Financial Engineering Estimation of Minimum Risk Hedge Ratio

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Abstract

In this paper, the financial engineering minimum risk-based portfolio hedging model is first analyzed. It is then followed by the investigation on various major estimation methods for the minimum risk hedge ratio. The results revealed in the current study show that the HR obtained by the ordinary least squares (OLS) model is maximal and the out-of-sample hedging performance is the best; however, the hedging effectiveness is not sufficiently stable for both the out-of-sample and in-sample estimation.

1. Introduction

The main purpose of the exchange of stock index futures is to avoid and resolve the systemic risks that the position of assets undergoes. That is, the hedge is carried out for the risk exposure of assets by using index futures contracts (which is also named "hedging" or "sea piano" (Wu et al., 1998)), and the future cash position is then substituted by the futures position temporarily (or the futures position is built to offset the potential risks due to the holding of cash position).

The effective use of hedging has always been the focus of academics and practitioners alike. The crucial role of the issue is the estimation of the hedge ratio (HR). Herein, the HR is defined as the relationship between the total value of futures contracts and that of stocks when the investors establish the transaction position in order to achieve the desired effect of hedging (Du, 2002), namely, HR = the total value of futures contracts / the total value of stocks. Therefore, the good or bad effect of hedging depends directly on whether the optimal hedge ratio (OHR) can accurately be calculated under various assumptions and objective functions.

A great deal of exploration involving the theoretical framework of the hedging of stock index futures is mainly devoted to the discussion of the OHR. Based on the modern portfolio hedging theory, Johnson (1960) proposed the OLS model with the minimum risk criteria. Using the Markowitz portfolio theory, Johnson (1960), Stein (1961) and Ederington(1979) regarded the futures position and the cash position as portfolios, and determined the optimal ratio between them in the condition of the minimum risk or maximum utility. According to the development of hedging,

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Ederington (1979) classified it into three categories, i.e., the simple hedging, the selective hedging and the portfolio hedging. Additionally, Kahl and Tomek (1983) advanced the Mean-Variance approach to account for the balance of benefits and risks. Howard and D'Antonio (1984) presented the optimal sharpe hedge ratio under the condition of the maximization of the utility function. Junkus and Lee (1985) empirically analyzed four kinds of the hedging strategies in light of the maximizing profits, eliminating risk, minimizing risk as well as maximizing utility. Bollerslev(1986) developed the GARCH model on the basis of the dynamics wave characteristics of financial time series. Ghosh (1993a) compared the OLS model and the EC model by empirically exploring the hedging strategies among the S&P500 Index, Dow Jones Industrial Average and NYSE Composite Index. Further, Ghosh (1993b) empirically investigated the hedging strategies of the French CAC40 stock index futures, UK FTSE100 stock index futures, Japan's Nikkei stock index futures, along with Germany's DAX stock index futures, and then concluded that the results obtained by the EC model was better than that derived by the OLS model. Chou et al. (1996) estimated and compared the hedge ratios between the conventional model and the EC model by employing the Japan's Nikkei Stock Average index futures with different time intervals. Lee et al (2009) examined four static and one dynamic hedging model by using the data from Taiwan, United States, Japan, Hong Kong , Singapore and Korean to find the optimal hedge ratios. Wang and Hsu(2010) empirically studied the hedge ratio stability of the Japan, Hong Kong and Korean index futures contracts during the Asian financial crisis and post-crisis. Krishan(2011) used daily data for the S&P CNX Nifty futures to estimate the effective hedge ratio and its hedging effectiveness of three models.

In the current paper, the author provides a deep analysis of the estimation of the minimum risk hedge ratio. For this purpose, the minimum risk-based portfolio hedging model is summarized.

2. Minimum Risk Hedge model

Based on the portfolio theory of hedging, Ederington (1979) further implemented the investigation on this topic, and indicated that the hedging objective is to minimize the variation over time in the portfolio value, and then the hedge ratio with minimum variance can be considered as the minimum risk hedge ratio, whose formula derivation is as follows:

At first, the rate of return R_s for hedgers during the period from beginning of t_0 to end of t_1 on the stock market can be given by

$$R_s = \frac{S_{t_1} - S_{t_0} + D}{S_{t_0}} \quad (1)$$

in which S_{t_0} and S_{t_1} are the stock values at the beginning of period t_0 and the end of period t_1, respectively. D is the dividends and bonuses form t_0 to t_1.

