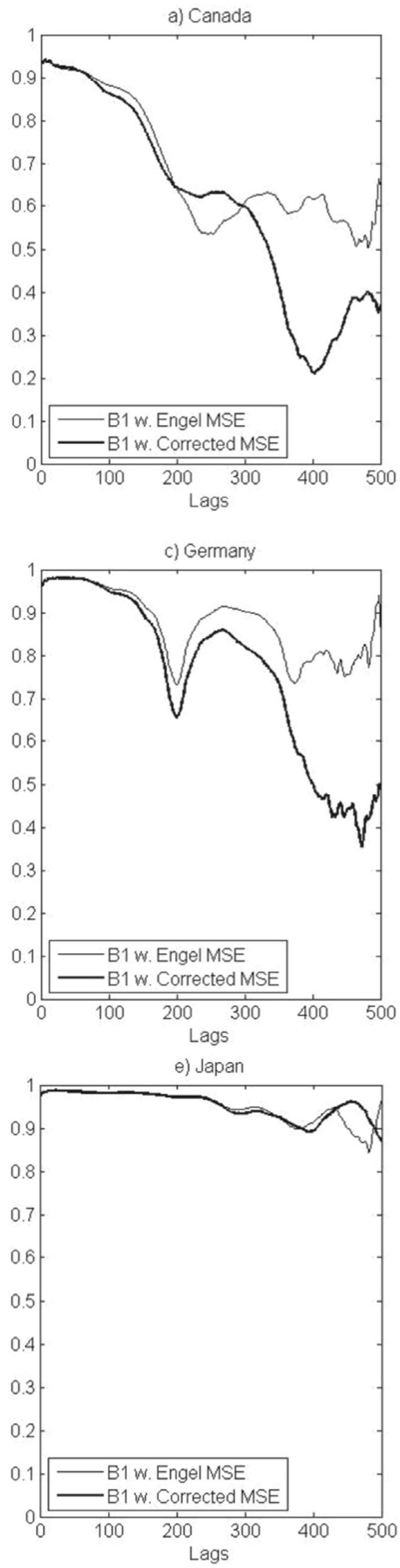

Fig. 3. The relative importance of tradables using CPI data.
Note: The figures plot $B1(k) = \frac{\mathrm{MSE}(x_t - x_{t-k})}{\mathrm{MSE}(x_t - x_{t-k}) + \mathrm{MSE}(y_t - y_{t-k})}$.

As a cross check, we examine the results using annual price indices derived from GDP deflators for the period between 1973 and 2013. To form the price indices we must allocate industries to traded and nontraded sectors. The literature in this area since DeGregorio, Giovannini and Wolf (1994) construct tradable/nontradables indices by assuming that tradables consist of manufacturing and agriculture. We follow their approach.[14]

We construct our price indices using sectoral GDP deflator data from the United Nations National Account database. These data are annual. The details are in Appendix C. To ensure comparability, we confine our attention to the six economies considered earlier.

The overall results, given in Appendix B, are broadly similar to those obtained using the CPI data. We find that changes in the relative price of tradables dominate real exchange rates movements at short and medium time horizons. After 10 years, they account for 80% of real exchange rate changes for all countries. In accordance with our CPI results, we find the importance of the relative price of tradables falls after three decades or more. The effect is again largest for Canada, Germany and France. Also we find no evidence that nontradables matter for Japan.[15]

5. Summing up

We have shown that the MSE measures used in Engel (1999) for real exchange rate accounting are biased. In this paper, we correct the MSE measure. Using the corrected measure, we re-examine the importance of nontradables price levels for real exchange rate movements at very long horizons over the floating period. In accordance with Engel (1999) and later work, we find that changes in the relative price of tradables dominate real exchange rate movements for horizons up to two decades. Over longer horizons, however, we find the relative price of nontradables play a more important role.

