Proceedings of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Guaranteed Dominant Pole Placement with Discrete-PID Controllers: A Modified Nyquist Plot Approach

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Abstract: Guaranteed dominant pole placement problem has already been considered in the literature (Journal of Process Control 19(2009):349–352). For the systems that are higher-order or have dead-time, pole placement procedure with PID controllers via modified Nyquist plot and root-locus has been proposed. Based on this idea, the dominant pole placement problem with discrete-PID controllers in z-domain is studied since it is important to take advantage of discrete-time domain representation during the pole placement procedure for time-delay systems. It is shown that modified Nyquist plot method is still valid in discrete-time domain and it is possible to find relevant discrete-PID controller parameters. Controller zeros are also considered in the study, since in the closed-loop controller zeros can disrupt the dominance. Success of the method demonstrated on example transfer functions.

Keywords: Process Control, Discrete-Time Control, Pole Placement, PID Controllers, Nyquist Plot.

1. INTRODUCTION

Pole placement approach in control system design is one of the most popular approaches to design feedback controllers because its design procedure is not complex and closed-loop system performance is predictable and can be changed easily as desired. Since the performance specifications of a closed-loop system are determined by the dominant pole locations, in general, it is expected for two of closed-loop poles to be in the dominant region and for other poles to be outside of the dominant region. Dominant pole placement design approach was introduced by Persson and Åström (Persson and Åström, 1993) and was further explained in (Åström and Hägglund, 1995).

In order to be able to place all of the system poles arbitrarily, order of the output feedback controller should be at least plant order minus one (Wang et al., 2009). However, it is important to provide dominant pole placement in a plant with PID controllers, since PID controllers are the well-known and mostly used controllers in industry because of their simple structure and acceptable robustness (Åström and Hägglund, 2006). Guaranteed dominant pole placement problem with continuous PID controllers is considered in (Wang et al., 2009). In their study, it is aimed to design a PID controller which guarantees dominance of closed-loop poles via root-locus and Nyquist plot approaches. If the system has a time delay, the pole placement technique includes a risk on dominant poles because of the possibility for closed-loop dominant poles to lose their meaning caused by the infinite spectrum of poles (Pavel et al., 2013). Due to the fact that the conventional pole placement methods are not generally appropriate for the time-delay systems, modified Nyquist plot design approach works well to guarantee dominance of the closed-loop system poles using PID controllers.

In the industrial automation systems, processes are usually controlled by computer based systems such as industrial PCs or PLCs. Since discrete-time control systems become an important subject to examine as a result of the advancing technology, it also provides a significant advantage to be able to find the discrete-PID controller parameters which guarantee dominant pole placement.

In this paper, it is aimed to apply the modified Nyquist plot approach for dominant pole placement to discrete-time control systems. During the explanation of the method, the order in the reference (Wang et al., 2009) is followed. Firstly, K_i and K_d parameters of PID controller are written in terms of the parameter K_p. After that modified Nyquist plot is introduced in the z-domain and the proposed controller design procedure is explained. The given method is then demonstrated on three different systems and PID controller parameters which guarantees dominant pole placement are found. Finally, the controller zeros are also considered in this paper because they may become close to the dominant closed-loop system poles for some calculated K_p values and in this case, effects of the dominant closed-loop poles can be almost neglected as mentioned in (Yinya et al., 2011).