Eq. (1) implies that the dividends and bonuses will be reinvested until at the end of period t_1, during which the rate of return is regarded as the risk-free rate of return.

Then, the rate of return R_f for hedgers on the stock index futures market during the same period is formulated by Eq.(2) presented below:

$$R_f = \frac{F_{t_1} - F_{t_0}}{F_{t_0}} \quad (2)$$

in which F_{t_0} and F_{t_1} are stock index futures contracts at the beginning of period t_0 and the end of period t_1, respectively.

If R_h is defined as a rate of return of portfolio consisting of long position of stock and short position of index futures, then, according to Eqs. (1) and (2), one obtains

$$R_h = \frac{(S_{t_1} - S_{t_0} + D) - N \times (F_{t_1} - F_{t_0})}{S_{t_0}} = R_s - N \times \frac{F_{t_0}}{S_{t_0}} \times \frac{F_{t_1} - F_{t_0}}{F_{t_0}} = R_s - h \times R_f \quad (3)$$

where N is the number of futures contracts-buying, and h is the hedge ratio (HR).

Note that the variance of the rate of return of portfolio R_h is written as

$$Var(R_h) = Var(R_s) + h^2 Var(R_f) - 2h \times Cov(R_s, R_f) \tag{4}$$

in which COV(·) represents the covariance.

In view of the fact that the efficiency of hedging is the highest when $Var(R_h)$ is minimal, the first-order and second-order derivatives of $Var(R_h)$ with respect to h are therefore achieved, respectively:

$$\frac{\partial Var(R_h)}{\partial h} = 2h Var^2(R_f) - 2Cov(R_s, R_f) \tag{5}$$

$$\frac{\partial^2 Var(R_h)}{\partial^2 h} = 2Var^2(R_f) \tag{6}$$

Evidently, the second-order derivative $\frac{\partial^2 Var(R_h)}{\partial^2 h}$ in Eq. (6) is larger than 0. Suppose that the first-order derivatives $\frac{\partial Var(R_h)}{\partial h}$ in Eq. (5) equals to 0, the minimum risk hedge ratio h^* is then obtained in the following expression:

$$h^* = \frac{Cov(R_s, R_f)}{Var^2(R_f)} \tag{7}$$

In this environment, the number of futures contracts-buying N in Eq. (3) can be found to be

$$N = h^* \cdot \frac{S_{t_0}}{F_{t_0}} \tag{8}$$

3. Estimation methodology

In addressing the evaluation of the minimum risk hedge ratio h^*, the historical data-based methods are, at present, widely used in mature markets. These methods for the estimation issue of h^* chiefly include: the ordinary least squares (OLS) model, the bivariate vector auto-regression (B-VAR) model, the error correction (EC) model and the generalized autoregressive conditional heteroskedasticity (GARCH) model, etc. Next, a brief description of how these techniques operate is provided.

3.1. OLS model

The ordinary least squares (OLS) model was first proposed by Johnson (1960). The central focus of this model is to carry out regression analysis through the difference approximation of the spot price and the futures price to achieve the least-squares fit. The estimation of the OLS model is a simple equation:

$$\Delta \ln S_t = \alpha + \beta \Delta \ln F_t + \varepsilon_t \tag{9}$$

in which $\Delta \ln S_t$ and $\Delta \ln F_t$ represent the rates of return on spot price and futures prices at t moment, respectively. α is the intercept term, and ε_t is the random error term. The slope β is just the minimum risk hedge ratio h^*, which yields

$$h^* = \beta = \frac{Cov(\Delta \ln S_t, \Delta \ln F_t)}{Var(\Delta \ln F_t)} \tag{10}$$

