A multi-period supply chain design with product life cycle considerations

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Abstract— **Effective design and management of supply chains assists in production and delivery of a variety of products at low cost, high quality, high service level, and short lead times. Thus, it is clear that these competitive criteria differ during the different phases of the product life cycle. To be effective, a firm's supply chain design strategy should be governed by objectives that are different from traditional objectives, have a strategic orientation , select the efficient supply chain partners, and dynamically match the supply chain strategies. This paper presents a two-phase mathematical programming approach for effective supply chain design with a total cost minimization, while considering the different product life cycle stages.**

Keywords— supply chain design; product life cycle; multi-criteria decision making.

I. INTRODUCTION

Because supply chain management revolves around efficient integration of suppliers, manufacturers, and distributors, it encompasses the firm's activities at many levels, from the strategic level through the tactical to the operational level. This paper focuses on the first level, ie the strategic level, its main objective design a multi-level supply chain while considering the different product life cycle stages

Typically, products can be categorized into three types, namely, functional, innovative, and hybrid [1]. Functional products' demand can be forecasted quite accurately and their market share remains fairly constant. They enjoy along life cycle with superficial design modification leading to different product types. Innovative products, however, are new products developed by organizations to capture a wider share of the market. They are significantly different from the available product types and are more adapted to the customer requirements. Finally, hybrid products can consist of either on different combinations of functional components, or on a mix of functional and innovative components.

Otherwise, all products have a life cycle, typically depicted as a curve of unit sales for a product category over time [2], which can be classified into four discrete stages: introduction, growth, maturity and decline. Figure 1 illustrates the four stages of the product life cycle. The competitive criteria generally differ during the different phases of product life cycle; for instance, availability and technology are needed at the ''introduction'' phase, and cost, quality and speed are needed at the ''maturity'' phase [3] .

The discussions in marketing and logistic literature universally conclude that the product life cycle stages, as well as the product type, have a great impact on appropriate supply chain design. Consequently, depending on the product life cycle stage, a firm should select its effective supply chain partners and dynamically match the supply chain strategies so as to satisfy the product requirements across multiple criteria and to maximize competitiveness over time. Indeed, the competitive criteria generally differ depending on the product type and the product life cycle phase, and the importance of these criteria is often subjective and depends on the decision maker. Based on Wang et al. [4], table 1 summarizes the supply chain strategy based on product type and product life cycle.

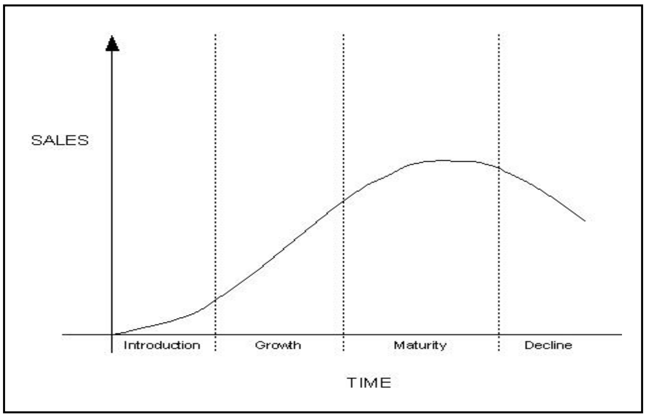

Fig.1. Marketable product life cycle

TABLE I THE SUPPLY CHAIN CLASSIFICATION BASED ON PRODUCT TYPE AND PRODUCT LIFE CYCLE

Product life cycle	Product type		
	Functional	Innovative	Hybrid
Introduction	Lean Supply chain	Agile Supply chain	Hybrid Supply chain
Growth	Lean Supply chain	Agile Supply chain	Hybrid Supply chain
Maturity	Lean Supply chain	Hybrid/Lean Supply chain	Hybrid Supply chain
Decline	Lean Supply chain	Hybrid/Lean Supply chain	Hybrid Supply chain

This study develops a mathematical model that incorporates the firm's PLC-oriented relative preferences for multiple purchasing, manufacturing and distributing criteria, so as to minimize the supply chain total cost. The proposed model incorporates subjective priorities for decision criteria in selecting the supply chain partners, with respect to cost, quality, R&D, service, and delivery performance, of supply, manufacturing and distributing arrangements

II. LITERATURE REVIEW

Aitken et al. [5] provided some important considerations about the relationship of life-cycle stages with supply-chain strategy. They discussed the matching of products to pipelines for maximizing competitiveness with respect to order-winner and market-qualifier product characteristics.

