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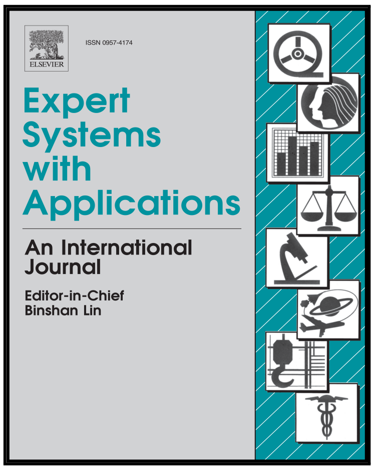

Ranking efficient DMUs using cooperative game theory

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Highlights

- We address the problem of ranking efficient DMUs in Data Envelopment Analysis.
- We rely on cooperative game theory in order to design a ranking.
- A game represents the increase in discriminant power when efficient DMUs are added.
- The Shapley value is adopted as the solution for this game.
- Proposal illustrated on datasets of the literature and compared with other methods.

Ranking efficient DMUs using cooperative game theory

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Abstract: The problem of ranking Decision Making Units (DMUs) in Data Envelopment Analysis (DEA) has been widely studied in the literature. Some of the proposed approaches use cooperative game theory as a tool to perform the ranking. In this paper, we use the Shapley value of two different cooperative games in which the players are the efficient DMUs and the characteristic function represents the increase in the discriminant power of DEA contributed by each efficient DMU. The idea is that if the efficient DMUs are not included in the modified reference sample then the efficiency score of some inefficient DMUs would be higher. The characteristic function represents, therefore, the change in the efficiency scores of the inefficient DMUs that occurs when a given coalition of efficient units is dropped from the sample. Alternatively, the characteristic function of the cooperative game can be defined as the change in the efficiency scores of the inefficient DMUs that occurs when a given coalition of efficient DMUs are the only efficient DMUs that are included in the sample. Since the two cooperative games proposed are dual games, their corresponding Shapley value coincide and thus lead to the same ranking. The more an efficient DMU impacts the shape of the efficient frontier, the higher the

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increase in the efficiency scores of the inefficient DMUs its removal brings about and, hence, the higher its contribution to the overall discriminant power of the method. The proposed approach is illustrated on a number of datasets from the literature and compared with existing methods.

Keywords: Efficiency assessment; Data Envelopment Analysis; Ranking efficient DMUs; Cooperative games; Shapley value.

1 Introduction

Data Envelopment Analysis (DEA) is a well-known non-parametric methodology to assess the relative efficiency of a sample of homogeneous Decision Making Units (DMUs) on the basis of the data about the input consumption and the output production. DEA models typically assign a normalized efficiency score to each DMU in order to distinguish between efficient and inefficient units. The efficient units are assigned the score 1, whereas the inefficient units obtain a score which reflects their degree of inefficiency. The ranking is, therefore, incomplete since the efficient DMUs cannot be discriminated in these terms. This is a permanent issue in DEA, and one that has received a lot of attention on the part of researchers along the years.

This paper begins with a thorough literature review in order to clarify the different efforts that have been made to overcome this limitation of the methodology. Next, a proposal in which the problem of ranking efficient DMUs in DEA is approached from a cooperative game perspective is presented. The idea underlying our approach is that the efficient DMUs are the ones that define the efficient frontier, and if some of them were not present, then the inefficient DMUs would increase their efficiency scores, and some of them could even appear to be efficient. However, some of the efficient DMUs may contribute to the definition of the efficient frontier more than others. That is to say, the contribution to the discriminant power of the efficient DMUs with respect to the correct assessment of the inefficient DMUs is not uniform.

The contribution of an efficient DMU can be measured by the reduction in the efficiency score of the inefficient DMUs that occurs when this efficient unit is included in the reference sample. In this sense, an important DMU is one that, whenever it is included in the sample, significant inefficiencies are uncovered, i.e. the efficiency scores of the inefficient DMUs decrease by a large amount. However, in the end, the efficient frontier is collectively determined by the whole set of efficient DMUs. Thus the idea of measuring the effect of the inclusion of a DMU in the sample can be extended to the inclusion of the different subsets of efficient DMUs. Therefore, crediting each of the efficient DMUs for their contribution to the overall discriminant power can be seen as a cooperative game. In this game, not only the individual contribution of a DMU, but also the contributions of all the groups which include it, are taken into account to determine the importance of this unit.

The ideas and concepts of cooperative game theory have been previously used in relation to DEA methodology as is described in the literature review section. In particular, in the present paper we rely on a well-known solution concept, the Shapley value (Shapley, 1953), which takes into account the average marginal contribution of a player to the worth of all coalitions in which it may participate.

The structure of this paper as follows. In Section 2, different DEA ranking and cooperative game DEA approaches are reviewed. Section 3 contains some preliminaries and an introductory example. The formalization of the approach proposed to rank the efficient DMUs is presented in Section 4. Section 5 reports some numerical examples on different datasets from the literature. Finally. the last section summarizes and concludes.

2 Literature review

There are two strands of DEA research that are relevant to this paper. One corresponds to the extensive literature in ranking DMUs using DEA. The other is the use of cooperative game theory in DEA. As regards the literature on ranking DMUs in DEA, several reviews exist, such as Adler et al. (2002), Jahanshahloo et al. (2008), and Hosseinzadeh Lofti et al. (2013). In general, four major categories of approaches can be distinguished. One category corresponds to methods based on cross-efficiency (XE). This type of methods uses Constant Returns to Scale (CRS) multiplier DEA formulations, and compute XE scores that assess the efficiency of each DMU by using not only the optimal multipliers computed for that DMU, but also those computed for every other DMU. Since alternative optimal multipliers can exist, many different alternative secondary goals have been proposed, e.g. aggressive/benevolent/neutral, best DMU rank, min/max deviation from ideal efficiency point, etc. Table 1 lists a number of representative methods in this category without intending to be exhaustive. In this type of methods a conventional DEA model (e.g. Charnes et al. 1978) is first solved to compute the efficiency of each DMU, which is then imposed as a constraint in the secondary goal DEA model. With these methods all the DMUs are ranked and not only the efficient DMUs. It may even occur that an efficient DMU is ranked below an inefficient one.

A second important category of DEA ranking methods is formed by those based on the computation of a Common Set of Weights (CSW) for all the DMUs, which can then be used to rank all DMUs. This type of methods also uses multiplier DEA formulations although they can assume either CRS or Variable Returns to Scale (VRS). Different criteria can be used to choose the CSW, e.g. compromise programming, regression analysis, deviation from weight profiles of efficient DMUs, etc. Table 2 lists a number of representative methods in this category.

