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Fatigue damage modelling of composite materials

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Abstract

This paper develops a mathematical model for fatigue damage evolution in composite materials. The characteristics of damage growth in composite materials are studied and compared with those of damage growth in homogeneous materials. Continuum damage mechanics concepts are used to evaluate the degradation of composite materials under cyclic loading. A new damage accumulation model is proposed to capture the unique characteristics of composite materials. The proposed model is found to be more accurate than existing models, both in modelling the rapid damage growth early in life and near the end of fatigue life. The parameters for the proposed model are obtained with experimental data. A numerical example is implemented to illustrate that the proposed model is able to accurately fit several different sets of experimental data.

Keywords: Fatigue; Damage; Composite; Life prediction

1. Introduction

Composite materials are widely used in numerous aerospace, automotive, and construction application because of their high specific stiffness and strength which benefits the material and provides potential energy savings. Composite materials were first used in aircraft engine rotor blades in the 1960s [1]. Since then, a great deal of research has been conducted to improve the properties of composite materials. Composite materials are also considered for many high-temperature applications in advanced aerospace vehicles and gas turbine engine components. The use of composite materials has been shown to reduce the life-cycle cost in some cases [2].

Some of these applications involve components that are subjected to cyclic loading. Cyclic loading causes damage and material property degradation in a cumulative manner. It is desirable that the composites and the constituents have sound fatigue-resistant properties during the service time of the component and the structure. It is important to evaluate the damage and degraded properties accurately to ensure that the structures operate with high reliability during their lives. It's also important to evaluate the performance of the structure in advance so that the maintenance or replacement of components can be scheduled before catastrophic failure. Such analysis should be able to predict the remaining life of the structure, given the previous loading history.

This paper aims to develop a versatile mathematical model to explain the fatigue behavior of composite materials. The difference in fatigue failure mechanism between composite materials and homogeneous materials, and the implication to fatigue life prediction are studied. Then, existing methods for damage evaluation in composite materials are compared and discussed. A new damage evaluation function is proposed to capture the unique characteristics of the failure of composite materials. Numerical examples are given to demonstrate the application of proposed function for composite materials subjected to cyclic loading, and to show that the versatility of the proposed model for different sets of experimental data.

2. Fatigue of composite materials

Fatigue is the main failure mechanism for structures under cyclic loading. Considerable research has been carried out for monolithic materials such as metals and progress has been made in devising fatigue-resistant materials as well as in developing methodologies for life

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prediction. For composite materials, fatigue analysis and consequent life prediction become difficult because the material properties of the constituents of the composite are quite different. The fatigue behavior of one constituent may be significantly affected by the presence of other constituents and the interfacial regions between the fibers and matrix. Fatigue properties of composites may vary significantly due to the large difference in the properties between the fibers and matrix of the composite and the composition of constituents.

Many experimental studies have been reported for obtaining the fatigue properties of different types of composite materials since 1967. Based on these experimental results, empirical *S–N* curves have been derived between stress and fatigue life. These relationships have been suggested for use in design [3]. Both linear and nonlinear *S–N* curves have been proposed based on the experimental results [4–9]. A nonlinear curve between strain and fatigue life is also used to predict the fatigue life of composite materials [10–12]. A linear relationship between the maximum stress S and the logarithm of N, the number of load cycles to fatigue failure, is widely used to fit the experimental data as

$$S = m \log N + b \tag{1}$$

where m and b are parameters related to material properties.

With the predicted fatigue life under constant cyclic loading, fatigue damage can be evaluated after a given number of cycles with a fatigue damage accumulation model, which is discussed in the next section. The composite is assumed to fail when the accumulated damage exceeds the critical level of damage.

3. Failure of composite materials

Under cyclic loading, damage will accumulate in composite materials and cause the fracture or functional failure of structures. For homogeneous, or monolithic materials with isotropic material properties, damage is accumulated at a low growth rate in the beginning, and a single crack propagates in a direction perpendicular to the cyclic loading axis. On the other hand, in composite materials, especially for those structures with multiple plies and laminates, the fracture behavior is characterized by multiple damage modes, such as crazing and cracking of the matrix, fiber/matrix decohesion, fiber fracture, ply cracking, delamination, void growth, and multidirectional cracking. These modes appear rather early in the fatigue life of composites.