Vonderembse [6] developed a typology for supply chain design based on product characteristics, customer expectations, and stage of the product life cycle. Chang [6] observed that supply-chain product-development strategy should depend on the particular phase of the product life cycle. This in turn affects core competencies and outsourcing synergies. They considered a multi-attribute quantitative approach for decision support in the supplier-selection context.

Narasimhan et al. [7] studied supplier selection in connection with the life-cycle stage of products. They proposed that in industrial purchasing contexts firms often procure a set of products from the same suppliers to benefit from economies of scale and scope. These products often are at different stages of their respective product life cycles. Moreover, firms consider multiple criteria in purchasing products, and the relative importance of these criteria varies depending on the product life cycle stage of a given product.

Punakivi and Hinkka [8] did not discuss modeling; they provided a logistics case study on transportation mode selection from the industry point of view. They identified the shortening of product and service life cycles as one of the four major influential trends, along with globalization, concentration on core competencies, and the growth and expansion of e-business in supply chain networks.

A useful table ranks the importance of the four criteria of convenience, price, quality, and speed in the industries of construction, electronics, machinery, and pharmaceuticals, respectively. Wang et al. [4] considered the supplier selection problem using a combination of the Analytic Hierarchy Process (AHP) and preemptive goal programming (PGP).

The uniqueness of the approach proposed in this paper for multi-period SCN design is that it incorporates efficiencies of individual supply chain processes, to prevent the decision maker from selecting inefficient processes in the corresponding product life cycle stage. Finally, this solution procedure is developed in alignment with the opinion of experts urging the need for efficient, agile, and compatible business processes for effective SCN design and operation. This paper addresses a multi-period supply chain design model for total cost minimization, throughout the product life cycle from its introduction on the market until its decline.

III. PROBLEM DEFINITION

The problem involves evaluating a set of potential actors on the basis of some performance criteria, in order to design effectively a supply chain network. In this paper, the objective is measured in terms of five criteria: R&D, cost, quality, service, response, and emissions.

To each criterion, a set of sub-criteria is associated. Figure 2 illustrates the hierarchical structure of the problem.

The solving methodology will be in two phases. Phase I evaluates all the alternatives in determining their efficiencies with respect to the product life cycle stage. It presents a multi-criteria decision making problem. The phase II, then involves the application of an integer programming model, which optimally selects candidates for SCN design, and identifies the optimal routing decisions for all entities in the network by integrating the efficiencies identified in phase I, demand, capacity requirements, and flow conservation constraints.

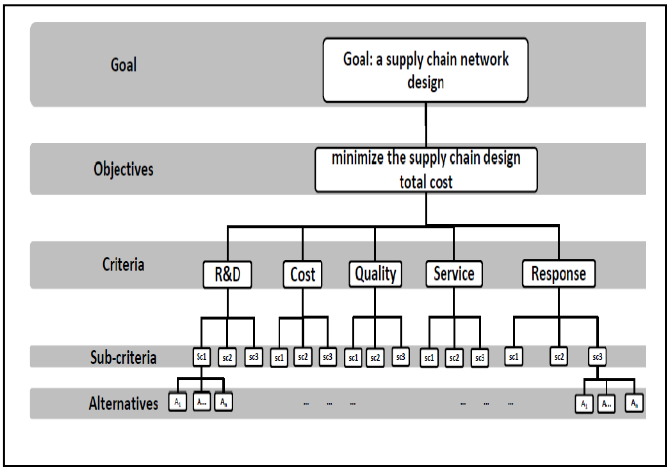

Fig.2. Hierarchical structure of the decision problem

A. Resolution methodology

1) Phase I : a multi-criteria decision making problem: The decision makers initiate phase I with the identification of the required business process types, which is followed by the consideration of potential candidates for each process.