Our result that nontradables matter for real exchange rates, albeit at very long horizons, is consistent with the evidence from other areas. The first piece of evidence is the finding that nontradables explain bilateral real exchange rate movements for fixed exchange rate regimes, see Mendoza (2000), Burstein et al. (2005), Chen et al. (2006) and Naknoi (2008). The second piece of evidence comes from the burgeoning literature comparing price levels using large micro price data sets. This work finds that price level differences across developed economies for tradable goods are smaller than for nontradables, see Crucini et al. (2005) and Crucini and Shintani (2008). The final, and in our view the most convincing, evidence is the sheer scale of the price level differences across developed and developing economies as revealed by the International Comparison Program. It is hard to see how these differences can be explained by differences in the relative price of tradables.[16]

Appendix A. The MSE bias

In this appendix, we derive the bias of the MSE measure proposed by Engel (1999) under the assumption that the series follow a random walk with drift.

Let $z_t = \mu + z_{t-1} + \varepsilon_t$, where ε_t is independently and identically distributed with mean zero and variance σ^2. Denote the first difference of z_t by $d_t(k) = z_t - z_{t-k}$. Then we can write

$$d_t(k) = k\mu + \sum_{i=0}^{k-1} \varepsilon_{t-i}$$

Its mean and variance are

$$E(d_t(k)) = k\mu \tag{A1-1}$$

and

$$E(d_t(k) - E(d_t(k)))^2 = E\left(\sum_{i=0}^{k-1} \varepsilon_{t-i}\right) = k\sigma^2. \tag{A1-2}$$

Since $E(d_t(k) - E(d_t(k)))^2 = E(d_t^2(k)) - E^2(d_t(k))$, we can write

$$E\left(d_t^2(k)\right) = E(d_t(k) - E(d_t(k)))^2 + E^2(d_t(k)) = k\sigma^2 + (k\mu)^2. \tag{A2}$$

The expected value of the second moment is the sum of the variance and the square of the mean. As discussed in the main text, we seek a MSE estimator whose expected value is the sum of the variance and the square of the mean as in (A2).

Engel's MSE uses the sample mean and variance measure proposed by Cochran (1988). Given T observations, Cochran proposed the sample mean of $d_t(k)$ as $\bar{d}(k) = \frac{k \cdot (z_T - z_1)}{T-k}$ so that the means across all k's are consistent each other. Its expected value is

$$E\left(\bar{d}(k)\right) = \frac{k \cdot E(z_T - z_1)}{T-k} = k \cdot E\left(\bar{d}(1)\right) = k\mu.$$

[14] DeGregorio, Giovannini and Wolf (1994) allocated industries to the traded sector by looking at the ratio of trade to total output for each industry.

[15] There are, however, some important differences between behavior of the CPI and GDP deflators that we explore in Appendix B.

[16] Chen, Choi and Devereux (2015) use price data from the International Comparison Program (ICP) and the International Comparison of Output and Productivity Program (ICOP) to show that nontradables explain most price level differences at a point in time across developed and developing economies.

This is the same as (A1-1). Thus $\bar{d}(k)(=k\bar{d}_t(1))$ is an unbiased estimator for the mean, $k\mu$.
Cochran(1988) also proposed the sample variance, $\hat{\sigma}^2(k)$ of $d_t(k)$ as follows:

$$\hat{\sigma}^2(k) = T_k \cdot \sum_{t=k+1}^{T} (d_t(k) - \bar{d}(k))^2 \quad \text{and} \quad T_k = \frac{T-1}{(T-k-1)(T-k)}.$$

We evaluate the expected value of the sample variance and show that the sample variance is also unbiased for $k\sigma^2$.

$$\begin{aligned} E(\hat{\sigma}^2(k)) &= T_k E\left(\sum_{t=k+1}^{T} (d_t(k) - k\bar{d}(1))^2\right) \\ &= T_k\left[E\left(\sum_{t=k+1}^{T} d_t^2(k)\right) - 2kE\left(\bar{d}(1)\sum_{t=k+1}^{T} d_t(k)\right) + (T-k)k^2E(\bar{d}^2(1))\right] \end{aligned} \tag{A3}$$

