Fuzzy Linear Regression Approach for Fail Quality of Industrial Machines

F. Torfi, M. Hafezi, *N. Hedayat, SH.Motamedi, S. Torfi

Abstract— The operational efficiency of construction engineering machinery is usually estimated either by the manufacturers through catalogues and curves or by the project mangers. One of the major problems for the soil project contractors is proper forecasting the fail quality of industrial machines. Experiences of the last few years in Iran have deemed these methods inappropriate to make accurate estimation of the efficiency.

The present paper used method with fuzzy regressions having independent variables or fuzzy dependent and independent variables to forecast fail quality of industrial machines in Iran. The study uses the Least-Squares Linear as an operational criterion. The data required to build the model were collected from observations and the performance efficiency of 20 operating machines in various projects in Iran. The Mathematica 7 and Lindo 8 soft wares were used to make and implement the model. Comparisons of the model's data with those provide by the manufacturers indicates a significant reduction of error on one hand and the ability of the model in accurately estimating the performance efficiency of the machineries on the other.

Keywords— Fuzzy linear regression، Fuzzy least-squares.

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I. INTRODUCTION

Library investigation on soil-related projects in Iran indicated a significant difference between the real-time and estimated efficiency values which cause major challenges in project planning and financing. This seems to be not exclusive to the Iranian cases as there are reported discrepancies between the real and estimated efficiencies elsewhere. The soil-related contractors in Iran have in recent years shown interests to models which could estimate the performance efficiency of machineries with a high degree of accuracy while having non-complex and easily applicable features. This has resulted in the introduction of various models which the present paper focuses on linear regression type for fail quality of industrial machines. Recent years has seen the development and application of the artificial intelligence networks or the Fuzzy models in engineering domains and manufacturing sector.

The followings are the reasons for the use of linear regression techniques for modeling the estimated operational efficiency. The artificial intelligent models require a variety of historical data for training and evaluation. In this case historical data include real-time efficiency machine under various operating and project conditions. Given the geographical spread of the projects, the collection of a vast amount of performance-related data is time-consuming, laborious and sometimes impossible task indeed. A fuzzy regression model however can be applied with a comparatively less data requirement.

Fuzzy Theory is a powerful tool, for decision making in fuzzy environment. Crisp methods work only with exact and ordinary data, so there is no place for fuzzy and vagueness data. Human has a good ability for qualitative data processing, which helps him or her to make decisions in fuzzy environment. In many practical cases, decisions are uncertain and they are reluctant or unable to make numerical input and output data. Torfi et al. [3] proposed a Fuzzy approach to evaluate the alternative options in respect to the user's preference orders in a fuzzy environment. In this paper, we apply their basic approximation operations in fuzzy least-squares estimator.

Fuzzy least squares linear regression was assumed to be a powerful tool for decision-making in fuzzy environment. Fuzzy regression analysis is a fuzzy (or possibility) type of classical regression analysis. It is applied under circumstances where evaluation of the functional relationship between the dependent and independent variables in a fuzzy environment is necessary.

Tanaka et al. [4] initiated a study in fuzzy linear regression analysis that considered the parameter estimation of models as linear programming problems. Based on the findings of Tanaka et al, further investigations were made, which took two approaches: the linear-programming-based methods [4]-[7] and fuzzy least-squares methods [8]-[10]. Most of these fuzzy regression models are analytically considered with fuzzy outputs and fuzzy parameters but non-fuzzy (crisp) inputs. This paper aims to study fuzzy linear regression models with fuzzy outputs, fuzzy parameters, and fuzzy inputs.

Sakawa and Yano [5] proposed a fuzzy parameter estimation model for the fuzzy linear regression (FLR) model as follows:

$$Y_i = A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}, \qquad j = 1,2,...,n.$$

Where both input data $X_{j1}, X_{j2}, \ldots, X_{jk}$ and output data Y_j are fuzzy. Three types of multi-objective programming problems were further formulated for the parameter estimation of FLR models along with a linear-programming-based approach. This multicriterial analysis of FLR models provided an appropriate method of parameter estimation by using the vagueness of the model via some indices of inclusion relations. Alternatively, a fuzzy least-squares approach directly uses information included in the input-output data set and considers the measure of best fitting based on distance under fuzzy consideration.