The mechanisms of crack initiation and crack growth are quite complex for composite materials. Even for unidirectional reinforced composites under the simple loading case such as tension along the direction of fibers, cracks can initiate at different locations and in different directions. Cracks can initiate in the matrix, perpendicular to the direction of loading. Cracks can also initiate in the interface along the directions of fibers between the fibers and matrix due to debonding. Many experimental fatigue tests have been carried out to study the crack growth in composites when there is only one dominant crack that is propagating. The crack propagates in the same plane and direction as the initial crack. The Paris law has been used to describe this fatigue crack propagation behavior. But this is limited to unidirectional aligned fiber reinforced composites. For more general laminates, a similar mode of crack propagation cannot be obtained even under simple loading. Thus, traditional fracture mechanics cannot be used for the fatigue analysis of composite materials.

The concept of damage accumulation may be used as a more suitable approach to predict the fatigue life of structures of composite materials. However, fatigue damage cannot be measured directly. Therefore, for quantitative evaluation of fatigue damage, Young's modulus or the stiffness of composite materials are often used to evaluate the fatigue damage due to cyclic loading. For example, fatigue damage can be defined in the term of Young's modulus as follows [13]:

$$D_1 = 1 - \frac{E}{E_0} \tag{2}$$

where D_1 is the accumulated fatigue damage, E_0 is initial Young's modulus of the undamaged material, and E is the Young's modulus of the damaged material. Thus the extent of damage can be quantified by measuring the Young's modulus of the material.

The experimental results show that the measured Young's modulus or stiffness just before complete failure of the specimen is not zero. The final accumulated damage is $1 - E_f/E_0$ instead of unity when the material fails, where E_f is the Young's modulus when fracture occurs. Therefore, a new damage parameter can be defined with the final Young's modulus E_f as

$$D = \frac{E_0 - E}{E_0 - E_f} \tag{3}$$

According to Eq. (3), the accumulated damage will be in the range between 0 and 1.

As previously mentioned, the complexity of composites leads to the presence of many modes of damage. These modes appear at the early stages of the fatigue life. The damage accumulates rapidly during the first few cycles. During this stage, microcracks initiate in multiple locations in the matrix. Debonding occurs at the weak interfaces between fibers and matrix. Also, some fibers with low strength may break during this stage. The next stage shows a slow and steady damage growth rate. Finally, the damage again grows rapidly during the last stage before the fracture occurs. Fig. 1 shows schematically the comparison of damage accu-

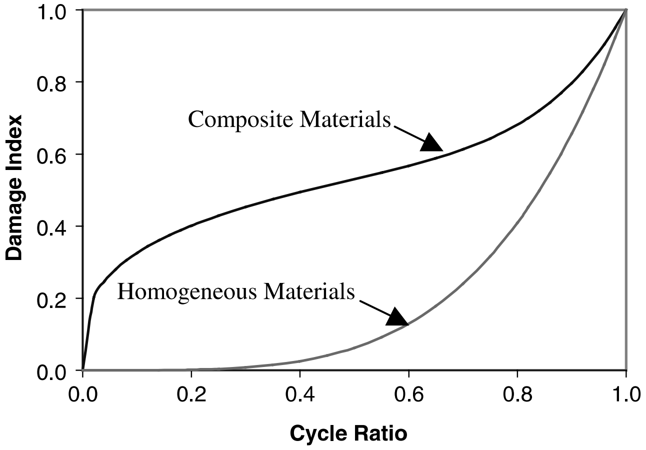

Fig. 1. Sketched fatigue damage accumulation.

mulation in composite materials and homogeneous materials as a function of fatigue cycle ratio. The Young's modulus measured from fatigue tests also shows the same characteristics of damage accumulation [9,14,15]. Fig. 1 is plotted in terms of damage index versus cycle ratio, where the damage index is defined in Eq. (3). The cycle ratio is the number of cycles at a given instant divided by the fatigue life.

