Extended design method for in-plane stability of haunched sway portal frames

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ABSTRACT

In current design rules the effect of a haunch on the sway in-plane stability of a steel portal frame only takes into account the influence of the haunch dimensions on the beam-to-column connection strength and stiffness. The effect of the haunch dimensions on the beam behavior, and thus on the frame behavior, is not included. The paper describes the effect of this phenomenon by regarding current design methods and comparing these with analytical solutions. The validity of the methods is covered by numerical simulations.

For a vertical beam loading, the larger the span of the portal frame, the higher the compressive force in the beam becomes. In addition, the longer the span of the frame, the smaller the critical buckling load of the beam becomes. This decreases the stability of the overall frame significantly. In fact, the compressive force in the beam of a portal frame has a significant effect on the additional stiffness the haunch provides to the column. Due to the adjusted center line of the haunch causing an eccentricity, an additional first order moment is generated. This additional internal moment reduces the additional stiffness the haunch provides. For some spans this may even cause the additional stiffness of the haunch to be negligible.

The research has given more insight, also on the effect of the shift of the compressive force in the beam, which depends on the geometry of the haunch. The study resulted in two simple correction factors for the current design rules, where these correction factors cover amplification factors for the original stiffness of the beam. The factors depend on the kind of loading (point load or equally distributed load) and on the haunch to rafter ratio (with regard to the length of the haunch as well as with regard to the height of the haunch).

Keywords: sway frames, haunches, in-plane stability

1 INTRODUCTION

In current design rules the effect of a haunch on the sway in-plane stability of a steel portal frame takes into account the influence of the haunch dimensions on the beam-to-column connection strength and stiffness. The effect of the haunch dimensions on the beam behaviour, and thus on the frame behaviour, is regarded as a mechanical issue and therefore not included in Eurocode 3. However, in practice simple hand rules are far more favourite than calculations based on extended mechanical analysis or a complex FEM calculation.

The main objective of the research described in this paper is to develop a simple calculation method to determine the in-plane stability of a portal frame with a haunch, including the effect of the haunch on the beam behaviour. The research examines existing methods for frames without haunches and determines if these methods still hold for frames with haunches.

First the generally applied method of Horne [1] as well as analytical models [2] are used to calculate the effect of the stiffness of the connection. Secondly, the rotational stiffness of the beam supports is regarded as a variable factor, depending on the loading on the beam (with or without a compressive force). This effect is examined by numerical calculations which are verified by a mechanical solution based on the energy method. And finally the effect of the haunch dimensions on the column behaviour is taken into account.
2 BEHAVIOUR AND CRITICAL LOAD OF FRAMES ACCORDING TO EC3

The research is carried out on a frame geometry and frame dimensions such that the frame is stiff enough to withstand large deformations, but weak enough to generate second order effects. According to [1] an amplification factor takes into account the second order effects.

\[ F_2 = \frac{\alpha_{cr}}{\alpha_{cr} - 1} \times F_1 \]  

(1)

where \( F_2 \) is the second order force
\( F_1 \) is the first order force
\( \alpha_{cr} \) is the ratio between Euler buckling load \( F_{cr} \) and the design load \( F_{Ed} \)

Eurocode 3 [1] provides several methods to calculate second order effects. When focusing on a one level orthogonal frame, Horne’s approximation formula as used in [1] can be expressed as follows:

\[ \alpha_{cr} = \left( \frac{H_{Ed}}{V_{Ed}} \right) \left( \frac{h}{\delta_{H,Ed}} \right) \]  

(2)

where \( H_{Ed} \) is the design value of the horizontal reaction force at the support of the frame, including the effect of fictive additional horizontal loads
\( V_{Ed} \) is the design value of the total vertical load on the structure
\( \delta_{H,Ed} \) is the horizontal displacement at the top of the frame
\( h \) is the height of the frame

Other methods, as the King’s method and the sway buckling length method, are not regarded in this paper, although reliable according to [2].
In the considered research the beam to column connection is realized by welded filler plates which stiffen the connection. The connection between the beam and the columns is considered flexible. The rotation of the beam will be slightly more than the rotation of the column. Fig. 1 represents the general mechanical model.