The first term of the bracket in (A3) is:

$$E\left(\sum_{t=k+1}^{T} d_t^2(k)\right) = \sum_{t=k+1}^{T} E\left(d_t^2(k)\right) = \sum_{t=k+1}^{T} (k\sigma^2 + (k\mu)^2) = (T-k)(k\sigma^2 + (k\mu)^2). \tag{A4}$$

The second term is:

$$\begin{aligned} E\left(\bar{d}(1)\sum_{t=k+1}^{T} d_t(k)\right) &= E\left(\mu + \frac{1}{T-1}\sum_{t=2}^{T}\varepsilon_t\right)\left(\sum_{t=k+1}^{T}\left(k\mu + \sum_{i=0}^{k-1}\varepsilon_{t-i}\right)\right) \\ &= E\left(\mu + \frac{1}{T-1}\sum_{t=2}^{T}\varepsilon_t\right)\left((T-k)k\mu + \sum_{t=k+1}^{T}\sum_{i=0}^{k-1}\varepsilon_{t-i}\right) \\ &= (T-k)k\mu^2 + \frac{1}{T-1}E\left(\sum_{t=2}^{T}\varepsilon_t\right)\left(\sum_{t=k+1}^{T}\sum_{i=0}^{k-1}\varepsilon_{t-i}\right) \\ &= (T-k)k\mu^2 + (T-k)\frac{1}{T-1}k\sigma^2. \end{aligned} \tag{A5}$$

And the last term in (A3) is:

$$E\left(\bar{d}^2(1)\right) = E\left(\mu + \frac{1}{T-1}\sum_{t=2}^{T}\varepsilon_t\right)^2 = \mu^2 + \frac{1}{(T-1)^2}E\left(\sum_{t=2}^{T}\varepsilon_t^2\right) = \mu^2 + \frac{1}{(T-1)}\sigma^2. \tag{A6}$$

Plugging (A4)–(A6) into (A3), we obtain

$$\begin{aligned} E(\hat{\sigma}^2(k)) &= k_n(T-k)\left((k\sigma^2 + (k\mu)^2) - 2\left((k\mu)^2 + \frac{1}{T-1}k^2\sigma^2\right) + \left((k\mu)^2 + \frac{1}{(T-1)}k^2\sigma^2\right)\right) \\ &= T_k(N-k)\left((k\sigma^2 + (k\mu)^2) - \left((k\mu)^2 + \frac{1}{T-1}k^2\sigma^2\right)\right) \\ &= T_k(T-k)\left(k\sigma^2 - \frac{1}{T-1}k^2\sigma^2\right) = k\sigma^2, \end{aligned} \tag{A7}$$

which is the same as the variance in (A1-2). Thus the Cochran's sample variance is an unbiased estimator.

Now we seek a MSE measure whose expected value is the sum of the variance and the square of the mean $(k\sigma^2 + (k\mu)^2)$ as in (A2). It is natural to consider the measure $\hat{\text{MSE}} = \frac{1}{(T-k)}\sum_{t=k+1}^{T} d_t(k)^2$ as a candidate for the mean square error. Indeed, it is unbiased for $k\sigma^2 + (k\mu)^2$.

The mean square error used by Engel (1999) is, however,

$$\text{MSE}^{\text{Engel}} = \hat{\sigma}^2(k) + (k\bar{d}(1))^2.$$

Using (A6) and (A7), we can see that its expected value is

$$E(\text{MSE}^{\text{Engel}}) = k\sigma^2 + (k\mu)^2 + \frac{1}{(T-1)}k^2\sigma^2.$$

Thus the difference between expected values of $\text{MSE}^{\text{Engel}}$ and $\hat{\text{MSE}}$ is

$$E(\text{MSE}^{\text{Engel}}) - E(\hat{\text{MSE}}) = \frac{1}{(T-1)}k^2\sigma^2.$$

Note that as k approaches T, the bias is expected to grow at a rate of k^2 and it is likely to explode.