The extension of the AHP process using OWA operators, proposed by [9] is then utilized to evaluate all the candidates of each process, at each product life cycle phase. However, these two procedures do not operate at the same level. The AHP is a global tool for creating a hierarchical model of the spatial decision problem, analyzing the whole process and evaluating each alternative. In the context of this paper, a typical four-level hierarchy of objectives, criteria, sub-criteria, and alternatives has been considered. The hierarchical structure of the decision problem is shown in the Figure 2. Using the OWA operators, we are able to model situations where the number of sub-criteria needed to satisfy a higher concept can be expressed in terms of linguistic quantifiers.

The above problem is solved separately for each of the three business process types at each product life cycle stage, and the solutions identify the efficiency scores; corresponding to the potential suppliers, manufacturers, and distributors ; to be utilized in the supply chain design model.

2) Phase II: the supply chain design: At each product life cycle stage; taking into account the efficiency scores obtained at the phase I, the capacity and the demand satisfaction constraints; the supply chain network design framework needs to identify the effective supply chain actors, as well as the deployment plans.

The proposed model allows identifying, at each product life cycle stage, the optimal routing of material from selected suppliers to manufacturers to warehouses by minimizing the supply chain total cost as well as the total CO2 emissions.

B. The model

1) Notation: To formulate the problem, the following parameters are used:

$CF_{i,t}$: Fixed cost to operate the supplier i at stage t
$CF_{j,t}$: Fixed cost to operate the producer j at stage t
$CF_{k,t}$: Fixed cost to operate the distributor k at stage t
$CO_{i,t}$: Fixed cost of opening the supplier i at stage t
$CO_{j,t}$: Fixed cost of opening the producer j at stage t
$CO_{k,t}$: Fixed cost of opening the distributor k at stage t
$CC_{i,t}$: Fixed cost of closing the supplier i at stage t
$CC_{j,t}$: Fixed cost of closing the producer j at stage t
$CC_{k,t}$: Fixed cost of closing the distributor k at stage t
$c_{ij,t}$: Unit production and transportation cost from supplier i to producer j, at stage t
$c_{jk,t}$: Unit production and transportation cost from producer j to distributor k, at stage t
$c_{kz,t}$: Unit production and transportation cost from distributor k to customer zone z, at stage t
$e_{i,t}$: Mean efficiency score of the supplier i, at stage t

Decision variables:

$X_{i,t}$ = 1 if the supplier i is selected at stage t, 0 otherwise
$X_{j,t}$ = 1 if the producer j is selected at stage t, 0 otherwise
$X_{k,t}$ = 1 if the distributor k is selected at stage t, 0 otherwise
$X_{ij,t}$: Quantity shipped from supplier i to producer j, at stage t, in Ton
$X_{jk,t}$: Quantity shipped from producer j to distributor k, at stage t, in Ton

$X_{kz,t}$: Quantity shipped from distributor k to customer zone z, at stage t, in Ton

2) The mathematical formulation: The effective multi-period supply chain network design problem, can be formulated as follows:

$$\begin{aligned}
\min f = \sum_t \Bigg[\Bigg[& \sum_{i=1}^{m} \mathrm{CF}_{i,t} * X_{i,t} \\
& + \sum_{j=1}^{n} \mathrm{CF}_{j,t} * X_{j,t} + \sum_{k=1}^{p} \mathrm{CF}_{k,t} * X_{k,t} \Bigg] \\
& + \Bigg[\sum_{i \in I} \mathrm{CO}_{i,t} * X_{i,t} * \left(1 - X_{i,t-1}\right) \\
& + \sum_{j \in J} \mathrm{CO}_{j,t} * X_{j,t} * \left(1 - X_{j,t-1}\right) \\
& + \sum_{k \in K} \mathrm{CO}_{k,t} * X_{k,t} * \left(1 - X_{k,t-1}\right) \Bigg] \\
& + \Bigg[\sum_{i \in I} \mathrm{CC}_{i,t} * X_{i,t} * \left(1 - X_{i,t+1}\right) \\
& + \sum_{j \in J} \mathrm{CC}_{j,t} * X_{j,t} * \left(1 - X_{j,t+1}\right) \\
& + \sum_{k \in K} \mathrm{CC}_{k,t} * X_{k,t} * \left(1 - X_{k,t+1}\right) \Bigg] \\
& + \Bigg[\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j,t} * X_{i,j,t} \\
& + \sum_{j=1}^{n} \sum_{k=1}^{p} c_{j,k,t} * X_{j,k,t} \\
& + \sum_{k=1}^{p} \sum_{z=1}^{q} c_{k,z,t} \\
& * X_{k,z,t} \Bigg] \Bigg] \qquad (1)
\end{aligned}$$