![Mechanical model of a beam with flexible supports/connections](image)

**Fig. 1.** Mechanical model of a beam with flexible supports/connections [3]

The connection stiffness can then be defined as follows:

\[ C_r = \frac{M}{\phi_r} \]  

(3)

where \( C_r \) is the rotational stiffness of the connection
\( M \) is the bending moment acting on the connection
\( \phi_r \) is the rotation caused by the flexibility of the connection

The exact stiffness of a connection is hard to determine. Therefore the stiffness of the connection is retrieved from a shell element model. This stiffness functions as a benchmark for the connection.
stiffness in the further research. The stiffness according to Eurocode regulations as well as a Dutch approximation methods [4] are safe values of the benchmark. The shell element solution leads to a little stiffer connection, see Fig. 2. However, the impact of this difference is negligible looking at the total frame behavior. For the further analysis only haunched cases are relevant. When stiffening is applied then the shell element analysis results in 11% higher stiffness towards EC3 (see [2]). One could say that EC3 is on the safe side.

![Connection stiffness](image)

*Fig. 2. Connection stiffness; comparison between several calculation methods for an unhaunched and unstiffened beam to column connection (see [2])*

To determine the buckling load with great precision a numerical analysis can best be used. With this method the total buckling behavior can be analyzed. For the vertical load two different situations can be distinguished: a point load scenario on the column and a uniformly distributed load scenario on the beams. In this study the governing buckling mode of the frame is a sway mode.

In case of the point load scenario the normal force distribution in the frame shows buckling can only occur in the column; no compressive forces are present in the beam. To calculate the stability of the sway buckling shape, the frame is divided in a column and a beam section. The beam provides support to the column in the form of a rotational spring. The rotational spring at the top of the column is again related to a combination of the stiffness of the beam and the stiffness of the connection. A schematization of the sway buckling mode is given in Fig. 3.

![Sway mode and schematization](image)

*Fig. 3. Schematization of a sway buckling mode for a point load scenario*

In case of the uniformly distributed load scenario the distribution of the vertical loads will lead to a different set of reaction forces. Both supports are fixed and therefore horizontal reactions occur. The rotational spring at the top of the column is again related to a combination of the stiffness of the beam and the stiffness of the connection. A schematization of the sway buckling mode is given in
Fig. 4. The most significant difference with the point load scenario is that a first order bending moment is generated at the connection.

3 STIFFNESS OF THE BEAM WITH A HAUNCH

Based on the numerical model for the connection stiffness described before the impact of the variable rafter stiffness at the haunches as well as the impact of a compressive force in the rafter can be studied further. A simple way to calculate the contributed stiffness from the beam is a hand calculation, but also other methods, such as the equilibrium method, the energy method, and finite element methods, are available. To cover the applicability of a numerical analysis (based on shell elements) for a parameter study, next mechanical analyses were performed:
- LEA of the beam without a compressive force
- LEA of the beam with a compressive force
- LBA of the beam with a compressive force

The geometry of the basic frame is given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Frame properties [mm]</th>
<th>column</th>
<th>beam</th>
<th>haunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>$L$</td>
<td>8000</td>
<td>30000</td>
</tr>
<tr>
<td>web height</td>
<td>$h_w$</td>
<td>352</td>
<td>373</td>
</tr>
<tr>
<td>web thickness</td>
<td>$t_w$</td>
<td>11</td>
<td>8.6</td>
</tr>
<tr>
<td>flange width</td>
<td>$b_f$</td>
<td>300</td>
<td>180</td>
</tr>
<tr>
<td>flange thickness</td>
<td>$t_f$</td>
<td>19</td>
<td>13.5</td>
</tr>
</tbody>
</table>

