Three different attitude measurements of spinning projectile based on magnetic sensors

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Abstract

Micro-inertial sensors currently could not provide long-time stability attitude information for the high spinning projectile because of drift errors. Meanwhile, the method of the navigation and attitude measurement with respect to the earth’s magnetic field is still auxiliary, and the attitude angle information cannot be got only by measuring the three-axis components of the geomagnetic field. In view of the flying characteristics of high spinning projectile, three different attitude measurements only using magnetic sensors are researched. Through comparative analyses, the calculating principle, system composition, applicable condition and error range of these methods are explained. Meanwhile, the semi-physical experiments are made to prove the effectiveness of the three attitude measurements. The experiment results indicate that only scalar arithmetic operations are required for these angular measurements and they have the day/night and all weather capability. The three different measurements have same angle error range within ±1° but different attitude updating rate.

1. Introduction

At present, the attitude measurement of a moving body is involved in many fields, for example, in aerial and marine vehicles [1–3], robots and human pose tracking [4]. Especially for the military application, it is important to test the projectile flight attitude accurately. Accurate measurement of angular motions of spinning projectiles with on-board sensors has been recognized as a daunting task. The fundamental requirements for such measurement are lightweight, small-size, and low power consumption. The currently available micro-inertial sensors have relatively low accuracy and the drift could cause remarkable attitude errors [5]. Successful attitude measurement with inertial sensors requires the expensive gyroscopes and accelerometers with exceptionally increased accuracy and complex filtering algorithm [6–8].

Geomagnetic field is a vector field as the earth’s natural resources. It provides the natural coordinate system for navigation with its rich element such as strength, inclination, declination and gradient. Recent progresses in magnetic sensor technologies have resulted in devices small enough, rugged enough, and sensitive enough to be useful in systems capable of making high-speed, high-resolution measurements of attitude relative to magnetic fields [9]. Because of its high reliability and anti-interference ability, attitude measurements with geomagnetic have become a hot spot in research of flying parameter measurement. Due to the fact that the three-component magnetic sensor cannot provide three independent equations, other methods are combined to calculate one of the three angles of yaw, pitch and roll to obtain another two. The problems make the magnetic sensor is still auxiliary in attitude measuring systems [10–13].

Thomas Harkins and David Hepner designed an attitude measuring system for spinning bodies, called...
“MAGSONDE” only with magnetic sensors. The “Zero Crossings Method” for the “MAGSONDE” has been provided in their report [14]. On this basis, an attitude angle measurement based on the ratio of extremum of two orthogonal magnetic sensors is introduced in this paper and this “Extremum Ratio Method” is extended to “Three Orthogonal Ratio Method”. Through the theory research and Semi-physical experiments, the three different attitude measuring methods that only using magnetic sensors were compared in detail. In all the three cases, we do not need to know the magnetic field strength, only scalar calibration curve and a magnetic field in the coordinate frame of fireball is showed in this paper. Orthogonal Ratio Method” is introduced in this paper and this “Extremum Ratio Method” is extended to “Three Orthogonal Ratio Method”. Through the theory research and Semi-physical experiments, the three different attitude measuring methods that only using magnetic sensors were compared in detail. In all the three cases, we do not need to know the magnetic field strength, only scalar calibration curve and a magnetic field in the coordinate frame of fireball is showed in this paper. 

2. Magnetic sensor configuration

Assuming that the gravity center of the spinning projectile is at the origin of the o-xyz coordinate system which fixed in body frame, its axis of rotation is on the x axis and its nose pointed in the +x direction. As showed in Fig. 1, the magnetic sensors $M_{ox}$, $M_{oy}$ and $M_{oz}$ locate respectively along the x axis, y axis and z axis. The sensor $M_{o1}$ locates in the o-xy plane and orients at a non-zero angle $\lambda$ from the spin axis x.