2. MODIFIED DISCRETE NYQUIST METHOD

2.1 Rearrangement of the Discrete PID Controller

A discrete-time transfer function of a system in general form can be given as follows ($G(z)$ is a proper transfer function).

$$G(z) = \frac{N(z)}{D(z)} \tag{1}$$

The PID controller in discrete-time domain can be given as follows.

$$C(z)=\frac{(K_p+K_i+K_d)z^2-(K_p+2K_d)z+K_d}{z\,(z-1)} \quad (2)$$

Closed-loop characteristic equation of the system related to Fig. 1 is given as below,

$$1+C(z)G(z)=0 \quad (3)$$

and the closed-loop system transfer function is then,

$$T(z)=\frac{N(z)\left((K_p+K_i+K_d)z^2-(K_p+2K_d)z+K_d\right)}{D(z)(z\,(z-1))+N(z)\left((K_p+K_i+K_d)z^2-(K_p+2K_d)z+K_d\right)} \quad (4)$$

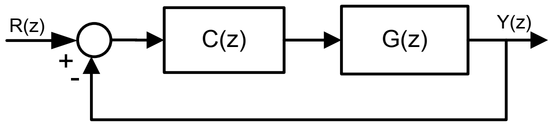

Fig. 1. Closed-loop control system.

It is assumed that the closed-loop system performance requirements are known, and hence, the closed-loop system dominant pole locations are also known. Therefore, the discrete-time PID controller parameters can be expressed depending on the parameter K_p. Assume that the dominant pole locations in the z-domain are given as $z_{1,2}=p\pm jq$. Then, if (3) is rewritten and one of the dominant pole locations is substituted, the following equality is achieved.

$$C(p+jq)=\frac{-1}{G(p+jq)} \quad (5)$$

If this complex equation is written by separating its real and imaginary parts, we obtain,

$$\frac{q}{p^2+q^2}K_d-\frac{q}{(1-p)^2+q^2}K_i=Im\left[\frac{-1}{G(p+jq)}\right] \quad (6)$$

and

$$\frac{-p+p^2+q^2}{p^2+q^2}K_d+\frac{-p+p^2+q^2}{(1-p)^2+q^2}K_i+K_p=Re\left[\frac{-1}{G(p+jq)}\right] \quad (7)$$

If (6) and (7) are solved together, following results are given.

$$K_i=C_1-\frac{(1-p)^2+q^2}{2(-p+p^2+q^2)}K_p \quad (8)$$

$$K_d=C_2-\frac{p^2+q^2}{2(-p+p^2+q^2)}K_p \quad (9)$$

where

$$C_1=\frac{(1-p)^2+q^2}{-2q}Im\left[\frac{-1}{G(p+jq)}\right]+\frac{(1-p)^2+q^2}{2(-p+p^2+q^2)}Re\left[\frac{-1}{G(p+jq)}\right] \quad (10)$$

$$C_2=\frac{p^2+q^2}{2q}Im\left[\frac{-1}{G(p+jq)}\right]+\frac{p^2+q^2}{2(-p+p^2+q^2)}Re\left[\frac{-1}{G(p+jq)}\right] \quad (11)$$

Here, the PID controller is written depending on one parameter thus it is possible to apply some of the well-known methods such as Nyquist plot. It is also important to remark that the given procedure is valid for all $G(z)$ systems regardless of the time delay.

In order to use the Nyquist method, the controller parameter K_p must be separated from the equation. For this purpose, (3) can be written in following form.

$$1+K_p\bar{G}(z)=0 \quad (12)$$

where

$$\bar{G}(z)=\frac{-N(z)(p^2+q^2-2pz+z^2)}{2(-p+p^2+q^2)\left((D(z)+(C_1+C_2)N(z))z^2-(D(z)+2C_2N(z))z+C_2N(z)\right)} \quad (13)$$

Since the time delay expression is hidden in $D(z)$, the transfer function $\bar{G}(z)$ is a proper rational transfer function.

2.2 Modified Nyquist Plot in Discrete-Time Domain

Nyquist plot of a system can also be drawn in discrete-time domain for discrete transfer functions. In this case, Nyquist path will be different than the Nyquist path in s-domain since stability regions are not same. In z-domain, the system is said to be stable if all of its poles are located inside the unit circle and unstable if any at least one of the poles is located outside of the unit circle.

It is possible to choose Nyquist path Γ_z as shown in Fig. 2.

