A robust optimization model for the design of a cardboard closed-loop supply chain

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Abstract

Environmental issues, legal requirements, and economic benefits of recycling activities lead to the development of reverse logistics operations and waste management. The current research considers a closed-loop supply chain for a cardboard recycling network including multiple suppliers and production stages. A mixed integer linear programming model is proposed to optimize the paper and cardboard recycling network, and a robust optimization approach is utilized to deal with demand uncertainty in this network. The model maximizes the total profit. It considers the operation, transportation, purchasing and holding inventory costs as well. Moreover, to illustrate the application of the proposed model, a real-world cardboard closed-loop supply chain design is investigated. The proposed model considers the option to open a new recycling center in the current network. The proposed model determines the optimal quantities of the waste paper, paper, sheet and cardboard that flow in this network. Finally, the computational analysis indicated that the proposed model provides efficient solutions for the studied cardboard closed-loop network.

Keywords: closed-loop supply chain, reverse logistics, waste management, robust optimization, paper and cardboard recycling.

1- Introduction

A substantial number of end-of-life products is released into the environment in the current century. Ezeah et al. (2013) stated that only 30-70% of waste generated in the cities of developing countries is collected for disposal. The uncollected waste is disposed into water or into open dumps along the streets, which comprises environmental and public health risks. Therefore, they suggested that the informal sector recycling activity could help waste management including resource conservation, litter control, and economic growth in the developing countries. The end-of-life products are prone to become dangerous wastes and may cause damage to the environment, if they are not managed properly. Over the last decades, the
economic benefits resulted in more attention to the issues such as waste reduction and recycling of end-of-life products. Thus, the manufacturers’ responsibility for the end-of-life management motivated them to consider the environmental effect in their traditional supply chain, leads to green supply chain management (Vahdani and Mohammadi, 2015).

The forward flow in a supply chain includes value-adding processes to produce the final product and deliver it to customers. Moreover, the backward or reverse flow includes activities concerning the collection of end-of-life products from customers to reduce environmental pollution and gain economic profit as well. The Closed Loop Supply Chain (CLSC) network includes the simultaneous forward and reverses logistic flows. Reverse logistics is one of the key concepts of a supply chain that makes it possible to reduce and reuse wastes (Kara and Onut, 2010). The American reverse logistics executive council defines the reverse logistics as ‘The process of planning, implementing, and controlling the efficient, and cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption or proper disposal’ (Barros et al., 1998). Actors in reverse flows may be members of forward flows such as logistic service providers, manufacturers, and retailers in a CLSC network. Further, optimizing the reverse supply chain network can result in cost benefits (Fleischmann et al., 2001). The location of different facilities such as collection centers, disposal centers, recycling centers, distribution centers, and manufacturing facilities affects the performance of the CLSC network. These factors highlight that proper management is necessary for the closed-loop supply chain design (Tan and Kara, 2007).

Many researchers have discussed the reverse and the CLSC system’s planning for end-of-life products in various industries such as carpet recycling (Biehl et al., 2007), vehicle recycling (Schultmann et al., 2006) and recycling of waste electrical and electronic equipment (Kara et al., 2007). Further, Shih (2001) proposed an optimization model for reverse logistic of electrical appliances. Zhang et al. (2014) proposed a multi-echelon multi-period solid waste disposal supply chain model. Their model objective function minimizes the total costs, including inventory, transportation, collection and operation costs, under capacity constraint. Moreover, Galves et al. (2015) studied a reverse logistics network design problem for a biogas plant in Nancy, France. They proposed an MILP model to minimize the total costs. He et al (2016) evaluated the impact of consumer free riding on carbon emissions in a product's life cycle across a dual channel closed loop supply chain. In a later study, Moshtagh and Taleizadeh (2017) proposed an integrated manufacturing and remanufacturing model considering factors
such as shortage, rework, and quality that are based on the return rate in a closed-loop supply chain.

Govindan et al. (2015) categorized various studies in the field of supply chain into four main classifications: reverse logistics, closed-loop supply chain, green and sustainability. Table 1 shows some of the recent studies in each classification. It can be inferred from Table 1 that most of the researchers employed a mathematical programming method to find an optimal plan.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Supply chain</th>
<th>Objective function</th>
<th>Product</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paksoy et al (2010)</td>
<td>reverse*</td>
<td>min costs*</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Kannan et al (2012)</td>
<td>closed-loop*</td>
<td>max profit*</td>
<td>single</td>
<td></td>
</tr>
<tr>
<td>Pishvaee and Razmi (2012)</td>
<td>green*</td>
<td>min social*</td>
<td>multi</td>
<td></td>
</tr>
<tr>
<td>Amin and Zhang (2013)</td>
<td>sustainability*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Frota Neto et al (2008)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Dehghanian and Mansour (2009)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
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<tr>
<td>Demirel and Gökçen (2008)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Üster et al (2007)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
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<tr>
<td>Du and Evans (2008)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Pishvaee et al (2010)</td>
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<td>max responsibility*</td>
<td>no</td>
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<tr>
<td>El-Sayed et al (2010)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
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<tr>
<td>Kadambala et al (2017)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
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<tr>
<td>Ma et al (2016)</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
<tr>
<td>Current research</td>
<td>reverse*</td>
<td>max responsibility*</td>
<td>no</td>
<td></td>
</tr>
</tbody>
</table>

* Asterisk in this table means that an article in a row has the feature mentioned in that column.

There are a lot of uncertain parameters in a real-world CLSC network (Vahdani et al., 2012). The uncertainty might arise from different players in a chain and several scenarios for parameters such as product demand, and returned product are recommended. Hence, a closed-loop supply chain management under uncertainty is one of the most important issues in the field of supply chain management.

Moreover, different approaches proposed to deal with the uncertainty can be categorized into four primary approaches: fuzzy programming; stochastic programming; stochastic dynamic programming, and robust optimization. The last one is a reliable technique that is used...
to deal with uncertainty in manufacturing systems. Hence, this approach addresses the uncertainty in the setting up different scenarios with the goal of finding a robust solution ensuring that all scenarios are 'close' to the optimum (Sahinidis, 2004). Consequently, many researchers used the stochastic and robust programming approaches to deal with uncertainty (Govindan et al., 2015). José Alem and Morabito (2012) mentioned two advantages for the robust optimization approach. They stated that the robust optimization is easier to solve than the stochastic programming approach and it does not need explicit knowledge about the probability distribution of uncertain parameters.

Table 2 summarizes some of the current supply chain models that studied the parameter uncertainty in their models. Safaei et al. (2010) considered a production and distribution network. Because of some stochastic factors such as operation time and unexpected delays, they proposed a multi-site, multi-period, multi-product hybrid mathematical-simulation approach that decreases the costs. Subulan et al. (2015) studied a CLSC under uncertainty and risk for a lead/acid battery supply chain in Turkey. They proposed a hybrid model based on the stochastic and possibilistic MILP formulations for their studied network. According to the studies in the field of closed-loop supply chains, the uncertainties involved in the reverse flow are higher than those in the forward flow of the supply chain (Fleischmann et al., 2001).

