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Several variants of Kalman Filter algorithm for power system harmonic estimation



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ABSTRACT

This paper presents the maiden application of a variant of Kalman Filter algorithm known as Local Ensemble Transform based Kalman Filter (LET-KF) for power system harmonic estimation. The proposed algorithm is applied for estimating the harmonic parameters of a power signal containing harmonics, sub-harmonics, inter-harmonics in presence of white Gaussian noise. These algorithms are applied and tested for both stationary as well as dynamic signals containing harmonics. The LET-KF algorithm reported in this paper is compared with the earlier reported Kalman Filter based algorithms like Kalman Filter (KF) and Ensemble Kalman Filter (EnKF) algorithms for harmonic estimation. The proposed algorithm is found superior than the reported algorithm for its improved efficiency and accuracy in terms of simplicity and computational features, since there are less multiplicative operations, which reduces the rounding errors. It is also less expensive as it reduces the requirement of storing large matrices, such as the Kalman gain matrix used in other KF based methods. Practical validation is carried out with experimentation of the algorithms with the real time data obtained from a large paper industry. Comparison of the results obtained with KF, EnKF and LET-KF algorithms reveals that the proposed LET-KF algorithm is the best in terms of accuracy and computational efficiency for harmonic estimation.

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Introduction

For the development of effective Power Quality (PQ) monitoring techniques, greater efforts are made by the researchers towards the development of less-complex and more efficient techniques for detection, classification, identification of power quality disturbances. Another key and challenging problem reported recently by the researchers related to power quality is the estimation of harmonic parameters for fundamental, harmonics, inter-harmonics and sub-harmonics components of voltage and currents signals. Accurate and efficient estimation of harmonics from the distorted voltage signals is an important issue for monitoring and analysis of power quality problems [1,2].

Harmonics are components of a distorted periodic waveform, whose frequencies are integer multiples of the fundamental frequency. In electrical power networks, the increasing use of nonlinear loads and power electronic based load devices has caused much more harmonic pollution, which significantly deteriorates the power quality [1]. In order to reduce the harmonic pollution, it is necessary to estimate the parameters of the harmonics. With the estimated parameters, such as amplitudes and phases, appropriate compensation system can be designed for improving the poor power quality performances [1,2].

For past few decades, various approaches have been proposed to estimate the parameters of these harmonics [1]. The Fast Fourier Transform (FFT) is a suitable approach for stationary signal, but it loses accuracy under time varying frequency conditions and also posses picket and fence problems [3–5]. The International Electro-Technical Commission (IEC) standard drafts have specified signal processing recommendations and definitions for harmonic, sub-harmonic and inter-harmonic measurement [4]. These standards recommend using Discrete Fourier Transform (DFT) for harmonic estimation with some windowing based issues but the DFT-based algorithms do not perform stably for systems with time varying frequency [5–7].

Many recursive algorithms are also proposed to solve harmonic estimation problem but each of them have several limitations in terms of accuracy, convergence and computational time. The Least Mean Square (LMS) based algorithms have the drawbacks for their poor convergence in addition to being failure in case of signal drifting and changing conditions. However, Recursive Least Square (RLS) group is the successful algorithms to some extent but the initialization for these algorithm parameters still remains a challenge in case of time varying dynamic signals. The accuracy is also limited for this class of algorithms [5–7].





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Nomenclature							
\hat{y}^f R y'^f C \wedge V Σ k_e I d	forecast observation ensemble perturbation matrix error covariance forecast observation ensemble $N \times N$ Eigen vector matrix diagonal matrix of corresponding Eigen values $M \times M$ orthogonal matrix $N \times M$ matrix Kalman gain matrix identity matrix product vector	$egin{array}{l} \overline{\mathbf{y}}^f \ \widetilde{\mathbf{x}} \ \widetilde{\mathbf{x}}^f \ \overline{\mathbf{x}}^{ff} \ X^{rf} \ H \ \mu_t \ k \ Ts \ ec{\xi} \end{array}$	mean of ensemble perturbation matrix update of state estimate ensemble mean $n \times N$ ensemble perturbation matrix observation matrix additive noise discrete time (sampling) index sampling period performance index				

Another extensively used algorithm is the Kalman filter, which is known for its simplicity, linearity and robustness. This algorithm is capable enough to estimate harmonic parameters in presence of noise and other non-linearities present in the harmonic signal [7–9]. However, the main limitation is that it requires prior information of the statistics of the harmonic signal and the initialization of the state matrix in an accurate and faster way is the main challenge. The dynamic variations present in the harmonic signals calls for some suitable and enhanced methods for accurate estimation of these harmonic components present in the signal [10–19].