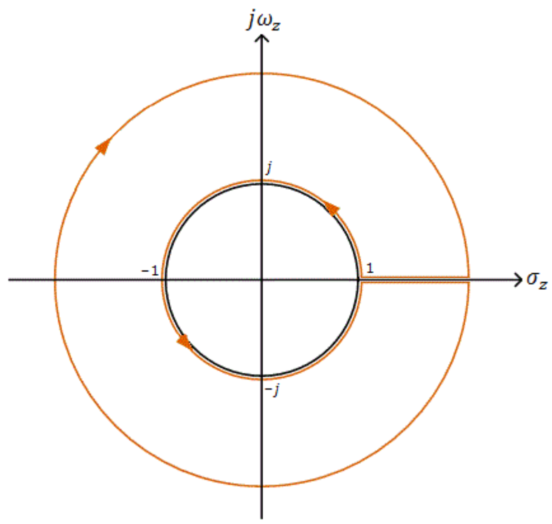

Fig. 2. Nyquist path in z-domain.

The Nyquist path given in Fig. 2 covers the whole z-plane outside of the unit circle which means instability region. In Nyquist stability analysis, it is expected to find the number of the unstable closed-loop system poles by just checking the number of unstable open-loop system poles and the number of the encirclements of the Nyquist plot around a critical point. It is possible to give the following formula.

$$Z=N+P \quad (14)$$

In (14) Z is the number of unstable closed-loop system poles, N is the number of encirclements and P is the number of unstable open-loop system poles.

However, here it is aimed to design a controller to guarantee the dominant pole placement. Thus, it is expected for the dominant closed-loop poles to be in the dominant region. For this reason, it is required to modify the Nyquist plot so that the Nyquist path encloses the circle that is located away from the dominant poles instead of the unit circle. If the distance between the determined closed-loop dominant poles and the origin is r then the Nyquist path should enclose the circle whose radius is r^m (m is usually 3-5) as in Fig. 3.

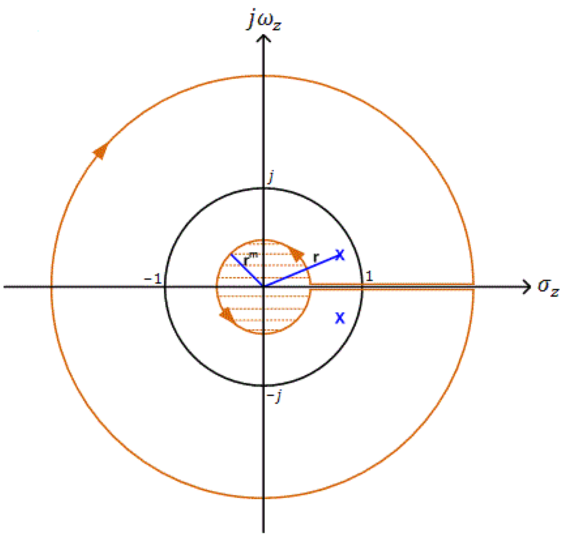

Fig. 3. Modified Nyquist path in z-domain.

In the figure, locations of the dominant poles are given as

$$z_{1,2} = p \pm jq = r\, e^{\pm j\omega T} \tag{15}$$

and the other closed-loop system poles are expected to be inside the small circle with radius r^m.

Since two of the closed-loop system poles should be in the dominant region, the value of Z should be two. In addition, from the open-loop system transfer function, the number of open-loop poles which are outside of the circle of radius r^m (P) is also known. Therefore, if the formula

$$N = 2 - P \tag{16}$$

is used, it is possible to find the number of encirclements to guarantee dominant pole placement.