A linear damage summation model was first used to evaluate the fatigue behavior of composite materials by Nicholas and Russ [16]. Halverson et al. [9] used a power function in terms of the cycle ratio to evaluate the remaining strength of the material and to calculate the fatigue life.

$$F_r = 1 - (1 - F_a)\left(\frac{n}{N}\right)^j \tag{4}$$

where F_r is the normalized remaining strength (normalized by the undamaged static strength), F_a is the normalized applied load (also normalized by the undamaged static strength), j is a material constant, n is the number of cycles of applied load. N is the fatigue life of a constant load. Then, according to the definition of damage in Eq. (2), the mathematical function for damage accumulation will also be a power function of cycle ratio.

$$D = (1 - F_a)\left(\frac{n}{N}\right)^j \tag{5}$$

Once the residual strength is computed, the degradation of the material can be described with Eq. (2) or (3). Other nonlinear damage accumulation functions have also been used [7,17]. These nonlinear damage accumulation functions are able to capture the characteristics of rapid damage growth either at the early stages of life or near the end of life, but not both. For example, the damage model of Subramanian et al. [11] explains the fast damage growth during early loading cycles but does not accurately describe the rapid damage growth close to the material fracture. Halverson et al. [9] model the characteristics of rapid damage growth at the end of fatigue life of the material. But the model is not accurate during the early loading cycles.

4. Proposed fatigue damage accumulation model

In this paper, a versatile new damage accumulation model is presented to describe the degradation of composite materials. This model accurately explains the rapid damage growth during both the early and final stages of life. The proposed function is of the form

$$D = q\left(\frac{n}{N}\right)^{m_1} + (1 - q)\left(\frac{n}{N}\right)^{m_2} \tag{6}$$

where D is the normalized accumulated damage; q, m_1 and m_2 are material dependent parameters; n is the number of applied loading cycles; and N is the fatigue life at the corresponding applied load level. The characteristics of rapid damage accumulation during the first few cycles can be captured with the first term, with $m_1 < 1.0$. The second term shows the fast damage growth at the end of fatigue life with $m_2 > 1.0$.

The parameters in Eq. (6) are defined in terms of fatigue life of interest as

$$q = \frac{A\left(\frac{N_0}{N}\right)^{\alpha}}{1 - (1 - A)\left(\frac{N_0}{N}\right)^{\alpha}} \tag{7}$$

$$m_1 = \left(\frac{N_0}{N}\right)^{\beta} \tag{8}$$

$$m_2 = \left(\frac{N}{N_0}\right)^{\gamma} \tag{9}$$

where N_0 is the reference fatigue life. The parameters α, β and γ are material dependent constants. These parameters can be obtained with fatigue experimental data. Once the damage indices are obtained during the fatigue tests, regression analysis can be carried out to obtain the parameters q, m_1 and m_2. Then, parameters α, β and γ can be calculated with Eqs. (7)–(9).

5. Experimental results and curve fitting

Two sets of experimental results are used here to demonstrate the application of the proposed damage accumulation function. The first set is with 810 O laminates and the second one is with a woven composite.

Subramanian et al. [11] obtained two sets of data for 810 O laminates, under two different loading levels (75% and 80% of ultimate strength of the laminate). These are symetric $(0,90_3)$ cross-ply laminates. Apollo fibers and HC 9106-3 toughened epoxy matrix are used. The fibers received 100% surface treatment and were sized with a thermoplastic material. The fatigue tests were performed at $R = 0.1$ and 10 Hz frequency [11]. The Young's moduli of the laminated composite were measured after

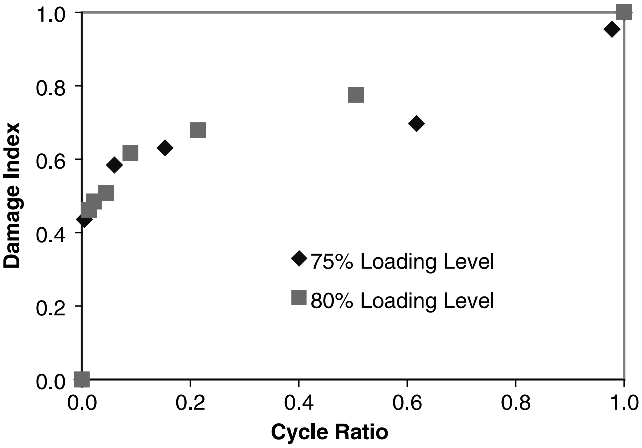

Fig. 2. Measured fatigue damage versus cycle ratio for 810 O laminates.