A variant Kalman filter, called Ensemble Kalman Filter (EnKF) [20] is proposed for accurate estimation of amplitude and phase of the harmonic components of distorted power system signal. The proposed method used sample covariance in Kalman gain instead of state covariance to avoid the singularity problem and computational feasibility for high-dimensional system [20]. But the prominent limitation of the most EnKF-based systems is perhaps the resource limited ensemble size [21–23]. This is true even for medium-size systems, with the model state vector size of the order of just tens of thousands, not to mention the large-scale applications [22,23]. This limitation calls for the use of the method, known as localization, which artificially reduces the influence of observations of spatial domain during the update [21]. The localization makes it possible to dramatically reduce the necessary ensemble size and create operational systems with as small as a hundred ensemble members or less [22]. Local Ensemble Transform Kalman Filter (LET-KF) as proposed by Szunyogh [23] has three features, (i) assimilation of all observations that may affect the analysis at a given local domain simultaneously, (ii) obtaining the analysis independently for each domain and (iii) introduction of changes when non-local observations are assimilated that improve the computational efficiency and add flexibility [21,23]. The authors have used the method for assimilation of large number of observational data for weather prediction and demonstrated its better performance in terms of accuracy and computational time [23]. However, there has been no attempt made to investigate its performance for harmonic estimation in power system.

In the view of the above following are the main objectives of the present work.

- (a) To study several variants of Kalman Filter algorithms for harmonic estimation.
- (b) Maiden application of Local Ensemble Transform based Kalman Filter (LET-KF) algorithm for estimating amplitudes and phases of the fundamental, harmonics, inter and sub harmonics in presence of Gaussian noises in power system signal.
- (c) To evaluate the comparative performances of KF [9,18], EnKF [20] and the proposed LET-KF algorithms to find the best harmonic estimator.

- (d) To check the accuracy and time of convergence for harmonic signal estimation with the proposed LET-KF algorithm.
- (e) To evaluate the performance of the proposed LET-KF algorithm for accurately estimating harmonic signal parameters on real time data obtained from a real time industrial data setup for harmonic estimation.

Several variants of Kalman Filtering (KF) algorithms applied for harmonic parameter estimation

In this section, several variants of KF algorithms, which are applied for harmonic estimation problems, are discussed. The details of KF and EnKF algorithms may be referred from [9,18,20]. The detail procedure of the LET-KF algorithm for harmonic estimation is also reported in this section.

Kalman Filter (KF)

The vector of unknown parameters X is taken and then KF algorithm is applied to update the weights as in Eq. (1). The KF discussed in this section is referred from [9,18].

$$G(k) = P(k/k - 1)H(k)^{T} \left(H(k)P(k/k - 1)H(k)^{T} + Q\right)^{-1}$$
(1)

where, G is the Kalman gain, H is the observation vector, P is the covariance matrix, and Q is the noise covariance of the signal. The covariance matrix is related with Kalman gain as given in the following equation.

$$P(k/k) = P(k/k - 1) - G(k)H(k)P(k/k - 1)$$
(2)

The updated estimated state vector is related with previous state vector as follows.

$$\widehat{X}(k/k) = \widehat{X}(k/k-1) + G(k)\Big(y(k) - H(k)\widehat{X}(k/k-1)\Big)$$
(3)

After updating the weight vector, amplitudes, phases of the fundamental and *n*th harmonic parameters and dc decaying parameters are found out using Eqs. (38)–(41).

Ensemble Kalman Filter (EnKF)

The EnKF discussed in this section is referred from [20]. This method is based on Monte Carlo approximation method of the Kalman filter, which avoids evolving the covariance matrix of the probability density function (pdf) of the state vector, *x* [20]. In this case, the distribution is represented by a sample, which is called an ensemble [20].

$$X = [x_1, x_2, \dots, x_N] \tag{4}$$

X is a $n \times N$ matrix, whose columns are the ensemble members, and it is called the prior ensemble. Ensemble members form a sample of the prior distribution [20]. As every EnKF step ties ensemble

members together so they are not independent. Signal data y(t) is arranged as a $m \times N$ matrix [20].