Table 2. Summary of the supply chain models that studied the parameter uncertainty

<table>
<thead>
<tr>
<th>Authors</th>
<th>Approach</th>
<th>Product</th>
<th>Period</th>
<th>Objective</th>
<th>Network</th>
<th>uncertain parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akbari and Karimi (2015)</td>
<td>Robust</td>
<td>single</td>
<td>multi</td>
<td>single</td>
<td>forward</td>
<td>capacity</td>
</tr>
<tr>
<td>Wei and Cai (2011)</td>
<td>Stochastic</td>
<td>single</td>
<td>multi</td>
<td>single</td>
<td>reverse</td>
<td>returned quantity</td>
</tr>
<tr>
<td>Ramezani et al (2013)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>demand quantity</td>
</tr>
<tr>
<td>Salema et al (2007)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>operation cost</td>
</tr>
<tr>
<td>Aghezzaf et al (2010)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>transportation</td>
</tr>
<tr>
<td>Pishvaee et al (2011)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Rahmani et al (2013)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Mirzapour et al (2011)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>Zeballos et al (2014)</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<tr>
<td>Ayvaz et al (2015)</td>
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<tr>
<td>Current research</td>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

* Asterisk in this table means that an article in a row has the feature mentioned in that column.

Besides a few researchers have studied the paper and cardboard recycling problem. Govindan and Soleimani (2017) presented a review of the reverse logistics and closed-loop
supply chain research in the *Journal of Cleaner Production*. They stated that the focus of the researchers was on the auto parts suppliers, vehicle manufacturer/remanufacturer, and electronic and computer products. Ervasti et al. (2016) conducted a global review of the terms and definitions to the paper recycling. These terms and definitions are different in the course of time and geographical region for the same product. Moreover, definitions of waste paper as a raw material, and basic definitions such as recycling rate and paper consumption are different as well. Therefore, they stated that there is a pressing need to create a uniform system for the paper recycling terms and definitions.

The waste paper needs immediate attention to reduce environmental pollution and different designs are proposed for the reverse and the closed-loop paper supply chain. Pati et al. (2008) suggested a model for a paper recycling network in India. Their considered objectives are optimizing the reverse logistics costs, the product quality improvement by increasing segregation at the source and the environmental benefits by improving the waste paper recovery. Kara and Onut (2010) formulated an MILP model for a paper recycling network in Turkey. They constructed a two-stage stochastic mixed-integer model with considering of the robust approach. Schweiger and Sahamie (2013) addressed the design of a paper recycling network including the external procurement, in-house recycling of paper and technology selection.

As mentioned above, increasing urbanization and industrial development led to an increasing gap between the demand and supply, and increased the environmental pollution. Hence, recycling is one of the key concepts embedded in the supply chain and formed the CLSC to reduce and manage wastes. The end-of-life paper products can be recycled and reused in the same or another cycle. Therefore, the life cycle for paper products restarts in the supply chain forming a closed-loop supply chain for the paper and cardboard products. Generally, material recycling plays an important role in different industries and different sectors of the society (Chen et al., 2016). The paper and cardboard recycling are one of the best secondary materials around the world. It is forming an environmentally sound raw material and a significant global trade commodity.

This paper presents a new framework to help the cardboard supply chain managers to design an optimal cardboard closed-loop supply chain. Additionally, more description of this approach is given in the next section. In this study, a mixed integer linear programming model is proposed to procure raw materials optimally, produce paper, sheet, and cardboard, and distribute the final product to customers. Furthermore, this research considers the demand uncertainty in some periods and utilizes a robust optimization approach to deal with it. The
The proposed model is not a generic one; it considers the internal and external flows for the paper-recycling network of a particular case under study. However, it can be employed for similar recycling paper networks.

The rest of this paper is organized as follows. In Section 2, the studied problem is described and a mixed integer linear programming model for the cardboard closed-loop network is outlined in Section 3. Section 4 presents a robust optimization model for the studied network. In Section 5, a case study is discussed, and the results are analyzed. Finally, Section 6 concludes the paper.

2-Model Description

A cardboard closed-loop supply chain design is considered in this research. In the studied network, four chains are connected and related to each other. This network is composed of a supply network, the internal and external cardboard production networks and a distribution network. These chains and their relationships are shown in Figure 1. The internal cardboard production network includes the facilities that work together to recycle waste papers and produce cardboard. The external cardboard production network is a parallel network with the internal network that procures raw materials if there is any demand. Generally, the external network competes with the internal network in the society. Additionally, the waste paper supply network collects waste paper, which includes the internal and external recycling centers. However, the priority of processing the waste paper in the next stages of the network is with the internal recycling center. The external recycling centers are employed when the internal recycling center cannot satisfy the demand.

Besides, the distribution network includes retailers and final consumers. Retailers may procure cardboard from different networks to pack their products. This cycle continues until a consumer discards the paper and/or cardboard. Finally, the paper and cardboard closed-loop restarts when a recycling center collects waste paper. In this research, different geographical regions are nominated as collection points in which the irrelevant and inappropriate waste papers separate and move to the disposal center. The appropriate waste paper enters to the paper production site as a recyclable raw material. However, the raw material for paper production sites can be purchased from an external recycling site as well.
Figure 2 illustrates the collection/recycling center operations and processes. The cardboard recycling process begins with the pulping. The second step consists of cleaning, screening, and removing contaminants from the pulp. In the third step, washing and flotation operations remove the ink. It performs depending on the requirements of the final product. Finally, the bleaching process removes color from the pulp and increases its brightness, and then it is ready for paper production.
In the next stage, the papers enter to the sheet production sites. This facility can supply paper from external paper production sites as well. Moreover, the produced sheet can be used in the internal cardboard production sites or can be sold to the external cardboard production sites. In the final stage, the produced cardboard moves to the retailers. The waste sheet and cardboard in the sheet and cardboard production process go back to the cycle as a raw material for the paper production sites. Figure 3 shows the paper and cardboard production lines.

The retailer’s demand, i.e. cardboard requested to pack the products, is uncertain. Moreover, the production system products move to the retailers or will be stored in the cardboard production sites to be sold in subsequent periods.

3- Model formulation

The proposed model is formulated as a mixed integer linear programming model. The mathematical model determines the optimal flows between different chains. Moreover, the proposed model determines the optimal quantity of supply and production in each period. The
operation costs, such as purchasing, disposal, recycling, transportation, inventory, and production are considered in the mathematical model. The model objective function maximizes the supply chain’s profit.

