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Disposal versus rework – Inventory control in a production system with random yield

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ABSTRACT

In a production environment where random yield plays a fairly significant role, a decision has to be made on how to handle products that do not satisfy given quality requirements. We consider a single-stage production system with a positive production time and random yield. To ensure that only high quality items are sold to the customer, a post-production quality control system has been put in place. We compare two different strategies for defective items: disposal or rework. Disposal is possible without any time delay whereas the rework process requires a positive rework time. While disposed-of items leave the production process, reworked products stay in the process and are assumed to be as good as products that are perfect when they are initially produced. The end products are stored in a warehouse to satisfy stochastic demand. We show how to determine the optimal base-stock level, which is very difficult because of unknown covariances between orders. Subsequently, an optimization model is proposed to support the planner's decision on which strategy to choose when it comes to whether to dispose of or rework defective items. By means of a sensitivity analysis we show which parameters directly affect this decision and give managerial insights. The analysis indicates that significant cost reductions can be obtained by choosing the best strategy for defective products.

1. Introduction

Customer service plays an especially important role in highly competitive markets where dissatisfaction about e.g., product quality leads to a loss of the customer's goodwill resulting in the customer selection of a new vendor. Thus, the plan must be to sell only high-quality products. A problem arises when the production process is not perfect, such that random yield losses occur. Random production yield is a common problem in the high-tech industry with complex production processes. For example, in the production of microchips, yields differ between 60% and the high 90% range depending on the manufacturer (Foremski, 2012). A second example is the production of curved glass for the display of a new cell phone series, where Samsung has to deal with yields down to less than 50% (McNutt, 2015). In such an environment, where sometimes more than every second item is defective, it is obvious that random production yield cannot be neglected. To guarantee that only high-quality products are sold to the customers, a quality control inspecting 100% of all produced items is required. The items passing the inspection are stocked in a warehouse to serve stochastic customer demand. This begs the question, how to handle all the defective products which should not be sold to the customer due to poor quality. Several opportunities arise: the products can either be scrapped (see e.g., Yano & Lee, 1995; Huh & Nagarajan, 2010; Inderfurth & Kiesmüller, 2015; Sonntag & Kiesmüller, 2017), sold as lower quality products for a lower price (see e.g., Gerchak, Tripathy, & Wang, 1996; Hsu & Bassok, 1999), reworked (see e.g., Wein, 1992; Grosfeld-Nir & Gerchak, 2004) or used otherwise.

In this paper, we study the strategic choice between scrapping and reworking which means that the planner can decide between these opportunities only at the beginning of the planning horizon. We assume that reworked items satisfy all quality requirements and are as good as items that are well made from the start and can be sold for the same price (see e.g., Inderfurth, Lindner, & Rachaniotis, 2005; Gotzel & Inderfurth, 2005; Buscher & Lindner, 2007). Reworking defective products to raise their quality might be desirable for a company for several reasons: First of all, rework might be reasonable for economic reasons. This is the case when defective products are of substantial value because of expensive input materials, e.g., in the high tech industry (Buscher & Lindner, 2007; Inderfurth et al., 2005) or when the time and cost for rework are

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lower than for the initial production of new items. Second, new legislation might force companies to reduce waste (Teunter & Flapper, 2003). Third, ecological aspects are gaining more and more attention and therefore influence the waste policy and the image of the company (Inderfurth et al., 2005). The image of a company has an influence on customer satisfaction and therefore on sales and the equity, which provide a competitive advantage especially in highly competitive markets where it is difficult to differentiate between the products (Chen, 2010; Teunter & Flapper, 2003). Flapper, Fransoo, Broekmeulen, and Inderfurth (2002) give an overview of industries mentioned in the literature where rework plays an important role for at least one of the specified reasons (e.g., the semiconductor and pharmaceutical industry).

We consider a single-stage production system producing batches of items with a known and constant production time independent of the batch size, and stochastic proportional yield, which leads to a random number of defective items. In a stochastic proportional yield model, the yield is a random multiple of the input (Henig & Gerchak, 1990). Subsequent to the production process, a quality control system is in place, inspecting all items with no time delay. Items satisfying the quality requirements are stocked in a warehouse to satisfy incoming stochastic customer demand, whereas defective items are either entirely disposed of or reworked. Note that once the planner has chosen one of these strategies, he cannot change it in the near future. The rework process – like the production process – corresponds to a known and constant rework time but is performed on a different machine. Thus, the production and the rework process require different resources. The rework process brings all defective items in a condition equal to that of perfectly produced products such that they can be stored in the warehouse as well. The warehouse has to initiate the production of a batch of items with varying lot size periodically to replenish stock.

The literature includes work on imperfect production systems where defective products are either scraped or reworked – totally or partially. Yano and Lee (1995) give a literature overview of different problem settings and approaches to solving imperfect production environments where yield losses are disposed of. In the following discussion, we will amplify two different streams of literature: make-to-stock models under random yield with disposal of defective items and make-to-order models under random yield with rework of defective items.

The literature on make-to-stock production systems, combining random yield settings with inventory control strategies, can be divided into two groups: production time zero or one (e.g., Henig & Gerchak, 1990; Bollapragada & Morton, 1999; Huh & Nagarajan, 2010) and arbitrary positive production times (Dettenbach & Thonemann, 2015; Inderfurth & Kiesmüller, 2015; Inderfurth & Vogelgesang, 2013; Sonntag & Kiesmüller, 2017). Production times of zero or one period (called zero production time in the following discussion) substantially reduce the complexity of the problem since no uncertainties of outstanding orders have to be considered, which means that the inventory position used to determine the order quantity is known. For zero production times, Henig and Gerchak (1990) and Bollapragada and Morton (1999) focus on the optimal order policy whereas Huh and Nagarajan (2010) concentrate on the optimization of the policy parameters in case of a linear inflation rule. A linear inflation rule is the commonly used heuristic order policy under random production yield since the optimal ordering policy is very complex (Henig & Gerchak, 1990).

Positive production times involve an uncertainty in the inventory position which makes the problem far more complex. Therefore, only a few authors have considered positive production times (e.g., Inderfurth & Vogelgesang, 2013; Dettenbach & Thonemann, 2015; Inderfurth & Kiesmüller, 2015; Sonntag & Kiesmüller, 2017). Inderfurth and Vogelgesang (2013) present concepts to determine safety stocks under different types of yield randomness. Dettenbach and Thonemann (2015) take into consideration multistage production systems with the aim of determining the location of quality inspections to obtain real-time yield information which reduces the required safety stock. They use dynamic programming for small and medium-sized problems and two heuristic approaches for larger problems. One of the heuristics is based on an idea of Ehrhardt and Taube (1987) and can lead to poor results depending on the parameter setting. The second heuristic is based on an idea of Huh and Nagarajan (2010) and leads to very good results but requires simulation since "it is difficult to calculate [...] analytically" (Dettenbach & Thonemann, 2015). Since the optimal order policy is difficult to determine and dynamic programming as well as simulation might involve high computation times, Inderfurth and Kiesmüller (2015) developed a new heuristic solution method. The so-called steady-state approach leads to very good results and is easy to implement in a spreadsheet. While Inderfurth and Kiesmüller (2015) present a single-stage production system, Sonntag and Kiesmüller (2017) extend the approach to analyze multi-stage production systems with in-between quality inspections. All these papers have in common that defective items are disposed of and therefore leave the production system.