The vector of unknown parameter/Ensemble as in Eqs. (5) and (6) is given by

$$X(k) = [X_1(k) \ X_1(k) \dots X_{2N-1}(k) \ X_{2N}(k) \dots X_{2N+2}(k)]^T$$
(5)

$$X = [A_1 \cos(\phi_1) A_1 \sin(\phi_1) \dots A_n \cos(\phi_n) A_n \sin(\phi_n) A_{dc} A_{dc} \alpha_{dc}]^{I}$$
(6)

The ensemble mean and covariance are [20]

$$E(X) = \frac{1}{Q} \sum_{k=1}^{Q} X(k)$$
(7)

$$C = \frac{GG^{\mathrm{T}}}{Q-1} \tag{8}$$

Here,

$$G = X - E(X) \tag{9}$$

The updated ensemble is then given by

$$\widehat{X} = X + CH^{T} \left(HCH^{T} + R \right)^{-1} (y - HX)$$
(10)

where, columns of *X* represent a sample from the prior probability distribution and columns of \hat{X} will form a sample from the posterior probability distribution [20]. The EnKF is now obtained by replacing

the state covariance *P* in Kalman gain matrix $K = PH^T (HPH^T + R)^{-1}$

by the sample covariance, *C* computed from the ensemble members (also called as ensemble covariance), where, *R* is a covariance matrix, which is always positive semi definite and usually positive definite, so the inverse of the above exists [20]. After the updating of the vector of unknown parameter using an Ensemble Kalman Filtering algorithm, amplitudes, phases of the fundamental and *n*th harmonic parameters and dc decaying parameters are obtained out using Eqs. (38)–(41).

Background theory of LET-KF algorithm

The background theory discussed in this section about LET-KF is taken from [21–23]. The main features of this method are well known for its more efficiency and more accuracy, since it has less multiplication operations that reduce the rounding errors [22]. This method is well known for less expense, because of the reduction of storage of large matrices that includes Kalman gain matrix (k_e). To describe the proposed LET-KF algorithm, let us consider the ensemble size to be N and represented by the local forecasted ensemble members by x_i^f , i = 1, 2..., N, each of which of length n.

$$\boldsymbol{x}_{i}^{f} = \begin{bmatrix} \boldsymbol{x}_{1}^{f}, \boldsymbol{x}_{2}^{f}, \dots, \boldsymbol{x}_{N}^{f} \end{bmatrix}$$
(11)

Then the forecasted ensemble mean is given by

$$\overline{x^f} = \frac{1}{N} \sum_{i=1}^{N} x_i^f \tag{12}$$

Also the $n \times N$ forecasted ensemble matrix is defined by

$$X^{f} = \frac{1}{\sqrt{N-1}} \left(x_{1}^{f}, x_{2}^{f} \dots x_{N}^{f} \right)$$

$$\tag{13}$$

Whereas the forecasted ensemble perturbation matrix is defined by

$$X^{\prime f} = \frac{1}{\sqrt{N-1}} \left(x_1^f - \overline{x^f}, x_2^f - \overline{x^f}, \dots, x_N^f - \overline{x^f} \right)$$
(14)

The Eigen value decomposition is used on a matrix of measured, real data, the inverse may be less valid when all eigenvalues are used unmodified. This is because as eigenvalues become relatively small, their contribution to the inversion is large. Those near zero or at the "noise" of the measurement system will have undue influence and could hamper solutions (detection) using the inverse. So, to avoid the problems with the Eigen value decomposition, the scaled and forecasted observation ensemble of the perturbation matrix can be introduced as y_i^f which can be represented by

$$v_i^f = h\left(x_i^f\right) \tag{15}$$

Now if the linear observation operator h = H is considered then the mean of this ensemble is $\overline{y^f} = H\overline{x^f}$ and the ensemble perturbations are represented by

$$\mathbf{y}_{i}^{f} = H\left(\mathbf{x}_{i}^{f}\right) - \overline{H(\mathbf{x}^{f})} = H(\mathbf{x}_{i}^{f}) - H(\overline{\mathbf{x}^{f}}) = H(\mathbf{x}_{i}^{f} - \overline{\mathbf{x}}^{f})$$
(16)

The ensemble perturbation matrix Y^{if} is defined by the columns y'_i , where, i = 1, 2, ..., N that can be written as

$$Y^{\prime f} = H X^{\prime f} \tag{17}$$

Now, a scaled and forecasted observation ensemble perturbation matrix is introduced that can be represented by

$$\hat{Y}^{\hat{f}} = R^{-1/2} Y'^{f} \tag{18}$$

The above equation has the effect of normalizing the observations so that it can be a dimensionless parameter with standard deviation. This prevents the possibility of losses in accuracy due to rounding errors. It can be rewritten as