2.3 PID Controller Design Procedures

In order to design a discrete PID controller via modified Nyquist plot approach, the following steps can be applied:

- First of all, locations of the dominant closed-loop system poles in z-domain should be determined according to expected closed-loop system performance criteria.
- After that the controller parameters should be written depending on the parameter K_p as explained earlier.
- Then, the open-loop system transfer function should be rearranged by using formula (13) so that the modified Nyquist plot can be drawn.
- The next step is to find the interval of the K_p value, which will provide the required encirclements, $\left(-1/p_1, -1/p_2\right)$ where p_1 and p_2 are the intersection points of the Nyquist plot and real axis.

Consequently, the PID controller parameters, that provide guaranteed dominant pole placement, in 3-D parameter space are found.

3. CASE STUDIES

3.1 First Order System with Time Delay

Consider a first order process with time delay,

$$G_1(s) = \frac{1}{s+1} e^{-0.3s}$$

and the corresponding discrete transfer function by taking the sampling time 0.1 seconds,

$$G_1(z) = \frac{0.09516}{z^3(z-0.90484)}$$

This system is expected to be controlled according to 5% overshoot and 6 seconds settling time performance criteria. Related closed-loop system dominant poles in the z-domain are given as,

$$z_{1,2} = 0.9332 \pm j0.0654 = 0.9355\, e^{\pm j0.07}$$

The PID controller parameters K_i and K_d can be found in terms of the parameter K_p as follows

$$K_i = 0.0493 + 0.07526K_p$$
$$K_d = -3.107 + 7.5364K_p$$

After that it is possible to separate the parameter K_p and obtain the new transfer function using (12).

$$\overline{G_1}(z) = \frac{0.8195\,(z^2 - 1.8664z + 0.87514)}{z^6 - 1.90484z^5 + 0.90484z^4 - 0.291z^2 + 0.59132z - 0.29566}$$

The magnitude of the dominants poles is $r = 0.9355$ and if m is chosen as 5, non-dominant poles should be inside the circle of radius $r^m = 0.9355^5 = 0.71646$. In order to find the required encirclements using (16), number of the poles of $\bar{G}_1(z)$, which are located in the dominant region, have to be found. If required calculations are done, all poles of $\overline{G_1}(z)$ are found to be in the dominant region so $P = 6$ and the required number of the encirclements is found as $N = -4$.

Finally, the Nyquist plot of $\overline{G_1}(r^m e^{j\omega T})$ is drawn and the interval in real axis which provides $N = -4$ is found. Fig. 4 shows the modified Nyquist plot.

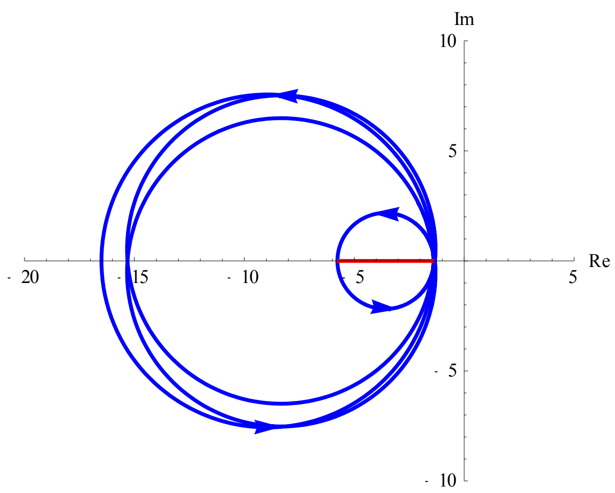

Fig. 4. Modified Nyquist plot of $\bar{G}_1(z)$.

In the Fig. 4, the red line represents the interval in which the number of the encirclements is $N = -4$. After that it is easy to find the K_p value range since p_1 and p_2 are determined through Nyquist plot. Therefore, we have,

$$K_p \in \left(\frac{-1}{p_1}, \frac{-1}{p_2}\right) = \left(\frac{-1}{-5.7637}, \frac{-1}{-1.3926}\right) = (0.1735,\ 0.718)$$

If the K_p parameter is chosen from the obtained interval, the closed-loop system poles are in the dominant region and the other poles are in the non-dominant region.