Unlike the limited literature on make-to-stock production systems with random yield and disposal of defectives, there exists a variety of literature on make-to-order systems where defective items are reworked. Most researchers investigate production and rework processes on the same machine and thus have to solve a scheduling problem (e.g., So & Tang (1995)) or a lot-sizing problem (see e.g., Liu & Yang, 1996; Teunter & Flapper, 2003; Inderfurth, Kovalyov, Ng, & Werner, 2007; Grosfeld-Nir & Gerchak, 2002; Wein, 1992). Jaber and Khan (2010) extend the production system with learning curves for the time to produce and rework items.

There is only one paper, by Gotzel and Inderfurth (2005), where an inventory-control policy in a production environment with random yield and stochastic demand is studied in which defective items are reworked. While Gotzel and Inderfurth (2005) assume that defective items can be temporarily stocked before rework, we consider a situation where defective items have to be reworked immediately. As an example, consider the steel industry where steel coming out of the furnace is inspected for quality. If it does not satisfy the given quality standards it has to be returned to a furnace. This reworking in a furnace is much faster when the steel is still hot and has had no time to cool down. In such a situation intermediate stock points are not favorable for economic reasons. Other examples can be found in industries such as the chemical, pharmaceutical or food industries, where items cannot be stored to wait for rework. Further, it is sometimes the case that there is no storage space available for items waiting for rework or that the company wants to reduce work in process inventory.

Allowing only one stockpoint in the system has consequences for the analysis, because no decision has to be made on the amount to be reworked. The rework quantity is dependent only on the production output, and therefore on the order quantity of a previous period and the realized yield. The number of defective items increases as the order quantity increases, thus the order quantity in a previous period determines the amount to be reworked in a later period. If a large number of reworked items arrive at the warehouse, the order quantity in the actual period can be reduced. Therefore, the actual order quantity depends on previous order quantities and thus covariances occur.

Aside from the fact that our model includes only one stockpoint we define the inventory position differently from Gotzel and Inderfurth (2005) and include only information about orders arriving during the risk period and thus only relevant information for the actual decision.

The contribution of this paper is as follows: (1) We show how to determine the base-stock level in a production environment where defective products are not disposed of but reworked and thus stay within the system. The induced covariances cannot be calculated easily and thus we propose an approximation. In a detailed numerical study we illustrate the excellent performance of our approximation. (2) We introduce a mathematical model representing production, quality control, disposal or rework as well as holding and backorder costs. The model can be used as a decision support tool for a planner when he or she has to decide if defective items should be disposed of or reworked. We show which parameters have an influence on this decision and give an idea on how robust the decision of the planner is regarding changes in the environment or the cost parameters. With the derived model we also gain some managerial insights.

The remainder of the paper is organized as follows: In Section 2, we describe the multi-stage production system and formulate the model. In Section 3, the steady-state approach is introduced. Since covariances between orders occur, which are not easy to calculate but cannot be neglected, we present an approximation and analyze its accuracy in Section 4. In Section 5, we present a mathematical model considering above-mentioned cost parameters, and analyze the effect of changes within the input parameters of the production system (5.3) and the cost parameters (5.4) on the decision whether to rework or dispose of defective items. Based on this analysis, we formulate managerial insights in Section 5.5. We conclude with a summary and suggestions for future research in Section 6.

2. Model formulation

We consider a single-stage production system producing batches of one single product with a constant production time of L_P periods per batch ($L_P > 0$). The production time can be independent of the batch size in, for example, the chemical industry where processing times are often independent of the amount being produced. Further, in the context of an MRP planning system, planned lead times are assumed to be constant, even though some variability exists, in order to enable coordinated decisions.

Due to deficiencies in the production, not all produced items are of perfect quality. Since it is not desirable to sell products of lower quality to the customers, a quality inspection subsequent to the production process is established. The inspection station checks the quality properties of all produced items with no time lag. We would like to note that the time for an inspection can be included in the production time since all items pass quality control. Thus, neglecting any delay for the inspection does not reflect a limitation in the model.

Items that satisfy the quality requirements are stocked in a warehouse to serve incoming stochastic customer demand. We assume that the demand across periods is independent and identically distributed (iid) and backlogged if it cannot be satisfied directly from stock. Inderfurth and Kiesmüller (2015) as well as Dettenbach and Thonemann (2015) and Sonntag and Kiesmüller (2017) analyzed single or multi-stage production systems where defective items are scrapped. In contrast to these contributions, we focus on a situation where defective items are reworked. The reworking process – like the production process – requires a rework time of L_R time units ($L_R > 0$) whereas the rework time can be either smaller, equal to or larger than the production time. After rework these products are stocked in a warehouse with the same quality as items that were perfect when first produced. Note that the rework process proceeds on a different machine from the one used in the production process, which means that different resources are required and they do not interfere with each other.

Fig. 1 illustrates the whole model composed of a production and a rework process, a quality-control process and a warehouse for the final product. The sequence of events in one period is as follows: First, the good items of the order placed L_P periods before as well as the reworked items of the order placed $L_P + L_R$ periods before are delivered. Subsequently, a new order is placed and demand occurs. At the end of the period, inventory holding and backorder costs are charged based on the inventory level.

We apply a stochastic proportional yield model which is commonly used to describe random yield due to an imperfect production process (Yano & Lee, 1995). In a stochastic proportional yield model the output $Y(Q)$ of the production process equals a positive fraction Z of the input Q such that $Y(Q) = Z \cdot Q$. In our model, the input for production is determined by the order quantity Q requested by the warehouse to refill stock and ensure that demand can be satisfied. $Z \in [0, 1]$ is a random variable called the yield factor with mean μ_Z and variance σ_Z^2 and is iid across the periods and independent of the demand distribution.

As already mentioned, at the beginning of each period, after receiving the batch of a prior order, the warehouse has to determine the required order quantity in order to minimize average holding and backorder costs. We consider a periodic review base-stock policy with a review period of one time unit and a base-stock level S. Such a policy is used because the optimal ordering policy for production systems with random yield does not possess a simple structure, even for production times equal to zero (Henig & Gerchak, 1990). In this paper, positive instead of zero production times are considered, which increases the complexity of the system and makes it even more difficult to determine the optimal policy structure. Therefore, we propose a heuristic ordering policy, which is easy to implement. The periodic review base-stock policy is such a candidate, and it is also optimal if there is no uncertainty in the yield. In such a situation the order quantity in period t equals the difference between the base-stock level S and the actual inventory position IP_t (Tempelmeier, 2006): $Q_t = S - IP_t$, where the inventory position is defined as the physical stock on hand minus backorders plus the outstanding order quantities.