A third major category of DEA ranking methods are those based on super-efficiency (SE). Such methods generally use envelopment DEA models and the key feature of this approach is that the DMU being ranked is dropped from the set of DMUs that define the technology. This can lead, in the case of extreme efficient DMUs, to SE scores larger than unity which can be used to rank those DMUs. Since for inefficient units these SE scores coincide with the conventional

Reference	Remarks
Doyle and Green (1994)	Secondary goals (aggressive/benevolent formulation)
Liang et al. (2008a)	Secondary goals (min total deviation from ideal efficiency point, min maximum deviation, min mean absolute deviation)
Liang et al. (2008b)	Nash equilibrium of XE game
Wu et al. (2009b)	Secondary goal (best rank); mixed-integer non-linear program
Wang and Chin (2010a)	Secondary goal (neutral formulation)
Wang and Chin (2010b)	Secondary goals (max total deviation from ideal efficiency point, max sum of square deviations from ideal efficiency point, max minimum deviation from ideal efficiency point)
Wang et al. (2011b)	Secondary goals (min distance from ideal DMU, max distance from anti-ideal DMU, max distance between ideal and anti-ideal DMUs, max relative closeness)
Örkcü and Bal (2011)	XE+Goal Programming (preemptive/non preemptive)
Jahanshahloo et al. (2011)	XE+TOPSIS
Wu et al. (2011, 2012b)	XE aggregation using entropy-based weights
Contreras (2012)	Secondary goal (enhanced best rank)
Wu et al. (2012a)	Weight-balanced XE
Alcaraz et al. (2013)	Compute ranking intervals; mixed-integer non-linear program

Table 1: Some XE DEA ranking methods

efficiency scores, the method is applied only to rank the efficient DMUs. Different metrics can be used to measure the distance of an efficient DMU to the corresponding SE frontier (obtained when the DMU is dropped from the set of DMUs that define the technology), e.g. radial, slacks-based measure, L1 norm, Tchebycheff norm, etc. Table 3 lists a number of representative methods of this category.

There is another group of DEA ranking methods that are based on the concept of cross-influence (XI). Similarly as in SE, a certain DMU is dropped from the set of DMUs that define the technology, but this time the emphasis is on the effect on all the DMUs, instead of on the effect on the SE score of that DMU. These methods generally use envelopment formulations and, in some cases, not only is a given DMU dropped from the sample that defines the technology, but a second DMU may also be dropped to measure and compare the corresponding effects on the efficiency scores of the DMUs or on some virtual DMU. This feature is important for the

Reference	Remarks
Despotis (2002)	Min convex combination of max and sum of deviations w.r.t. DEA efficiencies
Kao and Hung (2005)	Compromise programming using ideal efficiency vector
Liu and Peng (2008)	Common Weights Analysis (CWA) method
Chen et al. (2009)	Augmented Tchebycheff method using ideal efficiency vector
Wang et al. (2011a)	Regression analysis (min sum of square errors)
Ramón et al. (2012)	Min deviation from weight profiles of efficient DMUs (using l_1/ l_2/ l_∞ norm)
Sun et al. (2013)	Max efficiency subject to efficiency of ideal/anti-ideal DMU equal to unity
Amirteimoori et al. (2014)	Ranking index based on ratio of weighted sum of positive input and output differences
Ramazani-Tarkhorani et al. (2014)	Improved CWA method using lexicographic optimization
Carrillo and Jorge (2016)	Min sum of distances to ideal aggregated input-output point (using l_1/ l_∞ norm)
Ruiz and Sirvent (2016)	Min sum of distances to closest targets in common best practice frontier; compute ranking intervals

Table 2: Some CSW DEA ranking methods

Reference	Remarks
Andersen and Petersen (1993)	Radial, input or output-oriented model; infeasibility issues for VRS
Mehrabian et al. (1999)	Input-oriented Tchebycheff norm model
Tone (2001)	Slacks-based (Super SBM); input, output and non-oriented variants
Chen (2004)	Input-oriented; tackles infeasibility issues of VRS radial model
Jahanshahloo et al. (2004)	Non-oriented l_1 norm model
Wu and Yan (2010)	Linearized l_1 norm model
Rezai Balf et al. (2012)	Non-oriented Tecbycheff norm model
Jablonsky (2012)	Super SBM+extended Goal Programming

Table 3: Some SE DEA ranking methods

present research because, in our approach, subsets of efficient DMUs are dropped so that the cardinality of these subsets goes from one (i.e. dropping a single efficient unit) to as many as the number of efficient DMUs, and we will be interested in measuring the effect on the efficiency scores of the inefficient DMUs. Therefore, the proposed approach can be placed in this category. Table 4 lists a number of XI methods.

Reference	**Remarks**
Jahanshahloo et al. (2007)	Average influence of Strong-efficient (SE) DMUs on the rest
Du et al. (2010)	Impact of efficient DMUs on the inefficient DMUs plus impact of efficient DMUs on the impact of the other efficient DMUs
Shetty and Pakkala (2010)	Virtual DMU (average of inefficient DMUs); impact of efficient DMUs on the virtual DMU
Chen and Deng (2011)	Weighted measures of cross-dependence efficiency for efficient and for inefficient DMU
Hosseinzadeh Lofti et al. (2011)	Efficiency of aggregate DMU (sum of inputs and outputs of the rest of efficient DMUs); radial multiplier formulation
Izadikhah and Farzipoor Saen (2015)	Virtual DMU (average of inefficient DMUs); influence of efficient DMUs on the virtual DMU plus influence on the impact of other efficient DMUs on the virtual DMU

Table 4: Some XI DEA ranking methods

Finally, in addition to the major four categories mentioned above, Table 5 lists some of these other DEA ranking methods, e.g. based on Nash Bargaining games, cross-dominance relationships, optimistic/pessimistic efficiencies, TOPSIS, etc. Especially relevant in this category is the recent paper by Li et al. (2016) which have opened an interesting new research venue by applying Transferable Utility (TU) cooperative games to ranking efficient DMUs in DEA.