3.2. B-VAR model

Myers and Thompson (1989), and Herbst et al. (1993) reported that the results calculated by the OLS model are affected by the residual autocorrelation sequence, thus resulting in the development of the B-VAR model:

$$\Delta \ln S_t = C_s + \sum_{i=1}^{l} \alpha_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{l} \beta_{si} \Delta \ln F_{t-i} + \varepsilon_{st} \qquad (11a)$$

$$\Delta \ln F_t = C_f + \sum_{i=1}^{l} \alpha_{fi} \Delta \ln S_{t-i} + \sum_{i=1}^{l} \beta_{fi} \Delta \ln F_{t-i} + \varepsilon_{ft} \qquad (11b)$$

in which C_s, C_f are the intercept terms, $\alpha_{si}, \alpha_{fi}, \beta_{si}, \beta_{fi}$ are the regression coefficients, ε_{st}, ε_{ft} are the random error terms which are of statistical independence with identical distributions, and l is the optimal lag value that can eliminate the effect of residual autocorrelation.

Let $Var(\varepsilon_{ft}) = \sigma_{ff}$, $Cov(\varepsilon_{st}, \varepsilon_{ft}) = \sigma_{sf}$, the minimum risk hedge ratio h^* can then be written in the form

$$h^* = \frac{Cov(\Delta \ln S_t, \Delta \ln F_t | \Delta \ln S_{t-1}, \Delta \ln F_{t-1})}{Vav(\Delta \ln F_t | \Delta \ln S_{t-1}, \Delta \ln F_{t-1})} = \frac{\sigma_{sf}}{\sigma_{ff}} \qquad (12)$$

3.3. EC model

Engle and Granger(1987) conducted a study on the idea of the B-VAR model, and observed that the B-VAR approach ignores the cointegration between the spot price and the futures price, and then Ghosh (1993a), in accordance with the notion of cointegration theory proposed by Engle and Granger(1987), constituted the EC model where the non-stationary, long-run equilibrium and short-term dynamics are all accounted for:

$$\Delta \ln S_t = C_s + \omega_s Z_{t-1} + \sum_{i=1}^{l} \alpha_{si} \Delta \ln S_{t-i} + \sum_{i=1}^{l} \beta_{si} \Delta \ln F_{t-i} + \varepsilon_{st} \qquad (13a)$$

$$\Delta \ln F_t = C_f + \omega_f Z_{t-1} + \sum_{i=1}^{L} \alpha_{fi} \Delta \ln S_{t-i} + \sum_{i=1}^{L} \beta_{fi} \Delta \ln F_{t-i} + \varepsilon_{ft} \qquad (13b)$$

in which ω_s and ω_f are the error correction coefficients of the error-correction terms.

The minimum risk hedge ratio h^* evaluated by the EC model is computed by:

$$h^* = \frac{\sigma_{sf}}{\sigma_{ff}} \qquad (14)$$

4. Empirical results

Presently, China's first stock index futures-Shanghai and Shenzhen (HS) 300 index futures has been launched on April 16, 2010. Herein, the attempt is to construct an investment in the HS300 index, and to choose HS300 index futures to hedge in China's market. With this in mind, the empirical analyses in regard to the above four models were implemented to check the effectiveness of those different methods in the application of the emerging market in China, and the computation processes were performed with the aid of Eviews 5.0 statistical software package.

In this study, particular attention is paid to the investigations of the HS300 index futures and the HS300 index daily data. The sample interval has a total of 292 pairs of data points chosen from April 19, 2010 to June 30, 2011, which are shown in Fig. 1 and 2. First of all, 271 pairs of data points (from April 19, 2010 to May 31, 2011) grouped as in-sample data are used to estimate the minimum risk hedge ratio h^*, and then the rest of 21 pairs of data points (form June 1, 2011 to June 30, 2011) regarded as the out-of-sample data are employed to evaluate the performance of h^* in the future. During such a process, both the futures prices series and spot price series are taken logarithms, and the first-order phase difference is carried out to form return series. Note that the HS300 index futures data are available from Wenhua Finance(Wen and Liu, 2009), and the price series are taken from a daily closing price of the contract month. Because the contract has to be delivered on expiration date, the first day of the delivery month shifts