In case of random production yield and a positive production lead time it is not reasonable to include the outstanding order quantities, because the production output is uncertain. Therefore, we have to define the inventory position differently and suggest the use of the expected amount to be delivered instead of the outstanding order quantities, as long as we do not have information about the realized yield. This means that for all orders in the production process the yield is unknown and the expected amount to be delivered is included in the inventory position, while for all orders in the rework process realized yield is known and the estimates can be updated. Further, we only include information about the orders that will be delivered during the risk period (see also Kiesmüller, 2003). The risk period equals $L_P + 1$ review periods, because between review periods it is not possible to influence the amount of incoming items, either from production or from rework. Altogether, we get the following definition of the inventory position IP_t at the beginning of period t before ordering:

$$IP_t = \begin{cases} IL_{t-1} + Z_{t-L_P} Q_{t-L_P} + \sum_{l=1}^{L_P-1} \mu_Z Q_{t-l} + \sum_{l=L_P}^{L_P+L_R-1} (1-Z_{t-l}) Q_{t-l} \\ \quad + (1 - Z_{t-L_P-L_R}) Q_{t-L_P-L_R} + \sum_{l=L_R}^{L_P-1} (1-\mu_Z) Q_{t-l} & , \ L_P \geq L_R \\ IL_{t-1} + Z_{t-L_P} Q_{t-L_P} + \sum_{l=1}^{L_P-1} \mu_Z Q_{t-l} + \sum_{l=L_R}^{L_P+L_R-1} (1-Z_{t-l}) Q_{t-l} \\ \quad + (1 - Z_{t-L_P-L_R}) Q_{t-L_P-L_R} & , \ L_P < L_R \end{cases} \quad (1)$$

Note that the realized yield is modeled with a random variable (Z_t) because all possible values have to be considered. However,

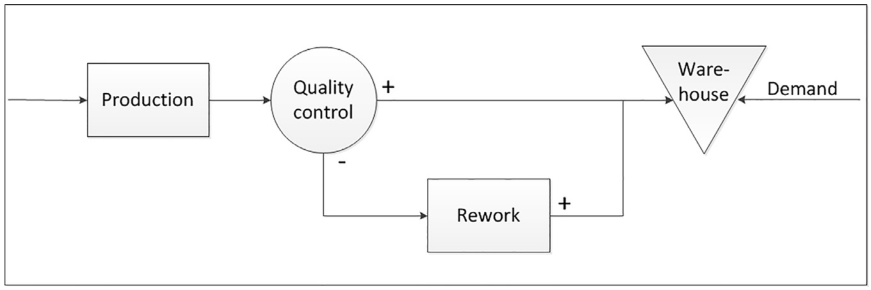

Fig. 1. A single-stage make-to-stock production system with random yield and rework.

if the model is applied in practice the random variables have to be replaced by the observed realized values. For example, if the planner has to decide on the order quantity, he uses the actual values for the yield (z_t), the demand (d_t) and the outstanding order quantities (q_t).

It is clear that we also have to distinguish between a situation where the production time is larger than or equal to the rework time, and one where the production time is smaller than the rework time. For the first case ($L_P \geq L_R > 0$), the inventory position equals the inventory level IL_{t-1} at the end of the previous period plus the sum of the following components: The first term $Z_{t-L_P}Q_{t-L_P}$ represents the delivered number of good items of the order placed in period $t-L_P$, because in the moment when production is finished, the yield $Z_{t-L_P}Q_{t-L_P}$ is known. The second term $\sum_{l=1}^{L_P-1}\mu_Z Q_{t-l}$ represents all orders still in production such that the yield is not known and therefore the expected amount to be delivered after production has to be estimated. The following term $\sum_{l=L_P}^{L_P+L_R-1}(1-Z_{t-l})Q_{t-l}$ equals outstanding quantities within the rework process, where yield is known because the production process for these items has already been completed. $(1-Z_{t-L_P-L_R})Q_{t-L_P-L_R}$ equals the number of delivered reworked items of the order placed L_P+L_R periods before. The last term $\sum_{l=L_R}^{L_P-1}(1-\mu_Z)Q_{t-l}$ is related to the orders in production and represents an estimate of the number of units which will not satisfy the quality requirements and therefore have to be reworked.

It is important to note that while all outstanding orders within the production process are included in the inventory position ($\sum_{l=1}^{L_P-1}\mu_Z Q_{t-l}$), only a part of the outstanding orders in the rework process is taken into account ($\sum_{l=L_R}^{L_P-1}(1-\mu_Z)Q_{t-l}$), because we consider only orders that arrive during the risk period. Thus, in contrast to Gotzel and Inderfurth (2005), who include all outstanding orders, we only include orders which will be delivered to the warehouse during the risk period of L_P+1 periods.

In cases where the rework time exceeds the production time ($L_R > L_P > 0$), the inventory position consists of the same elements with one difference: the last term, the defective quantities thus so far unknown, which will enter the rework process in future periods, does not appear. If the production time is smaller than the rework time, no order exists where the quantity of defective units is not yet known but will be delivered during the risk period of length L_P+1.

We can merge some terms in (1) and reformulate the inventory position as

$$IP_t = \begin{cases} IL_{t-1} + \sum_{l=L_P+1}^{L_P+L_R}(1-Z_{t-l})Q_{t-l} + \sum_{l=L_R}^{L_P} Q_{t-l} + \sum_{l=1}^{L_R-1}\mu_Z Q_{t-l} & , L_P \geq L_R \\ IL_{t-1} + \sum_{l=L_R}^{L_P+L_R}(1-Z_{t-l})Q_{t-l} + Z_{t-L_P}Q_{t-L_P} + \sum_{l=1}^{L_P-1}\mu_Z Q_{t-l} & , L_P < L_R \end{cases} \quad (2)$$

with $\sum_{i=a}^{b} x_i = 0$ for $b < a$.

Since the inventory position includes the expected number of units to be delivered, the moment when the realized yield is observed the inventory position has to be updated as follows.

$$IP_{t+1} = IP_t - D_t - (\mu_Z - Z_{t+1-L_P})Q_{t+1-L_P} \quad (3)$$

It can happen that the realized yield (Z_{t+1-L_P}) is much larger than expected (μ_Z), which increases the inventory position without placing an order. In extreme cases, which occur vary rarely, it is possible that the inventory position before ordering is already above the base-stock level. Therefore, the ordering policy in case of random yield has to be adjusted as follows:

$$Q_t = \begin{cases} S - IP_t, & IP_t < S \\ 0, & IP_t \geq S \end{cases} \quad (4)$$

Note that we do not apply a linear inflation policy as often used in the random yield literature with disposal of defective items, because all ordered units will arrive. There is only a difference in the observed lead time, because some of the units have to be reworked. Inflation factors are necessary if units not satisfy quality requirements are disposed of.

The inventory position in formula (2) is not only used to determine the order quantity, but it is also used in our model to determine the inventory level IL_{t+L_P} at the end of period $t+L_P$:

$$IL_{t+L_P} = IP_t - \sum_{l=0}^{L_P} D_{t+l} + \mu_Z Q_t - R_t - \sum_{l=1}^{min\{L_P,L_R\}-1} R_{t-l} \quad (5)$$

The term $\sum_{l=0}^{L_P} D_{t+l}$ equals the demand that occurs between the beginning of period t and the end of period $t+L_P$. The second term reflects the estimated amount to be delivered in period $t+L_P$ from the order, placed in period t, where the yield is unknown when production starts. The next terms are related to the updates of the inventory position as shown in Eq. (3) which are done in each period, when yield is realized. We call the difference between expected and realized yield for a given order quantity Q_t the forecast error, which is defined as

$$R_t = \mu_Z Q_t - Z_t Q_t. \quad (6)$$

It is obvious that the inventory level as given in (5) is a function of the base-stock level S and hence we denote it as $IL(S)$ in the following discussion. The higher the base-stock level S, the higher the stock-on-hand and the lower the backorder quantities and vice versa. Our objective is to determine a base-stock level S, which minimizes the average holding and backorder cost $C(S)$:

$$C(S) = hE[(IL(S))^+] + bE[(-IL(S))^+] \quad (7)$$

where h denotes the unit holding and b the unit backorder costs, and $(M)^+$ is defined as $max\{0, M\}$.