3.2 Second Order System with Time Delay

Consider a second order process with time delay,

$$G_2(s) = \frac{1}{s^2+2s+4} e^{-0.4s}$$

and the corresponding discrete transfer function ($T_{samp} =$ 0.1 secs),

$$G_2(z) = \frac{0.004667z+0.004366}{z^4(z^2-1.783z+0.8187)}$$

If the aim is to control the system so that the overshoot is less than 10% and the settling time is less than 6 seconds. Related closed-loop system dominant poles in z-domain are given as,

$$z_{1,2} = 0.932 \pm j0.085 = 0.93587\ e^{\pm j0.09095}$$

The PID controller parameters K_i and K_d are written in terms of the parameter K_p as follows

$$K_i = 0.21688 + 0.1055\ K_p$$
$$K_d = 4.9061 + 7.7991\ K_p$$

If the transfer function with the PID controller is rewritten,

$$\overline{G_2}(z) = \frac{0.041558\ (z+0.93551)\ (z^2-1.864z+0.87585)}{z^8-2.783z^7+2.602z^6-0.8187z^5+0.0239z^3-0.02343z^2-0.02z+0.02142}$$

The magnitude of the dominant poles is calculated as $r = 0.93587$ and if m is chosen as 3, non-dominant poles should be inside the circle of radius $r^m = 0.93587^3 = 0.81968$. The number of poles of $\overline{G_2}(z)$ inside dominant region is found to be $P = 2$. As a result, it is required to find K_p interval that provides no encirclements.

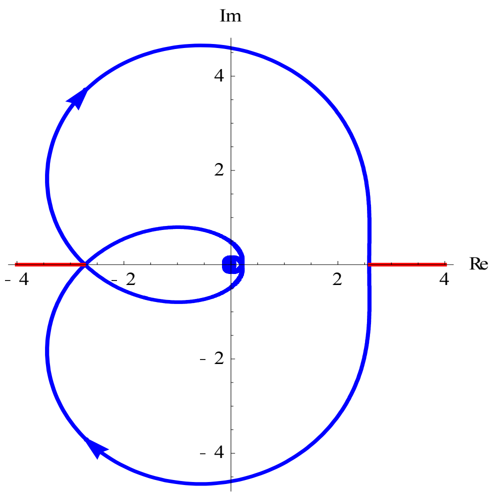

Fig. 5. Modified Nyquist plot of $\bar{G}_2(z)$.

With the help of the Nyquist plot of $\bar{G}_2(r^m e^{j\omega T})$ in Fig. 5, the feasible K_p value range is calculated.

$$K_p \in \left(\frac{-1}{2.58558}, \frac{-1}{-2.72851}\right) = (-0.3867,\ 0.3665)$$

Here, it is again guaranteed that closed-loop poles are in the dominant region and the other poles are in the non-dominant region for this range of K_p.

3.3 Higher Order System with Time Delay

Last example transfer function is a fourth order process with time delay and one zero located on the left half s-plane.

$$G_3(s) = \frac{(s+3)}{(s+1)^2(s+4)^2} e^{-2s}$$

The corresponding discrete transfer function with 0.4 seconds sampling time is,

$$G_3(z) = \frac{0.00556z^3+0.009967z^2-0.00213z-0.000415}{z^5(z^4-1.744z^3+1.031z^2-0.2361z+0.01832)}$$

The closed-loop system dominant poles which provide 5% overshoot and 14 seconds settling time is calculated as below.

$$z_{1,2} = 0.8856 \pm j0.1067 = 0.892\ e^{\pm j0.12}$$

The PID controller parameters in terms of the parameter K_p are

$$K_i = 0.2 + 0.1361\ K_p$$
$$K_d = -5.61 + 4.424\ K_p$$

Using the same procedure, the number of the poles of $\overline{G_2}(z)$ inside dominant region is found to be $P = 5$ (m is chosen as 3). Therefore, it is required to find the interval in which the number of the encirclements is $N = -3$. Fig. 6 shows the Nyquist plot of $\bar{G}_3(r^m e^{j\omega T})$.