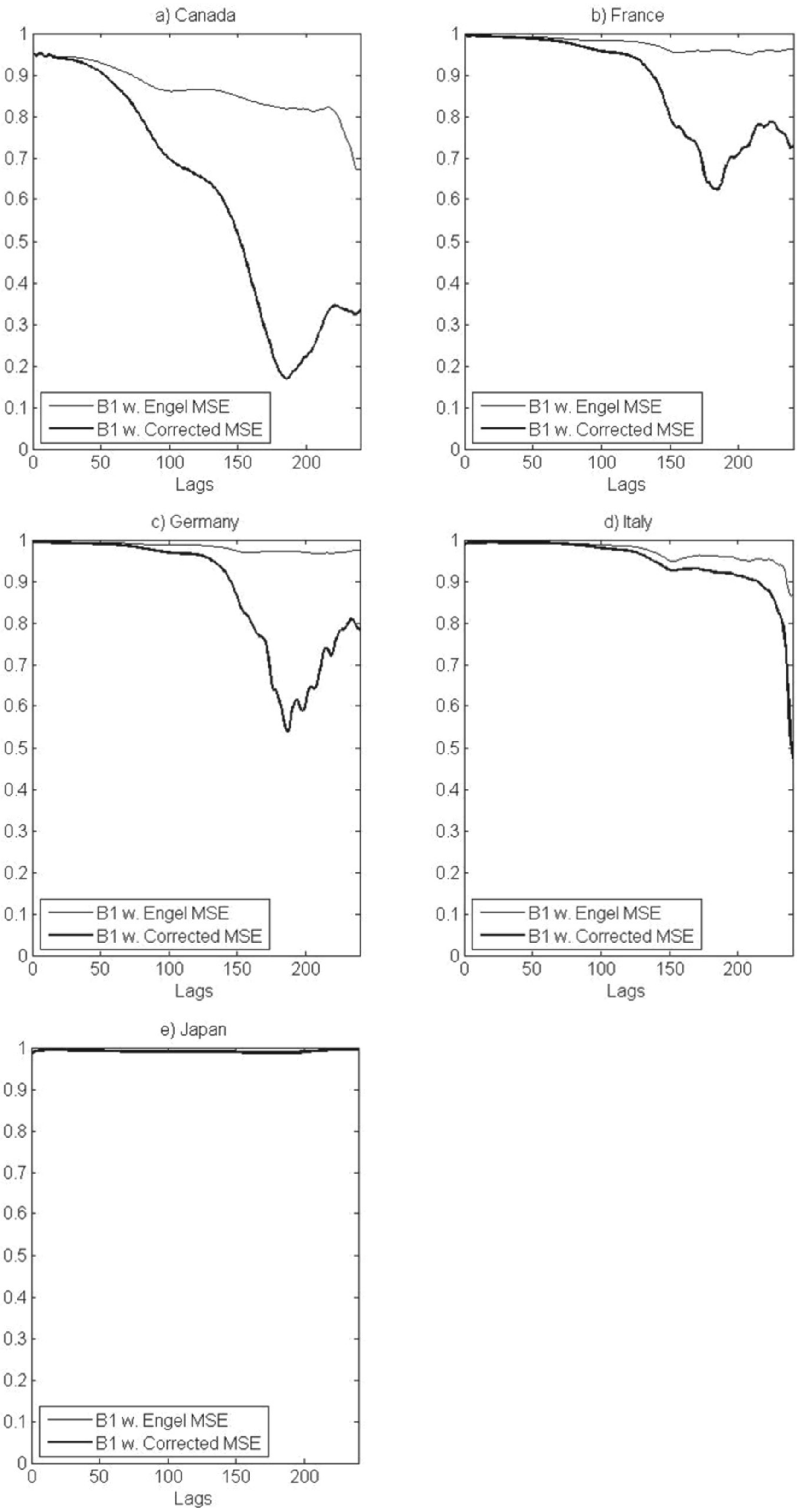

Fig. B.1. The relative importance of tradables using CPI data for 1973–1995.
Note: The figures plot $B1(k) = \frac{\mathrm{MSE}(x_t - x_{t-k})}{\mathrm{MSE}(x_t - x_{t-k}) + \mathrm{MSE}(y_t - y_{t-k})}$.