Determining the optimal base-stock level in case of random yield is not easy in the presence of positive production times, even without reworking defective items. For production systems where imperfect products are scraped instead of being reworked, previous approaches in the literature involve high computation times due to the application of Markov chains (Dettenbach & Thonemann, 2015; Inderfurth & Kiesmüller, 2015), simulation (Dettenbach & Thonemann, 2015; Inderfurth & Kiesmüller, 2015) or stochastic dynamic programming (Gotzel, 2010). Therefore, Inderfurth and Kiesmüller (2015) introduced an approximate steady-state approach for a single-stage production system where defective items are disposed of. The performance of the approach has been shown to be excellent while it is easy to implement in a spreadsheet. Because of the excellent performance of the approach and the absence of efficient solution methods for production systems where defective items are reworked, we adapt the idea for the production system described above.

As a starting point for this approach, formula (7) can be rewritten as

$$C(S) = h\int_0^\infty x\varphi_{IL}(x)dx - b\int_{-\infty}^0 x\varphi_{IL}(x)dx \tag{8}$$

where φ_{IL} reflects the probability density function of the inventory level IL defined in (5). To determine the base-stock level S which minimizes the average holding and backorder cost, the distribution of the inventory level IL with the density function φ_{IL} is required.

3. Determining the base-stock level

To calculate the average cost for a given base-stock level, Inderfurth and Kiesmüller (2015) showed that, for symmetric demand distributions, a normal distribution with mean μ_{IL} and variance σ_{IL}^2 is a suitable approximation of the inventory level. For asymmetric demand distributions Inderfurth and Kiesmüller (2015) as well as Sonntag and Kiesmüller (2016) showed that other distribution functions for modelling the inventory level are suitable. In this paper, we will not focus on asymmetric demand distribution because the analysis and the insights are similar.

For a normally distributed inventory level, the optimal base-stock level is given by the following newsboy equation (Inderfurth & Kiesmüller, 2015):

$$P(IL \geq 0) = \frac{b}{b+h} \tag{9}$$

We will fit a normal distribution on the first two moments of the inventory level, which means we need to derive expressions for the moments. The mean inventory level μ_{IL} can be determined directly from (5) and (4).

Lemma 1. *Under a strictly linear control rule (which means: $Q_t = S - IP_t$), the mean inventory level μ_{IL} in a production system with positive production and rework times is given as:*

$$\mu_{IL} = S - (L_P + 1)\mu_D - (1-\mu_Z)\mu_Q \tag{10}$$

where μ_D and μ_Q reflect the mean demand and the mean order quantity, respectively.

Proof. For the proof see Appendix A. □

In order to derive an expression for the variance of the inventory level, we need the moments of the forecast error as defined in (6). While the mean of the forecast error equals zero ($\mu_R = E[R_t] = E[\mu_Z Q_t - Z_t Q_t] = 0$), the variance does not. Sonntag and Kiesmüller (2017) demonstrate that the following equation holds:

$$\sigma_R^2 = (\sigma_Q^2 + \mu_Q^2)\sigma_Z^2 \tag{11}$$

Knowing the first two moments of the forecast error, the second central moment of the inventory level – the variance – can be determined.

Lemma 2. *Under a strictly linear control rule, the variance of the inventory level σ_{IL}^2 in a production system with positive production and rework times can be calculated as*

$$\sigma_{IL}^2 = (L_P+1)\sigma_D^2 + L_R\sigma_R^2 + (1-\mu_Z)^2\sigma_Q^2 \tag{12}$$

where σ_D^2, σ_R^2 and σ_Q^2 reflect the variances of the period demand, the forecast error and the order quantity.

Proof. For the proof see Appendix B. □

It is clear that the mean and the variance of the order quantity are required to determine the mean and the variance of the inventory level. The mean order quantity equals the mean demand ($\mu_Q = \mu_D$) since in the long run all demands have to be satisfied. To determine the variance of the order quantity, σ_Q^2, a recursive equation for the order quantity can be obtained:

$$Q_t = \begin{cases} D_{t-1}, & L_P \geq L_R = 1 \\ D_{t-1} + (1-\mu_Z)Q_{t-1} - (1-\mu_Z)Q_{t-L_R}, & L_P \geq L_R > 1 \\ D_{t-1} + (1-\mu_Z)Q_{t-1} - (1-Z_{t-L_R})Q_{t-L_R} + (\mu_Z - Z_{t-L_P})Q_{t-L_P}, & L_R > L_P > 0 \end{cases} \tag{13}$$

For the proof see Appendix C.

It is clear that for rework times larger than one time unit, the order quantity Q_t in period t depends on the order quantities in previous periods. To explain this phenomenon, remember that yield is random and estimates are included in the inventory position for the amount that is delivered L_P periods after production starts and the quantity delivered after $L_P + L_R$ periods when the rework is finished. When yield is realized these estimates are updated. If in one period the realized yield of the production process is larger than expected, more items than expected arrive in the warehouse and fewer items have to be reworked. Therefore the items arrive earlier than expected. This means that the order quantity in one period depends on the order quantities of previous periods and covariances occur when the variance of the order quantity has to be determined.

It is obvious that, if rework time is one period, the variance of the order quantity equals the variance of the demand: $\sigma_Q^2 = \sigma_D^2$. Moreover, for larger rework times the variance of the order quantity depends on the covariances of orders as follows.

$$VAR[Q_t] = \begin{cases} VAR[D_{t-1}] + VAR[(1-\mu_Z)Q_{t-1}] + VAR[(1-\mu_Z)Q_{t-L_R}] \\ -2(1-\mu_Z)^2 COV[Q_{t-1}, Q_{t-L_R}], & L_P \geq L_R > 1 \\ VAR[D_{t+1}] + VAR[(1-\mu_Z)Q_{t-1}] \\ +VAR[(1-Z_{t-L_R})Q_{t-LR}] + VAR[(\mu_Z - Z_{t-L_P})Q_{t-L_P}] \\ -2COV[(1-\mu_Z)Q_{t-1}, (1-Z_{t-L_R})Q_{t-L_R}] \\ +2COV[(1-\mu_Z)Q_{t-1}, (\mu_Z - Z_{t-L_P})Q_{t-L_P}] \\ -2COV[(1-Z_{t-L_R})Q_{t-LR}, (\mu_Z - Z_{t-L_P})Q_{t-L_P}], & L_R > L_P > 0 \end{cases} \tag{14}$$

To determine the covariances, we need to know corresponding joint probability distribution of the two random variables considered. These joint distributions are unknown and therefore the covariances cannot be calculated easily. In the following discussion, we show how the covariances between order quantities can be approximated to get a good estimate. The numerical study reveals that the performance of the approximation is excellent.

4. Approximation of the covariances between orders

For all the subsequent analyses we focus on rework times that are smaller than or equal to the production times ($1 < L_R \leq L_P$). Nevertheless, an analysis similar to the one for production times exceeding rework times can be adapted for the opposite relation.

We approximate the covariance between order quantities by using the recursive equation of the order quantity as given in (13). For every L_R, the recursive equation of the order quantity is plugged into the formula for the covariance in (14). Unfortunately, we end up with different formulae for the covariances for different values of L_R (for details see Appendix D).

Table 1
Covariance factors.