Li et al. (2016) have proposed a TU cooperative game approach in which in the reduced DEA models to be solved, a certain subset of efficient DMUs is dropped from the sample. In their method the characteristic function measures the effect of removing these efficient units from the sample on the super-efficiency scores of the efficient units. In the present paper, a different TU cooperative game approach is proposed. In our proposal, the characteristic function measures the effect of including a subset of efficient DMUs' effect on the efficiency scores of the inefficient DMUs. The idea is that when the efficient DMUs are not present in the reference sample (do not cooperate), then the inefficient DMUs are overrated with some of them possibly passing as efficient. As more and more efficient DMUs cooperate, and are added in the reference sample, the efficiency scores assigned to the inefficient DMUs decrease and get closer to their original efficiency score which is the score computed when all efficient DMUs (the grand coalition), are considered. Therefore, the characteristic function of the TU cooperative game proposed in this paper measures the increase in the discrimination power of DEA associated to each subset of efficient DMUs that agree to cooperate and join the reference sample.

Reference	Remarks
Torgersen et al. (1996)	Importance of efficient DMUs as benchmarks for inefficient DMUs
Wang et al. (2007)	Geometric average of optimistic and pessimistic efficiencies
Wu et al. (2009a)	Nash Bargaining game using efficiency and XE
Salo and Punkka (2011)	Compute ranking intervals and dominance relationships
Hosseinzadeh Lofti et al. (2011)	TOPSIS integration of ranking results from different methods
Jahantighi et al. (2013)	TOPSIS integration of ranking results from different methods
Wang and Li (2014)	Improved Nash Bargaining game using efficiency and min XE
Oukil and Amin (2015)	Maximum cross-appreciation
Li et al. (2016)	Use Shapley value of super-efficiency TU cooperative game

Table 5: Some other DEA ranking methods

As regards cooperative games approaches in DEA that are not related to the ranking of DMUs, the reader is referred to Lozano et al. (2016), where a number of TU games approaches, as well as Nash Bargaining games approaches are reviewed (see also Lozano et al. (2015), and Hinojosa et al. (2016)). Among the papers reviewed in Lozano et al. (2015) that apply TU games and, in particular, the Shapley value, the one that is more relevant to this research is Li and Liang (2010), which proposes a cooperative game in which the players are the input and output variables and the grand coalition is the total set that includes all the inputs and outputs. Measuring the marginal contribution of each input and output to the efficiency change ratio of the DMUs, the importance of the different variables can be determined. Therefore, in such an approach, the reduced DEA model that is solved for each coalition drops certain subsets of input and output variables but the whole set of DMUs is always used.

3 Preliminaries and introductory example

A cooperative game in characteristic function form is a pair (N, v), where N is a finite set, the set of players, and the characteristic function, v, is a function, $v : 2^N \longrightarrow \mathbb{R}$, with $v(\emptyset) = 0$. A subset S of N is called a coalition. The number $v(S)$ can be regarded as the worth of the coalition S in the game v.

A cooperative game is monotonic if $v(S) \geq v(T)$, whenever $S \supseteq T$.

The dual game v^d of a cooperative game v is defined by $v^d(S) = v(N) - v(N \setminus S)$, for all $S \in 2^N$. Note that $(v^d)^d = v$ for every game v.

A function f which assigns to every cooperative game, v, a (possibly empty) subset, $f(v)$, of $\mathbb{R}^n$ is called a solution concept. If for each cooperative game, v, $f(v)$ is a singleton, then f is called a value. A value, f, is said self-dual if $f(v) = f(v^d)$.

A well-known value from cooperative TU game theory is the Shapley value. The Shapley value, $\varphi(v)$, of v is defined, for each $k \in N$, by

$$\varphi_k(v) = \sum_{S \subseteq N \setminus \{k\}} \frac{s!(n-s-1)!}{n!} \left(v(S \cup \{k\}) - v(S)\right), \tag{1}$$

where n is the cardinality of N and s is the cardinality of S.

It is well-known that the Shapley value is a self-dual value (see Kalai and Samet (1987)). Moreover, the Shapley value, φ, is the unique solution concept which satisfies the following four appealing properties:

EFF (Efficiency): For each cooperative TU game, v, $\sum_{k \in N} \varphi_k(v) = v(N)$.

NPP (Null Player Property): For each cooperative TU game, v, and each null player[1], $k \in N$, $\varphi_k(v) = 0$.

SYM (Symmetry): For each cooperative TU game, v, and each pair $k, l \in N$ of symmetric[2] players, $\varphi_k(v) = \varphi_l(v)$.

[1] A player $k \in N$ is a null player if, for each $S \subset N$, $v(S \cup k) = v(S)$.

[2] Two players, k and l, are symmetric if, for each coalition $S \subseteq N \setminus \{k, l\}$, $v(S \cup \{k\}) = v(S \cup \{l\})$.

ADD (Additivity): For each pair of cooperative TU games, v and w, $\varphi(v+w) = \varphi(v) + \varphi(w)$.

EFF requires that φ allocates the total worth of the grand coalition, $v(N)$, among the players. NPP means that the players that contribute zero to every coalition, should receive nothing. SYM asks φ to treat equal players equally. Finally, ADD is an useful property which will play and important role in the application of the present paper, as shown below.

In our approach a TU cooperative game will be defined and the Shapley value will be adopted as the solution concept in order to derive a ranking of the efficient units in a DEA model. For the sake of simplicity we present our analysis for the standard input-oriented CCR model (Charnes et al. (1978)), however, the procedure presented in this paper can be applied independently of the specific DEA model used to asses the efficiency of the DMUs.

The CCR DEA model is defined as follows. Suppose there are m independent DMUs, $j \in M = \{1, 2, \ldots, m\}$, each of them consumes k different inputs, $i \in I = \{1, 2, \ldots, k\}$, in quantities x_{ij}, to generate h different outputs in quantities y_{rj} ($r \in H = \{1, 2, \ldots, h\}$).

The efficiency of a given DMU, $j_0 \in M$, can be computed as follows:

$$\begin{array}{rlll} E_{j_0}(M) = \min & \theta_{j_0} & & \\ s.t. & \sum_{j\in M} \lambda_j x_{ij} \le \theta_{j_0} x_{ij_0} & \forall i \in I & \\ & \sum_{j\in M} \lambda_j y_{rj} \ge y_{rj_0} & \forall r \in H & \\ & \lambda_j \ge 0 & \forall j \in M & \\ & \theta_{j_0} \quad \text{free} & & \end{array} \qquad (2)$$

DMU $j_0 \in M$ is efficient if $E_{j_0}(M) = 1$ and the deviation variables in the reformulated model below, $s^-_{ij_0}$, $i \in I$, and $s^+_{rj_0}$, $r \in H$, are both zero:

$$\begin{array}{rll} \min & \theta_{j_0} - \varepsilon\left(\sum_{i\in I} s^-_{ij_0} + \sum_{r\in H} s^+_{rj_0}\right) & \\ s.t. & \sum_{j\in M} \lambda_j x_{ij} = \theta_{j_0} x_{ij_0} - s^-_{ij_0} & \forall i \in I \\ & \sum_{j\in M} \lambda_j y_{rj} = y_{rj_0} + s^+_{rj_0} & \forall r \in H \\ & \lambda_j \ge 0 & \forall j \in M \\ & s^-_{ij_0} \ge 0 & \forall i \in I \\ & s^+_{rj_0} \ge 0 & \forall r \in H \\ & \theta_{j_0} \quad \text{free}, & \end{array} \qquad (3)$$

where ε is a non-archimedian constant.