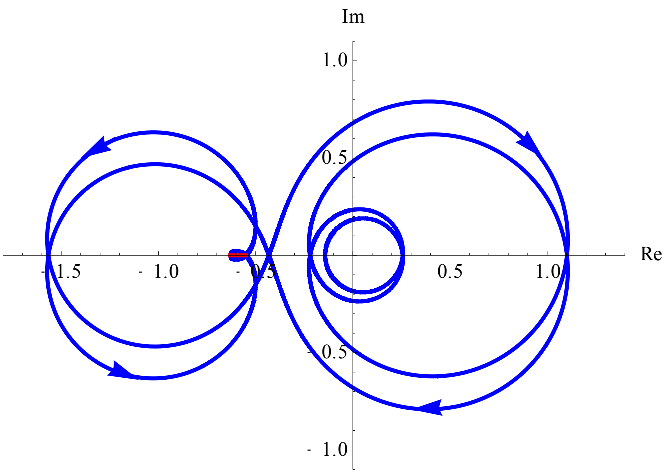

Fig. 6. Modified Nyquist plot of $\bar{G}_3(z)$.

It is possible to find a narrow K_p value range as can be seen from the Nyquist plot.

$$K_p \in \left(\frac{-1}{-0.62854}, \frac{-1}{-0.55133}\right) = (1.591,\ 1.8138)$$

Note that it is not always possible to find such a K_p range depending on the determined performance criteria and "m" value. In some cases, in order to guarantee dominant pole placement for selected "m" value, it may be required to change the performance criteria.

4. EFFECTS OF PID CONTROLLER ZEROS

The given method guarantees the dominant pole placement in terms of closed-loop system poles. However, for some K_p values in the calculated interval, controller zeros can affect the system performance adversely. Zeros of the PID controller can be located near to the dominant closed-loop system poles and this case is already discussed in (Yinya et al., 2011). Since any K_p value in the calculated interval is acceptable, instead of a random selection, it is possible to choose the Kp value considering the locations of the zeros of the controller.

In this study, PID controller zeros are also taken in account because of the reasons explained above. It is required to keep the controller zeros away from the dominant region if possible or since it is not always possible to locate the zeros as required a pre-filter can be used as studied in (Yinya et al., 2011). Therefore, in the calculated K_p range, locations of the zeros are investigated and it is tried to reduce the effects of controller zeros by finding such a subinterval in which the PID controller zeros are also located outside of the dominant region.

4.1 First Order System with Time Delay

For the first order system example, the poles (red) and zeros (blue) change for $K_p \in (0.1735,\ 0.718)$ as given in Fig 7. The dotted circle of radius r^4 represents the border between dominant and non-dominant region.

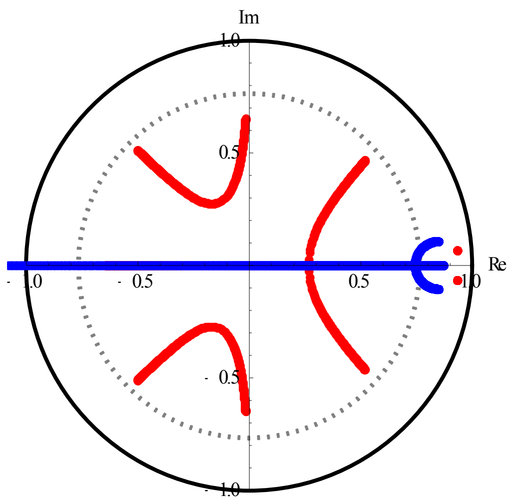

Fig. 7. Variation curve of the poles and zeros (Example 4.1).