3.1 LEA of the beam without a compressive force

As stated before, the goal of the research was to come up with a simple hand calculation rule, comparable to the one for beams in portal frames without haunches. The general mechanical model for beam elements with connections is as illustrated in Fig.1. The haunch increases the rotational stiffness of the beam. According to [3] this additional stiffness can safely be covered by a factor $\eta_1$, where this factor shall be used as an amplification factor for the original unstiffened rotation stiffness, see equation 4.

$$ C_{rafter} = \frac{6 \times \eta_1 \times EI}{L} $$

where $C$ is the adapted rotational stiffness at the end of the rafter,
$\eta_1$ is the stiffness correction factor,
$EI$ is the bending stiffness of the unhaunched beam section,
$L$ is the length of the beam.
So, to use the rule of thumb for a beam with a haunch, the equation to determine the rotational spring stiffness of the beam has to be adapted, see equation (4). The results of all in [2] elaborated calculation methods are shown in Table 2. As can be seen all calculations of the rotational stiffness are close to or similar to the benchmark.

| Table 2. Rotational stiffness of a haunched beam using several calculation methods |
|---------------------------------|----------------|------------------|----------------|----------------|
| method                         | rotation [Rad]| moment [kNm]    | stiffness C_r [Nm/Rad] | deviation % |
| hand calculation               | -              | -               | 13.20 10^9            | 0.76         |
| equilibrium method             | 0.015405       | 100             | 13.20 10^9            | 0.76         |
| energy method                  | -              | -               | 13.20 10^9            | 0.76         |
| FEM beam elements              | 0.10208        | 669             | 13.20 10^9            | 0.54         |
| FEM shell elements             | 0.01533        | 100             | 13.10 10^9            | 0            |

Table 3 shows that the average stiffness correction factor is dependent on the haunch length ratio and the haunch width ratio. The factor is independent of the length and the cross section of the beam.

| Table 3. Factor η_1 for several haunch dimensions related to the beam dimensions |
|---------------------------------|----------------|----------------|----------------|----------------|
| Relative haunch length in %     | 2.5            | 5.0            | 10.0           | 15.0           | 20.0           |
| Relative haunch height in %     |                |                |                |                |                |
| 50                              | 1.072          | 1.145          | 1.297          | 1.477          | 1.615          |
| 100                             | 1.093          | 1.195          | 1.417          | 1.712          | 1.965          |
| 150                             | 1.108          | 1.228          | 1.507          | 1.903          | 2.275          |
| 200                             | -              | 1.253          | 1.573          | 2.058          | 2.583          |
| 250                             | -              | 1.270          | 1.625          | 2.183          | 2.760          |
| 300                             | -              | 1.283          | 1.665          | 2.283          | 2.947          |

3.2 LEA of the beam with a compressive force
The center line of the beam changes due to the haunch dimensions. The linear elastic calculation of the stiffness will change when a normal force is applied at the supports. This normal force will cause a first order moment on the beam (see Fig. 5). The eccentricity depends on the haunch height in relation to the beam height.

Fig. 5. Beam with a haunch where the compressive force is applied at the heart line of the cross-section of the beam

The factor η_1 does not hold for a situation of a beam with a haunch and a compressive force. A correction factor η_2 is included, where this factor can be described as follows (based on [2]):

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where \( h \) is the height of the column (height of the frame)
\( L \) is the length of the beam (span of the frame)
\( e \) is the maximum eccentricity of the haunch towards the beam
\( a \) is the length of the haunch
\( \eta_1 \) is the stiffness correction factor
\( \eta_2 \) is the adapted stiffness correction factor

### 3.3 LBA of the beam with a compressive force

The second order effect of a beam with a haunch is determined in an equal way as for a beam without a haunch. The additional stiffness the haunch provides is covered with the stiffness factor \( \eta_2 \).