Denote by N the set of efficient DMUs.

For each $T \subseteq M$, let $E_{j_0}(T)$ be the efficiency score of the DMU $j_0 \in T$ when the sample of DMUs under study are the DMUs in T. Such an efficiency score is assessed by using the standard input-oriented CCR model specified in (2).

We analyze how cooperative game theory can be used to discriminate between efficient DMUs. The idea is to rank the efficient DMUs according to the results obtained by applying a solution concept to the appropriate cooperative transferable utility (TU) game. Our proposal is to use the Shapley value as a solution concept that can properly assign an score representing the importance of the DMUs in this setting .

Li et al.(2016) precede us in modeling the situation as a TU game and in using the Shapley value as a tool to rank the efficient DMUs. In their model, as in ours, the set of agents (players) is the set of efficient DMUs, N and the characteristic function measures, for each coalition, $S \subset N$, the super-efficiency of the efficient DMUs in S.

Formally, the game defined by Li et al. (2016) is a cooperative TU game, (N, v), where the characteristic function, for each $S \subseteq N$, is

$$v(S) = \sum_{k \in S} (E_k(M \setminus S) - 1). \tag{4}$$

Example 1: *Consider the dataset with* $m = 6$ *independent DMUs* ($M = \{A, B, C, D, E, F\}$) *shown in Table 6. It corresponds to a two-input* ($I = \{1, 2\}$)*, single constant output* ($H = \{1\}$) *scenario.*

DMUs	Inputs		Output	Efficiency score
$j = 1, 2, \ldots, 6$	x_{1j}	x_{2j}	y_j	$E_j(M)$
A	1	8	1	1.00
B	2	4	1	1.00
C	4	2	1	1.00
D	8	1	1	1.00
E	5	5	1	0.60
F	3	10	1	$0.\widehat{54}$

Table 6: Dataset and efficiency scores

Figure 1 plots the dataset specified in Table 6. Note that DMUs A, B, C and D are efficient, while E and F are inefficient. Therefore the set of efficient DMUs is $N = \{A, B, C, D\}$. *The efficiency scores,* $E_j(M)$, *of the two inefficient DMUs are shown in Table 6, and their efficient projections are labeled* E^0 *and* F^0 *in Figure 1. The input consumption of* E^0 *and* F^0 *are, respectively,* $(3.0, 3.0)$ *and* $(1.\widehat{63}, 5.\widehat{45})$.

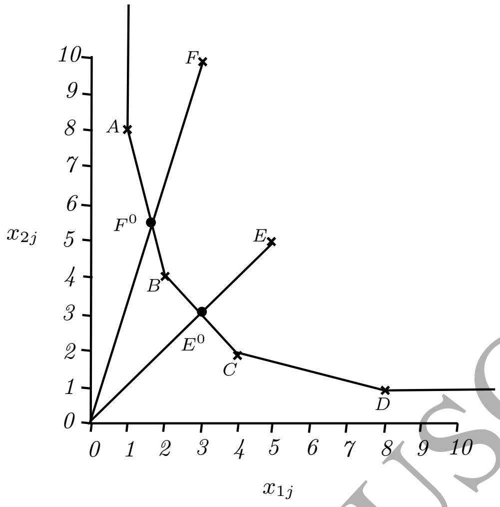

Figure 1. Original efficient frontier and inefficient DMUs projections.

For instance, for the coalition of efficient DMUs, $S = \{B, C\}$, *the super-efficiency of* B *and* C *is computed assuming that they are not included in the reference sample. This is shown in Figure 2. Note that the efficient frontier defined by the corresponding modified reference sample is defined by DMUs A and D. In Li et al. (2016) approach, only the DMUs in* S *are projected on this modified efficient frontier. Such projections are labeled* B^1 *and* C^1 *in Figure 2. The input consumption of* B^1 *and* C^1 *are, respectively,* $(3, 6)$ *and* $(6, 3)$. *Taking into account that the observed input consumption of these DMUs are* $(2, 4)$ *and* $(4, 2)$, *respectively, the super-efficiency scores of* B *and* C *are* $E_B(M \setminus S) = 1.5$ *and* $E_C(M \setminus S) = 1.5$. *Therefore, the characteristic value of coalition* S *in the cooperative game in Li et al. (2016) is obtained as* $v(S) = (E_B(M \setminus S) - 1) + (E_C(M \setminus S) - 1) = (1.5 - 1) + (1.5 - 1) = 1$.

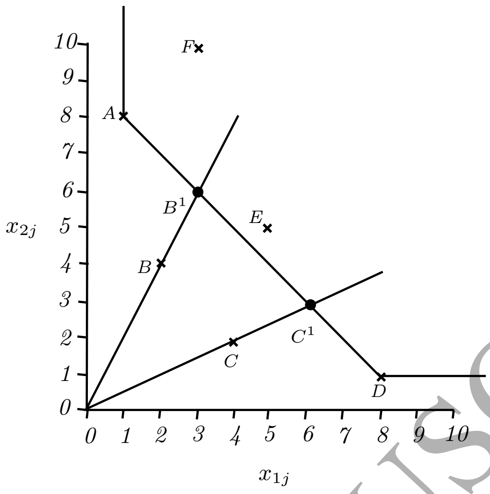

Figure 2. Li et al. (2016) modified efficient frontier and efficient DMUs projections for $S = \{B, C\}$.

Table 7 shows the value of the characteristic function for each possible coalition of efficient DMUs. The Shapley value of the game and the ranking of the efficient DMUs is shown in Table 8.