Indices

\begin{align*}
i &\quad \text{set of retailers} & i=1...I \\
n &\quad \text{set of final consumers} & n=1...N \\
j &\quad \text{set of internal and external collection-recycling centers} & j=1...J \\
f &\quad \text{set of candidate collection-recycling centers} & f=1...F \\
p &\quad \text{set of internal and external paper production sites} & p=1...P \\
r &\quad \text{set of sheet production sites} & r=1...R \\
c &\quad \text{set of internal and external cardboard production sites} & c=1...C \\
d &\quad \text{set of disposal centers} & d=1...D \\
t &\quad \text{set of periods} & t=1...T \\
\end{align*}

Model Parameters

- Supply parameters

\begin{align*}
C_{njt} &\quad \text{purchasing cost of waste paper from the final consumer } n \text{ in the internal recycling center } j \text{ in period } t \\
C_{nft} &\quad \text{purchasing cost of waste paper from the final consumer } n \text{ in the candidate recycling center } f \text{ in period } t \\
C_{cjt} &\quad \text{purchasing cost of waste paper from the external cardboard production site } c \text{ in the internal recycling center } j \text{ in period } t \\
C_{cft} &\quad \text{purchasing cost of waste paper from the external cardboard production site } c \text{ in the candidate recycling center } f \text{ in period } t \\
C_{pjr} &\quad \text{purchasing cost of raw materials from the external recycling center } j \text{ in the paper production site } p \text{ in period } t \\
C_{pre} &\quad \text{purchasing cost of paper from the external paper production site } p \text{ in the sheet production site } r \text{ in period } t \\
E_1 &\quad \text{minimum purchase of waste paper from the external cardboard production site} \\
M_1 &\quad \text{maximum purchase of waste paper from the external cardboard production site} \\
E_2 &\quad \text{minimum purchase of waste paper from the final consumer} \\
M_2 &\quad \text{maximum purchase of waste paper from the final consumer} \\
E_3 &\quad \text{minimum purchase of recycled raw materials from the external recycling center} \\
M_3 &\quad \text{maximum purchase of recycled raw materials from the external recycling center} \\
E_4 &\quad \text{minimum purchase of paper from the external paper production site} \\
M_4 &\quad \text{maximum purchase of paper from the external paper production site} \\
\end{align*}

- Production parameters

\begin{align*}
C_{jrt} &\quad \text{the cost of collecting waste paper in the internal recycling center } j \text{ in period } t \\
C_{frt} &\quad \text{the cost of collecting waste paper in the candidate recycling center } f \text{ in period } t \\
\end{align*}
$D_{jd}$ distance between the internal recycling center $j$ and the disposal center $d$

$D_{fd}$ distance between the candidate recycling center $f$ and the disposal center $d$

$D_{jp}$ distance between the internal recycling center $j$ and the paper production site $p$

$D_{fp}$ distance between the candidate recycling center $f$ and the paper production site $p$

$D_{rc}$ distance between the sheet production site $r$ and the cardboard production site $c$

$D_{cp}$ distance between the cardboard production site $c$ and the paper production site $p$

$TC_{jd}$ transportation cost of inappropriate waste paper from the internal recycling center $j$ to disposal center $d$ in period $t$

$TC_{fd}$ transportation cost of inappropriate waste paper from the candidate recycling center $f$ to disposal center $d$ in period $t$

$TC_{jp}$ transportation cost of recycled raw materials from the internal recycling center $j$ to the internal paper production site $p$ in period $t$

$TC_{fp}$ transportation cost of recycled raw materials from the candidate recycling center $f$ to the internal paper production site $p$ in period $t$

$TC_{rt}$ transportation cost of sheet from the sheet production site $r$ to the cardboard production site $c$ in period $t$

$TC_{cp}$ transportation cost of waste cardboard from the cardboard production site $c$ to the internal paper production site $p$ in period $t$

$C_{dt}$ disposing cost in the disposal center $d$ in period $t$

$C_{pt}$ processing cost in the internal paper production site $p$ in period $t$

$C_{rt}$ processing cost in the sheet production site $r$ in period $t$

$C_{ct}$ processing cost in the internal cardboard production site $c$ in period $t$

$Max_j$ maximum capacity of the internal collection-recycling center $j$

$Max_f$ maximum capacity of the candidate collection-recycling center $f$

$Min_f$ minimum capacity of the candidate collection-recycling center $f$

$Max_p$ maximum capacity of the internal paper production site $p$

$Max_r$ maximum capacity of the sheet production site $r$

$Max_c$ maximum capacity of the internal cardboard production site $c$

$Max_d$ maximum capacity of the disposal center $d$

$Max_n$ maximum number of allowed new recycling centers

$h_{pj}$ inventory holding cost in the internal recycling center $j$ in period $t$

$h_{pf}$ inventory holding cost in the candidate recycling center $f$ in period $t$

$h_{ct}$ inventory holding cost in the cardboard production site $c$ in period $t$

$q_{pj}$ maximum storing capacity in the internal recycling center $j$

$q_{pf}$ maximum storing capacity in the candidate recycling center $f$

$q_{pc}$ maximum storing capacity in the internal cardboard production site $c$

- **Rates**
  - $\beta$ rate of irrelevant waste in the collection-recycling center
  - $\lambda_1$ disposal rate of collected waste paper from consumer (CWPC)
  - $\lambda_2$ disposal rate of collected waste paper from the external cardboard production sites (CWPP)
\( \alpha_1 \) cardboard production rate  
\( \alpha_2 \) sheet production rate

- **Distribution parameters**  
  \( d_i \) cardboard demand in the retailer \( i \) in period \( t \)  
  \( F_{ct} \) unit price for the cardboard  
  \( F_{rt} \) unit price for the sheet

**Decision variables**

- **Purchase decision variables**
  
  \( W_{njt} \) purchased waste paper from the final consumer \( n \) by the internal recycling center \( j \) in period \( t \)  
  \( W_{nft} \) purchased waste paper from the final consumer \( n \) by the candidate recycling center \( f \) in period \( t \)  
  \( W_{cjt} \) purchased waste paper from the external cardboard production site \( c \) by the internal recycling center \( j \) in period \( t \)  
  \( W_{cft} \) purchased waste paper from the external cardboard production site \( c \) by the candidate recycling center \( f \) in period \( t \)

- **Flow decision variables**
  
  \( Y_{jdt} \) transferred inappropriate waste paper from the internal recycling center \( j \) to the disposal center \( d \) in period \( t \)  
  \( Y_{fdt} \) transferred inappropriate waste paper from the candidate recycling center \( f \) to the disposal center \( d \) in period \( t \)  
  \( X_{jpt} \) transferred recycled raw materials from the internal and external recycling center \( j \) to the internal paper production site \( p \) in period \( t \)  
  \( X_{ftp} \) transferred recycled raw materials from the candidate recycling center \( f \) to the internal paper production site \( p \) in period \( t \)  
  \( V_{rpt} \) transferred waste sheet from the sheet production site \( r \) to the internal paper production site \( p \) in period \( t \)  
  \( V_{cpt} \) transferred waste cardboard from the internal cardboard production site \( c \) to the internal paper production site \( p \) in period \( t \)  
  \( X_{prt} \) transferred paper from the internal and external paper production site \( p \) to the sheet production site \( r \) in period \( t \)  
  \( X_{rcr} \) transferred sheet from the sheet production site \( r \) to the internal and external cardboard production site \( c \) in period \( t \)  
  \( X_{ci} \) production quantity in the cardboard production site \( c \) in period \( t \)  
  \( X_{ri} \) supply quantity of the final product to the retailer \( i \) in period \( t \)