Fuzzy least-squares are fuzzy extensions of ordinary least-squares. In this paper, two types of fuzzy least-squares are proposed as the parameter estimation for the FLR model is proposed as follows:

$$Y_i = A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}, \qquad j = 1,2,...,n\,.$$

In this paper, attempt is made to apply an extension of their approaches with triangular fuzzy numbers. The proposed methodology consists the extension of approximate-distance fuzzy least-squares (ADFL) estimator. The proposed method is assumed to be appropriate alternative approaches to fail quality of industrial estimation of machine in industrial projects.

The remainder of this paper is structured as follows: The next section introduce the method used to compute the fail quality of industrial Estimation of machines. Section 3 provides application of the algorithm to collected data set. In section 4 illustrated this method in detail for the specifically-defined problem of this paper then Computational results are represented. Conclusions and future researches which are presented in section 5.

II. Fuzzy Least-Squares Linear Regression

The rationale for the Fuzzy Theory is briefly reviewed before developing fuzzy Least-squares Linear Regression as follows:

A. Fuzzy arithmetic

First, we briefly review the rationale for the Fuzzy Theory before the development of fuzzy Least-squares Linear Regression as follows:

Definition 3.1. A Fuzzy set M in a universe of discourse *X* is characterized by a membership function $\mu_M(x)$ which associates with each element *x* in *X*, a real number in the interval [0, 1]. The function value $\mu_M(x)$ is termed the grade of membership of *x* in M [12]. The present study uses triangular Fuzzy numbers. A triangular Fuzzy number, M, can be defined by a triplet $M = (\alpha, \beta, \delta)_T$. Its conceptual schema and mathematical form are shown by Eq. (1).

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & x \le \alpha \\ \dfrac{x-\alpha}{\beta-\alpha} & \alpha \prec x \le \beta \\ \dfrac{\delta-x}{\delta-\beta} & \beta \prec x \le \delta \\ 1 & x \succ \delta \end{cases} \qquad (1)$$

Definition 3.2. Let $M = (\alpha, \beta, \delta)_T$ and $N = (\chi, \gamma, \lambda)_T$ be two triangular Fuzzy numbers, then the vertex method is defined to calculate the distance between them, as Eq. (2):

$$d_T^2(X,Y) = (\alpha_x - \alpha_y)^2 + (\beta_x - \beta_y)^2 + (\delta_x - \delta_y)^2 \qquad (2)$$

The basic operations on Fuzzy triangular numbers are as follows [13]:

For approximation of multiplication [13]:

$$(\alpha,\beta,\delta)_T \times (\chi,\gamma,\lambda)_T \cong (\alpha\times\chi, \beta\times\gamma, \delta\times\lambda)_T \qquad (3)$$

For addition:

$$(\alpha,\beta,\delta)_T + (\chi,\gamma,\lambda)_T \cong (\alpha+\chi, \beta+\gamma, \delta+\lambda)_T \qquad (4)$$

Given the above-mentioned Fuzzy theory, the proposed Fuzzy Least-squares Linear Regression Approaches is then defined as follows:

B. Developed version of the approximate-distance fuzzy least-squares

This is basically an extension of and improvement on the model applied by Yang and Lin [11] above which is expressed by the FLR model as follows:

$$FLR: \quad Y_i = A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}, \quad j = 1,2,\ldots,n, \quad (5)$$

Where outputs $Y_j = \left(\alpha_{Y_j}, \beta_{Y_j}, \delta_{Y_j}\right)_T$, inputs $X_{ji} = \left(\alpha_{X_{ji}}, \beta_{X_{ji}}, \delta_{X_{ji}}\right)_T$ and parameters $A_j = \left(\alpha_{a_j}, \beta_{a_j}, \delta_{a_j}\right)_T \ \forall i = 1,2,\ldots,k, \quad j = 1,2,\ldots,n$ so that the notion $M = (\alpha, \beta, \delta)_T$ is triangular fuzzy number.

The difficulty in treating model (5) of fuzzy input-output data is that $A_i X_{ji}$ may not be of triangular fuzzy number. Although the product of two triangular fuzzy numbers may not be a triangular fuzzy number, Dubois and Prade [14] presented an approximation form. Based on this analytical framework, Yang and Ko [10] further developed the model presented by Dubois and Prade and suggested an approximation type of fuzzy least-squares. What follows here is the application of approximation to present an algorithm for parameter estimation of the FLR model (5).