S	$v(S)$	S	$v(S)$	S	$v(S)$
{A}	1	{A,C}	1.25	{A,B,C}	$3.\widehat{53}$
{B}	0.25	{A,D}	2	{A,B,D}	3.7
{C}	0.25	{B,C}	1	{A,C,D}	4
{D}	1	{B,D}	1.25	{B,C,D}	$6.\widehat{09}$
{A,B}	2.7	{C,D}	4	{A,B,C,D}	$8.\widehat{4}$

Table 7: Characteristic function of Li et al. (2016) approach

Efficient DMUs	Li et al. (2016)	
	Shapley value	Ranking
A	1.6246633	4
B	1.90496633	3
C	2.221633	2
D	2.69318182	1

Table 8: Li et al. (2016) ranking of efficient DMUs

4 A proposal for ranking the efficient DMUs

The approach that is proposed in this paper also defines a cooperative TU game where the players are the efficient DMUs. The modified reference sample in this case includes the inefficient units and the efficient units in coalition S. We concentrate on the efficiency score of the inefficient DMUs in $M \setminus N$, that is, the DMUs that are projected on the modified efficient frontier are the inefficient units.

Note that for each inefficient DMU, $j_0 \in M \setminus N$, and each subset of efficient DMUs, $S \subseteq N$, the inequality $E_{j_0}(M \setminus N) \geq E_{j_0}((M \setminus N) \cup S))$ holds. Therefore, if only the efficient DMUs in $S \in N$ are added to the set of inefficient DMUs, $M \setminus N$, then the efficiency score of an specific inefficient DMU, $j_0 \in M \setminus N$, decreases by an amount equal to $E_{j_0}(M \setminus N) - E_{j_0}((M \setminus N) \cup S)$.

In our framework, from the perspective of any inefficient DMU, $j \in M \setminus N$, the worth of any coalition of efficient DMUs, $S \subset N$, can be computed by the difference $E_j(M \setminus N) - E_j((M \setminus N) \cup S)$, which is the reduction on the efficiency score that DMU j suffers when the DMUs in coalition S cooperate by joining the set of inefficient DMUs, $M \setminus N$, to form the reference sample.

Therefore, for each inefficient DMU, $j \in M \setminus N$, a cooperative TU game v_j can be defined, where the efficient DMUs are considered the set of players, and the characteristic function is such that $v_j(\emptyset) = 0$ and, for each nonempty coalition $S \subseteq N$,

$$v_j(S) = E_j(M \setminus N) - E_j((M \setminus N) \cup S).$$

Alternatively, a different approach could have been adopted for the definition of the cooperative TU game from the point of view of each inefficient DMU ($j \in M \setminus N$). Thus, the worth of any coalition of efficient DMUs, $S \subset N$, could alternatively be measured by the difference $E_j(M \setminus S) - E_j(M)$, which is the increment on the efficiency score that DMU j enjoys when the DMUs in coalition S leave the whole reference sample, M. Hence, analogously, for each inefficient DMU $j \in M \setminus N$, a different cooperative TU game can be defined where the efficient DMUs are also considered the set of players, and the corresponding characteristic function is such that $v_j^*(\emptyset) = 0$ and, for each nonempty coalition $S \subseteq N$,

$$v_j^*(S) = E_j(M \setminus S) - E_j(M).$$

Notice that, for each inefficient DMU, $j \in M \setminus N$, both games, v_j and v_j^*, are monotonic.

Interestingly, it turns out also that v_j and v_j^* are dual games, since

$$v_j^d(S) = v_j(N) - v_j(N \setminus S) =$$

$$= (E_j(M \setminus N) - E_j(M \setminus N \cup N)) - (E_j(M \setminus N) - E_j(M \setminus N \cup N \setminus S)) =$$

$$= (E_j(M \setminus N) - E_j(M)) - (E_j(M \setminus N) - E_j(M \setminus S)) =$$

$$= E_j(M \setminus S) - E_j(M) = v_j^*(S).$$

Then, since the Shapley value is a self-dual value, then the Shapley value of either (N, v_j) or (N, v_j^*) can be used as a reference in order to rank the efficient DMUs in N under a specific inefficient DMU (j) point of view. The ranking obtained is the same because $\varphi(v_j) = \varphi(v_j^*)$.

Nevertheless, the ranking of the efficient DMUs should not be seen under the perspective of a single inefficient DMU, but all the inefficient DMUs should be considered as a whole. Thus, our proposal is to introduce the cooperative TU game v corresponding to the sum of the games specified above, that is, for each coalition $S \subseteq N$:

$$v(S) = \sum_{j \in M \setminus N} v_j(S) = \sum_{j \in M \setminus N} (E_j(M \setminus N) - E_j((M \setminus N) \cup S)) \tag{5}$$

Moreover, we can also consider the game v^*, defined as

$$v^*(S) = \sum_{j \in M \setminus N} v_j^*(S) = \sum_{j \in M \setminus N} (E_j(M \setminus S) - E_j(M)). \tag{6}$$

Since v and v^* are also dual games, then the Shapley value of either (N, v) or (N, v^*) can equivalently be used to discriminate between the different efficient DMUs. The ranking obtained by using any of the two games is the same.

It is worth pointing out that the efficiency scores of all the efficient DMUs that are included in the sample is always unity and hence the change in their efficiency scores is always zero, independently of which other efficient DMUs are considered. This is why the characteristic functions are defined as the change in the efficiency scores of the inefficient DMUs, because those of the efficient DMUs included in the sample are zero.

Approaching DEA ranking as a cooperative game makes sense because in DEA the Production Possibility Set is inferred collectively from the sample. Thus, it can be thought as the result of a cooperative endeavor of the DMUs. The cooperative aim of the DMUs is implicit in their providing the data about their input consumption and output production. If the DMUs did not want to cooperate in the benchmarking they just need to keep the information private. The grand coalition is formed by design, i.e. because we are interested in computing the true efficiencies and that requires including all the efficient DMUs in the sample. Since the aim of DEA efficiency assessment is to identify the best practices and the corresponding

efficient frontier and as the inefficient DMUs contribute nothing in that regard they are not invited to participate in the game. The game is played only between the efficient DMUs, which are the ones that shape the efficient frontier.

As in any TU cooperative game, the proposed approach considers that there exists a commodity which stores utility and which can be transferred among the players. The characteristic function value of the grand coalition represents the total inefficiency uncovered by DEA. Thus, the implicit commodity used in this case is the corresponding benchmark credit, i.e. the credit for contributing to uncovering the total inefficiency present in the observations. It is clear that, since the efficient frontier is defined by the efficient DMUs, these are the DMUs that deserve the credit. It is not obvious, however, how to distribute that credit among the efficient DMUs. That is what the proposed solution (namely, the Shapley value) does. The credit allocation obtained with the Shapley value is used to rank the efficient DMUs. Moreover, the property of additivity, which is inherent to the Shapley value as a cooperative game theory solution, is key to aggregate the effects of all inefficient DMUs.