- **Inventory decision variables**
  
  \( I_{ct} \) inventory in the cardboard production site \( c \) in period \( t \)  
  \( I_{jt} \) inventory in the internal recycling center \( j \) in period \( t \)  
  \( I_{ft} \) inventory in the candidate recycling center \( f \) in period \( t \)

- **Binary decision variables**
\[ \delta_f \text{ if the candidate recycling center } f \text{ is opened, 0 otherwise} \]
\[ \pi_{jt} \text{ if the external recycling center } j \text{ is selected for purchase of raw materials in period } t, 0 \text{ otherwise} \]
\[ \pi_{ct} \text{ if the external cardboard production site } c \text{ is selected for purchase of waste paper in period } t, 0 \text{ otherwise} \]
\[ \pi_{pt} \text{ if the external paper production site } p \text{ is selected for purchase of paper in period } t, 0 \text{ otherwise} \]
\[ \pi_{nt} \text{ if the final consumer } n \text{ is selected for purchase of waste paper in period } t, 0 \text{ otherwise} \]

3-1 Objective function

The model maximizes the profit for the studied network. The first and second terms in equation (2) are revenue from the sheet and cardboard production, respectively. Other terms in equation (2) are logistics costs including purchasing, production, disposal, recycling, transportation, and holding inventory costs.

\[ \text{Max } Z. \]  

\[ Z = \sum_{i=1}^{R} \sum_{j=1}^{C} \sum_{t=1}^{T} F_{it} X_{rect} + \sum_{c=1}^{m} \sum_{e=1}^{E} \sum_{t=1}^{T} F_{ct} X_{ret} - \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{t=1}^{T} (C_{nt} W_{nt} + \sum_{j=1}^{J} C_{nj} W_{nj}) + \sum_{c=1}^{m} \sum_{e=1}^{E} \sum_{t=1}^{T} \sum_{c=1}^{m} C_{ej} W_{ej} + \sum_{j=1}^{J} \sum_{t=1}^{T} C_{jt} W_{jt} + \sum_{k=1}^{K} \sum_{t=1}^{T} (\sum_{c=1}^{m} C_{ct} + \sum_{c=1}^{m} C_{ct}) + \sum_{p=1}^{P} \sum_{t=1}^{T} D_{pt} X_{pt} + \sum_{r=1}^{R} \sum_{c=1}^{m} \sum_{t=1}^{T} C_{cr} + \sum_{t=1}^{T} \sum_{r=1}^{R} D_{rt} X_{rt} + \sum_{c=1}^{m} \sum_{t=1}^{T} h_{ct} I_{ct} + \sum_{j=1}^{J} \sum_{t=1}^{T} (\sum_{j=1}^{J} h_{jt} I_{jt} + \sum_{j=1}^{J} h_{jt} I_{jt}) \]  

3-2 Constraints

- Flow conservation

\[ (1 - \alpha) \sum_{t=1}^{T} X_{ret} = \sum_{p=1}^{P} V_{cpt} \]  
\[ \forall t, c \]  

\[ \alpha t \sum_{t=1}^{T} X_{ret} = X_{ct} \]  
\[ \forall t, c \]
Constraints (3), (5), (7) and (10) impose the flow conservation at the cardboard production site \( c \), sheet production site \( r \), paper production site \( p \) and disposal center \( d \), respectively. Constraint (4) gives the cardboard production rate at the cardboard production site \( c \), and constraint (6) is the sheet production rate at the sheet production site \( r \). Constraints (8) and (9) are the flow conservation that states raw materials inventory at the internal recycling center \( j \) and the candidate recycling center \( f \) in each period, respectively.

The recycling and production rates are defined differently in each geographical region and each course of time. For example, paper recovery in research from Japan is defined as follows (Ervasti et al., 2016).

\[
\text{(paper recovery)} = \text{(recovered paper supply)} + \text{(deinked market pulp)} - \text{(imports of recovered paper)} + \text{(exports of recovered paper)}
\]

Further, paper recovery in research from the United States is calculated as follows (Ervasti et al., 2016).

\[
\text{(paper recovery)} = \text{(consumption of recovered paper at paper and board mills)} + \text{(other uses of recovered paper)} + \text{(recovered paper exports)} - \text{(recovered paper imports)}
\]

However, in the studied closed-loop network, the waste paper includes collected waste paper from final consumers (CWPC) and collected waste paper from external cardboard production sites (CWPP). Referring to Figure 4, recycling and production rates are given as:

\[
\text{(Recycling rate)} = \frac{\text{(appropriate waste paper)}}{\text{(collected waste paper)}}
\]

\[
\text{(Paper production rate)} = \frac{\text{(output paper quantity)}}{\text{(input raw material quantity)}}
\]

Moreover, as Figure 4 shows, not all collected waste paper in the recycling center are relevant. The irrelevant waste rate in a recycling center \( \beta \) should be separated from the collected
wastes. Additionally, not all the relevant wastes in a recycling center are appropriate. The collected waste paper from final consumers is inappropriate at the rate of $\lambda_1$, and the collected waste cardboard from production sites is not recyclable at the rate of $\lambda_2$. Besides, raw materials that enter to the sheet and cardboard production sites are not transformable to the final product entirely. The waste sheet and waste cardboard rates are $(1-\alpha_1)$ and $(1-\alpha_2)$, which go back to the paper production sites. In accordance with Figure 4, the definitions of the rates for the studied cardboard CLSC network are as follows.

(Irrelevant waste rate $\beta$) = (irrelevant waste) / (collected waste paper)
(Cardboard production rate $\alpha_1$) = (output cardboard quantity) / (input sheet quantity)
(Sheet production rate $\alpha_2$) = (output sheet quantity) / (input paper quantity)
(Disposal rate of collected waste paper CWPC $\lambda_1$) = (inappropriate waste paper in CWPC) / (collected waste paper from CWPC)
(Disposal rate of CWPP $\lambda_2$) = (inappropriate waste paper in CWPP) / (collected waste paper from CWPP)

![Figure 4. Classification of collected waste paper](image)

- **Demand constraint**

\[
\sum_{c=1}^{m} (X_{ct} + I_{c(t-1)} - I_c) = \sum_{i=1}^{l} d_t, \quad \forall t \tag{11}
\]

Constraint (11) expresses the flow balance for the demand points. The flow entering demand points in each period will be equal to the production quantity in the same period plus the inventory from the former periods. It ensures satisfying the customer’s demand.