$$Y^{f^T} R^{-1} Y^{f} = \widehat{Y^{fT}} \widehat{Y^{f}}$$

$$\tag{19}$$

So in this case the Eigen value decomposition becomes

$$\mathbf{Y}^{f_T}\mathbf{Y}^f = \mathbf{C} \wedge \mathbf{C}^T \tag{20}$$

The multiplication performance can be avoided by $Y^{\hat{f}T}Y^{\hat{f}}$ because it avoids the possible loss of accuracy. Instead of using the Eigen value decomposition, the singular value decomposition (SVD) can be used that is represented by

$$Y^{fT} = C\Sigma V^T \tag{21}$$

where, *C* is the $N \times N$ eigenvector matrix as in Eq. (11) and Σ is the $N \times M$ matrix satisfying $\wedge = \Sigma \Sigma^T$ and *V* is an $M \times M$ orthogonal matrix. Then (22) can be used for updating the ensemble perturbation matrix.

$$X'^{a} = X'^{f} C (I + \Lambda)^{-1/2}$$
(22)

Since \wedge is a diagonal matrix, $I + \wedge$ is also diagonal, the computation of the matrix $(I + \wedge)^{-1/2}$ is easy. To update the state estimate, we can use the Eq. (23) as

$$K_{e} = X'^{f} Y'^{f^{T}} \left(Y'^{f} Y'^{f^{T}} + R \right)^{-1} = X'^{f} \widehat{Y'^{f}} \left(\widehat{Y'^{f}} \widehat{Y'^{f}}^{T} + I \right)^{-1} R^{-1/2}$$

= $X'^{f} C \Sigma (\Sigma^{T} \Sigma + I)^{-1} V^{T} R^{-1/2}$ (23)

The last line obtained using SVD and V is orthogonal. To avoid storing the Kalman gain matrix k_e , it can be build up as the product vector

$$d = \Sigma \left(\Sigma^T + I \right)^{-1} V^T R^{-1/2} \left(y - \overline{y^f} \right)$$
(24)

Now from right to left and use the equation to update the state estimates.

$$\widetilde{\mathbf{x}} = \overline{\mathbf{x}^f} + X^{\prime f} C d \tag{25}$$

LET-KF based harmonic estimation

Any distorted waveform can be represented as the sum of harmonic components of higher order frequencies. These higher order frequencies are integral multiple of fundamental frequency in case of harmonics. Mathematically any harmonic signal can be modeled as [6,7,9,18,20,24–26]

$$\mathbf{y}^{f}(t) = \sum_{n=1}^{N} \nu_{n} \operatorname{Sin}(\omega_{n}t + \varphi_{n}) + \nu_{dc} \exp(-\alpha_{dc}t) + \mu_{t}$$
(26)

where, *N* is the number of harmonic order, ω_n is the angular frequency, φ_n is the phase of the harmonic signal. Further, the angular frequency can be written as $\omega_n = n2\pi f_1$, f_1 is the fundamental frequency, μ_t is the additive noise, and $v_{dc} \exp(-\alpha_{dc} t)$ is the probable decaying term. The discrete time version of (26) can be represented as (27)

$$\mathbf{y}^{\prime f}(k) = \sum_{n=1}^{N} \nu_n \operatorname{Sin}(\omega_n k T_s + \varphi_n) + \nu_{dc} \exp(-\alpha_{dc} k T_s) + \mu_k$$
(27)

where, T_s is the sampling period. The decay term can be approximated using the first two terms of Taylor series expansion and also after neglecting higher order terms

$$\nu_{dc} \exp(-\alpha_{dc} kT_{S}) = \nu_{dc} - \nu_{dc} \alpha_{dc} kT_{S}$$
(28)

Substituting (28) in (27) and (28) becomes

$$\mathbf{y}^{\prime f}(k) = \sum_{n=1}^{N} \nu_n \operatorname{Sin}(\omega_n k T_S + \varphi_n) + \nu_{dc} - \nu_{dc} \alpha_{dc} k T_S + \mu_k$$
(29)

The cause of nonlinearity in this model is due to the phase component of the sinusoidal signal. In this paper, LET-KF is used for the estimation of amplitudes and phases of signal. For the estimation of amplitudes and phases, (29) can be rewritten as

$$y^{\prime f}(k) = \sum_{n=1}^{N} [\nu_n \sin(\omega_n k T_S) \cos \phi_n + \nu_n \cos(\omega_n k T_S) \sin \phi_n + \nu_{dc} - \nu_{dc} \alpha_{dc} k T_S + \mu_k]$$
(30)

Hence, for the estimation purpose, the signal can be expressed in parametric form as

$$\mathbf{y}^{f}(\mathbf{k}) = H(\mathbf{k})\mathbf{X}^{f} \tag{31}$$

$$H(k) = [\sin(\omega_1 kT_S) \ \cos(\omega_1 kT_S) \dots \sin(\omega_n kT_S) \ \cos(\omega_n kT_S) \dots 1 - kT_S]^T$$
(32)