Regarding the intuition that the two proposed cooperative games give the same Shapley value, note that the two cooperative games proposed and their corresponding Shapley values aim at measuring the contribution of each efficient DMU as regards its influence in shaping the efficient frontier. That influence is related to the geometry of the efficient frontier in the vicinity of that efficient DMU. Hence, it is not surprising that its computation is not affected by whether we study the sensitivity of the efficient frontier in one direction (gradually removing the efficient DMUs until only the inefficient ones are present) or the other (starting with only the inefficient DMUs and gradually adding the efficient ones).

Moreover, the two proposed approaches are, in terms of the computation of their corresponding Shapley value, totally equivalent. There are no differences either in terms of the way the characteristic function is computed: in both cases it refers to the change in the efficiency scores of all the DMUs in the modified reference sample, which coincides with the change in the efficiency scores of the inefficient DMUs since the efficient DMUs do not change their status and thus maintain their unity efficiency scores.

The computation of the Shapley value for a small number of players is rather simple. For a large number of players the computational burden associated to the Shapley value is high. Experiments carried out with TUGlabExtended (Mirás and Sánchez, 2008), a free MATLAB package, indicate that the Shapley value of games of up to 15 players can be solved in around 15 minutes. For a larger number of players approximate estimation methods should be used (see for instance Castro et al. (2009)).

Example 1 (continued): *For the empty coalition, $S = \emptyset$ the reference sample considered in the proposed approach consists of just the inefficient DMUs in $M \setminus N = \{E, F\}$. In this case, shown in Figure 3, both DMUs appear to be efficient. The more efficient DMUs belong to the coalition S, the more the modified efficient frontier approximates the true efficient frontier, and the modified efficiency scores approximate the true efficiency scores.*

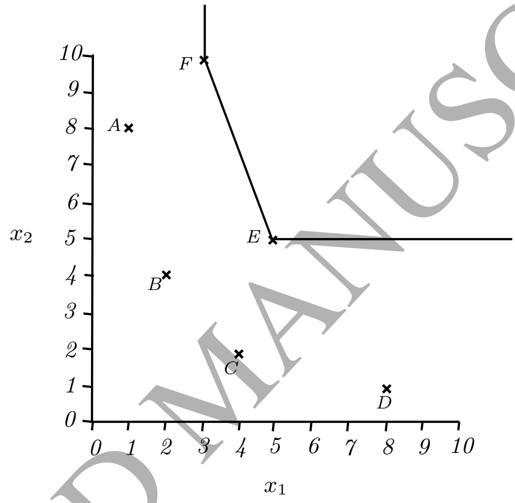

Figure 3. Modified efficient frontier when $S = \emptyset$.

For the coalition $S = \{B, C\}$ the reference sample consists of the inefficient DMUs in $M \setminus N = \{E, F\}$ plus the efficient DMUs in the coalition. Thus, the modified efficient frontier is defined by the DMUs in S, as shown in Figure 4.

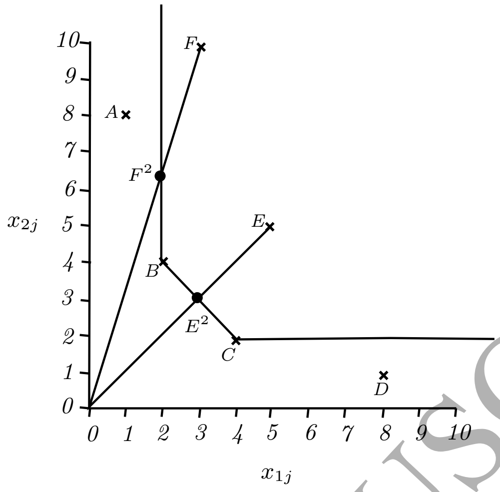

Figure 4. Modified efficient frontier and inefficient DMUs projections in our approach for $S = \{B, C\}$.

The interpretation is that DMUs A and D outside coalition S do not cooperate in joining the reference sample. The corresponding projections on the modified efficient frontier are labeled E^2 *and* F^2 *in Figure 3. The input consumption of* E^2 *and* F^2 *are, respectively,* $(3.0, 3.0)$ *and* $(2.0, 6.\hat{6})$. *Note that, in the case of the inefficient DMU E, the projection is the same as* E^0 *in Figure 1, and therefore the modified efficiency score is the same, that is* 0.6. *However, for the inefficient DMU F, the modified efficiency score is* $0.\widehat{6}$, *and therefore has decrease with respect to the unity efficiency score computed in the case of* $S = \emptyset$. *Therefore, in our approach, the characteristic value of coalition S in the cooperative game is* $(1-0.6)+(1-0.\hat{6}) = 0.4+0.\hat{3} = 0.7\widehat{3}$.

Table 9 shows the value of the charateristic function of each possible coalition of efficient DMUs and Table 10 shows the Shapley value in our game and the ranking of efficient DMUs.

S	$v(S)$	$v^*(S)$	S	$v(S)$	$v^*(S)$	S	$v(S)$	$v^*(S)$
{A}	0.2857	$0.\widehat{12}$	{A,C}	$0.708\widehat{3}$	$0.1\widehat{87}$	{A,B,C}	$0.8\widehat{545}$	$0.8\widehat{545}$
{B}	$0.5\widehat{3}$	$0.146\widehat{21}$	{A,D}	0.4077	$0.\widehat{12}$	{A,B,D}	$0.\widehat{78}$	0.6101
{C}	$0.2\widehat{4}$	$0.0\widehat{6}$	{B,C}	$0.7\widehat{3}$	$0.4468\widehat{45}$	{A,C,D}	$0.708\widehat{3}$	0.32121
{D}	0	0	{B,D}	$0.\widehat{6}$	$0.146\widehat{21}$	{B,C,D}	$0.7\widehat{3}$	0.5688
{A,B}	$0.6\widehat{545}$	$0.61\widehat{01}$	{C,D}	$0.2\widehat{4}$	0.2	{A,B,C,D}	$0.8\widehat{545}$	$0.8\widehat{545}$

Table 9: Characteristic functions for both games

Efficient DMUs	Our approach	
	Shapley value	Ranking
A	0.24332088	2
B	0.38152642	1
C	0.19730925	3
D	0.03238889	4

Table 10: Our ranking of efficient DMUs

Summarizing, while Li et al. (2016) rank the efficient DMUs based on how these efficient DMUs increase their super-efficiency when they are removed from the reference sample, disregarding the effect on the inefficient units, the approach proposed in this paper focuses on the inefficient units, and measures the reduction in their efficiency scores that occurs as more and more efficient DMUs cooperate and join the reference sample. In other words, while the approach in Li et al. (2016) comes from and is inspired by the super-efficiency concept, the approach proposed in this paper is based on the concept of the increase in the discriminant power of DEA contributed by each efficient DMU.