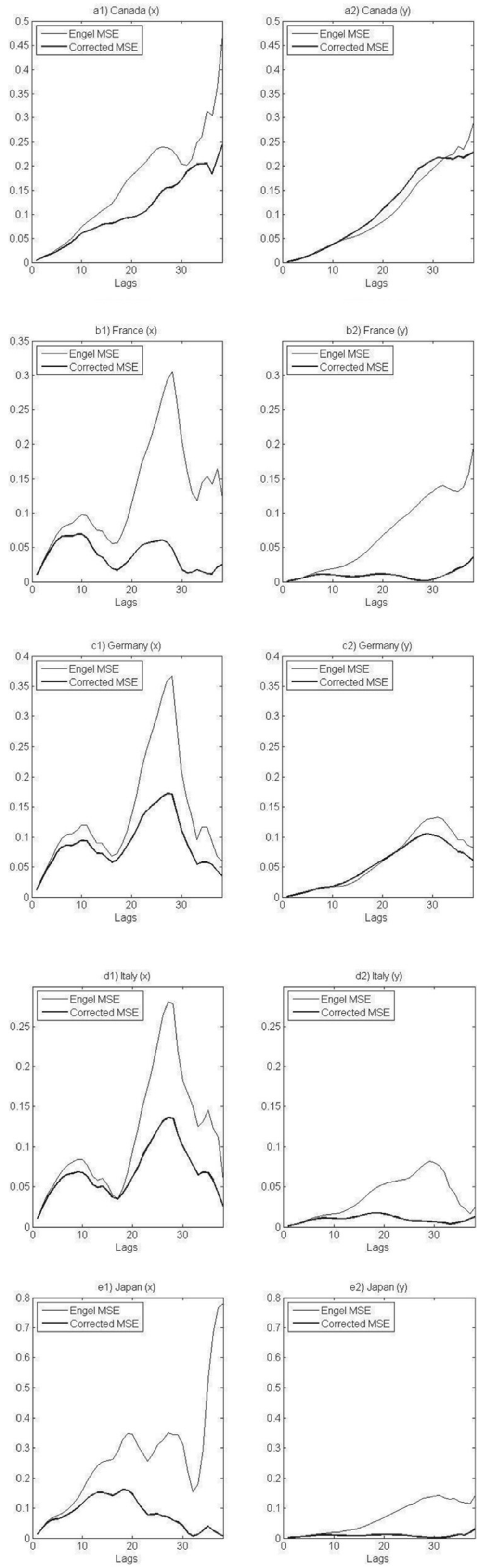

Fig. B.2. Comparing the MSE measures using GDP deflator data.