- **Capacity constraints**

\[
\sum_{s=1}^{N} W_{sij} + \sum_{c=\max+1}^{C} W_{cji} \leq Max_j, \quad \forall t, j \tag{12}
\]
\[
\sum_{n=1}^{N} W_{nf} + \sum_{c=m+1}^{C} W_{cf} \geq \Min_f \times \delta_f \quad \forall t, f
\]
(13)

\[
\sum_{n=1}^{N} W_{nf} + \sum_{c=m+1}^{C} W_{cf} \leq \Max_f \times \delta_f \quad \forall t, f
\]
(14)

\[
\sum_{j=1}^{J} X_{jft} + \sum_{f=1}^{F} X_{jft} + \sum_{r=1}^{R} V_{rpt} + \sum_{c=1}^{C} V_{cpt} \leq \Max_p \quad \forall t, p
\]
(15)

\[
\sum_{p=1}^{P} X_{pt} \leq \Max_r \quad \forall t, r
\]
(16)

\[
\sum_{r=1}^{R} X_{rect} \leq \Max_c \quad \forall t, c
\]
(17)

\[
\sum_{j=1}^{J} Y_{jdt} + \sum_{f=1}^{F} Y_{jdt} \leq \Max_d \quad \forall t, d
\]
(18)

\[
I_{ct} \leq q_c \quad \forall c, t
\]
(19)

\[
I_{jt} \leq q_j \quad \forall j, t
\]
(20)

\[
I_{ft} \leq q_f \quad \forall f, t
\]
(21)

Equations (12), (15), (16), (17) and (18) show the maximum available capacity of the internal chain recycling center, and the paper, sheet, and cardboard production site and the disposal center, respectively. Constraints (13) and (14) present the capacity limit of candidate recycling center \(f\) and constraints (19)-(21) limit holding inventory in each facility.

- **Candidate facilities**
  \[
  \sum_{f} \delta_f \leq M_f
  \]
  (22)

Constraint (22) sets the maximum number of new recycling centers that can be activated.

- **Purchasing constraints**
  \[
  \sum_{j=1}^{J} W_{cjt} + \sum_{j=1}^{J} W_{cjt} \geq E_{1}\pi_{ct} \quad \forall t, c
  \]
  (23)

\[
\sum_{j=1}^{J} W_{cjt} + \sum_{j=1}^{J} W_{cjt} \leq M_1\pi_{ct} \quad \forall t, c
\]
(24)

\[
\sum_{j=1}^{J} W_{nft} + \sum_{j=1}^{J} W_{nft} \geq E_{2}\pi_{nt} \quad \forall t, n
\]
(25)

\[
\sum_{j=1}^{J} W_{nft} + \sum_{j=1}^{J} W_{nft} \leq M_2\pi_{nt} \quad \forall t, n
\]
(26)

\[
X_{jpt} \geq E_{3}\pi_{jt} \quad \forall t, j
\]
(27)

\[
X_{jpt} \leq M_3\pi_{jt} \quad \forall t, j
\]
(28)
Constraints (23)- (30) force the minimum and maximum amount of waste paper, raw material and paper that can be supplied by external cardboard production sites, consumers, external recycling center and paper recycling, respectively.

- **Non-negative and binary decision variables.**
  \[ W_{njt}, W_{njt}, W_{cjt}, W_{jft}, Y_{jft}, Y_{jft}, X_{jft}, X_{jft}, V_{rpt}, V_{rpt}, X_{rct}, X_{cjt}, bk_{nt} \geq 0 \quad \forall t, n, j, f, d, p, r, c \]
  \[ \delta f, \pi_{pt}, \pi_{cjt}, \pi_{rct}, \pi_{jct} \in [0,1] \]

**4-Robust model**

Several disciplines are proposed to deal with the uncertainty in the supply chain. When a system faces scenarios for the system parameters, one can utilize the notion of robustness to control the system perturbations. The robust optimization approach is a reliable technique to deal with uncertainty (Sahinidis, 2004). In this research, demand is uncertain and it is assumed that demand scenarios are given. Then the robust optimization approach is employed to deal with the demand uncertainty in the studied network. The robust optimization approach presented by Mulvey et al. (1995) involves two types of robustness: 'solution robustness' and 'model robustness'. For the solution robustness type, the solution remains close to the optimal and for the model robustness type, the solution is almost feasible in a set of scenarios. In the robust model, the objective function included a penalty function for both model and solution robustness.

The robust optimization model includes the structural and control constraints. Structural constraints are free of any noise and it is formulated as an MILP model. On the other hand, control constraints face noisy data. Thus, two sets of variables known as design and control variables are considered in the framework of the robust optimization. The design variables are adjusted once and the control variables change according to the occurrence of uncertain scenarios in the model optimization process. The structure of the robust optimization model is as follows.

\[ \text{Min} \quad c^T x + d^T y \]
where \( x \) is a vector of design variables and the coefficient matrix \( A \) is free of noise. Besides, \( B \) and \( C \) are coefficient matrices subject to noise and \( y \) is a vector of control variables. The first constraint is the structural constraint and the second one is the control constraint.

Assume that this problem includes a set of scenarios \( \Omega = 1..S \) in which each scenario is associated with a set of control constraints and a probability of occurrence \( P_s \) for scenario \( s \) where \( \sum P_s = 1 \). Here, \( y_s \) defines a control variable for each scenario \( s \in \Omega \) and \( \delta_s \) defines an error vector that will measure the infeasibility allowed in the control constraint under scenario \( s \in \Omega \).

Then, a robust optimization model is formulated as follows:

\[
\text{Min } \sigma(x, y_1, y_2, ..., y_s) + \omega \rho(\delta_1, \delta ..., \delta_s) \quad (36)
\]

\[
s.t. \quad Ax = b \quad (37)
\]

\[
B_s x + C_s y + \delta_s = e_s \quad \forall s \in \Omega \quad (38)
\]

\[
x, y \geq 0 \quad \forall s \in \Omega \quad (39)
\]

The second term of the above objective function formulates the model robustness concepts and indicates that some scenarios may result in infeasible designs on a set of input parameters, where \( \omega \) applies as the infeasibility weight of a scenario. Yu and Li (2000) presented a more suitable formulation for the first term of this objective function, which is as follows.

\[
\text{Min } \sum_{s=1}^{S} P_s \zeta_s + \lambda \sum_{s=1}^{S} P_s \left[ \left( \zeta_s - \sum_{s'=1}^{S} P_{s'} \zeta_{s'} \right) + 2\theta_s \right] \quad (40)
\]

\[
s.t. \quad \zeta_s - \sum_{s'=1}^{S} P_{s'} \zeta_{s'} + \theta_s \geq 0 \quad \forall s \in \Omega \quad (41)
\]

\[
\theta_s \geq 0 \quad \forall s \in \Omega \quad (42)
\]

In the model given in equation (39), \( \zeta_s \) is the original minimizing objective function in the optimization model. It expresses that if the amount of \( \zeta_s \) is greater than \( \sum_{s'=1}^{S} P_{s'} \zeta_{s'} \), then \( \theta_s \) is equal to zero, whereas if the amount of \( \sum_{s'=1}^{S} P_{s'} \zeta_{s'} \) is greater than \( \theta_s \), then \( \sum_{s=1}^{S} P_s \zeta_s - \zeta_s = \theta_s \).