Example 2 (Illustrative dataset from Li et al. (2016)): *The dataset corresponds to $m = 8$ independent DMUs ($M = \{A, B, C, D, E, F, G, H\}$), which use two inputs ($I = \{1, 2\}$) to produce two outputs ($H = \{1, 2\}$), as shown in Table* 11.

DMUs $j = 1, 2, \ldots, 8$	Inputs		Outputs		Efficiency score $E_j(M)$
	x_{1j}	x_{2j}	y_{1j}	y_{2j}	
A	2	7	3	1	1
B	2	12	4	1	1
C	5	5	2	1	1
D	10	4	2	1	1
E	3	6	1	1	1
F	10	6	1	1	0.75
G	4	12	2.5	1	0.5625
H	5	11	3.5	1	0.733108

Table 11: Dataset and efficiency scores

DMUs A, B, C, D and E are efficient, whereas DMUs F, G and H are inefficient. According to our proposal, Table 12 *shows the value of the characteristic function of each possible coalition of efficient DMUs.*

S	$v(S)$	$v^*(S)$	S	$v(S)$	$v^*(S)$	S	$v(S)$	$v^*(S)$
{∅}	0	0						
{A}	0.677576	0.309749	{B,D}	0.350917	0.092649	{A,D,E}	0.930198	0.461832
{B}	0.1	0	{B,E}	0.25	0.005682	{B,C,D}	0.575893	0.259316
{C}	0.166667	0.024194	{C,D}	0.25	0.259316	{B,C,E}	0.56131	0.056446
{D}	0.103448	0.092649	{C,E}	0.166667	0.056446	{B,D,E}	0.595954	0.098331
{E}	0	0.005682	{D,E}	0.225806	0.098331	{C,D,E}	0.25	0.276816
{A,B}	0.677576	0.704392	{A,B,C}	0.856061	0.728586	{A,B,C,D}	0.94871	0.954392
{A,C}	0.856061	0.358438	{A,B,D}	0.897946	0.787725	{A,B,C,E}	0.861743	0.850944
{A,D}	0.897946	0.393082	{A,B,E}	0.695076	0.704392	{A,B,D,E}	0.930198	0.787725
{A,E}	0.695076	0.378499	{A,C,D}	0.94871	0.704392	{A,C,D,E}	0.954392	0.854392
{B,C}	0.49256	0.024194	{A,C,E}	0.861743	0.603475	{B,C,D,E}	0.644643	0.276816
						{A,B,C,D,E}	0.954392	0.954392

Table 12: Characteristic functions for both games

Table 13 *shows the Shapley value and the ranking according to our approach compared with that in Li et al. (2016).*

Efficient DMUs	Li et al. (2016)		Our approach	
	Shapley value	Ranking	Shapley value	Ranking
A	2.1517	1	0.53601263	1
B	1.5705	2	0.11725638	4
C	0.7827	3	0.12878097	3
D	0.6753	4	0.13878705	2
E	0.3881	5	0.03355497	5

Spearman rank correlation coefficient: 0.6

Table 13: Ranking of the efficient DMUs

5 Numerical Illustrations

In this section we will illustrate the proposed approach by using different datasets from the literature. In what follows, our results are compared with those obtained by some of the different approaches reviewed in Section 2. We provide the Spearman rank correlation coefficient in each case. For all the examples we have chosen to use the game denoted by v for computing the

Shapley value.

Real-world dataset from Li et al. (2016)

Table 14 shows a the real-world dataset corresponding to 14 Chinese city commercial banks in 2012. For each bank, three inputs (Fixed assets, Employee's pay and General expenses) and two outputs (Profit and Loans) are considered.

DMUs $j = 1, \ldots, 14$	Banks	Inputs			Outputs		Efficiency score $E_j(M)$
		Fixed assets	Employee's pay	General expenses	Profit	Loans	
1	Jiujiang	775,310	283,893	644,280	2,193,085	26,392,924	1
2	Chengdu	1,120,795	679,087	1,674,877	3,262,998	91,799,733	0.853694
3	Baotou	5,086,018	420,404	2,867,703	2,845,104	57,320,923	0.441529
4	Hangzhou	1,114,333	1,239,207	3,187,168	4,474,572	148,341,428	0.973852
5	Huishang	1,178,797	1,155,569	2,433,518	5,680,038	159,941,475	1
6	Harbin	5,810,688	332,044	2,466,586	3,789,997	85,298,079	0.715444
7	Jiangsu	3,203,353	2,864,359	6,617,403	9,625,877	342,827,271	0.817987
8	ZHHR	150,570	207,802	877,996	636,046	21,152,061	1
9	Tianjin	1,267,597	296,940	1,820,934	3,298,603	118,767,291	1
10	Chongqing	1,015,688	142,024	1,577,106	2,518,447	75,256,873	1
11	Dalian	1,154,635	619,940	2,327,875	2,253,330	98,880,747	0.773406
12	Hankou	1,088,335	95,372	1,221,129	2,414,775	57,757,208	1
13	Nanjing	2,172,992	635,475	2,721,211	4,980,404	121,962,186	0.827474
14	Ningbo	2,397,820	624,031	3,529,395	5,098,041	142,564,629	0.768847

Table 14: Dataset and efficiency scores

Table 15 shows the ranking of the six efficient DMUs (1, 5, 8, 9, 10 and 12) in our approach and in the Li et al. (2016) approach.

Efficient DMUs	Li et al. (2016)		Our approach	
	Shapley value	Ranking	Shapley value	Ranking
1	0.6335	4	0.11618148	3
5	0.5525	5	0.13473162	2
8	0.0453	6	0.02417655	6
9	0.8168	3	0.05180832	5
10	1.2692	2	0.14934275	1
12	1.4497	1	0.05388128	4

Spearman rank correlation coefficient: 0.314

Table 15: Ranking of the efficient DMUs

As we can see, although both Li et al. (2016) and the proposed approach use Shapley value, the corresponding rankings differ because their respective cooperative TU games are different. The one in Li et al. (2016) is based on the super-efficiency of the efficient DMUs while the approach proposed in this paper is based on the contribution of the efficient DMUs to the increase in the discriminant power of DEA.

Dataset from Jahanshahloo et al. (2011)

Jahanshahloo et al. approach (2011) is based on cross-efficiency (see Table 1). Table 16 shows the dataset considered and the efficiency scores.