different number of cycles of tensile fatigue loading, and the corresponding damage indices were computed according to Eq. (3). The damage indices are plotted in Fig. 2. First, one set of data is used to obtain the values of parameters of the proposed model. Then, the proposed model is used with the computed parameters to model another set of experimental data.

The damage indices under the 75% loading level are used to obtain the parameters q, m_1 and m_2, in the damage accumulation function of Eq. (6). Then, the parameters, α, β and γ, are obtained with Eqs. (7)–(9). The experimental results and the proposed model are plotted in Fig. 3. It can be seen that the proposed function models the experimental results very well with the coefficient of determination $R^2 = 0.9949$. The parameters for the proposed model are shown in Table 1.

Parameters q, m_1 and m_2 for the damage model at the 80% loading level are obtained with the values of α, β and γ obtained at the 75% loading level. Fig. 4 shows the comparison between the damage measured at 80% loading level and the predicted damage with proposed model with the parameters obtained at 75% loading level. It can be seen that the predicted results agree well with the experimental results in this example for the tested composite material. The coefficient of determination (R^2) for this agreement is 0.9644. The values of the parameters for the proposed damage function are also shown in Table 1.

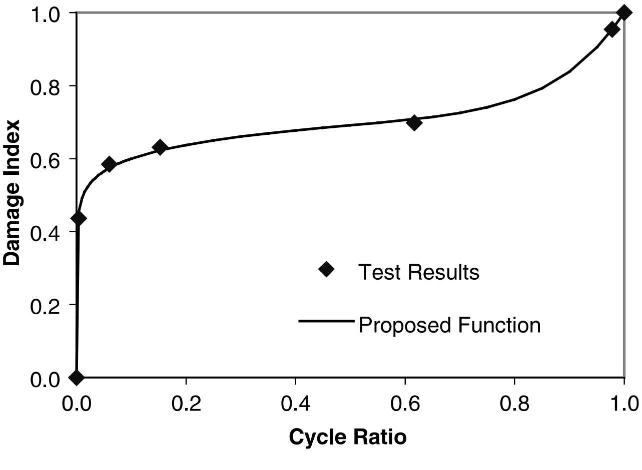

Fig. 3. Experimental observation and model prediction of damage index for 810 O laminates (75% loading level).

Table 1
Parameters of the proposed function for 810 O laminates

Loading level	q	m_1	m_2	α	β	γ
75%	0.7380	0.0909	9.1938	0.4147	0.4343	0.4018
80%	0.7789	0.1119	7.5871			

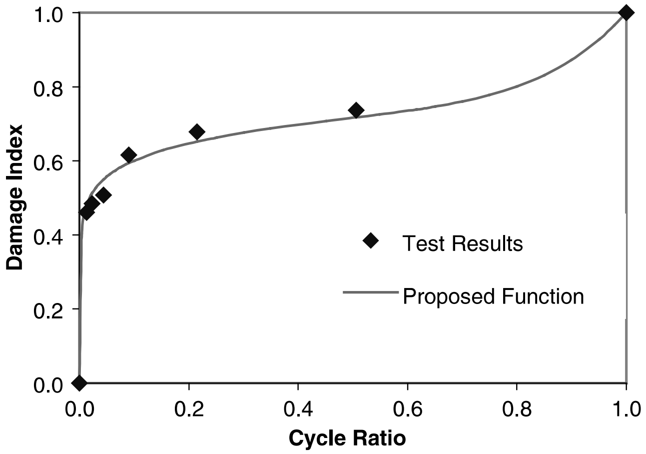

Fig. 4. Experimental observation and model prediction of damage Index for 810 O laminates (80% loading level).