By assuming $M = (\alpha, \beta, \delta)_T$ and $N = (\chi, \gamma, \lambda)_T$ to be two triangular Fuzzy numbers; therefore, by using the basic operations on Fuzzy triangular numbers, it will be possible to express an approximation of multiplication and addition as follows: $A_0 + A_1 X_{j1} + \cdots + A_k X_{jk} \cong \left(\tilde{\alpha}_j, \tilde{\beta}_j, \tilde{\delta}_j\right)_T$

where

$$\tilde{\alpha}_j = \tilde{\alpha}_{a_0} + \sum_{p=1}^{k} \left(\tilde{\alpha}_{a_p} * \tilde{\alpha}_{x_{jp}}\right)$$

$$\tilde{\beta}_j = \tilde{\beta}_{a_0} + \sum_{p=1}^{k} \left(\tilde{\beta}_{a_p} * \tilde{\beta}_{x_{jp}}\right)$$

$$\tilde{\delta}_j = \tilde{\delta}_{a_0} + \sum_{p=1}^{k} \left(\tilde{\delta}_{a_p} * \tilde{\delta}_{x_{jp}}\right)$$

Since $A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}$ is of approximate triangular fuzzy number, the distance d_T^2 is defined on two triangular fuzzy numbers. Thus, the following objective function is considered:

$$U(A_0, A_1, \ldots, A_k) = \sum_{j=1}^{n} d_T^2\left(Y_j, A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}\right)$$

$$= \sum_{j=1}^{n} \frac{1}{3}\left[\left(\alpha_{y_j} - \tilde{\alpha}_j\right)^2 + \left(\beta_{y_j} - \tilde{\beta}_j\right)^2 + \left(\delta_{y_j} - \tilde{\delta}_j\right)^2\right]$$

The minimization of $U(A_0, A_1, \ldots, A_k)$ over A_i subject to

$$0 \le \alpha_{a_i} 1, \quad 0 \le \beta_{a_i} \le 1, \quad 0 \le \delta_{a_i} \le 1, \quad , i = 0,1,2,\cdots,k$$

is called the developed version of the approximate-distance fuzzy least-squares method.

The regression analysis is commonly presented in actual practical cases where there is heterogeneity of observations. In order to overcome the heterogeneous problem, the cluster wise fuzzy regression analysis [15] is adopted. Fuzzy clustering that has been widely studied and applied in a variety of substantive areas [15]-[18], is applied to ward off the heterogeneous problems. Cluster wise fuzzy regression embeds fuzzy clustering into fuzzy regression model fitting at each step in the iterations. Given a data set $\{(X_{j1}, \ldots, X_{jk}, Y_j), j = 1,2,\ldots,n\}$, a cluster wise FLR model is fitted to the data set $Y_j = A_0 + A_1 X_{j1} + \cdots + A_k X_{jk}, \quad j = 1,2,\ldots,n,$ Let $\mu_{ij} \in [0,1]$ with $\sum_{i=1}^{c} \mu_{ij} = 1 \quad \forall j = 1,.2,\ldots,n$. The notation μ_{ij} is used to represent the membership of the jth data point $(X_{j1}, \ldots, X_{jk}, Y_j)$ belonging to the ith class. After embedding μ_{ij} to the objective function J, one has a cluster wise objective function as follows:

$$U(\mu, A_{0i}, A_{1i}, \ldots A_{ki}) = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m d_T^2\left(Y_j, A_{0i} + A_{1i} X_{j1} + \cdots + A_{ki} X_{jk}\right)$$

Where, $m \ge 0$ is the index of fuzziness. According to the Lagrange multiplier, one has the necessary condition for μ_{ij} with

$$\mu_{ij} = \left(\sum_{p=1}^{c} \frac{\left(d_{LR}^2\left(Y_j, A_{0i} + A_{1i} X_{j1} + \cdots + A_{ki} X_{jk}\right)\right)^{1/(m-1)}}{\left(d_{LR}^2\left(Y_j, A_{0p} + A_{1p} X_{j1} + \cdots + A_{kp} X_{jk}\right)\right)^{1/(m-1)}}\right)^{-1}, \quad i = 1,2,\ldots c, \quad j = 1,2,\ldots n$$

Outliers always have immense effects in model fitting, especially in regression. e.g., they decrease the accuracy of estimation. Thus, robust regression and outlier detection become an important consideration. Robustness seems to be more important in fuzzy regression. However, the cluster wise fuzzy regression as applied here presents the restriction of membership functions with $\sum_{i=1}^{c} \mu_{ij} = 1 \quad \forall j = 1,.2,\ldots,n$ so that the results will be deteriorated due to outliers and noise. An easy way of modifying this cluster-wise fuzzy regression into detect and tolerate noise and outliers is to apply the idea of Dave's noise cluster [19]. A noise cluster is the one, which contains noise points or outliers so that all the points have equal a priori probability of belonging to a noise cluster [11]. Assume that the cluster (c + 1) is a noise cluster, then the objective function can be expressed as follows:

$$U^0(\mu, A_{0i}, A_{1i}, \ldots A_{ki}) = \sum_{i=1}^{c}\sum_{j=1}^{n} \mu_{ij}^{m} d_{ij}^{2}$$

where

$$d_{ij}^2 = \begin{cases} d_T^2(Y_j, A_{0i} + A_{1i}X_{j1} + \cdots + A_{ki}X_{jk}), & i = 1,2,\ldots c, \quad j = 1,2,\ldots n \\ \tau^2 & i = 1,2,\ldots c, \quad j = 1,2,\ldots n \end{cases}$$

(6)

$$\tau^2 = \lambda \left(\frac{\sum_{i=1}^{c}\sum_{j=1}^{n} d_{ij}^2}{nc} \right), \quad \lambda \succ 0 \quad a\,cons\tan t.$$

The necessary condition for minimization of U^0 over μ is

$$\mu_{ij} = \left(\sum_{p=1}^{c+1} \frac{(d_{ij}^2)^{1/(m-1)}}{(d_{pj}^2)^{1/(m-1)}} \right), \quad i = 1,2,\ldots c+1, \quad j = 1,2,\ldots n. \qquad (7)$$

C. Fuzzy membership function

We deliberately transform the existing precise values to five-levels, Fuzzy linguistic variables very low (VL), low (L), medium (M), high (H), and very high (VH). Among the commonly used Fuzzy numbers, triangular and trapezoidal fuzzy numbers are likely to be the adoptive ones due to their simplicity in modeling easy interpretations. Both triangular and trapezoidal fuzzy numbers are applicable to the present study. We assume that a triangular fuzzy number can adequately represent the five-level Fuzzy linguistic variables, thus, is used for the analysis hereafter.

As a rule of thumb, each rank is assigned an evenly spread membership function that has an interval of 0.30 or 0.25. Based on these assumptions, a transformation table can be found as shown in Table 1. For example, the Fuzzy variable, very low has its associated triangular Fuzzy number with the minimum of 0.00 mode of 0.10 and maximum of 0.25. The same definition is then applied to another Fuzzy variable Low, Medium, High, and Very High [13].

TABLE 1. Transformation for Fuzzy membership functions.

Rank	grade	Membership function
Very low (VL)	1	(0.00,0.10,0.25)
Low (L)	2	(0.15,0.30,0.45)
Medium (M)	3	(0.35,0.50,0.65)
High (H)	4	(0.55,0.70,0.85)
Very high (VH)	5	(0.75,0.90,1.00)

III. Application

A. Data collection

The first priority for developing the linear regression model is collection of data which are usually gathered from the expert's views on factors that influence the machine.

The second phase involved measurements of the real-time efficiency of 20 machines engaged in 18 industrial projects during a one year period. Since some of the crucial factors of efficiency (such as the operator's skills and project) are measured qualitatively and as such being partially subjective and value-ridden, data collection was carried out by experts to improve the reliability and validity components.

The industrial activities under various topographic and climatic conditions of Iran were selected randomly to include the variability requirements. The main performance criteria were the volume of industrial during a working shift and the volume that was loaded by the loaders and transported by trucks. By dividing the total earth moved by the number of shift work hours, the performance efficiency was calculated as can be seen in table 3.

TABLE 2. The main performance criteria

NO	CRITERIA	STATUS
1	Total Service Life Time (Hour)	0-1500
2	Service and Maintenance Condition	Good-Average-Rather poor-Poor
3	Operator Technical Skills	Good-Average-Rather poor-Poor
4	General Condition of Operator During Performance	Good-Average-Rather poor-Poor
5	Site Management Condition	Good-Average-Rather poor-Poor
6	Number of Days Operation is in Progress	Between 0 ~100
7	Time of Operation	MORNING,AFTERNOON,NIGHT
8	Average Temperature During Operation (^{0}C)	-15 ~ 45

A. *Developing the linear regression model for machine efficiency*

Independent variables which have entered the model for estimating the performance efficiency of machines included factors such as those described in table 2. The operator technical skills (X3) has four different states such as, Good-Average-Rather poor-Poor. From these four varieties, the type "Good" is considered to be the best state (VH), followed by Average (H), rather poor (M) and Poor (L). This method was also applied for other independent variables.