Then, a scenario-based model is proposed as given in Section 4. The control decision variables and auxiliary parameters are as follows.

- **Robust model auxiliary parameters**
\[ d_{it} \] cardboard demand in retailer \( i \) in period \( t \) under scenario \( s \)
\[ \lambda \] the weight of risk
\[ \omega \] infeasibility weight
\[ P_s \] the probability of scenario \( s \)

- **Control decision variables**
  \[ X_{it}^s \] supply quantity of final product to retailer \( i \) in period \( t \) under scenario \( s \)
  \[ I_{ct}^s \] inventory in the cardboard production site \( c \) in period \( t \) under scenario \( s \)
  \[ \delta_{it}^s \] infeasibility penalty for retailer \( i \) in period \( t \) under scenario \( s \)
  \[ \theta_s \] linearization variable under scenario \( s \)

The model objective function can be rewritten as equation (43).

\[
\text{Max} \sum \limits_s P_s RL^s - \lambda \sum \limits_s P_s (RL^s - \sum \limits_{s \in S} (RL^s - \sum \limits_{s \in S} P_s RL^s) + 2 \theta_s) - \omega \sum \limits_i \sum \limits_t \sum \limits_s P_s \delta_{it}^s
\]  

where \( RL^s \) is defined as equation (2) in Section (3-1) for various scenarios.

The first and second terms in equation (42) are the mean value and the variance of the total profit, respectively. The later term measures the model robustness for infeasibility related to the control constraint (44) under the scenario \( s \). The demand constraint (10) in the model is replaced by the following constraint:

\[
\sum \limits_{c \in S} (X_{ct} + I_{ct(t-1)} - I_{ct}) = \sum \limits_i (d_{it} - \delta_{it}^s) \quad \forall t, s
\]  

According to the balance equation (44), the term supply quantity \( X_{ct} \) for each period and under various scenarios is equal to \( \sum \limits_{c \in S} (X_{ct} + I_{ct(t-1)} - I_{ct}) \). Finally, constraint (45) is a linearization constraint added to the robust model. Similarly, constraints (3)-(10) and (12)-(31) in the MILP model, which is given in Section 3, are considered in the robust optimization model.

\[
RL^s - \sum \limits_{s \in S} P_s RL^s + \theta_s \geq 0
\]

**5- Case Study**

The paper recycling is an important recycling scheme applied to the waste materials. The recycled paper is the most important raw material for the paper and cardboard production industry (Ervasti et al., 2016). Additionally, the pulp, paper, and cardboard production processes use the recycled paper. In Section 3, a mixed integer linear programming model is proposed to procure raw materials optimally, produce paper, sheet, and cardboard, and
distribute the final product to customers in a cardboard closed-loop supply chain network. Here, the demand for the final product is uncertain in different periods, and a robust optimization approach is adopted to deal with it. Demand estimation in the studied supply chain depends on the classes of customers. Traditionally, some customers order the internal cardboard network, and their demand for cardboard is predictable. However, an order from a new or an occasional customer may be received for the network. It depends on the conditions such as seasonal demand or failure in their original cooperating network. Thus, some demand scenarios are identified. A scenario-based robust optimization can model it. This approach represents uncertainty through the setting up of various scenarios aiming to find a robust solution to ensure that all scenarios are 'close' to the optimum.

In this section, a real-world industrial case from a cardboard production industry in the Amol city in Iran is given. Through the historical data and surveys, three scenarios for cardboard demand namely, bad; moderate; and good with a probability of occurrence of 0.35, 0.25 and 0.4 are considered, respectively. In the bad scenario, the demand is 10 percent less than the moderate scenario. However, in the good scenario, cardboard demand can be 15% more than the moderate state.

The CLSC network is composed of three types of recycling centers. The waste paper can be supplied by the existing internal recycling center or can be purchased from two external recycling centers. Moreover, three options are available and one recycling center is allowed to be activated. There is one disposal center for disposing inappropriate waste paper. Likewise, paper can be supplied from the existing paper production sites or can be purchased from external paper production sites. Generally, the quality of waste paper supplied by the external cardboard production sites is better than the one of the waste paper collected by the recycling centers. In the next stage, the corrugated sheet is produced. Finally, one of the two internal cardboard production sites produces the cardboard, and the corrugated sheet can be sold to the external cardboard production sites as well.

Here, manufacturers of the industrial products are assumed as retailers that request cardboard to pack their products. In this model, 12 retailers are the main customers of the chain. Besides, consumers are segmented into seven regions for which the studied CLSC collects the waste paper. The network decisions are performing for six periods. The unit cost for the given parameters is Rials per ton. The model parameters are estimated as follows:

- The cost of purchasing waste paper from external cardboard production sites is 4,200,000, and the cost of purchasing paper from external paper production sites 1 and 2 are 9,500,000 and 8,500,000, respectively. Similarly, the cost of purchasing raw material
from external recycling centers 1 and 2 are 5,000,000 and 5,300,000, respectively. These cost differences may be due to the different distance between recycling centers and the production facilities.

- The unit operation cost for the disposal, internal recycling, and candidate recycling centers 1, 2 and 3 are 200,000, 1,200,000, 1,400,000, 1,500,000, and 1,200,000, respectively. Moreover, the unit operation cost for the internal paper, sheet, and cardboard production sites are 1,500,000, 1,200,000 and 1,500,000, respectively.

- The unit transportation cost to the different facilities is 3,000. Moreover, the distance between internal and candidate recycling centers 1, 2 and 3 and the disposal center are 3, 10, 20 and 15 km and from the internal paper production sites are 25, 25, 37 and 30 km, respectively. Moreover, the distance between the sheet and paper production sites and cardboard production sites are 20 and 21.5 km, respectively.

- The price for a unit of the sheet is 11,500,000 and for a unit of cardboard is 13,000,000.

- The unit inventory holding cost of the recycling site and cardboard production site are 33,300 and 90,000, respectively.

- The irrelevant waste rate $\beta$ in the recycling center is estimated to be about 28 percent. In addition, the waste paper that is back to the cycle is not appropriate entirely. The collected waste paper from the final consumers are inappropriate with the rate of $\lambda_1=0.17$, and it is $\lambda_2=8$ for the collected waste paper from external cardboard production sites. Moreover, the sheet production rate ($\alpha_2$) and the cardboard production rate ($\alpha_1$) are about 0.94 and 0.90, respectively. Other parameters are given in Tables 3-6.