Kumar and Talreja [15] conducted tension–tension fatigue experiments on the AS4/PR500 5 harness satin weave composite laminates t a frequency of 10 Hertz and R ratio of 0.1 to study the fatigue behavior of the composite material. Tension fatigue tests were conducted on the symmetric ($0/90_{2w}$) laminates. Two types of specimen, unaged and 6000 h aged, were tested. Young's modulus of the material was measured after different numbers of fatigue cycles. The damage indices were obtained with Eqs. (2) and (3) from the measured Young's moduli, and are plotted in Fig. 5.

The proposed function is now applied to the experimental results of Fig. 5. The parameters of proposed function for the damage indices in Fig. 5 are obtained with least squares method. The parameters are shown in Table 2 for the results of both aged and unaged specimen. The proposed function and experimental results are compared in Fig. 6. It can be seen from Fig. 6 that the proposed function does an excellent job of capturing the characteristics of the damage evolution in composite materials. The values of the coefficients of determination

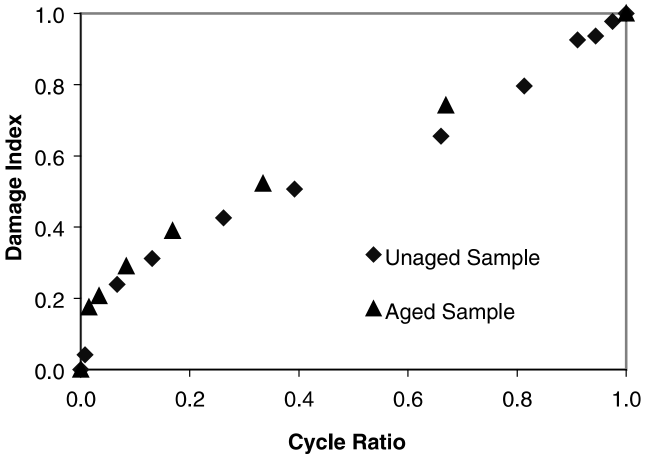

Fig. 5. Calculated damage from measured Young's moduli of AS4/PR500 woven composite.

Table 2
Parameters of the proposed function for the AS4/PR500 woven composite

Material	q	m_1	m_2
Unaged specimen	0.7755	0.4633	4.1076
Aged specimen	0.7143	0.3535	2.0350

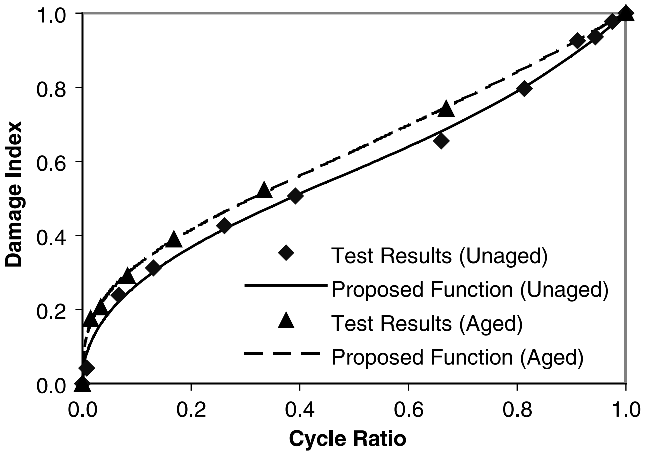

Fig. 6. Experimental observation and model prediction for the AS4/PR500 woven composite.

R^2 are 0.9696 and 0.9973 for unaged and aged samples, respectively.

6. Conclusion and discussion

The characteristics of damage accumulation in composite materials are studied in this paper. The experimental results of damage growth show that there are three different stages during the damage evolution in composites under fatigue loading. During the first stage, fatigue damage grows rapidly due to the occurrence of multiple damage modes within the material. The damage increases steadily and slowly during the second stage. During the third and final stage, the damage again grows rapidly due to the fracture of fibers.

A nonlinear model is proposed to capture the unique characteristics of damage evolution in composite materials subject to fatigue loading. Parameters of the proposed model are obtained with experimental data. With the obtained parameters, the fatigue damage can be evaluated at any time. The numerical examples show that the proposed damage function can model the experimental results very well. Fatigue damage indices during the service time can be obtained with proposed model more accurately.