![Beam with a haunch where the compressive force is applied at the center line of the cross-section of the beam resulting in a different buckling shape](image)

For the equally distributed scenario an additional mechanism shall be considered, above the influence of the increased beam stiffness. The beam is vulnerable for buckling and therefore the rotational stiffness has to be reduced. However, buckling of the beam does not occur at the design load but at the load at which the total frame buckles. The calibration of the critical buckling load is thus an iterative process, where the determination of the rotational stiffness goes as follows:

1. First iteration step:
   - The stiffness of the beam is calculated for the design load scenario.
   - The critical buckling load of the column is determined with the adapted rotational spring stiffness at the top of the column (hinged support at the column foot).

2. Second step:
   - The stiffness of the beam is calculated for the design load scenario times the buckling factor \( \alpha_{cr} \) of the column, determined in the previous step.
   - The critical buckling load of the column is determined with the again adapted rotational spring stiffness at the top of the column.

3. Et cetera.

This leads to a proper approximation of the rotational spring stiffness at the top of the columns, which can be taken into account for further calculations. The second order effect is determined with

\[
\varphi_{2^{nd\, order}} = \frac{\alpha_{cr}}{\alpha_{cr} - 1} \times \varphi_{1^{st\, order}}
\]

where \( \varphi_2 \) is the second order rotation
\( \varphi_1 \) is the first order rotation
\( \alpha_{cr} \) is the ratio between critical buckling load \( F_{cr} \) and design load \( F_{Ed} \)

So the rotational spring stiffness becomes
where $C_r$ is the adapted rotation stiffness at the end of the rafter
$\eta_2$ is the adapted stiffness correction factor
$EI$ is the bending stiffness of the unhaunched beam section
$L$ is the length of the beam
$\alpha_{cr}$ is the ratio between critical buckling load $F_{cr}$ and design load $F_{ed}$

The first order rotational stiffness of a beam with a haunch and a normal force is presented in section 3.1. The buckling effect and its result are presented in Table 4.

### Table 4. Comparison buckling factor $\alpha_{cr}$ and rotational stiffness $C$ of a haunched beam

<table>
<thead>
<tr>
<th>method</th>
<th>$\alpha_{cr}$</th>
<th>$C$ [Nmm/Rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>hand calculation</td>
<td>30.48</td>
<td>$11.53 \times 10^9$</td>
</tr>
<tr>
<td>equilibrium method</td>
<td>30.48</td>
<td>$11.53 \times 10^9$</td>
</tr>
<tr>
<td>energy method</td>
<td>-</td>
<td>$11.60 \times 10^9$</td>
</tr>
<tr>
<td>FEM beam elements</td>
<td>30.38</td>
<td>$11.50 \times 10^9$</td>
</tr>
<tr>
<td>FEM shell elements</td>
<td>30.83</td>
<td>$11.44 \times 10^9$</td>
</tr>
</tbody>
</table>

## 4 PARAMETER STUDY

A parameter study is carried out to create a proper view of the behaviour of different portal frames (see Table 5). The span length, the height and the beam section vary. Every model is investigated for the two different load scenarios.

The portal frames considered in the parameter study are for a single type of haunch: the haunch is constructed by parts of the beam and the haunch length is 10% of the span of the frame. The maximum height of the haunch is equal to the height of the beams. This covers the design of haunches as described in [5] and [6].

### Table 5. Parameter combinations (√ = carried out, X= not carried out)

<table>
<thead>
<tr>
<th>height [m]</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>span [m]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>30</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>40</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>beam section</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IPE 300</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>IPE 400</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>IPE 500</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>IPE 600</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

The numerical model is composed by shell elements SHELL181, using ANSYS [4]. Properties of this element type include residual stresses, a large deformation capacity and a large rotation capacity. The FEM calculations are performed for the entire parameter study. For each frame the factor $\alpha_{cr}$ is calculated using the shell element model. The results of the parameter study are shown in Fig. 7. The parameter combinations 1 to 6 are explained in Table 6. As illustrated in Fig. 7 the effect of the haunch is largely dependent on the type of loading and the slenderness of the beam. Remarkable is that, within the considered parameter study, the impact of haunches is very small in the case of equally distributed loading on frames with very slender beams (low ratio beam height versus span length).
Further, it should be mentioned that a haunch reduces the physical buckling length of a column, see Fig. 8. The reduction of the buckling length of the column is dependent on the height of the haunch. In general the effect of this reduction is small, but for relatively large haunches the effect was taken into account.