L_R	A_{L_R}
4	$[-(1-\mu_Z)\sum_{k=0}^{\infty}(-(1-\mu_Z))^k]$
5	$[-(1-\mu_Z)]$
6	$[-(1-\mu_Z)-(1-\mu_Z)^3\cdot\sum_{k=0}^{\infty}(-(1-\mu_Z))^k\cdot f_{k+1}]$
7	$[-(1-\mu_Z)\cdot\sum_{k=0}^{\infty}((1-\mu_Z)^2)^k]$
8	$[-(1-\mu_Z)-(1-\mu_Z)^3-2(1-\mu_Z)^5+(1-\mu_Z)^6-5(1-\mu_Z)^7\cdots]$
9	$[-(1-\mu_Z)-(1-\mu_Z)\cdot\sum_{k=1}^{\infty}2^{k-1}((1-\mu_Z)^2)^k]$
10	$[-(1-\mu_Z)-(1-\mu_Z)^3-2(1-\mu_Z)^5-5(1-\mu_Z)^7\cdots]$
⋮	⋮

The covariances, and therefore the variance of the order quantity, can be calculated nearly exactly for rework times of two and three periods. For rework times larger than three, the covariances can be approximated very well by $(\sigma_Q^2-\sigma_D^2)\cdot A_{L_R}$, whereas A_{L_R} depends on the mean yield μ_Z in different ways for each L_R (for details see Appendix E).

Lemma 3. *Under a strictly linear control rule, the variance of the order quantity σ_Q^2 in a production system with positive production and rework times ($L_P \geq L_R > 0$) can be approximated as*

$$\sigma_Q^2 \approx \begin{cases} \sigma_D^2, & L_R = 1 \\ \frac{\sigma_D^2}{1-2(1-\mu_Z)^2\left[1-\frac{1}{2-\mu_Z}\right]}, & L_R = 2 \\ \frac{\sigma_D^2}{1-2(1-\mu_Z)^2}, & L_R = 3 \\ \frac{\sigma_D^2(1+2(1-\mu_Z)^2\cdot A_{L_R})}{1-2(1-\mu_Z)^2+2(1-\mu_Z)^2\cdot A_{L_R}}, & L_R > 3 \end{cases} \quad (15)$$

with the factors A_{L_R} as given in Table 1 and f_k denoting the Fibonacci numbers starting with $f_0=0,\ f_1=1$ and $f_k=f_{k-1}+f_{k-2}$ ($\forall k\geq 2$).

Proof. Formula (15) directly follows from (14). For details on Table 1 see Appendix E. □

Before we make further use of the above formulae within the solution approach, it is important to validate the quality of the approximation of the variance of the order quantity as given in Eq. (15).

The aim of this study is to investigate the performance of the approximations of the covariances relating to the optimal base-stock levels and costs. We analyze several instances with respect to the effect of different production and rework times and different coefficients of variation for both demand and yield factor under two different cost ratios $b/(b+h)$. We compare the results of the steady-state approach with the optimal solution determined by simulation. The parameter setting is presented in Table 2:

The critical ratio $b/(b+h)$ equals 0.9 or 0.95. The demand parameters are similar as in Inderfurth and Kiesmüller (2015). Demand is normally distributed with a fixed mean ($\mu_D = 20$) whereas the coefficient of variation varies between 0.1 and 0.3.

Table 2
Numerical values of the input parameters.

	$L_P = 5$	$L_P = 10$
$b/(b+h)$	{0.9; 0.95}	
μ_D	20	
ρ_D	{0.1; 0.2; 0.3}	
μ_Z	{0.5}; {0.8;0.9}	
ρ_Z	{0.1; 0.2; 0.3; 0.4; 0.5}; {0.1;0.2;0.3}	
L_R	{1; 3; 4; 5}	{1; 5; 9; 10}

If higher coefficients of variation for the demand are requested, a gamma distribution is suitable which is not considered in the following discussion. Furthermore, we assume a symmetric ($\mu_Z = 0.5$) as well as an asymmetric ($\mu_Z = \{0.8; 0.9\}$) beta distributed yield factor. Using these mean yields we are able to model productions where on average half of all products are defective, as in the Samsung example in the introduction, or situations where fewer yield losses occur (e.g., Intel's microchip production Foremski, 2012). In the symmetric case, we allow for values for the coefficient of variation of the yield between 0.1 and 0.5 whereas for the asymmetric case, the coefficient of variation varies between 0.1 and 0.3.

We distinguish between a production time L_P of five and ten periods. Depending on the production length, we consider a system, where the rework time is very short ($L_R = 1$), half of the length of the production time ($L_R = 3$ for $L_P = 5$ and $L_R = 5$ for $L_P = 10$), just one period smaller ($L_R = 4$ for $L_P = 5$ and $L_R = 9$ for $L_P = 10$) or of equal length as the production time ($L_R = L_P = 5$ and $L_P = L_R = 10$). In total we analyze 240 instances of symmetric yield and 288 instances of asymmetric yield.

For the steady-state approach the base-stock levels were calculated solving formula (8) with respect to S, whereas the base-stock level was rounded up to the next integer. Therefore, only discrete values occur for S. We compare the base-stock levels with the optimal solution determined via simulation by increasing S stepwise by one unit until the minimum costs are reached. This procedure is possible because the cost function is convex in S (Huh & Nagarajan, 2010). Each simulation run represented 5000 periods with a 1000-period warm-up phase. To guarantee high accuracy, a sequential sampling procedure was used where the number of simulation runs was determined such that the half width of the 95% confidence interval of the average cost per period was smaller than 0.5% of the corresponding sample average. The simulation-based optimal base-stock level is the one minimizing the simulated cost based on formula (8).

Tables 3 and 4 show the results for symmetric and asymmetric yield. The first column gives the number of instances within the full factorial design in which the base-stock level of the steady-state approach is equal to the optimum S_{Sim} determined via simulation. The second column ($S_{Sim}+/-1$) indicates that the base-stock level is one unit below or above the optimum. The third column reveals that the deviation from optimum is larger than one unit.

For symmetric yields in Table 3 the approach leads to excellent results independent of the production time L_P. We would like to mention that in only six instances the base-stock level is underestimated, which is worse than overestimating it because one unit backordered is more expensive than one unit of additional inventory.

For asymmetric yields in Table 4, the results are similar even though the number of instances with a deviation from the simulated solution increased. It is obvious that, for a cost ratio $b/(b+h)$ of 0.9, the approach leads to very good results for short as well as for long production times – independent of all other parameter settings. For a cost ratio of 0.95, deviations from optimum can be greater than one unit with a maximum of four. The results differ only slightly for increased production time. Nevertheless, since the base-stock level increases for longer production times, a higher absolute variation from the optimal solution has only small effects, especially when it comes to cost.

Therefore, after showing the effect of the approximations on the base-stock level, we analyzed the effect on the corresponding average inventory holding and backorder costs. We simulated the average cost C^* for the base-stock levels S_{SS} calculated with the steady-state approach and compared the results with the minimum average cost C^*_{Sim} obtained by simulation. The percentage

Table 3
Quality of base-stock level under symmetric yield.

		$L_P = 5$			$L_P = 10$				
		S_{Sim}	$S_{Sim}+/-1$	Larger	S_{Sim}	$S_{Sim}+/-1$	Larger		
L_R	1	19	11	0	17	13	0	1	L_R
	3	19	11	0	20	10	0	5	
	4	17	13	0	24	6	0	9	
	5	17	13	0	12	16	2	10	
ρ_D	0.1	22	18	0	28	12	0	0.1	ρ_D
	0.2	24	16	0	26	14	0	0.2	
	0.3	26	14	0	19	19	2	0.3	
$b/(b+h)$	0.90	32	28	0	33	26	1	0.90	$b/(b+h)$
	0.95	40	20	0	40	19	1	0.95	
ρ_Z	0.1	14	10	0	16	8	0	0.1	ρ_Z
	0.2	19	5	0	16	8	0	0.2	
	0.3	12	12	0	12	11	1	0.3	
	0.4	13	11	0	14	9	1	0.4	
	0.5	14	10	0	15	9	0	0.5	

Table 4
Quality of base-stock level under asymmetric yield.