The vector of unknown parameter is represented by

$$X^{f} = \left[X_{1}^{f}(k) \; X_{2}^{f}(k) \dots X_{2N-1}^{f}(k) \; X_{2N}^{f}(k) \dots X_{2N+1}^{f}(k) \; X_{2N+2}^{f}(k) \right]$$
(33)

$$X^{d} = \left[v_1 \cos \phi_1 \ v_1 \sin \phi_1 \dots v_n \cos \phi_n \ v_n \sin \phi_n \dots v_{dc} \ v_{dc} \alpha_{dc} \right]^T$$
(34)

The forecasted ensemble mean, forecasted ensemble matrix and forecasted ensemble perturbation matrix of ensemble vector are computed using (12)-(14) from the above equations. The updated ensemble is then given by

$$\tilde{x} = \overline{x^f} + X^{\prime f} C d \tag{35}$$

Estimated signal is given by

$$\tilde{y} = H(k)\tilde{x(k)} \tag{36}$$

Estimation error becomes

$$e(k) = y^{f}(k) - y(k)$$
(37)

After updating, the vector of unknown parameters using the LET-KF algorithm, amplitudes and phases of the fundamental and *n*th harmonic parameters including dc decaying parameters can be computed using the following expression (38)–(41):

$$\nu_n = \sqrt{\left(X_{2N}^{f^2} + X_{2N+2}^{f^{2N}}\right)} \tag{38}$$

$$\phi_n = \tan^{-1} \frac{X_{2N}^g}{X_{2N-1}^{ef}} \tag{39}$$

$$v_{dc} = X_{2N+1}^{f} \tag{40}$$

$$\alpha_{dc} = \begin{pmatrix} X_{2N+2}^{f} \\ \overline{X_{2N+1}^{f}} \end{pmatrix}$$
(41)

LET-KF algorithm steps for harmonic estimation:

Step-1:	Initialize Amplitude, Phase and Frequency of
	fundamental and harmonic components, dc decaying
	components and forecasted ensemble vector (Local
	Members of the Ensemble).
Step-2:	Generate the power signal containing fundamental
	and higher order harmonics satisfying several
	conditions as- one period of the signal sampled at
	2.5 kHz rate and also conform to 200-ms windowing
	in practice as per IEC recommendation referred in [4].
Step-3:	Discretize and Model the signal in parametric form
	using (31).
Step-4:	Initialize the number of unknown parameters/
	ensembles (X'^{f}) and specify error covariance matrix (R) .
Step-5:	Evaluate estimation error using $(35)-(37)$.
Step-6:	Calculate forecasted ensemble mean, forecasted
	ensemble matrix and forecasted ensemble perturbation
	matrix of ensemble vector using $(12)-(14)$.
Step-7:	Obtain the estimate of forecasted ensemble vector
	using (35).
Step-8:	If final iteration is not reached, go to step-5.
Step-9:	Estimate amplitude and phase of fundamental and
	harmonic components and dc decaying components
	using (38)–(41) from final estimate of the forecasted
	ensemble vector.

Simulation, results and discussions

Stationary signal corrupted with random noise and DC decay

To evaluate the performance of the proposed LET-KF algorithm for estimating the harmonic amplitudes and Phases for harmonic, sub harmonics and inter harmonics, a discretized signal having a fundamental frequency of $f_1 = 50$ Hz, third harmonic frequency $f_3 = 150$ Hz, fifth harmonic frequency $f_5 = 250$ Hz, seventh harmonic frequency $f_7 = 350$ Hz and eleventh harmonic frequency $f_{11} = 550$ Hz is generated using MATLAB. The stationary power signal consisting of 1st, 3rd, 5th, 7th and 11th order of harmonics is given in Eq. (32). This type of signal is typically present in industrial load comprising of power electronic devices, Variable Frequency Drives (VFD's) and arc furnaces [9,18,20].

$$\begin{aligned} x(t) &= 1.5 \sin(2 \times pi \times f_1 \times t + 80^\circ) \\ &+ 0.55 \sin(2 \times pi \times f_3 \times t + 70^\circ) \\ &+ 0.2 \sin(2 \times pi \times f_5 \times t + 45^\circ) \\ &+ 0.15 \sin(2 \times pi \times f_7 \times t + 36^\circ) \\ &+ 0.1 \sin(2 \times pi \times f_{11} \times t + 30^\circ) + 0.5 \exp(-5t) + \mu_n \end{aligned}$$
(42)