According to coordinate system rotation matrix rules, the field strength along the sensitive axes of the four sensors are given by [15]

\[
\begin{align*}
M_{ox} &= |\vect{M}| \cos \psi \cos \sigma_m \\
M_{oy} &= -|\vect{M}| \cos \gamma \sin \psi \cos \sigma_m + |\vect{M}| \sin \gamma \sin \sigma_m \\
M_{oz} &= |\vect{M}| \sin \gamma \sin \psi \cos \sigma_m + |\vect{M}| \cos \gamma \sin \sigma_m \\
M_{o1} &= |\vect{M}| \cos \psi \cos \sigma_m \cos \lambda - |\vect{M}| \cos \gamma \sin \psi \cos \sigma_m \sin \lambda \\
&\quad + |\vect{M}| \sin \gamma \sin \sigma_m \sin \lambda
\end{align*}
\]

(1)

where $\vect{M}$ is the strength and direction of geomagnetic field. The angle between $\vect{M}$ and x axis is designated as $\sigma_m$, $\psi$ is the sum of the declination and real yaw. The projectile roll angle is described by $\gamma$. There are three unknown parameters in (1), but no three independent equations. Therefore, one or more attitude angles must be known from other ways in order to calculate the rest attitude angles [5].

3. Theory analysis for three different methods

According to the flight characteristic of the high spinning projectile, some basic hypotheses are as follows:

1. Velocity vector is in the firing plane all the time [16], that is, $\psi$ is invariable.
2. $\sigma_m$ changes much slowly with time relative to the roll rate.

With the above hypothesis, the “Zero Crossings Method” was anew explained below. Based on it, the other two new methods were introduced in this section.

3.1. Zero crossings method

The normalized field strength along the sensitive axis for two non-orthogonal sensors $M_{oy}$ and $M_{o1}$ throughout several roll cycles is plotted in Fig. 2 with $\sigma_m = 45^\circ$, $\psi = 30^\circ$ and $\lambda = 60^\circ$. Denoting the two pairs of roll angles at the zero crossings for the two sensors as ($\gamma_{sy_a}, \gamma_{sy_b}$) and ($\gamma_{st_1}, \gamma_{st_2}$). By (1), with fixed $\psi$ and $\lambda$, the value of ratio $R = (\gamma_{st_2} - \gamma_{st_1})/(\gamma_{sy_b} - \gamma_{sy_a})$ only depends on $\sigma_m$ [14,17]. The corresponding relation of ratio $R$ and $\sigma_m$ is showed in Fig. 3.

The combination of the $R - \sigma_m$ calibration curve and a parity check completely specifies the angle $\sigma_m$ between the projectile axis and the magnetic field [17].

3.2. Extremum Ratio Method

Not only the ratios of zero crossing, but also the ratios of maximums and minimums of the two magnetic sensors have corresponding relationship with $\sigma_m$ and they are proved as follows: $\sigma_m$ and $\psi$ change slowly with time compared with $\gamma$, so when $M_{oy}$ and $M_{o1}$ reach the maximum or the minimum, that is $dM_{sy}/dt = 0$ and $dM_{o1}/dt = 0$, we always have

$$
\cos \sigma_m \sin \psi \sin \gamma + \sin \sigma_m \cos \gamma = 0
$$

(2)

It is seen from (2) that $M_{oy}$ and $M_{o1}$ reach the maximums and minimums at the same time, respectively. Denoting the ratios of the maximums and minimums of two magnetic sensors as $R_{max} = (M_{o1max}/M_{symax})$ and $R_{min} = (M_{o1min}/M_{symin})$. When $M_{sy}$ or $M_{o1}$ reaches the extreme values, the attitude angles must satisfy (2), so combining (1) and (2), the relationships between $R_{max}$, $R_{min}$ and $\sigma_m$ are as follows:

$$
R_{max} = \sin \lambda + \frac{\cos \psi \cos \lambda \cos \sigma_m}{\sqrt{\sin^2 \sigma_m + \sin^2 \psi \cos^2 \sigma_m}}
$$

(3)
\[
R_{\text{min}} = \sin \lambda - \frac{\cos \psi \cos \lambda \cos \sigma_m}{\sqrt{\sin^2 \sigma_m + \sin^2 \psi \cos^2 \sigma_m}}
\]

(4)

The flight projectile allows changing \( \gamma \) and \( \sigma_m \) with respect to each \( \psi \).

That is, for a fixed yaw angle and the corresponding range of \( \sigma_m \), the curves of the extremum ratio of the two magnetic sensors can be made out by making the projectile rotating one circle with different \( \sigma_m \). Fig. 4 shows the curves of the ratios \( R_{\text{max}}, R_{\text{min}} \) versus \( \sigma_m \) when \( \psi = 30^\circ \) and \( \lambda = 60^\circ \).

During the flight, the ratio of extremum could be got from the outputs of the sensors, then the \( \sigma_m \) in this occasion could be got from the curve of calibration according to the calculated ratios.