		$L_P = 5$			$L_P = 10$				
		S_{Sim}	$S_{Sim}+/-1$	Larger	S_{Sim}	$S_{Sim}+/-1$	Larger		
L_R	1	19	16	1	21	15	0	1	L_R
	3	21	13	2	23	10	3	5	
	4	22	12	2	18	14	4	9	
	5	17	15	4	20	13	3	10	
ρ_D	0.1	29	14	5	27	16	5	0.1	ρ_D
	0.2	24	21	3	27	18	3	0.2	
	0.3	26	21	1	28	18	2	0.3	
$b/(b+h)$	0.90	45	27	0	47	25	0	0.90	$b/(b+h)$
	0.95	34	29	9	35	27	10	0.95	
μ_Z	0.8	38	33	1	39	32	1	0.8	μ_Z
	0.9	41	23	8	43	20	9	0.9	
ρ_Z	0.1	21	27	0	27	21	0	0.1	ρ_Z
	0.2	33	15	0	32	15	1	0.2	
	0.3	25	14	9	23	16	9	0.3	

cost difference of instance i was then calculated as

$$\delta_i = \frac{C^*(S_{SS}) - C^*_{Sim}}{C^*_{Sim}} \cdot 100\% \quad (16)$$

and the maximum relative difference of N instances was computed as

$$\delta_{max} = max_{i=1,\ldots,N}\delta_i \quad (17)$$

and the average relative difference of N instances as

$$\bar{\delta} = \frac{1}{N}\sum_{i=1}^{N}\delta_i. \quad (18)$$

Table 5 shows the average and maximum percentage cost deviation from the optimal solution for a production time of five and ten periods under symmetric yield ($\mu_Z = 0.5$).

The calculations reveal that the approximation of the covariance and the variance as given in (15) shows excellent performs especially for high production time. The results for an asymmetric yield are similar when it comes to an optimal solution with an average percentage cost deviation of 0.15% for a production time of five periods and 0.09% for a production time of ten periods, the maximum deviation over all instances equalling 2.5% and 1.4%, respectively.

Due to the satisfying results, formula (15) can be used to approximate the variance although the covariances are unknown. Note that the higher the mean yield the lower the influence of the covariances because they are multiplied with the term $(1-\mu_Z)^2$ (compare formula (14)) and the approximation itself also consists of several such terms.

Table 5
Average and maximum percentage deviation from optimal costs for $L_P = 5$ and $L_P = 10$.

		$L_P = 5$		$L_P = 10$			
		$\bar{\delta}$	δ_{max}	$\bar{\delta}$	δ_{max}		
L_R	1	0.08	0.64	0.08	0.58	1	L_R
	3	0.10	0.72	0.04	0.30	5	
	4	0.16	1.22	0.02	0.18	9	
	5	0.09	0.60	0.06	0.34	10	
ρ_D	0.1	0.21	1.22	0.06	0.58	0.1	ρ_D
	0.2	0.07	0.45	0.03	0.36	0.2	
	0.3	0.04	0.24	0.06	0.34	0.3	
$b/(b+h)$	0.90	0.12	1.22	0.05	0.58	0.90	$b/(b+h)$
	0.95	0.09	1.12	0.05	0.42	0.95	
ρ_Z	0.1	0.17	1.22	0.09	0.42	0.1	ρ_Z
	0.2	0.04	0.45	0.02	0.18	0.2	
	0.3	0.15	0.72	0.07	0.39	0.3	
	0.4	0.10	0.45	0.05	0.58	0.4	
	0.5	0.07	0.64	0.02	0.12	0.5	

5. Disposal versus rework

Since we are able to compute the optimal base-stock policy, we can compare two strategies for defective products. The first strategy results in the disposal of all defective items whereas the

Table 6
Formulae for rework or disposal.

	With rework ($i = wR$)	With disposal ($i = nR$)
$\mu_{Q,i}$	μ_D	μ_D/μ_Z
$\sigma^2_{Q,i}$	$\sigma^2_D, \quad L_R = 1$	$(\rho^2_Z\mu^2_D + \sigma^2_D)/(\mu^2_Z - \sigma^2_Z)$
	$\frac{\sigma^2_D}{1-2(1-\mu_Z)^2\left[1-\frac{1}{2-\mu_Z}\right]}, \quad L_R = 2$	
	$\frac{\sigma^2_D}{1-2(1-\mu_Z)^2}, \quad L_R = 3$	
	$\frac{\sigma^2_D(1+2(1-\mu_Z)^2\cdot A_{L_R}}{1-2(1-\mu_Z)^2+2(1-\mu_Z)^2\cdot A_{L_R}}, \quad L_R > 1$	
$\sigma^2_{R,i}$	$(\sigma^2_{Q,wR} + \mu^2_{Q,wR})\sigma^2_Z$	$(\sigma^2_{Q,nR} + \mu^2_{Q,nR})\sigma^2_Z$
$\sigma^2_{IL,i}$	$(L_P + 1)\sigma^2_D + L_R\sigma^2_{R,wR} + (1-\mu_Z)^2\sigma^2_{Q,wR}$	$(L_P + 1)\sigma^2_D + L_P\sigma^2_{R,nR}$
S_i	$(L_P + 1)\mu_D + (1-\mu_Z)\mu_{Q,wR}\Phi^{-1}(b/(b+h))\sigma_{IL,wR}$	$(L_P + 1)\mu_D + \Phi^{-1}(b/(b+h))\sigma_{IL,nR}$
$\mu_{IL,i}$	$S_{wR} - (L_P + 1)\mu_D - (1-\mu_Z)\mu_{Q,wR}$	$S_{nR} - (L_P + 1)\mu_D$

second strategy considers that all defective items are reworked. Note that whether to dispose of or rework imperfect products is a one-time decision. Once the planner has made a decision as to which strategy to choose, this decision cannot be changed, e.g., from batch to batch.

First, we focus on the differences in the model and therefore in the formulae depending on what has been decided about what to do with imperfect products. Afterwards a mathematical model incorporating different cost parameters, e.g., production and inspection cost, is presented. This model is used in the numerical analysis in Section 5.2 to examine how sensitive the decision concerning disposal or rework is to various input and cost parameters.

In a production system where defective products are disposed of, not all ordered items arrive at the warehouse. Unlike the model for rework, in the case of disposal, defective items leave the production system. Therefore, an ordering policy as presented in (4) is not suitable because in every period fewer products than required are received. Therefore, a linear inflation policy, which has been shown to perform very well (Huh & Nagarajan, 2010), is commonly used. The order quantity in this case equals:

$$Q_t = \begin{cases} F(S - IP_t), & IP_t < S \\ 0, & IP_t \geq S \end{cases} \tag{19}$$

F is called the linear inflation factor and is often defined as the reciprocal of the mean yield: $F = 1/\mu_Z$ (see e.g., Bollapragada & Morton, 1999; Huh & Nagarajan, 2010; Inderfurth & Vogelgesang, 2013; Inderfurth & Kiesmüller, 2015). The yield inflation factor takes into account that defective items are disposed of, which reduces the output of the production process. It compensates for fewer items with better quality.

Using the definition of the order quantity in (19), the steady-state formulae for the case with disposal rather than rework are required, which were derived by Inderfurth and Kiesmüller (2015) and Sonntag and Kiesmüller (2017). In Table 6, we summarize the formulae for the cases with rework (wR) and with no rework (nR).