### Table 3. Purchasing cost of the waste paper in each recycling site (*10³)

<table>
<thead>
<tr>
<th>Consumer</th>
<th>Internal recycling site</th>
<th>Candidate recycling site1</th>
<th>Candidate recycling site2</th>
<th>Candidate recycling site3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3100</td>
<td>3800</td>
<td>3300</td>
<td>4100</td>
</tr>
<tr>
<td>2</td>
<td>3500</td>
<td>3500</td>
<td>3600</td>
<td>3400</td>
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<tr>
<td>3</td>
<td>3200</td>
<td>3300</td>
<td>3000</td>
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<tr>
<td>4</td>
<td>3400</td>
<td>3450</td>
<td>4100</td>
<td>3160</td>
</tr>
<tr>
<td>5</td>
<td>3150</td>
<td>3200</td>
<td>3200</td>
<td>3320</td>
</tr>
<tr>
<td>6</td>
<td>3650</td>
<td>4400</td>
<td>4350</td>
<td>4150</td>
</tr>
<tr>
<td>7</td>
<td>3300</td>
<td>3400</td>
<td>3470</td>
<td>3400</td>
</tr>
</tbody>
</table>

### Table 4. Maximum raw material supply in each period (tons)

<table>
<thead>
<tr>
<th>Facility</th>
<th>Supply capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumer</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>Candidate recycling center</td>
<td>Minimum capacity</td>
</tr>
<tr>
<td>---------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 5. capacity limit for candidate recycling centers (tons)

<table>
<thead>
<tr>
<th>Facility3</th>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal recycling center</td>
<td>150</td>
</tr>
<tr>
<td>Disposal center</td>
<td>50</td>
</tr>
<tr>
<td>Paper production site</td>
<td>130</td>
</tr>
<tr>
<td>Sheet production site</td>
<td>250</td>
</tr>
<tr>
<td>Cardboard production site</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 6. capacity of internal facilities (tons)

The optimization software Lingo 15.0 efficiently solved the robust optimization model in less than one minute. The computational results are shown in Tables 7-11. The planning horizon, in this case is composed of six sequential periods. Table 5 shows the optimal quantity of production, inventory, supply, and under-fulfilment for various scenarios of cardboard demand in each period. In equation (44), the production quantity is independent of scenarios. Hence, Table 7 shows that the production quantities are the same for different scenarios in each period. According to the demand balance given in equation (44), the cardboard supply is equal to the cardboard demand minus the under-fulfilment in each period and under each scenario. Further, the supply of cardboard in each period is equal to the production quantity in the same period plus the inventory from the previous period.

The cardboard inventory in the cardboard production site is estimated to be 20 tons at the beginning of the planning horizon. The proposed optimal production plan handles various demand scenarios. For instance, the final product demand in the first scenario of the third period is 156.42 tons; however, the suggested robust optimal production is 173.8 tons. Therefore, the model proposes to hold inventory in the first scenario. The cardboard inventory in this period is equal to 36.61 tons. It is composed of the difference between the production and the demand in the current period, 17.38 tons, and holds the cardboard inventory from the previous period, 19.23 tons.
Because the aim of this study is to obtain an optimal robust solution for the cardboard closed-loop supply chain network, the studied network uses the maximum production capacity ‘211.5 tons’ to produce the final product in the first period. As Table 7 shows, the cardboard demand is higher than the maximum cardboard production capacity for different scenarios in the first period. Therefore, under-fulfilment occurs corresponding to the different scenarios in this period, and no inventory is hold for the cardboard in the first period.

The final product inventory in each period will be used in the coming periods, but under-fulfilment quantity is a lost sale. Therefore, the robust optimization model suggests holding inventory to reduce under-fulfilment quantities. Finally, the stored inventory in the sixth periods will be used as the initial inventory for the next six sequential periods of production planning. As Table 7 indicates when the bad or moderate demand scenarios occur, there is no under-fulfilment.

In the current network, there is a recycling center known as the internal recycling center. However, opening a new recycling center is suggested. The new network structure is called a proposed network. The waste paper for recycling operations can be supplied from two external facilities including seven final consumers and two external cardboard production sites. Table 8 gives the waste paper quantity purchased from various sources in each period. The maximum collection capacity for the internal recycling center in the cardboard closed-loop supply chain network is about 150 tons. As Table 6 shows, the internal recycling center procures waste paper from consumers 1, 5, 6, 7, and both of the external cardboard production facilities in different periods. Here, the internal recycling center works on its maximum collection capacity.

Because the maximum collection capacity of the internal recycling center is less than the average cardboard demand in the CLSC network, one candidate-recycling center is allowed to open. Hence, the model suggested to open the candidate recycling center 3 according to some factors such as distance, transportation, and collection costs. The waste paper from the consumers 2, 3 and 4 will be collected by the candidate recycling center 3. In each period, 18.9 and 12.24 tons of inappropriate waste paper need to be transferred from the internal and new recycling centers to the disposal center, respectively.

Table 9 shows the amount of entered waste sheet and cardboard to the paper production sites in each period. It is expected that 10 percent of the sheets and 6 percent of the cardboards is wasted in the current network. Moreover, Table 9 gives the amount of recycled waste paper that transfer from the internal and external recycling centers to each paper production site. As reported in Table 9, the paper production sites use the maximum capacity of external recycling facilities. Further, about 28 percent of collected waste paper by the internal recycling centers
are irrelevant and must be separated and sent to disposal centers ($\beta=28\%$). On the other hand, some related waste papers are inappropriate and should be sent to the disposal center. The amounts of waste paper purchased by the internal and new recycling centers are 130 and 72 tons. However, these quantities drop to 78.3 and 39.6 tons after separating irrelevant wastes and inappropriate waste paper.

Because of the limited paper production capacity, the sheet production site procures paper from the external paper production sites as well. Here, paper for the sheet production site can be procured from the internal and external paper production sites. Table 8 shows the quantity of produced paper that transfer to the sheet production site in each period. The sheet production site works on its maximum capacity, as shown in Table 10. For instance, in the first period, 206.4 and 43.6 tons of paper transfer from the internal and external paper production sites to the sheet production site, respectively.
Table 7. The optimal decisions for the proposed network (tons)

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
<th>Period 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td>235.17</td>
<td>261.3</td>
<td>300.5</td>
<td>173.07</td>
<td>192.3</td>
<td>221.15</td>
</tr>
<tr>
<td>Production</td>
<td>211.5</td>
<td>211.5</td>
<td>211.5</td>
<td>192.3</td>
<td>192.3</td>
<td>173.8</td>
</tr>
<tr>
<td>Inventory</td>
<td>19.23</td>
<td>36.61</td>
<td>57.61</td>
<td>76.51</td>
<td>100</td>
<td>4.99</td>
</tr>
<tr>
<td>Supply</td>
<td>231.5</td>
<td>231.5</td>
<td>231.5</td>
<td>173.07</td>
<td>192.3</td>
<td>156.42</td>
</tr>
<tr>
<td>Under-fulfilment</td>
<td>3.67</td>
<td>29.8</td>
<td>69</td>
<td>28.85</td>
<td>26.07</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Table 8. Waste paper procured by recycling centers (tons)

<table>
<thead>
<tr>
<th>Facility</th>
<th>Period 1</th>
<th>Period 2</th>
<th>Period 3</th>
<th>Period 4</th>
<th>Period 5</th>
<th>Period 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Final consumer</td>
<td>1</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
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<td>2</td>
<td>27</td>
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<td>6</td>
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<tr>
<td></td>
<td>7</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Cardboard production site</td>
<td>1</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 9. Waste paper transferred from different sources (tons)