If the required equation (17) is solved to find the K_p value range in which controller zeros are also outside of the dominant region for $m = 5$, it is seen that such a K_p value does not exists.

$$|z_{1,2}| = \left|\frac{2K_d + K_p \mp \sqrt{-4K_dK_i + {K_p}^2}}{2\,(K_p + K_i + K_d)}\right| < r^m \qquad (17)$$

If calculations are done for $m = 4$ as seen from the Fig. 7 there is a solution which is calculated as,

$K_p \in (0.5087,\ 0.5285)$

If K_p is chosen from this interval, it is possible to show that the performance of the closed-loop system is not affected by the PID controller zeros much.

Fig. 8 shows the closed-loop step response of the first order system with time delay both for $K_p = 0.2$ and $K_p = 0.51$ together. For $K_p = 0.2$ the closed-loop system has 6.4% overshoot, 6.0 seconds settling time and has an inverse overshoot which is not pleasant. Whereas, for $K_p = 0.51$ the closed-loop system has 5.0% overshoot and 5.7 seconds settling time, which means the performance criteria are met.

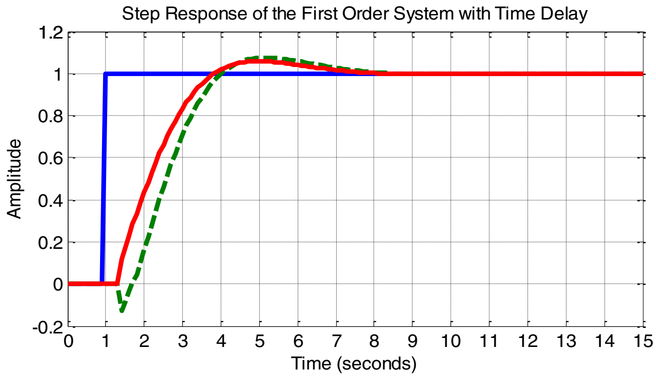

Fig. 8. Closed-loop response of the first order system.

4.2 Second Order System with Time Delay

For the given second order system, it can be shown that it is not possible to place the closed-loop system zeros out of the dominant region. Furthermore, PID controller zeros will be out of the unit circle in most cases as can be seen from Fig. 9. Here, it is better to choose value of the K_p parameter such that controller zeros will be located inside the unit circle so that it is allowed to design a pre-filter to improve the closed-loop system response (A pre-filter design is not considered in this paper).

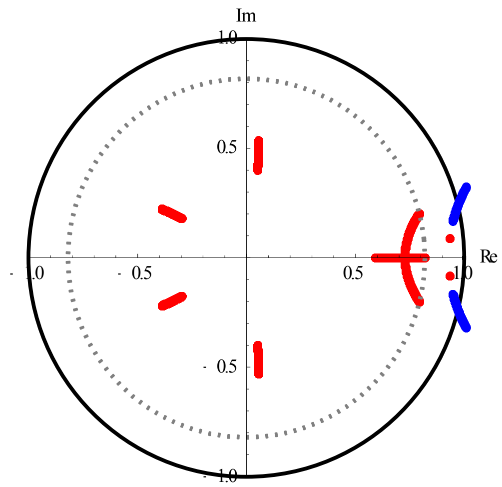

Fig. 9. Variation curve of the poles and zeros (Example 4.2).

Fig. 10 shows the unit step response of the system for $K_p = 0.35$ (system has 6.4% overshoot and 5.95 seconds settling time) and for $K_p = -0.35$ (system has 6.4% overshoot and 6.45 seconds settling time) which is also inside the calculated interval.