These formulae show, that the variance of the order quantity differs a lot. Since the variance of the order quantity influences the variance of the forecast error as well as the variance of the inventory level, it has a large effect on the base-stock level S.

As already mentioned, we approximate the inventory level with a normal distribution. For a normally distributed inventory level, the average inventory holding and backorder cost can be calculated as in Sonntag and Kiesmüller (2017):

$$H(S_i, i) = (h+b)\left[\sigma_{IL,i}\cdot\varphi\left(\frac{-\mu_{IL,i}}{\sigma_{IL,i}}\right) + \mu_{IL,i}\cdot\left(1 - \Phi\left(\frac{-\mu_{IL,i}}{\sigma_{IL,i}}\right)\right)\right] - b\cdot\mu_{IL,i} \quad, \ i \in \{wR, nR\} \tag{20}$$

with the corresponding mean and variance of the inventory level as presented in Table 6.

5.1. Mathematical model

To support the decision on whether a rework station should be integrated or defective items should be scrapped, a cost model considering production and quality control costs, possible rework or disposal costs as well as holding and backorder costs is introduced to calculate the average cost per period. For simplicity, we introduce the binary variable X, which indicates whether defective products are reworked or disposed of:

$$X = \begin{cases} 1, & rework \\ 0, & disposal \end{cases} \tag{21}$$

We consider variable production costs P per period, charged with p for every produced item ($p \geq 0$). On average, the production volume equals the mean order quantity $(\mu_{Q,i})_{i\in\{wR,nR\}}$ in each period. Thus, we get

$$P(i) = p\cdot L_P\cdot\mu_{Q,i}, \quad i \in \{wR, nR\}. \tag{22}$$

We do not consider fixed production costs because only a single product is produced and thus no set-up costs are required to initialize the machine in advance of each production run. Concerning quality control costs, we neglect fixed costs for implementing such a control station because these costs are not relevant for the decision as to whether products should be scrapped or reworked. A quality control process exists in both cases. Variable quality control costs A are charged with parameter a for each item produced ($a \geq 0$):

$$A(i) = a\cdot\mu_{Q,i}, \quad i \in \{wR, nR\} \tag{23}$$

If defective products are reworked, variable costs $R(i)$ occur with parameter r ($r \geq 0$)for every defective item $((1-\mu_Z)\mu_{Q,wR})$:

$$R(i) = X\cdot r\cdot L_R\cdot(1-\mu_Z)\mu_{Q,i}, \quad i = wR \tag{24}$$

Instead, if defective products are scrapped, disposal costs g ($g \geq 0$) are charged for each defective and thus disposed of item $((1-\mu_Z)\mu_{Q,nR})$:

$$G(i) = (1 - X)\cdot g\cdot(1-\mu_Z)\mu_{Q,i}, \quad i = nR \tag{25}$$

Finally, inventory holding and backorder costs H are charged as in formula (20) with $h \geq 0$ and $b \geq 0$:

$$H(S_i, i) = h\int_0^\infty x\varphi_{IL,i}dx - b\int_{-\infty}^0 x\varphi_{IL,i}dx, \quad i \in \{wR, nR\} \tag{26}$$

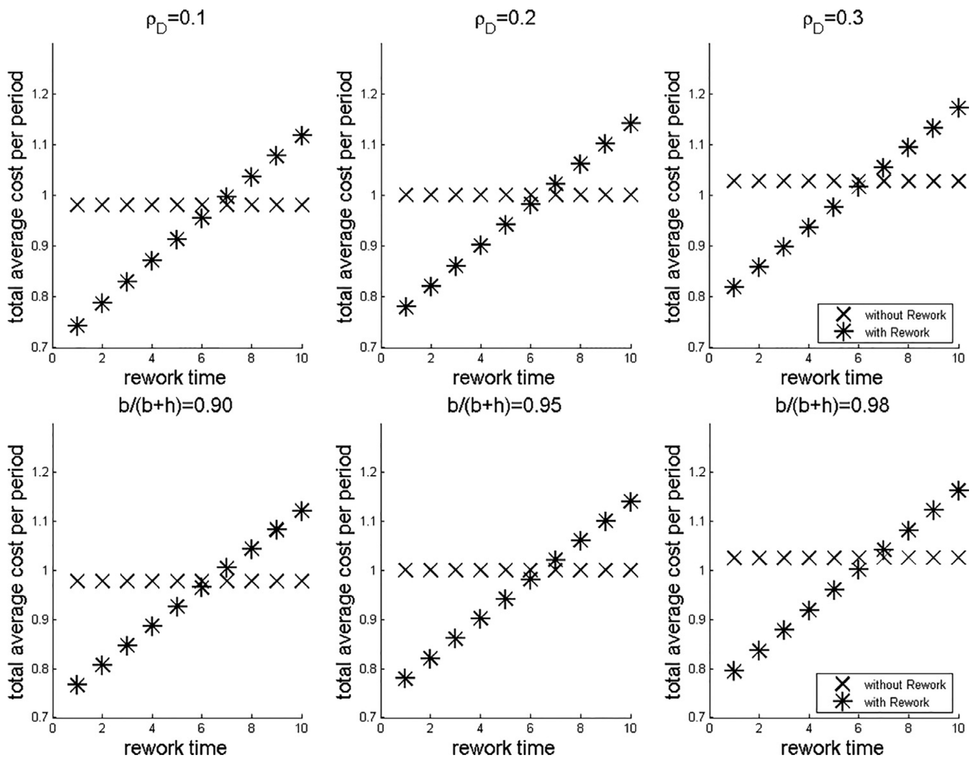

Fig. 2. Effect of input parameter variations (ρ_D and $b/(b+h)$) on total cost.

Summarizing all the different cost terms, we get the following cost function for $i \in \{wR, nR\}$:

$$\begin{aligned} TC(S_i, i) &= P(i) + A(i) + R(wR) + G(nR) + H(S_i, i) \\ &= p \cdot L_P \cdot \mu_{Q,i} \\ &\quad + a \cdot \mu_{Q,i} \\ &\quad + X \cdot r \cdot L_R \cdot (1 - \mu_Z)\mu_{Q,wR} \\ &\quad + (1 - X) \cdot g \cdot (1 - \mu_Z)\mu_{Q,nR} \\ &\quad + h \int_0^{\infty} x\varphi_{IL,i} dx - b \int_{-\infty}^{0} x\varphi_{IL,i} dx \end{aligned} \tag{27}$$

In the next section, we analyze different production systems where all defective products are either scrapped or reworked.

5.2. Numerical analysis

To analyze the effect of different input parameters on the decision concerning whether defective items should be scrapped or reworked, we use one example as a benchmark and change one parameter after another. This analysis indicates which input parameters are critical and therefore should be accorded greater attention. As a benchmark we consider a production system with a production time L_P of ten periods and a rework time L_R which can vary between one and ten periods. The mean demand μ_D and the corresponding coefficient of variation are 20 and 0.2, respectively. The cost ratio $b/(b+h)$ is set to 0.95. The mean yield μ_Z equals 0.8 with a coefficient of variation ρ_Z of 0.3, which indicates that yield losses are not negligible but improvable.

The setting for the cost parameters is as follows: variable production cost p are charged with one unit for each produced item, quality control cost a equal the production cost. If defective items are disposed of, cost g of two units per item occur. Instead, if defective products are reworked, variable cost r of three units per item are charged. Thus, we get a rework to disposal cost ratio of 1.5 and a production to rework cost ratio of 1/3.