DMUs	Inputs		Output	Efficiency score
$j = 1, 2, \ldots, 6$	x_{1j}	x_{2j}	y_j	$E_j(M)$
1	13	1	1	1
2	6	3	1	1
3	2	6	1	1
4	1	10	1	1
5	9	5	1	0.638
6	4	8	1	0.682

Table 16: Dataset and efficiency scores.

Jahanshahloo et al. model obtain different rankings by considering different optimal weights. Table 17 shows two of this rankings and one of them coincides with the ranking obtained in our approach.

Eficcient DMUs	Jahanshahloo et al. (2011)		Our approach	
	Rank 1	Rank 2	Shapley value	Rank
1	4	3	0.0723	3
2	2	1	0.3023	1
3	1	2	0.2743	2
4	3	4	0.0310	4

Spearman rank correlation coefficients: 0.6 and 1, respectively

Table 17: Ranking of the efficient DMUs

Dataset from Chen (2004)

Chen's method is also based on super-efficiency (see Table 3). The following dataset corresponds to 20 Japanese companies. For each company three inputs (asset, equity and number of employees) and one output (revenue) are considered.

Efficiency scores in Chen's method are assessed by the BCC model. Therefore there are five efficient companies (DMUs 1, 2, 6, 8 and 18). Table 18 shows the dataset and both the rankings obtained by Chen (2014) and those obtained in our approach based on the BCC model.

DMUs $j = 1, \dots, 20$	Company	Inputs			Output	Efficiency score	Chen (2004)	Our approach	
		Assets	Equity	Employee	Revenue	$E_j(M)$	Rank	Shapley value	Rank
1	Mitsui and Co.	50,905.3	5,137.9	40,000	106,793.2	1	3	0.12607307	5
2	Itochu Corp.	51,432.5	2,333.8	5,775	106,184.1	1	1	0.39291098	2
3	Mitsubishi Corp.	67,553.2	7,253.2	36,000	104,656.3	0.74248			
4	Toyota Motor Corp.	112,698.1	47,177	183,879	97,387.6	0.4108			
5	Marubeni Corp.	49,742.9	2,704.3	5,844	91,361.7	0.91739			
6	Sumitomo Corp.	41,168.4	4,351.5	30,700	86,921	1	5	0.12992915	4
7	Nippon Telegraph and Tel.	133,008.8	47,467.1	138,150	74,323.4	0.26865			
8	Nissho Iwai Corp.	35,581.9	1,274.4	19,461	66,144	1	4	0.20041857	3
9	Hitachi Ltd.	73,917	21,914.2	328,351	60,937.9	0.40528			
10	Matsushita Electric Indl.	60,639	26,988.4	282,153	58,361.6	0.47569			
11	Sony Corp.	48,117.4	13,930.7	177,000	51,903	0.54156			
12	Nissan Motor	52,842.1	9,583.6	39,467	50,263.5	0.47975			
13	Honda Motor	38,455.8	13,473.8	112,200	47,597.9	0.62931			
14	Toshiba Corp.	46,013	8,023.3	198,000	40,492.7	0.45933			
15	Fujitsu Ltd.	39,052.2	8,901.6	188,000	40,050.3	0.53631			
16	Tokyo electric power	110,055.8	12,157.7	50,558	38,869.5	0.18567			
17	Nec Corp.	38015	6,517.4	157,773	36,356.4	0.50901			
18	Tomen Corp.	16,696	676.1	3,654	30,205.3	1	2	1.45464323	1
19	Japan Tobacco.	17,023.6	10,816.6	31,000	29,612.2	0.98076			
20	Mitsubishi Electric Corp.	31,997	4,129.6	116,479	28,982.2	0.5218			

Spearman rank correlation coefficient: 0.6

Table 18: Dataset, efficiency scores and ranking of the efficient DMUs

Illustrative dataset from Chen and Deng (2011)

The method used in Chen and Deng (2001) is also based on the concept of cross-influence (see Table 4). In this case the data corresponds to 8 DMUs where two inputs and two outputs are considered. Five DMUs (DMUs A, B, C, D and F) are efficient basing on the BCC model used by Chen and Deng (2011).

Table 19 shows the dataset and the results obtained by our approach compared with that in Chen and Deng (2011).

DMUs	Inputs		Outputs		Efficiency score	Chen and Deng (2011)	Our approach	
$j = 1, \ldots, 6$	x_{1j}	x_{2j}	y_{1j}	y_{2j}	$E_j(M)$	Rank	Shapley value	Rank
A	2	12	4	1	1	3	0.15312117	2
B	2	8	3	1	1	1	0.26838633	1
C	5	5	2	1	1	4	0.13876517	3
D	10	4	2	1	1	2	0.12971967	4
E	10	6	1	1	0.750			
F	3.5	6.5	1	1	1	5	0.02591667	5
G	4	12	2.5	1	0.625			
H	5	11	3.5	1	0.909			

Spearman rank correlation coefficient: 0.7

Table 19: Dataset, efficiency scores and ranking of the efficient DMUs

6 Final remaks

In this paper a new method for ranking efficient DMUs in DEA has been proposed. It is based on a well-known concept solution of cooperative game theory, namely the Shapley value, applied to any of a pair of dual cooperative TU games. The players of both TU games are the efficient DMUs and their characteristic functions measure the worth of a coalition in terms of the amount of inefficiency uncovered by the inclusion of the corresponding efficient DMUs in the reference sample. Thus, for the grand coalition, the initial DEA efficient frontier is formed and the inefficient DMUs are projected onto it. In contrast, if the efficient DMUs do not cooperate, then the inefficient DMUs are projected onto a modified efficient frontier thus overestimating their efficiency score. As the efficient DMUs cooperate and join the reference sample, the modified efficiency scores become lower and closer to the initial efficiency scores. This increase in the discriminant power gained with the cooperation is what the characteristic functions of the proposed TU games take into account.

Although the proposed approach has been formulated for a certain (radial, input-oriented, CRS) DEA model, it can also be applied independently of the specific DEA model used to assess the efficiency of the DMUs. The proposed approach has been tested on a number of datasets from the literature and compared with the results of other methods with which sometimes coincide but others differ. This is not unusual. As reviewed in Section 2, there are many different DEA ranking methods and they generally differ for a specific dataset. It is the rationale of each method what supports its validity and, in the case of the approach proposed in this paper, the rationale is based on the solid theoretical foundation of cooperative game theory. Particularly, it is founded on the idea that the participation of each of the efficient DMUs contribute to a certain extent to define the DEA efficient frontier and hence it should be accordingly credited.

As topics for further research, we may consider the possibility of using other TU game solution concepts, such as the nucleolus. The possibility of using other non-coalitional cooperative game approaches, such as Nash bargaining, may also be investigated.

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