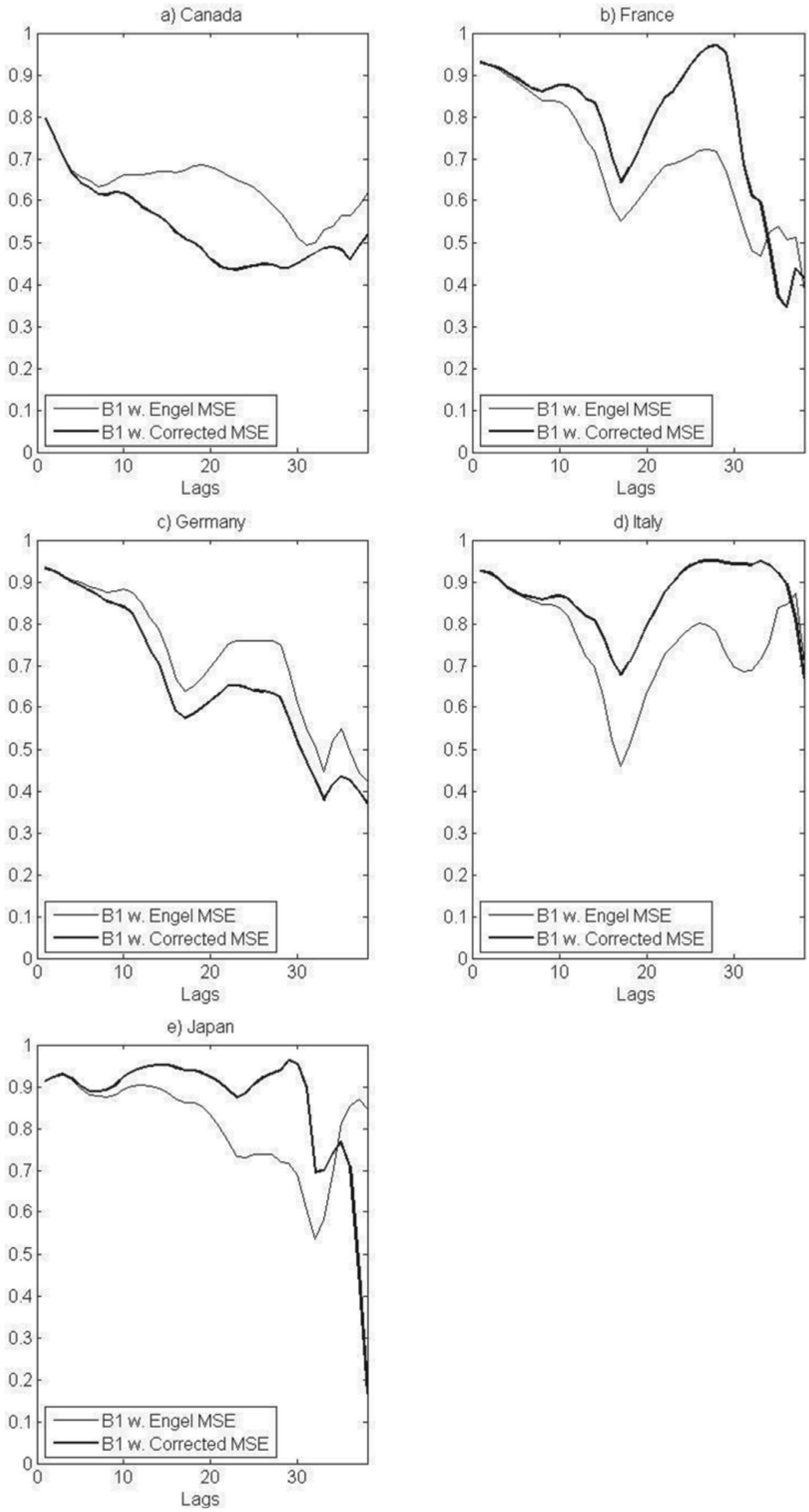

Fig. B.3. The relative importance of tradables using GDP deflator data.
Note: The figures plot $B1(k) = \frac{\mathrm{MSE}(x_t - x_{t-k})}{\mathrm{MSE}(x_t - x_{t-k}) + \mathrm{MSE}(y_t - y_{t-k})}$.

Appendix B. Additional empirical results

We provide two additional empirical results using Engel's (1999) original data for the floating exchange rate period and traded/nontraded price indices derived from sectoral GDP deflators.

B.1. The results for 1973–1995 using Engel's (1999) data

Fig. B.1 repeats Fig. 3 in the text but using Engel's original data from 1973–1995. Using the Engel's MSE measure, we find his result that tradables dominate at all horizons for all economies except Canada. With the corrected MSE measure, however, the share of tradables falls for France and Germany. These results indicate that the MSE bias matters with short spans.