s.t

Efficiency constraints:

$$e_{i,t} X_{i,t} \geq e_{exp,I,t} X_{i,t} \quad \forall\, i, t \qquad (2)$$

$$e_{j,t} X_{j,t} \geq e_{exp,J,t} X_{j,t} \quad \forall\, j, t \qquad (3)$$

$$e_{k,t}Z_{j,t} \geq e_{exp,K,t}\, X_{k,t} \quad \forall\, k,t \tag{4}$$

Capacity limits constraints:

$$\sum_{j=1}^{n} X_{i,j,t} \leq cap_{i,t} * X_{i,t} \quad \forall\, i,t\ t \tag{5}$$

$$\sum_{k=1}^{p} X_{j,k,t} \leq cap_{j,t} * X_{j,t} \quad \forall j\,,t \tag{6}$$

$$\sum_{z=1}^{q} X_{k,z,t} \leq cap_{k,t} * X_{k,t} \quad \forall k\,,t \tag{7}$$

Flow conservation constraints

$$\sum_{i=1}^{m} X_{i,j,t} - \sum_{k=1}^{p} X_{j,k,t} = 0 \quad \forall\, j\,,t \tag{8}$$

$$\sum_{j=1}^{n} X_{j,k,t} - \sum_{z=1}^{q} X_{k,z,t} = 0 \quad \forall\, k\,,t \tag{9}$$

Total market demand satisfaction constraint:

$$\sum_{k=1}^{p} X_{k,z,t} \geq D_{z,t} \quad \forall\, z,t \tag{10}$$

Non-negativity constraints

$$X_{i,j,t} \geq 0\, \forall\; i,j\,,t \tag{11}$$

$$X_{j,k,t} \geq 0\; \forall\, j,k,tt \tag{12}$$

$$X_{k,z,t} \geq 0\; \forall\; k,z,tt \tag{13}$$

$$X_{i,t} \in \{0,1\}\, \forall i,tt \tag{14}$$

$$X_{j,t} \in \{0,1\}\, \forall j,tt \tag{15}$$

$$X_{k,t} \in \{0,1\}\, \forall k,tt \tag{16}$$

$i = 1,\ldots,m,$
$j = 1,\ldots,n,$
$k = 1,\ldots,p,$
$z = 1,\ldots,q$

The developed multi-period supply chain model aims to select effective suppliers from a candidate set of suppliers, as well as to locate a given number of effective producers, and distributors, to satisfy the demand requirements at the customer zones at each product life cycle stage, in order to minimize the overall supply chain design cost, subject to supplier, producer and distributor capacity restrictions, and efficiency scores.

We assume that the customer zone locations and their specific demand estimates are given in advance. The potential supplier, producer and distributor locations as well as their capacities are also known. For each selected actor, a decision must be made on the total units of products that need to be transported from the selected supplier, to the open producer to the open distributor, and the total units of products that need to be distributed from the open distributor to the customer zones.

In the above formulation, the objective function minimizes the supply chain total cost, which includes fixed and variable costs. The variable costs consist on the purchasing costs, the production costs, and the distribution costs. While the fixed costs consist on the facilities opening costs, operating costs, and closing costs. The opening costs occur on the period t+1 only if the corresponding facility was closed on the previous period t. Simultaneously, for the closing costs, they occur on a period t only if the corresponding facility was opened on the period t-1. These two costs are related to the dynamic nature of the problem.

Constraint sets (2)-(4) prohibit the selection of ineffective actors. Constraint sets (5)-(7) stipulate that all shipments from a supplier, a plant, or a distribution center, must not exceed its maximum capacity. Constraint sets (8)-(9) indicate a conservation of flow at each facility, while (10) requires that all the market demand must be met. A non-negativity on each shipment is imposed by constraint sets (11)-(13). Constraint sets (14)-(16) restrict every facility to be either open or closed.