3.3. Three Orthogonal Ratio Method

The “Extremum Ratio Method” makes use of the ratio of the extremum of the two non-orthogonal magnetic sensors \( M_{s1} \) and \( M_{sy} \) to get the attitude information. The thought
can be extended to the ratio calculation of three pairwise-orthogonal magnetic sensors.

The output of the magnetic sensor $M_{sz}$ is a constant in a spin cycle under the assumptions of the flight characteristic of the high spinning projectile. When $M_{sz}$ reaches the maximum or the minimum, $dM_{sz}/dt = 0$, and we have

$$\cos \sigma_m \sin \psi \cos \gamma - \sin \sigma_m \sin \gamma = 0$$

(5)

From (2) and (5), we know that $M_{sy}$ and $M_{sz}$ respectively reach the maximums and minimums at different time. The time interval of each extremum is just right a quarter of a roll cycle. Fig. 5 shows the ratio relationships of the $M_{sx}/M_{sz}$ and $M_{sy}/M_{sz}$ in A, B, C and D points respectively when $\psi = 30^\circ$ and $\sigma_m = 80^\circ$.

Denoting the ratios of three pairwise-orthogonal magnetic sensors at the extreme value points of $M_{sy}$ and $M_{sz}$ as $R_{xy_{\text{max/min}}} = (M_{sx}/M_{sy_{\text{max/min}}})$ and $R_{xz_{\text{max/min}}} = (M_{sx}/M_{sz_{\text{max/min}}})$. Combining (2), (5), and (1), we can get

$$R_{xy_{\text{max/min}}} = R_{xz_{\text{max/min}}} = \pm \frac{\cos \psi \cos \sigma_m}{\sqrt{\sin^2 \sigma_m + \sin^2 \psi \cos^2 \sigma_m}}$$

(6)
It is seen from (6) that there is the one-to-one correspondence between the ratios $R_{yx_{\min}}$, $R_{xz_{\min}}$ and $\sigma_m$ for each fixed $\psi$. So, we can also calculate the $\sigma_m$ by the curve of calibration according to $R_{yx_{\max}}$ and $R_{xz_{\max}}$. Due to the zero crossing exists between the maximum and the minimum, $\sigma_m$ at the zero crossing time could be obtained by the interpolation method [18]. According to (1), when $M_{sy} = 0$ or $M_{sz} = 0$, we have

$$\tan \gamma = \sin \psi \cot \sigma_m$$

$$\cos \gamma = - \sin \psi \cot \sigma_m$$

Substituting known $\psi$ and calculated $\sigma_m$ into (7) and (8), then $\gamma$ in these zero crossing time could be solved. In projectile’s flight, $\psi$ is considered as invariable, but there are errors caused by this approximation method. With $\gamma$ and (1), yaw angle could be corrected by the following equation

$$\tan \psi = (M_{sz} \cdot \sin \gamma - M_{sy} \cdot \cos \gamma)/M_{sx}$$

(9)

4. Comparative analysis

All of these three different attitude measuring methods are based on the mathematics corresponding relations between $\sigma_m$ and the certain ratios of these magnetic sensors. Every of the three methods has its own characteristics in some ways with a link in principle. Their differences should be discussed below through some comparative analysis.

4.1. System composition and attitude updating rate

Both “Zero Crossing Method” and “Extremum Ratio Method” must use four magnetic sensors in order to realize all the three attitude angles measurement. The four magnetic sensors contain a non-orthogonal installation magnetic sensor, which makes some difficulties in the installation of the sensors and the system building. In contrast, the “Three Orthogonal Ratio Method” only needs three pairwise-orthogonal magnetic sensors to achieve the same effect.

In a proper rotation cycle, the “Zero Crossing Method” has only one ratio, so it can work out only one group of attitude angles. The “Extremum Ratio Method” can get a pair of extreme value ratio, accordingly, it can get two groups of attitude angles. There are four characteristic ratios of the “Three Orthogonal Ratio Method” in a spin cycle. Its attitude updating rate is four times of the former and twice of the latter.

4.2. Launch window range

There is the necessity of certain magnetic sensor being orthogonal to the field during a roll cycle (zero crossing point) in all the three different methods. That limits the range of the magnetic aspect launch angles within which the particular sensor configuration is able to operate. This applicability region is called the “Magsonde window” [14,19]. Both of “Zero Crossing Method” and “Extremum Ratio Method” require the $M_{sy}$ and $M_{sz}$ to have the zero point. Meanwhile, the “Three Orthogonal Ratio Method” requires the $M_{sy}$ and $M_{sz}$ to have the zero point.