B.2. Price indices derived from sectoral GDP deflator data

Fig. B.2 repeats Fig. 2 in the text with the annual GDP deflator data. When viewing the results from the sectoral price data it should be borne in mind that these data are annual, that they cover overall GDP rather than just consumption and that deflators are conceptually different price indices as compared to the CPI. Fig. B.2 shows that there are marked differences between the behavior of the x_t and y_t variables constructed from CPI and that from GDP deflator data. In accordance with the CPI data, the MSE for the relative price of tradables tends to decline at long horizons. The exception is Canada. In contrast to the CPI data, however, the relative price of nontradables declines at long horizons for two countries – Germany and Italy.

The difference also arises from the potential bias in the Engel (1999) MSE measure for the x_t and y_t variables. Unlike the CPI case, the bias does not always increase at long horizons for the relative price of tradables. The bias falls at long horizons for France, Germany and Italy. On the other hand, the bias for the y_t variable increases in the cases of France and Japan.

Fig. B.3 gives the real exchange rate accounting results using the GDP deflator data. The results for Canada and Germany show that the relative importance of tradables falling over time. By 30 years, the share is 46% for Germany and Canada. By then, tradables account for 69% for France. Viewed in this way, the GDP deflator results are broadly consistent with the findings from the CPI data.

The real exchange rate accounting findings using the GDP deflators differ from the CPI findings in one important respect. As with the CPI data, the Engel MSE ratio overstates the importance of tradables for Canada and Germany at long horizons. But it understates the importance of tradables for the other countries. The GDP deflator results demonstrate that the direction of the bias in the ratio of relative MSE's is an empirical question and can differ across data sets.

Appendix C. Data – sources and methods

C.1. Price indices derived from CPI data

Engel's (1999) price data cover up to 1995. We extend his data to 2015. Engel calculated his tradable (given by the price index for goods) and nontradables (given by the price index for services) price indices with CPI data. His data has the following CPI classifications: all items, all goods less food, food, services less rent and rent. These classifications are no longer published by the OECD. Fortunately, other sources are available as described below.

There is one notable difference in our approach. Engel (1999) obtained his expenditure weights using a regression procedure. We obtain our weights directly from the OECD. Table C.1 compares our weights with those of Engel (1999). We use 1995 weights because these are close to the mid-point of our data.[17] Our estimates show a higher nontradable weight in the price index.

Data sources

USA: The US is the base economy. Price indices for commodities and services are available from the BLS using the Consumer Price Index for all urban consumers at http://www.bls.gov/cpi/.

Table C.1
Weights for nontradables in the overall price index.

	Engel (1999)	OECD 1995
Canada	0.39	0.48
France	0.25	0.47
Germany	0.28	0.45
Italy	0.24	0.47
Japan	0.31	0.48
USA	0.46	0.58

Source: Engel (1999) Table A1, 534 for column two. The weights in column three are from: http://stats.oecd.org/Index.aspx?DataSetCode=MEI_PRICES.

[17] Our results are largely unchanged if we use the Engel (1999) weights.

Canada: The OECD (at http://stats.oecd.org/Index.aspx?DataSetCode=MEI_PRICES) provides price indices for services less housing and for housing. We combined these indices using 1995 weights to obtain a service price index. We then derived the commodity price index as a residual.

France, Germany and Italy: We use the Harmonized Index of consumer prices: Goods (Overall Index Excluding Services) and overall services. These data are available from January 1996. We obtained the price indices from FRED at the Federal Reserve Bank of St Luis (at: https://research.stlouisfed.org/fred2/).

Exchange rates: from the OECD (at http://stats.oecd.org/index.aspx?queryid=169).