From the 810 O laminate, it can be seen that the parameters obtained at one loading level can be used to model the damage accumulation under another different loading level. The numerical example shows very good agreement between the proposed function and the measured damage from experimental tests. The proposed model is seen to work well for both cross-ply and woven composite laminates.

It is critical to model the damage evolution accurately for fatigue life prediction. The proposed damage accumulation model appears to accomplish this with excellent accuracy for the materials considered. This helps to increase the confidence in model-based prediction of fatigue life.

References

[1] Middleton DH. Composite materials in aircraft structures. Longman Scientific & Technical, New York, J. Wiley, 1990.

[2] Jones RM. Mechanics of composite materials. Ann Arbor, MI: Edwards Brothers; 1999.

[3] Harris B, Gathercole N, Reiter H, Adam T. Constant-stress fatigue response and life-prediction for carbon-fiber composites. In: Cardon AH, Fukuda H, Reifsnider K, editors. Progress in durability analysis of composite systems. Rotterdam: A.A. Balkema Publisher; 1995. p. 63–74.

[4] Yang J. Fatigue and residual strength degradation for graphite/epoxy composite under tension-compression cyclic loading. J Compos Mater 1978;12(1):19–39.

[5] Nicholas S. An approach to fatigue life modeling in titanium-matrix composites. Mater Sci Eng A 1995;200:29–37.

[6] Himmel N, Kienert M, Maier M. Fatigue characterization of general purpose thermoplastic composites subject to elementary mechanical loading. In: Cardon AH, Fukuda H, Reifsnider K, editors. Progress in durability analysis of composite systems. Rotterdam: A.A. Balkema Publisher; 1995. p. 81–6.

[7] Joseph E, Perreux D. A model for predicting the fatigue damage of filament wound pipes. In: Cardon AH, Fukuda H, Reifsnider K, editors. Progress in durability analysis of composite systems. Rotterdam: A.A. Balkema Publisher; 1995. p. 91–9.

[8] Tamuzs V, Gruseckis I, Andersons J. The effect of overload on residual durability of laminated GRP. In: Cardon AH, Fukuda H, Reifsnider K, editors. Progress in durability analysis of composite systems. Rotterdam: A.A. Balkema Publisher; 1995. p. 147–52.

[9] Halverson HG, Curtin WA, Reifsnider KL. Fatigue life of individual composite specimens based on intrinsic fatigue behavior. Int J Fatigue 1997;19(5):269–77.

[10] Halford GR, Lerch BA, Saltsman JF, Arya VK. Proposed framework for thermomechanical fatigue life prediction of metal-matrix composites (MMCs). In: Sehitoglu H, editor. Thermomechanical fatigue behavior of materials, ASTM STP 1186. Philadelphia: American Society for Testing and Materials; 1993. p. 176–94.

[11] Subramanian S, Reifsnider KL, Stinchcomb WW. A Cumulative damage model to predict the fatigue life of composite laminates including the effect of a fiber-matrix interphase. Int J Fatigue 1995;17(5):343–51.

[12] Reifsnider K, Case S, Duthoit J. The mechanics of composite strength evolution. Compos Sci Technol 2000;60:2539–46.

[13] Lemaitre L, Chaboche JL. Mechanics of solid materials. London: Cambridge University Press; 1990.

[14] Chawla KK. Composite materials—science and engineering. 2nd ed. New York: Springer-Verlag NewYork Inc.; 1998.

[15] Kumar R, Talreja R. Fatigue damage evolution in woven fabric composites. 41st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conferences and Exhibit, Atlanta, 2000.

[16] Nicholas S, Russ R. Elevated temperature fatigue behavior of SCS-6/Ti–24Al–11Nb. Mater Sci Eng A 1992;153:514–9.

[17] Fukuda H, Katoh H, Yasuda J. Low velocity impact damage of carbon fiber reinforced thermoplastics. In: Cardon AH, Fukuda H, Reifsnider K, editors. Progress in durability analysis of composite systems. Rotterdam: A.A. Balkema Publisher; 1995. p. 139–45.