Solving (1) for the roll angles at which $M_{sy} = 0$ and $M_{sz} = 0$ yields, we get (7) and (8). There is no constraint for $\sigma_m$ in (7) and (8). For sensor $M_{sz}$, solving (1) for the roll angles at which $M_{sz} = 0$:

$$\sin(\gamma - \theta) = -\frac{\cos \psi \cot \lambda}{\sqrt{\tan^2 \sigma_m + \sin^2 \psi}}$$

The existence criterion for $\gamma - \theta$ of $|\sin(\gamma - \theta)| \leq 1$ leads to the requirement that $|\cos \psi \cos \sigma_m| \leq |\sin \lambda|$. In generally, $\lambda < 90^\circ$:

If $\sin \lambda \geq |\cos \psi|$, $\sigma_m \in [0^\circ, 180^\circ]$. 

If $\sin \lambda < |\cos \psi|$, 

$$\arccos \left(\frac{\sin \lambda}{|\cos \psi|}\right) \leq \sigma_m \leq \arccos \left(-\frac{\sin \lambda}{|\cos \psi|}\right).$$

With the above analysis, we find whatever is $180^\circ$ the value of $\sigma_m$, the sensors $M_{sy}$ and $M_{sz}$ always have the zero crossing point. So, the “Three Orthogonal Ratio Method” does not be limited by “Magsonde window”. At the same time, the other two methods only suit for a particular launch widow which depends on the range of possible $\sigma_m$ during the flight.

4.3. Measurement Blind Area

When the axis of rotation is parallel to the local geomagnetic vector ($\sigma_m = 0^\circ$ or) during the flight, the outputs of $M_{sy}$ and $M_{sz}$ stay at zero value. When the axis of rotation is in a very small angle area around the local geomagnetic vector, the outputs of the $M_{sy}$ and $M_{sz}$ are such small that the useful signals basically are submerged in the measurement noise. All of the three attitude measuring methods cannot have effective role in such an area called “Measurement Blind Area”. Because of the extremely short time in which the projectile stays in the “Measurement Blind Area”, the influence on the whole of measurement can be ignored. In practical projects, the problem can be solved by the forecast and filter algorithm [20,21].

4.4. Error analysis

The reasons for the errors of these above attitude measurements are analyzed as follows:

(1) Suppose that $\psi$ is invariable in the flight for the estimation, which result in errors of $\sigma_m$ and $\gamma$. This kind of error exists in all the three attitude measuring methods. The second correct method for eliminating the influence of the hypothesis in these calculation methods could be carried out by repeating calculation steps with the corrected $\psi$ instead of the hypothetical one.

(2) In the three methods, the interpolation method is adopted in the calculation process of $\sigma_m$, i.e. using
Fig. 6. Errors of the three attitude angles for Zero Crossing Method. There is one group of calculated attitude angles in a proper rotation cycle.

Fig. 7. Errors of the three attitude angles for Extremum Ratio Method. There are two groups of calculated attitude angles in a proper rotation cycle.

Fig. 8. Errors of the three attitude angles for Three Orthogonal Ratio Method. There are four groups of calculated attitude angles in a proper rotation cycle.
interpolation to estimate the value of $\sigma_m$ in the pre-calibrated curve. This produces the error of $\sigma_m$ and then leads to the error of $\gamma$ when use (7) and (8) to estimate the angle. Meanwhile, the “Zero Crossing Method” in a proper rotation cycle only can get one $\sigma_m$. So, the faster changes of the $\sigma_m$ in the flight leads the bigger error of this method. The attitude update rate of the “Three Orthogonal Ratio Method” is higher than the other two methods, accordingly, the error of the $\sigma_m$ is the smallest.

(3) In the curve of the sensors output, the precision of interpretation of zero point is lower than the accuracy of extreme value estimate. The results of the “Zero Crossing Method” are two symmetrical values, the parity check needs in its calculation process. The two problems make the calculating precision of this method relatively low.

Through the above the error analysis, we have a conclusion like this: in the same conditions, the attitude error of the “Zero Crossing Method” is the biggest among the three different methods, the precision of “Three Orthogonal Ratio Method” is the best.

4.5. Numerical simulation analysis

According to the (1)–(9), the numerical simulation is carried out under the given conditions: $\psi = 30^\circ$, $\alpha = 80^\circ$.
and γ changes with the angular velocity 2 rps. The above three methods are used to estimate all the attitude angles. The calculated results of each method are illustrated in Figs. 6–8.