C. *Solution methodology*

1) problem reformulation: The multi-period nature of the problem is related to the first objective function formulation (1) which involves the closing and reopening costs. The formulation of these costs includes nonlinear components and makes the objective function problem quadratic.

However, the binary quadratic problems are known to be NP-hard problems which are also practically difficult to solve [10]. Basically, the available solution procedures for the quadratic programming problem may be classified as attempting either to solve the problem directly or to transform it into an equivalent linear mixed-integer program, and then solve the latter problem.

In this paper we will use the second solution procedure, namely the linearization of the mathematical model using the method proposed by Fortet [11]. The method consists on linearizing a 0-1 polynomial programming problem by replacing each polynomial term $X_i * X_j$ with a single additional variable and two auxiliary linear constraints.

Therefore, the financial function is rewritten as:

$$\begin{aligned}\min f = \sum\nolimits_{t\in T} \Big[\sum\nolimits_{i\in I} & \mathrm{CF_{i,t}} * \mathrm{X_{i,t}} \\ & + \sum\nolimits_{j\in J} \mathrm{CF_{j,t}} * \mathrm{X_{j,t}} + \sum\nolimits_{k\in K} \mathrm{CF_{k,t}} * \mathrm{X_{k,t}}\Big] \\ & + \Big[\sum\nolimits_{i\in I} \mathrm{CO_{i,t}} * (\mathrm{X_{i,t}} - \mathrm{XO_{i,t}}) \\ & + \sum\nolimits_{j\in J} \mathrm{CO_{j,t}} * (\mathrm{X_{j,t}} - \mathrm{XO_{j,t}}) + \sum\nolimits_{k\in K} \mathrm{CO_{k,t}} \\ & * (\mathrm{X_{k,t}} - \mathrm{XO_{k,t}})\Big] \\ & + \Big[\sum\nolimits_{i\in I} \mathrm{CC_{i,t}} * (\mathrm{X_{i,t}} - \mathrm{XC_{i,t}}) \\ & + \sum\nolimits_{j\in J} \mathrm{CC_{j,t}} * (\mathrm{X_{j,t}} - \mathrm{XC_{j,t}}) + \sum\nolimits_{k\in K} \mathrm{CC_{k,t}} \\ & * (\mathrm{X_{k,t}} - \mathrm{XC_{k,t}})\Big] \\ & + \Big[\sum\nolimits_{i\in I} \sum\nolimits_{j\in J} \mathrm{c_{i,j,t}} * \mathrm{X_{i,j,t}} \\ & + \sum\nolimits_{j\in J} \sum\nolimits_{k\in K} \mathrm{c_{j,k,t}} * \mathrm{X_{j,k,t}} \\ & + \sum\nolimits_{k\in K} \sum\nolimits_{z\in Z} \mathrm{c_{k,z,t}} * \mathrm{X_{k,z,t}}\Big] \qquad (17)\end{aligned}$$

With

$$XO_{i,t} = X_{i,t} * X_{i,t-1} \quad \forall\ \mathrm{i, t} \qquad (18)$$

$$XO_{j,t} = X_{j,t} * X_{j,t-1} \ \forall\ \mathrm{j, t} \qquad (19)$$

$$XO_{k,t} = X_{k,t} * X_{k,t-1} \ \forall\ \mathrm{k,t} \qquad (20)$$

$$XC_{i,t} = X_{i,t} * X_{i,t+1} \ \forall\ \mathrm{i, t} \qquad (21)$$

$$XC_{j,t} = X_{j,t} * X_{j,t+1} \ \forall\ \mathrm{j, t} \qquad (22)$$

$$XC_{k,t} = X_{k,t} * X_{k,t+1} \ \forall\ \mathrm{k, t} \qquad (23)$$

For linearization, the following constraints are added to the previous problem constraints:

Linearization constraints

$$XO_t \le X_t \quad \forall i, j, k \qquad (24)$$

$$XO_t \le X_{t-1} \quad \forall i, j, k \qquad (25)$$

$$XO_t \ge X_t + X_{t-1} - 1 \quad \forall i, j, k \qquad (26)$$

$$XC_t \le X_t \quad \forall i, j, k \qquad (27)$$

$$XC_t \le X_{t-1} \quad \forall i, j, k \qquad (28)$$

$$XC_t \ge X_t + X_{t-1} - 1 \quad \forall i, j, k \qquad (29)$$

The set of equations (18)-(23) represent the replacement of the product of the binary variables by a new additional variable. The constraint sets (24)-(29) was added to force the equality between the product of the binary variables and the new additional variable.