directly to the next contract in order to develop a continuous price series. Notice also that the HS300 index daily data come from the Qianlong market analysis system.
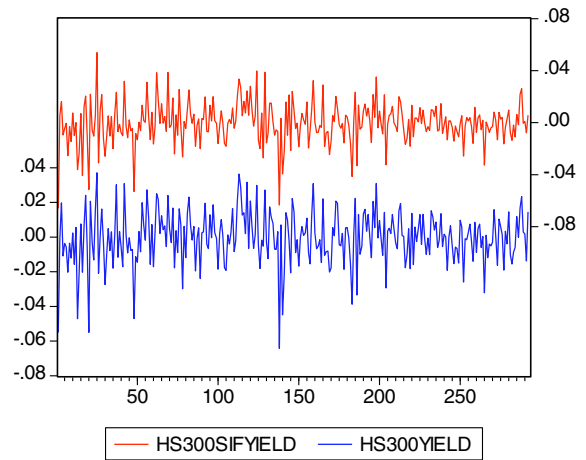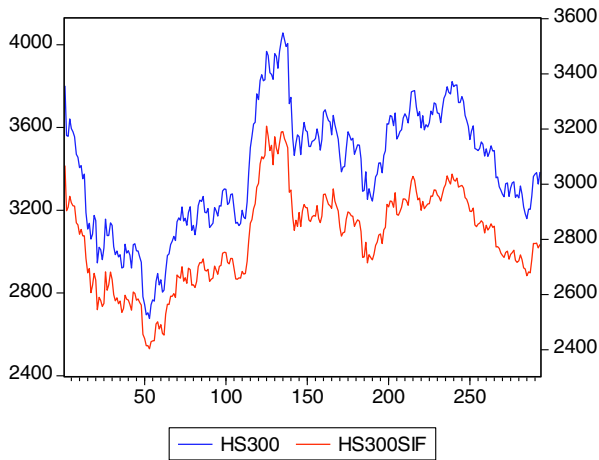

Fig. 1 Series for HS300 Index and HS300 Index Futures prices

Fig. 2 Series for HS300 Index and HS300 Index Futures returns

Table 1 Descriptive statistics for returns of HS300 Index and HS300 Index Futures

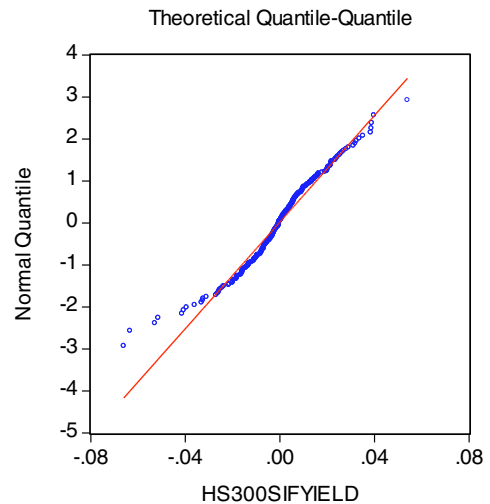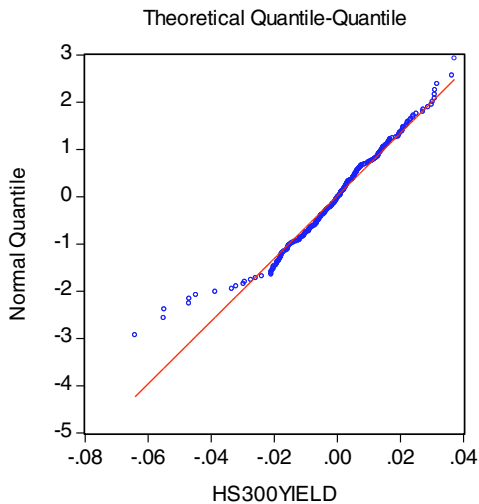|	HS300 Index Futures Return	HS300 Index Return
Mean	-0.000405	-0.000334
Median	-0.000464	0.000279
Maximum	0.053872	0.037109
Minimum	-0.066015	-0.064164
Std. Dev.	0.016056	0.015319
Skewness	-0.404459	-0.619724
Kurtosis	5.231460	4.786683
Jarque-Bera	68.54412	57.52968
Probability	0.000000	0.000000
Sum	-0.118142	-0.097647
Sum Sq. Dev.	0.075019	0.068289
Observations	292	292
Correlation	0.945918	
Probability	<0.0001	