In view of the results of the numerical simulation, the attitude angle errors of the three methods are all within ±1°, and the estimated accuracy of the “Three Orthogonal Ratio Method” is obviously higher than the others. Meanwhile, for all these methods, the error of the angle σ_m is smaller than the errors of γ and ψ. These numerical simulation results coincide with the above error analysis.
5. Experiments and results

Based on the theory algorithms research, the experiments were performed to verify and compare these three attitude measurements. A semi-physical device was designed for the simulation experiments. The hardware components diagram of the device is shown in Fig. 9. Fig. 10 illustrates this homemade prototype.

The prototype is mainly comprised of a pair of Honeywell’s HMC1021/1043 magnetometers (single-axis/three-axis magnetic sensors with ±6 Gauss measurement range and about 8 μ Gauss noise level), a Texas Instruments’ ADS8365 (16-Bit, 250 kbps, 6-Channel, Simultaneous Sampling analog-to-digital converter), a CYGNAL’s C8051F320 (Full Speed USB, 16 kbps flash MCU), and an ATMEL’s AT45DB642D (64 Mb serial-interface flash memory with SPI interface). The signals of the magnetic sensors was acquired and stored in the flash memory. Then, through the USB port, these data was transferred to computer for attitude calculation. Fig. 11 shows the system block diagram of the experimental setup.

During the experiments, the hardware prototype was mounted on three axis turntable, with the axes of the prototype aligned with the three rotation axis of the turntable (see Fig. 12). Firstly, the calibration curves of the each method were made with fixed $\psi = 30^\circ$. Then, semi-physical simulation experiments was processed under the condition that the device rotates with a constant roll rate and $\sigma_m$ was stay at 80° with $\psi = 30^\circ$. These above three attitude measuring methods were used to estimate all the three attitude angles. Fig. 13 shows the real output signals of the four magnetic sensors after filter. The calculating errors of $\sigma_m$ for each method are showed in Fig. 14. Table 1 shows the detail results of the semi-physical simulation experiments. The outputs of the turntable were considered as the reference of the attitude angles.

It is seen from the results of experiment that the three different attitude measuring methods discussed all could keep the attitude errors less than 1 degree. The error of the “Zero Crossing Method” is relatively large and the attitude updating rate of the “Three Orthogonal Ratio Method” is the highest. These results are the same as the numerical simulation discussed in Section 4.5. Compared with results of numerical simulations, the attitude calculating errors of the actual experiments are little bigger. Although the digital filter [22] is used, the noise of the magnetic sensors and the installation error are the main probable reason of the errors for experiments. In addition, different sensors have different characteristics which lead to the error of $\psi$ when (9) is used to estimate it. The results of experiments prove the effectiveness of the three attitude measuring methods.

6. Conclusion

In the paper, the theories of three different attitude measurements for spinning projectile only using magnetic sensors are discussed. All of the three methods which only need scalar arithmetic operations have a series of merits and would not be affected by the weather or light. Through the comparison, the differences between these methods in several sides are analyzed in detail. The semi-physical simulation experiment is done and the results show that the measurement system with these three methods is valid and the errors of the attitude angles are in the range of permission of the attitude measurements. The “Three Orthogonal Ratio Method” has some advantages in precision and attitude updating rate compared with the other two methods. These results indicate that the three different measuring methods are effective for high spinning projectile. The resulting angle data from these methods can be used with diagnostic tools for flight characteristic of projectile. The research in the paper provides a new theoretical basis for the attitude measurement and navigation with the geomagnetic.

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Table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\sigma_m$ Error (°)</th>
<th>$\psi$ Error (°)</th>
<th>$\gamma$ Error (°)</th>
<th>Attitude update rate</th>
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<td>Zero crossing</td>
<td>±1</td>
<td>±1</td>
<td>±1</td>
<td>1 Point per cycle</td>
</tr>
<tr>
<td>Extremum ratio</td>
<td>±0.5</td>
<td>±1</td>
<td>±0.8</td>
<td>2 Point per cycle</td>
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<td>Three orthogonal ratio</td>
<td>±0.2</td>
<td>±0.6</td>
<td>±0.8</td>
<td>4 Point per cycle</td>
</tr>
</tbody>
</table>

References


D. Li, X.Z. Bu, Attitude measurement on high spinning projectile using magnetic sensors and accelerometers 2 (2008) 106–112.