The aforementioned signal is corrupted by 5% of Gaussian random noise (μ_n) with zero mean and unity variance. All the amplitudes given are in per unit (p.u) values and phases are in degrees. For improving the performance in KFalgorithm, many extensions and modifications are reported in the literature [8–20]. The basic objective of these modifications is less deviations along with faster convergence and accurate calculations. The algorithms are implemented in MATLAB 2009a installed in a PC with 2.25 GHz Intel CPU and 2 GB RAM and the results obtained are reported based on 50 Hz nominal fundamental frequency of the harmonic signal. The performance index (ξ) which is defined as a measure of accuracy in estimation is represented by (43)-

$$\xi = \frac{\sum_{k=1}^{N} \left[y^{\prime f}(k) - y(\widetilde{k}) \right]^2}{\sum_{k=1}^{N} \left[y^{\prime f}(k) \right]^2} \times 100$$
(43)

where, \hat{y}^f is actual forecasted observation ensemble perturbation Matrix and $y^{\widehat{f}(k)}$ is the estimated and forecasted ensemble perturbation matrix. In this case, the significance of the performance index ξ is that it provides the accuracy of the estimation algorithm. Small value of ξ corresponds to more accurate estimation and the more value of ξ corresponds to significantly less accurate estimation.

Figs. 1 and 2 represent the amplitude and phase estimation plot of the harmonic signal containing fundamental, 3rd, 5th, 7th and 11th harmonics. The estimated signals are very close to actual value for each harmonics in case of amplitude and phase estimation using proposed LET-KF algorithm.

For evaluating the performance of proposed LET-KF algorithm along with other two recently reported algorithms, the comparative MSE is reported in Figs. 3 and 4. These figures represent the amplitude and phase MSE of estimation for the harmonic signal containing fundamental, 3rd, 5th, 7th and 11th harmonics using KF, EnKF and LET-KF algorithms. It is clearly evident from the figures that the performance of proposed LET-KF algorithm is better than KF and EnKF algorithms in case of amplitude and phase estimation for the signal with harmonics. The proposed LET-KF algorithm converges faster than that of KF and EnKF algorithms.

Fig. 5 represents the frequency estimation plot of the fundamental harmonic signal using KF, EnKF and LET-KF algorithms. The proposed algorithm outperforms over the existing KF and EnKF algorithms as is vivid in the figure. The estimated frequency output of the fundamental harmonic signal is almost matching with 50 Hz fundamental signal frequency. The deviations observed in case of LET-KF are less than that of two recently reported KF and EnKF algorithms.

To investigate the performance of the proposed LET-KF algorithm for sub harmonic signals, a signal as given by (34) is created in MATLAB. The proposed LET-KF algorithm is applied and then amplitude and phase are estimated. The test signal considered in case of sub harmonic is of $f_{sub} = 20$ Hz.



Fig. 1. Estimated amplitude vs time plot of the harmonic signal using LET-KF algorithm.



Fig. 2. Estimated phase vs time plot of the harmonic signal using LET-KF algorithm.



Fig. 3. Amplitude MSE vs time plot of the harmonic signal using EnKF, KF and LET-KF algorithms.



Fig. 4. Phase MSE vs time plot of the harmonic signal using EnKF, LET-KF and KF algorithms.



Fig. 5. Estimated frequency vs time plot of the fundamental signal using KF, LET-KF and EnKF algorithms.

$$x(t) = 0.5\sin(2 \times pi \times f_{sub} \times t + 75^{\circ}) + \mu_n \tag{44}$$

Fig. 6 represents the amplitude of estimation plot along with actual signal of the sub harmonic signal in presence with Gaussian noise obtained with LET-KF algorithm. It is found that the estimation error achieved with the proposed algorithm for the sub harmonic signal is very much reduced and almost matches with the actual signal.



Fig. 6. Estimated amplitude vs time plot of the sub harmonic signal using LET-KF algorithm.

Next the performance of the algorithms is investigated on interharmonic as given by (45) and generated in MATLAB. The signal considered for testing is of two different frequencies at, $f_{inter-1} = 180$ Hz and $f_{inter-2} = 230$ Hz.

$$\begin{aligned} x(t) &= 0.25 \sin(2 \times pi \times f_{inter-1} \times t + 65^\circ) + \mu_n \\ x(t) &= 0.35 \sin(2 \times pi \times f_{inter-2} \times t + 10^\circ) + \mu_n \end{aligned} \tag{45}$$

Fig. 7 represents the amplitude estimation plot along with actual signal containing inter harmonics using LET-KF algorithm at 180 Hz frequency. The estimation signal obtained with the proposed LET-KF almost matches with the actual inter harmonic signal.