<table>
<thead>
<tr>
<th>Facility</th>
<th>Internal paper production site 1</th>
<th>Internal paper production site 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period 1</td>
<td>Period 2</td>
</tr>
<tr>
<td>Internal recycling center</td>
<td>78.3</td>
<td>78.3</td>
</tr>
<tr>
<td>Cardboard production site1</td>
<td>5.7</td>
<td>4.47</td>
</tr>
<tr>
<td>Cardboard production site2</td>
<td>25.6</td>
<td>25.6</td>
</tr>
<tr>
<td>Sheet production site1</td>
<td>30.3</td>
<td>30.3</td>
</tr>
<tr>
<td>External recycling center1</td>
<td>20.2</td>
<td>20.2</td>
</tr>
<tr>
<td>External recycling center2</td>
<td>39.6</td>
<td>39.6</td>
</tr>
<tr>
<td>Candidate recycling center3</td>
<td>39.6</td>
<td>39.6</td>
</tr>
</tbody>
</table>
Table 10. Quantity of paper transferred to the sheet production site (tons)

<table>
<thead>
<tr>
<th>Facility</th>
<th>Period1</th>
<th>Period2</th>
<th>Period3</th>
<th>Period4</th>
<th>Period5</th>
<th>Period6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal paper prod. Site</td>
<td>78.30</td>
<td>78.3</td>
<td>78.3</td>
<td>78.3</td>
<td>117.9</td>
<td>78.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>128.10</td>
<td>126.87</td>
<td>125.7</td>
<td>128</td>
<td>87.06</td>
</tr>
<tr>
<td>External paper prod. Site</td>
<td>13.60</td>
<td>14.83</td>
<td>16</td>
<td>13.7</td>
<td>15.04</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Moreover, the produced sheets in the internal network are used in the internal cardboard production sites or can be sold to external cardboard production sites. This decision depends on factors such as the maximum cardboard site capacity, the cardboard demand, and earning from selling the sheet and cardboard products. Table 11 shows the amount of transferred sheet to the internal or external cardboard production sites in each period. In the first period, the average demand is more than the produced sheet. Consequently, all of the produced sheet is transferred to internal cardboard production sites. In the next periods, the produced sheet is allocated to the internal or external cardboard production sites.

Table 11. Quantity of sheet transferred to the cardboard production sites (tons)

<table>
<thead>
<tr>
<th>Facility</th>
<th>Period1</th>
<th>Period2</th>
<th>Period3</th>
<th>Period4</th>
<th>Period5</th>
<th>Period6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal cardboard prod. site</td>
<td>130</td>
<td>130</td>
<td>130</td>
<td>130</td>
<td>130</td>
<td>130</td>
</tr>
<tr>
<td>External cardboard prod. site</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The amount of expected profit in the scenario-based model depends on the scenario realization. Figure 5 illustrates the average profit in the proposed network when one of the three different demand scenarios occurs. As shown, regardless of any realization of demand scenarios, the robust solution yields an acceptable profit.

Figure 5. The average profit in a scenario-based model vs. robust counterpart
In the proposed MILP model, some of the demand scenarios may lead to infeasible results. Here, the penalty weight $\omega$ is applied as the model infeasibility under occurrence of the scenarios. Table 12 evaluates the proposed robust model with various $\omega$ values. As one can see, the under-fulfilment, which represents the model robustness, decreases with an increase in the $\omega$ value. This means that for the larger value of $\omega$, the expected under-fulfilment drops to 239.99 tons. This value is the sum of under-fulfilment in the entire planning horizon. As Table 12 shows, the best value for $\omega$ in the studied problem is 120, because the lowest amount of under-fulfilment and the best solution is reported at this point. It can be seen that in the worst case about 6.4% of cardboard demand is not satisfied. Therefore, in this case, $\omega$ is set to 120, and $5.9*10^9$ Rials is the obtained objective function value for the robust optimal solution.

Table 12. The trade-off between weight and model robustness

<table>
<thead>
<tr>
<th>Weight($\omega$)</th>
<th>Under-fulfilment(tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-119</td>
<td>244.98</td>
</tr>
<tr>
<td>$\omega \geq 120$</td>
<td>239.99</td>
</tr>
</tbody>
</table>

Figure 6 compares the reported inventory and under-fulfilment quantity for the current and proposed networks. It can be inferred from Figure 6 that the cardboard inventory for both networks is not different in all of the given demand scenarios. However, the under-fulfilment quantity decreases, when a new recycling center is activated. In other words, opening a new recycling center increases the cardboard production quantity, and the studied network can respond to more customers. Likewise, the total profit changes from $5.7*10^9$ to $5.9*10^9$ Rials by activating a new recycling center.
6- Discussions and concluding remarks

The end-of-life products may become dangerous wastes and cause damage to the environment if they are not properly managed. Therefore, attention to issues such as waste reduction, and recycling products has increased in the recent decades. Material recycling plays a major role in different industries and different sectors of the society, and recycled paper and cardboard are one of the best secondary materials. Moreover, many researchers are interested and discussed the reverse and closed-loop supply chain design in various industries. The interest in the waste paper recycling has risen in the past few years, and researchers proposed various designs for the closed-loop cardboard supply chain.

In this research, a multi-period mixed integer linear programming model is proposed for a cardboard CLSC network. In the traditional supply chain planning, each facility has an independent operational plan. However, this study integrated the supply chain activities to obtain an optimal operation plan in the cardboard CLSC network. It determines the optimal flow between different facilities to satisfy the demand for the cardboard and maximize the supply chain total profit. Further, this research considered demand uncertainty and utilized the robust optimization approach to deal with uncertainty. Moreover, the presented model evaluated a set of real-world data. The results indicated that the proposed model provides an efficient plan for the cardboard closed-loop supply chain network.

Because the maximum collection capacity of the internal recycling center is less than the average cardboard demand, opening a new recycling center is suggested. In the studied...
network, the cardboard production increases, if a new recycling center is opened and as a result, the amount of under-fulfilment decreases. Thus, the model is set to reduce the under-fulfilment and increase the profit. Moreover, an alternative strategy to deal with under-fulfilment in the studied cardboard network is the overtime production.

It is an interesting finding that the ratio of defectives and losses in the operations and processes of the CLSC network under study were drastically high. For instance, the raw material collected in the recycling centers includes 28 percent of irrelevant wastes, and again about 17 percent are inappropriate waste paper. Identifying the roots of these losses may increase the CLSC network profit and productivity. However, any considerable work on the modeling of this aspect of the closed-loop networks is not reported in the literature.

Considering green, sustainable, and environmental issues along with the total costs minimization in recycling the waste paper is an interesting ground for future works. In addition, it is strongly recommended to explore the other sources of uncertainty in the parameters of the CLSC network such as the operating costs. Moreover, according to the nature of the uncertainty, there are opportunities to deal with uncertainty utilizing the fuzzy or stochastic programming to design and analyse of a CLSC network.

References