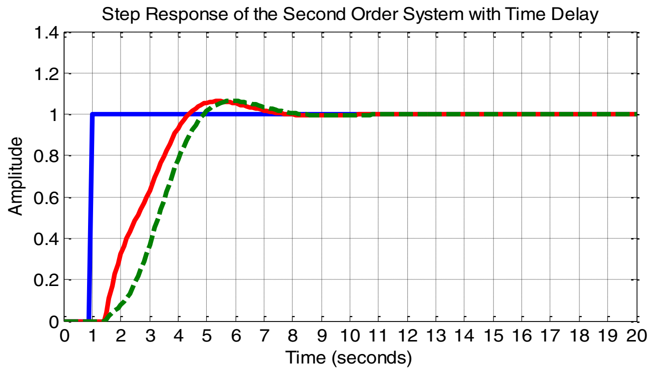

Fig. 10. Closed-loop response of the second order system for example 4.2 .

4.3 Second Order System with Time Delay

Similarly, the interval that results in controller zeros placed in nondominant region for $m = 3$, is obtained

$K_p \in (1.591,\ 1.777)$

Root loci of the poles and zeros for the calculated interval can be drawn as in Fig. 11.

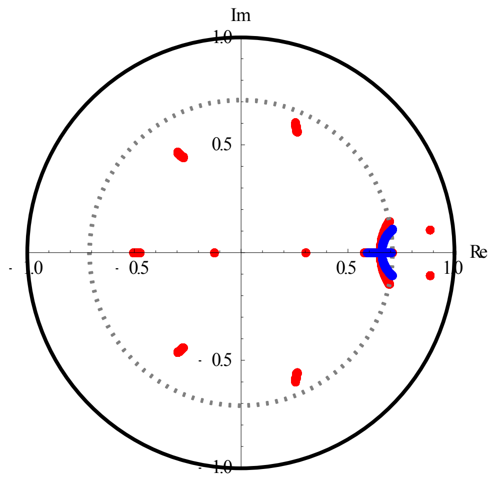

Fig. 11. Variation curve of the poles and zeros (Example 4.3).

Fig. 12 shows the unit step response of the last system for $K_p = 1.6$ (system has 5.2% overshoot and 15.7 seconds settling time) and for $K_p = 1.8$ (system has 5.7% overshoot and 15.5 seconds settling time). For this particular system, calculated range is not very large; therefore, system response does not change much in the K_p value range.

Here, it is observed from the results that with the proposed method, dominant pole placement is performed successfully. In addition, it is shown that the controller zeros in dominant region or outside of the unit circle affect the system response adversely.

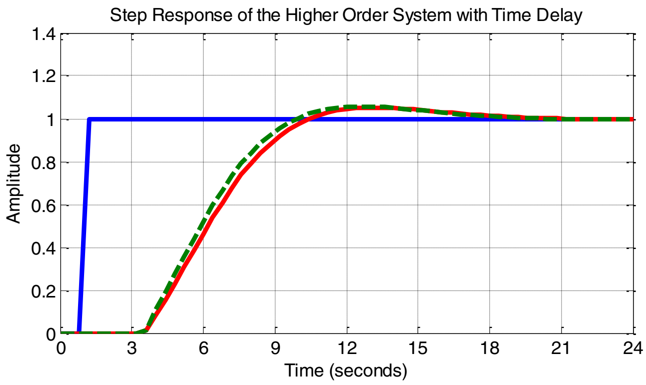

Fig. 12. Closed-loop response of the higher order system.

5. CONCLUSION

In this paper, guaranteed pole placement with discrete PID controllers based on the idea given in (Wang et al., 2009) is considered. Modified Nyquist plot approach is used to obtain discrete PID controller parameters. The design procedure is explained and controller parameters are obtained for three different systems. Moreover, effects of the PID controller zeros in the closed-loop system are also discussed and subintervals for the K_p parameter, in which the controller zeros are also located away from the dominant poles, is found. It is shown that the modified Nyquist plot approach works well to guarantee dominant pole placement in z-domain.

It is important that although the given method works well in discrete-time domain, it is not always possible to find feasible K_p intervals.

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