Table 1 presents the comparative performance of KF, EnKF and proposed LET-KF algorithm for estimating harmonic parameters for fundamental, 3rd, 5th, 7th and 11th order harmonics along with sub and inter harmonics. The estimated values obtained with



Fig. 7. Estimated amplitude vs time plot of the inter harmonic signal using LET-KF algorithm.

all three algorithms for each of amplitude and phase is reported with their computational times as well. It is evident from the table that the performance of the proposed LET-KF algorithm is better than any of the other two algorithms in terms of accuracy and computational time.

Table 2 presents the values of the Performance Index (ξ) obtained with the proposed LET-KF and other two algorithms in presence of no noise, 10 dB and 40 dB noises respectively under simulated environment. The performance index achieved with the proposed LET-KF algorithm on three different signals to noise ratio is again the best amongst all the three algorithms.

During the LET-KF estimation process the highest amplitude error deviation (%) is observed in case of inter harmonics signal, which is 5.4% and is significantly lesser than that with KF and EnKF algorithms. Whereas, the highest phase deviation obtained with the LET-KF for seventh harmonic signal is 0.71%, which is the least. The overall estimation performance of LET-KF algorithm is significantly better than other two in terms of accuracy, convergence and computational time.

Dynamic signal

Dynamic signals are time dependant signals whose parameters such as amplitude, phase and frequency varies with respect to time. The proposed LET-KF algorithm is evaluated for dynamic signal given by (46) and (47). The amplitude, phase and frequency are estimated by the algorithm one by one and performance is evaluated. The dynamic performance is investigated for three different frequencies, such as 1 Hz, 3 Hz and 6 Hz and different amplitudes as referred in (47) below [20,25,26].

$$y(t) = \{1.5 + a_1(t)\}\sin(\omega_\circ t + 80^\circ) + \{0.5 + a_3(t)\}\sin(3\omega_\circ t + 60^\circ) + \{0.2 + a_5(t)\}\sin(5\omega_\circ t + 45^\circ) + \mu_n$$
(46)

$$\begin{array}{l} a_{1} = 0.15 \sin 2\pi f_{1}t + 0.05 \sin 2\pi f_{5}t \\ a_{3} = 0.05 \sin 2\pi f_{3}t + 0.02 \sin 2\pi f_{5}t \\ a_{5} = 0.025 \sin 2\pi f_{1}t + 0.005 \sin 2\pi f_{5}t \\ f_{1} = 1.0 \text{ Hz} \\ f_{3} = 3.0 \text{ Hz} \\ f_{5} = 6.0 \text{ Hz} \end{array} \right\}$$

$$(47)$$

Fig. 8 presents the amplitude estimation plot of the dynamic signal obtained using LET-KF algorithm. It can be observed that the estimated values of the amplitude of the dynamic signal achieved with the LET-KF closely match with the actual signal.

Table 1

Performance of KF, EnKF, and proposed LET-KF algorithm for harmonic parameter estimation including sub and inter harmonics.

Algorithm	Parameters	Sub	Fund	3rd	Inter-1	Inter-2	5th	7th	11th	CT(sec)
Actual	Frequency	20	50	150	130	180	250	350	550	-
	Amp (V)	0.2	1.5	0.5	0.1	0.15	0.2	0.15	0.1	
	Phase (deg)	75	80	60	65	10	45	36	30	
KF [9,18]	Amp (V)	0.2045	1.4988	0.5051	0.1107	0.1635	0.2143	0.1578	0.1105	0.4512
	Error (%)	2.25	0.080	1.02	10.70	9.000	7.150	5.200	10.50	
	Phase	75.5544	79.9715	60.8927	65.8817	10.4093	44.9312	36.5543	30.8157	
	Error (%)	0.7392	0.035	1.4878	1.35646	4.0930	0.15289	1.5397	2.7190	
EnKF [20]	Amp (V)	0.2021	1.5100	0.5049	0.0996	0.1592	0.2142	0.1556	0.0946	0.1573
	Error (%)	1.050	0.066	0.980	0.400	6.133	7.100	3.733	5.400	
	Phase	75.2547	79.8782	60.1708	65.8750	10.1409	45.1201	36.4381	30.7478	
	Error (%)	0.3396	0.15225	0.2846	1.3461	1.4090	0.2668	1.2169	2.4926	
LET-KF	Amp (V)	0.2018	1.4998	0.5048	0.0997	0.1534	0.2115	0.1477	0.0945	0.0753
	Error (%)	0.900	0.0133	0.960	0.300	2.2666	5.750	1.533	5.400	
	Phase	75.2417	79.9661	60.1679	65.0259	10.0459	44.9312	36.2566	29.8180	
	Error (%)	0.3222	0.042	0.27983	0.03985	0.4590	0.15280	0.71278	0.60667	

Table 2 Comparison of performance index (ξ).