IV. Numerical results

In Sections 2 fuzzy least-squares method have been constructed for the estimation of an FLR model with fuzzy input-output data. In this section some numerical examples are given. From the results in Table 3,4 and 5 it is seen that the parameter estimates and sum of squares of residuals (SSR) from the approximate-distance and interval-distance fuzzy least-squares methods are almost the same.

TABLE 3. Parameter estimates and SSR for the model

No.	Apprximate- distance ($\widetilde{Y}$)		
	$\widetilde{\alpha}_y$	$\widetilde{\beta}_y$	$\widetilde{\delta}_y$
1	0.00	0.10	0.25
2	0.00	0.10	0.25
3	0.35	0.50	0.65
4	0.00	0.10	0.25
5	0.35	0.50	0.65
6	0.00	0.10	0.25
7	0.15	0.30	0.45
8	0.00	0.10	0.25
9	0.00	0.10	0.25
10	0.15	0.30	0.45
11	0.15	0.30	0.45
12	0.00	0.10	0.25
13	0.15	0.30	0.45
14	0.00	0.10	0.25
15	0.00	0.10	0.25
16	0.35	0.50	0.65
17	0.00	0.10	0.25
18	0.15	0.30	0.45
19	0.00	0.10	0.25
20	0.15	0.30	0.45

TABLE 4. catalog productivity and actual productivity

No.	catalog productivity ($\hat{Y}$)	actual productivity (Y)
1	**VL**	**VL**
2	**VL**	**VL**
3	**H**	**L**
4	**L**	**VL**
5	**VH**	**L**
6	**VL**	**VL**
7	**VL**	**VL**
8	**L**	**L**
9	**L**	**VL**
10	**L**	**VL**
11	**VL**	**VL**
12	**VL**	**VL**
13	**M**	**L**
14	**H**	**M**
15	**L**	**L**
16	**L**	**VL**
17	**L**	**VL**
18	**VL**	**VL**
19	**L**	**L**
20	**L**	**L**

TABLE 5. SSR FOR THE MODEL

No.	SSR based on Apprximate- distance $d_T^2\left(Y,\widetilde{Y}\right)$	$d_T^2\left(Y,\hat{Y}\right)$
1	**0**	**0**
2	**0**	**0**
3	**0.04**	**0.04**
4	**0**	**0.034167**
5	**0.04**	**0.1475**
6	**0**	**0**
7	**0.034167**	**0.034167**
8	**0.034167**	**0.034167**
9	**0**	**0.034167**
10	**0.034167**	**0**
11	**0.034167**	**0.034167**
12	**0**	**0**
13	**0**	**0.04**
14	**0.1475**	**0.340833**
15	**0.034167**	**0.034167**
16	**0.1475**	**0.04**
17	**0**	**0.034167**
18	**0.034167**	**0.034167**
19	**0.034167**	**0.034167**
20	**0**	**0**
Sum	0.614169	0.915836

At the same time measuring the real-time performance efficiencies of a fleet of 20 machines, their efficiency was calculated from the manufacturers' manual data. The efficiencies of these machines were also calculated by the linear regression models. Calculation of the efficiencies of three methods including the manuals, regression models and their comparisons with the real-time performance of the machines, the deviation in three methods were obtained the results of which are described in the following figure 1.

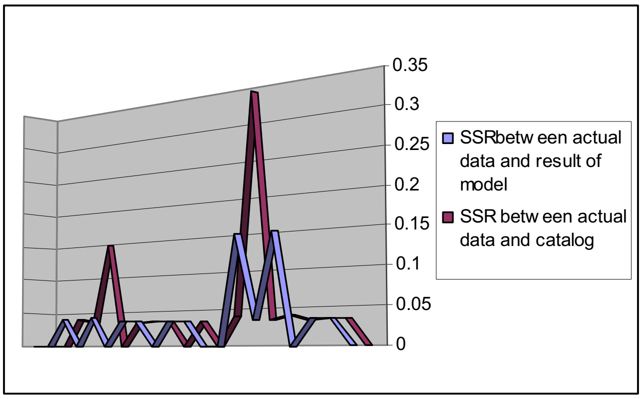

Figure 1. SSR between actual data and catalog, with actual data and regression model.

V. CONCLUSIONS

In the present study, a Developed version of the approximate-distance fuzzy least-squares is suggested for solving the problem. The application of the fuzzy logic in this study provided a solution which seems to be closer to the real world. Under the circumstances where the inputs, outputs and parameters are vague and stochastic, the fuzzy linear regression is preferred to other methods. Furthermore, it would be viable to apply this method in future investigations by using trapezium fuzzy numbers, LR numbers and other related factors to address and solve similar industrial problems.

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