As a result of the parameter study, a comprehensive design procedure can be created to come up with an easy way to determine the in-plane stability of a haunched portal frame. A distinction is made between frames loaded by vertical loads at the column position and frames loaded by an equally distributed vertical load on the beam. In the last case a compressive force in the beam will influence the frame stability. For the cases investigated several methods that can be used for calculating the impact of the connection stiffness hold. To be able to use a simple calculation method comparable to the method of Horne, the effect of a haunch on the frame behaviour is proposed to be taken into account by an amplification factor for the original beam stiffness.

5 RESULT

As a result of the parameter study, a comprehensive design procedure can be created to come up with an easy way to determine the in-plane stability of a haunched portal frame. A distinction is made between frames loaded by vertical loads at the column position and frames loaded by an equally distributed vertical load on the beam. In the last case a compressive force in the beam will influence the frame stability. For the cases investigated several methods that can be used for calculating the impact of the connection stiffness hold. To be able to use a simple calculation method comparable to the method of Horne, the effect of a haunch on the frame behaviour is proposed to be taken into account by an amplification factor for the original beam stiffness.
For the point load scenario the effect of the haunch on the in-plane stability can be taken into account by a beam stiffness amplification factor \( \eta_1 \) which is verified for haunches made from the same material and the same web and flange thickness of the beam. The factor is independent on the length and cross-section of the beam and is then used to multiply the original beam stiffness. This results in a higher buckling resistance of the beam and thus the rotational spring stiffness at the top of the fictional column increases. The remaining procedure stays the same, where:

- The rotational stiffness of the connection is changed by the haunch. Eurocode 3 regulations cover this influence.
- The stiffness of the beam is changed by the haunch. An amplification factor \( \eta_1 \) for the beam stiffness shall be used to take into account the effect of the increased beam stiffness in the haunches.
- The buckling length of the column can be reduced; it can be set to the length between the center of the haunch and the bottom of the column (instead of the center of the beam and the bottom of the column).

For the equally distributed load scenario the effect of the haunch on the in-plane stability can be taken into account by a beam stiffness amplification factor \( \eta_2 \) which is also valid for haunches made from the same material and the same web and flange thickness of the beam. For the equally distributed scenario an additional mechanism shall be considered, above the influence of the increased beam stiffness. The beam is vulnerable for buckling and therefore the rotational stiffness has to be reduced. The calibration of the critical buckling load is an iterative process which leads to a proper approximation of the rotational spring stiffness at the top of the columns, which can be taken into account for further calculations.

6 SUMMARY AND CONCLUSIONS

In this paper an efficient and simple method to take into account the effect of a haunch on the frame stability is described. The method implements the influence of the haunch dimensions on the frame behaviour by an amplification factor \( \eta_1 \) for the original beam stiffness. The effect of the horizontal compressive force in the beam in the case of an equally distributed beam load results in an additional first order bending moment in the beam. The effect of this phenomenon on the frame stability is covered by an adapted amplification factor \( \eta_2 \) for the original beam stiffness.

Within the research limits regarding the \( L/h \) ratio and haunch/beam length ratio, the existing (simple) methods to determine the in-plane stability of portal frames can still be used. Numerical studies have shown that with the adapted stiffness, hand calculations are still safe enough for verification of the sway stability of haunched portal frames. The amplification factor is dependent on the dimensions of the frame and the dimensions of the haunches. For a relative large ratio between span length and beam height the impact of a haunch on the frame stability may become very small.

REFERENCES


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