Fig. 3 Schematic of HS300 Index return

Fig. 4 Schematic of HS300 Index Futures return

Table 1 shows that the standard deviation of the return for the HS300 Index is approximately close to that for the HS300 index futures, meaning that the return risks are close to each other. It can also been seen that both the distributions of the two series show a positive bias with the peak and heavy-tailed behaviour. The Jarque-Bera statistics in Table 1 and the quantiles in Fig. 3 and 4 reveal that both of the two series are not normally distributed. Additionally, the results of the correlation coefficient indicate that there is a big correlation between the index futures and the stock portfolio returns.

In conformity with the theory of cointegration, the cointegration relationship exists only when the two series have the same order single sequence. Therefore, the integration test can be performed for the series and its first-order difference sequence by using the ADF method.

Table 2 Results of ADF test

Series	S_t	F_t	ΔS_t	ΔF_t
ADF test statistic	-1.697374**	-1.736041**	-16.77628	-17.77222
Prob.	0.4315	0.4119	0.0000	0.0000

Note: The critical value is -2.572023 with the confidence level of 10%, -2.871263 with the confidence level of 5%, and -3.452674 with confidence level of 1%. ** indicates that the confidence level of 1% is significant.

It can be shown from Table 2 that the values for the two series in the ADF test are both greater than that of 10% of the critical value. This suggests that the unit root null hypothesis is not rejected and the two series are non-stationary. After fulfilling the first-order difference, the values for the two series in the ADF test are less than that of 1% of the critical value, and then the unit root null hypothesis can be rejected. Therefore, two series are in line with the $I(1)$ process and meet the prerequisite for the cointegration. Next, the simple cointegration regression is carried out and the unit root for the residual after regression is tested. Since the ADF statistic is -17.93521, which is less than -3.452674 (1% of the critical value) with the confidence level of 1%, the residual after the cointegrating regression is a stationary series. It may then be taken for granted that the cointegration relationship exists between S_t and F_t.

For the obtained test results, it is mentioned that the sample data should fulfil the requirement of the autocorrelation, heteroscedasticity, cointegration and other prerequisites for the above models. In the sequel, h^* will

be calculated on the basis of in-sample data by utilizing the above four models, and the performance of h^* with the out-of-sample data will also be tested.

Here, the index of the hedging performance defined by Ederington (1979) is obtained as

$$He = 1 - \frac{Var(H)}{Var(U)} \qquad (18)$$

in which $Var(U)$ is the variance obtained before hedging, $Var(H)$ is variance gained after hedging.

5. Conclusions

(1) The minimum risk hedge ratio h^* computed by various models is slightly different as a whole, and is less than the result (equalling to 1) obtained by the simple hedging. In other words, the value of futures contracts is less than the stock market. This manifests that the hedging strategy posed by these methods are less costly than the simple hedging strategy. The results also show that h^* estimated by the EC model is the smallest, and that evaluated by the OLS model is the greatest. Owing to the adding of the error correction term, h^* estimated by the EC model is smaller than that determined by the OLS, B-VAR and GARCH models, respectively, which implies that h^* is overvalued without considering the cointegration.

(2) The variances σ_{in}^2 and σ_{out}^2 obtained after hedging for the in-sample and out-of-sample estimation are both far less than those without hedging, indicating that the hedging can effectively reduce the portfolio risks. For σ_{in}^2, it is the smallest when using the B-VAR model, but for σ_{out}^2, it is the smallest while employing the OLS model. It is further worthwhile to note that both σ_{in}^2 and σ_{out}^2 are the largest with the help of the EC model.

Acknowledgements

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