Fig. 8. Estimated amplitude (A1) vs time plot of the dynamic harmonic signal using LET-KF algorithm.

Real time validation of proposed LET-KF algorithm

To investigate the performance of the proposed LET-KF algorithm under real time environment for estimating harmonics in power system, a voltage signal is recorded across a Variable Frequency Drive (VFD) panel that is used for controlling the speed and torque of the induction motor at Hindustan Paper Corporation Limited (HPCL) industry located at Panchgram in Cachar district of Assam, India, through an experimental setup with power quality analyzer depicted in Fig. 9. The distorted voltage signal is acquired through USB connecting port of a Power Quality Analyzer (PQA) and sent to the laptop for analysis through the proposed LET-KF algorithm and other two algorithms, KF and En-KF.

Specifications of the instrument used are:

- 1. Laptop (Maker HP):1.5 GHz, 2 GB RAM, Intel Pentium3 Processor.
- 2. Power Quality Analyzer (Maker Fluke):
 - True RMS Voltage (AC/DC): 5-1250 V.
 - True RMS Current (AC/DC): 5–5000 A.
 - Frequency Range: 40 Hz to 15.9 kHz.
- PC Connectivity: USB Port and open choice pc communication software.
- 4. Variable Frequency Drive (AC) Make- Siemens.
 - Model No MICROMASTER-440.
 - Rating 250 KW/750 RPM.



Fig. 9. Real time recording of harmonic data at HPCL across a VFD panel.



Fig. 10. Estimated voltage vs sample plot of the real time harmonic signal using LET-KF.

Table 3

Comparison of the performance index (ξ) for real time data using KF, EnKF, and proposed LET-KF algorithm.

Algorithm	Amplitude	Phase	Computational Time (Sec)
KF	9.5245	12.2562	0.6715
EnKF	5.4165	8.4512	0.3565
LET-KF	2.2518	5.3654	0.1752

- 2 Analogue Inputs, 6 Digital Inputs, 2 Analogue Outputs, 3 Relay Outputs.
- Input Voltage 3ph 380–480 V ± 10%, 47–63 Hz.
- 5. 3-Phase Induction Motor Make Siemens.
 - Rating 260 KW, 458 A, 988 RPM, 415 V.
 - Frame KDW-400.
 - Connection-Delta.
 - Rotor Voltage 790 V, 200 A, 50 Hz.

The real time distorted voltage data is recorded across the VFD panel while the motor is at running condition. Fig. 10 depicts the estimation performance of proposed LET-KF algorithm under real time data. The estimated voltage almost matches with the actual voltage signal along with lesser deviation. Hence, the results obtained with real time data from a real time system validate the performance under theoretical aspects. As per IEC 61000-4-30 for computing power quality parameters 10 cycles in a 50 Hz system, which is 200 ms windowing at a sample time of 0.4 ms has been used for the experiment.

Table 3 presents the values of performance index obtained with proposed LET-KF, and other two algorithms using the real time data recorded at HPCL along with the computational time. The performance of the proposed LET-KF algorithm is found to be superior to both algorithms, KF and EnKF.

Conclusion

A new variant of Kalman filter, Local Ensemble Transform based Kalman Filter (LET-KF), is applied for the first time for the estimation of amplitude and phase of a time varying fundamental signal, its harmonics, sub harmonics and inter harmonics corrupted with white Gaussian noise. The harmonic parameters are estimated using the proposed LET-KF and other two variants of Kalman Filter, i.e. KF and EnKF algorithms, for evaluating their comparative performance. The experimentation is made on both static and dynamic signals with different orders of harmonics. The performance index and results obtained with all the three algorithms reveals that the proposed LET-KF algorithm is the best amongst all the three algorithms in terms of accuracy and computational time in estimating harmonic, sub-harmonic and inter harmonics. In addition, the computational time achieved with the proposed LET-KF is the least due to the simplicity of the algorithm with less multiplicative operations. It is also less expensive, as it does not require the storing of large Kalman gain matrices like in other KF based methods. Further, the real time experimentation on the data obtained at a large paper industry has validated the superior performance of the proposed LET-KF algorithm as compared to other recently reported two KF based algorithms.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.ijepes.2015.12. 028.

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