C.2. Price indices derived from GDP deflators

We derived our data from the United Nations National account database (at http://unstats.un.org/unsd/snaama/Introduction.asp). The data cover the period from 1973 to 2013. We assume that tradables consist of agriculture, mining and manufactures while nontradables consist of the rest of the economy. The price indices are the implicit deflators of these sectors. We use 1995 shares in valued added to construct the overall price index.

The drawback of the UN data is that they are highly aggregated. For example, they include utilities with mining and manufacturing. Their advantage is that they are provided by national statistical authorities directly to the UN reflecting the most recent GDP revisions.[18]

References

Asea, P., Corden, W.M., 1994. The Balassa–Samuelson model: an overview. Rev. Int. Econ. 2, 191–200.
Balassa, B., 1964. The purchasing power parity doctrine. J. Political Econ. 72, 584–596.
Bergin, P.R., Glick, R., Taylor, A.M., 2006. Productivity, tradability, and the long-run price puzzle. J. Monetary Econ. 53, 2041–2066.
Betts, C., Kehoe, T., 2006. US real exchange rate fluctuations and relative price fluctuations. J. Monetary Econ. 53, 1297–1326.
Betts, C., Kehoe, T., 2008. Real Exchange Rates and the Relative Price of Nontraded Goods. University of Minnesota Unpublished.
Bhagwati, J.N., 1984. Why are services cheaper in the poor countries? Econ. J. 94, 279–286.
Burstein, A., Eichenbaum, M., Rebelo, S., 2005. Large devaluations and the real exchange rate. J. Political Econ. 113, 742–784.
Cassel, G., 1916. The present situation of the foreign exchanges. Econ. J. 26, 62–65.
Cassel, G., 1918. Abnormal deviations in international exchanges. Econ. J. 28, 413–415.
Cassel, G., 1922. Money and Foreign Exchange after 1914. Macmillan, New York.
Chen, L., Choi, S., Devereux, J., 2006. Accounting for US regional real exchange rates. J. Money Credit Bank. 38, 229–244.
Chen, L., Choi, S., Devereux, J., 2015. Explaining price level difference: new evidence on the Balassa–Samuelson effect. South. Econ. J. 82, 81–99.
Cochrane, J.H., 1988. How big is the random walk in GNP? J. Political Econ. 96, 893–920.
Crucini, M., Telmer, C., Zachariadis, M., 2005. Understanding European real exchange rates. Am. Econ. Rev. 95, 724–738.
Crucini, M., Shintani, M., 2008. Persistence in law of one price deviations: evidence from micro-data. J. Monetary Econ. 55, 629–644.
De Gregorio, J., Giovannini, A., Wolf, H.C., 1994. International evidence on tradables and nontradables inflation. Eur. Econ. Rev. 38, 1225–1244.
Engel, C., 1999. Accounting for U.S. real exchange rate changes. J. Political Econ. 107, 507–538.
Ghironi, F., Melitz, M.J., 2005. International trade and macroeconomic dynamics with heterogeneous firms. Q. J. Econ. 120, 865–915.
Mendoza, E., 2000. On the Instability of Variance Decompositions of the Real Exchange Rate Across Exchange-rate-regimes: Evidence from Mexico and the United States, NBER Working Paper Series, No. 7768.
Naknoi, K., 2008. Real exchange rate fluctuations, endogenous tradability and exchange rate regimes. J. Monetary Econ. 55, 645–663.
Rogoff, K., 1996. The purchasing power parity puzzle. J. Econ. Lit. 34, 647–668.
Samuelson, P.A., 1964. Theoretical notes on trade problems. Rev. Econ. Stat. 23, 145–154.
Taylor, A., Taylor, M., 2004. The purchasing power parity debate. J. Econ. Perspect. 18, 135–158.

[18] An alternative data set is the ten sector data base at the University of Groningen (at: http://www.rug.nl/research/ggdc/data/10-sector-database). These data cover from 1970 to 2010. For the most part, there is a close agreement with the UN source. The exceptions are Japan and the US.