IV. NUMERICAL RESULTS

This section presents a small-scale supply chain design problem adapted from a real-life situation. The purpose is neither to show any advantage of the modeling process by comparing with other MIP models, nor to exhibit the efficiency of problem solving by benchmarking the computation time to other algorithms. The figure 3 shows the sales distribution in Ton for the 3 different customer zones.

The mathematical model is solved to identify the optimum solution. Table 2 illustrates the optimal deployment plans for the supply chain network for the introduction phase. Using the same data values as previously, table 3 gives the optimal supply chain network for the maturity phase. The model demonstrates that the proposed approach cannot only adopt the supply chain strategy according to the degree of concern at different phases, but also consider the trade-off effect to avoid selecting inefficient actors in the correspondent product life cycle stage. From the results listed in table 3 for the maturity phase, it is interesting to note that supplier 2, which was considered to be inefficient in the introduction phase, was selected as an efficient actor in the maturity phase, and similarly for the producer 3 and the distributor 3.

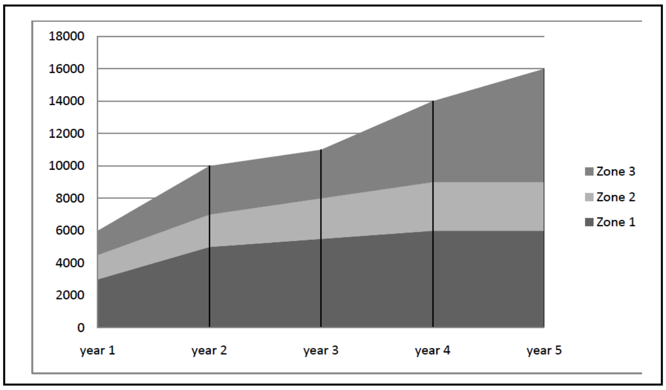

Fig.3. Sales distribution of the product in Ton

TABLE II OPTIMAL SUPPLY CHAIN NETWORK FOR THE INTRODUCTION STAGE

	P_1	P_2	D_1	D_2	Z_1	Z_2	Z_3
S_3	2000	12000					
S_4		2000					
P_1			2000				
P_2			1500	12500			
D_1						3500	
D_2					8000		4500
Demand					>=8000	>=3500	>=4500

TABLE III OPTIMAL SUPPLY CHAIN NETWORK FOR THE MATURITY STAGE

	P_1	P_3	D_2	D_3	Z_1	Z_2	Z_3
S_2		2000					
S_3	2000	2000					
S_4		10000					
P_1			2000				
P_3			4000	10000			
D_2					6000		
D_3					2000	3500	4500
Demand					>=8000	>=3500	>=4500

IV. CONCLUSION

Effective supply chain management envisioned as a solution to meet the constantly changing needs of the customer at low cost, high quality, short lead times, and high variety.

In this paper, a mathematical programming approach with product life cycle and green procurements considerations is proposed for effective supply chain network design. In phase I of the decision making process, a multi-criteria decision making model is utilized, based on an aggregation model using an extension of the AHP process by the OWA operators, to evaluate the performance of suppliers, manufacturers and distributors. The efficiency scores obtained in this phase will be useful for the efficiency constraints of the phase II. Consequently, this approach has its practical meaning in aggregating supply chain performance and assessing the supply chain partners under different phases of product life cycle. In phase II, a mixed integer programming problem is utilized to design the supply chain network and identify the optimal routing decisions. The case of production and delivery of multiple products that are in different product life cycle stages is another interesting issue that needs to be considered.

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