In the following section, we run a sensitivity analysis to illustrate which parameters are critical and should therefore receive more attention than others.

5.3. Variations in the input of the production environment

First, we change the values of the coefficients of variation of the demand, ρ_D, as well as the mean μ_Z and the coefficients of variation ρ_Z^2 of the yield. Additionally, we look at the effect of changes in the cost ratio $b/(b+h)$, which follows from service level agreements with the customers. Note that while one of the parameters is changed, all the other parameters are fixed. Obviously, changes in demand can occur over time due to the addition of new customers or varying demand quantities of existing customers. Changes in the yield arise from an improved production system, producing less defective items. The service level is agreed by contract with the customer. There are situations conceivable where a customers willingness to pay for high service increases and therefore he signs a contract, which guarantees a higher service level.

The considered scenarios are as follows: $\rho_D \in \{0.1, 0.2, 0.3\}$, $b/(b+h) \in \{0.90, 0.95, 0.98\}$, $\mu_Z \in \{0.7, 0.8, 0.9\}$ and $\rho_Z \in \{0.1, 0.3, 0.5\}$. Thus, we analyze the effect of single parameter changes – either a decrease or an increase – compared with the benchmark scenario. To make the results comparable, we calculated the total average cost per period relative to the cost of the production system without rework in the benchmark scenario.

Fig. 2 shows the results for a variation of the demand variability and the cost ratio $b/(b+h)$. It can be seen that demand variations as well as variations in the cost ratio $b/(b+h)$ have almost no effect on the decision whether to dispose of or rework defective products. The point of indifference between both strategies always lies between a rework time of six and seven periods and thus depends entirely on the length of rework times. Because of the slope of the cost when reworking defectives, the decision whether to rework or scrap defective items becomes even more important

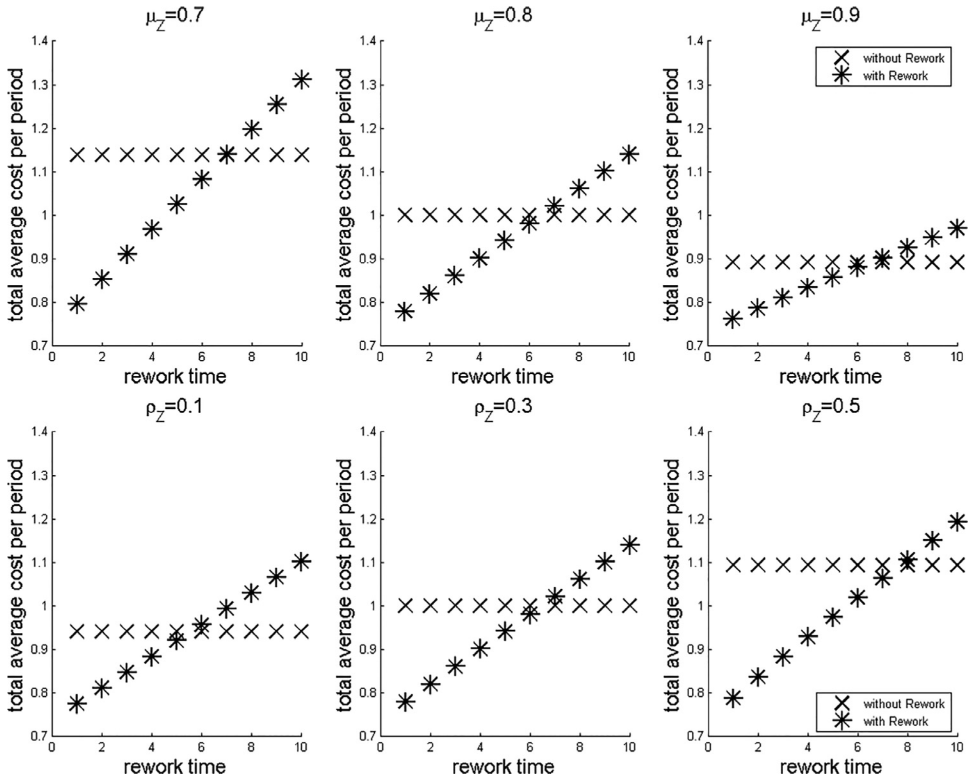

Fig. 3. Effect of input parameter variations (μ_Z and ρ_Z) on total cost.

because a wrong decision leads to a high amount of additional cost.

Fig. 3 shows the results for a variation of the mean and the coefficient of variation of the yield.

If the results are compared with the results in Fig. 2, it is obvious that the yield parameters have a greater impact. On one hand, an increasing mean yield goes in line with decreasing yield losses, which reduces total cost. On the other hand, the higher the mean yield, the less valuable is a costly rework process, represented by a decreasing slope. In other words, the greater the yield, the smaller the difference between the cost of reworking and of disposing of defective items.

As well as the mean yield, the coefficient of variation of the yield also has a large effect. We can see that, compared with the other parameters where the decision whether to rework or not was independent of the parameter setting, the variation of the yield forces the decision of a planner. For a coefficient of variation of the yield of 0.1, the point of indifference lies between a rework time of five and six periods: for a coefficient of variation of 0.2, between a rework time of six and seven periods; and for a coefficient of variation of 0.3, between a rework time of seven and eight periods. Thus, with higher yield variability reworking remains the best strategy even for larger rework times. High yield variability makes it difficult to estimate the yield losses. Thus, the probability of stock-outs during the risk period increases. The shorter the rework time, the earlier initially imperfect items arrive in the warehouse, reducing the probability that customer demand cannot be fulfilled.

The analysis illustrates the fact that the demand variability as well as the target cost ratio $b/(b+h)$ should not affect the decision of a planner on whether to scrap or rework defective items. In other words, the model is robust against variations in the demand and changes in the required cost ratio. Changes in these parameters affect only total cost. The planner should instead decide based on the mean and the variance of the yield whether it is worthwhile to rework defective items or not. While the point where the planner is undecided between rework and disposal is not affected by the demand parameters, the cost ratio or the mean yield, this point changes for different coefficients of variation of the yield. In this case, the decision on how to handle imperfect items depends heavily on the ratio between production time and rework time.

5.4. Variations in costs

We now focus on the robustness of the decision whether to rework or not if cost parameters change. Specifically, we change the ratio of rework and disposal cost r/g, the ratio of production and rework cost p/r and quality control cost a. The cost parameters may increase if the products become more and more complex over time due to new functionality. On the other hand, the cost parameters may decrease due to learning effects and improvements in the production, rework or quality control processes.

For all three cost parameters we analyze three scenarios as in the previous section:
$r/g \in \{1.25, 1.5, 1.75\}$, $p/r \in \{1/6, 1/3, 1/2\}$ and $a \in \{0.5 \cdot p, p, 1.5 \cdot p\}$. Fig. 4 shows the results.

Of course, the cost ratio of rework and disposal has no effect on total cost for the model where defective products are disposed of. However, with increasing cost ratio the slope of the total cost function increases if the products are reworked. The decision whether to dispose of or rework defective items is also affected by the rework cost parameter. The higher the rework cost per item and period relative to the disposal cost, the lower the rework time L_R has to be to make rework profitable. The longer the rework times, the higher the cost during the reworking process for bringing imperfect products to a condition equivalent to that of items that were produced correctly in the first place.

If we look at the production to rework cost ratio, reworking becomes more favorable with increasing production costs even if the rework time equals the production time. The reason for this is that

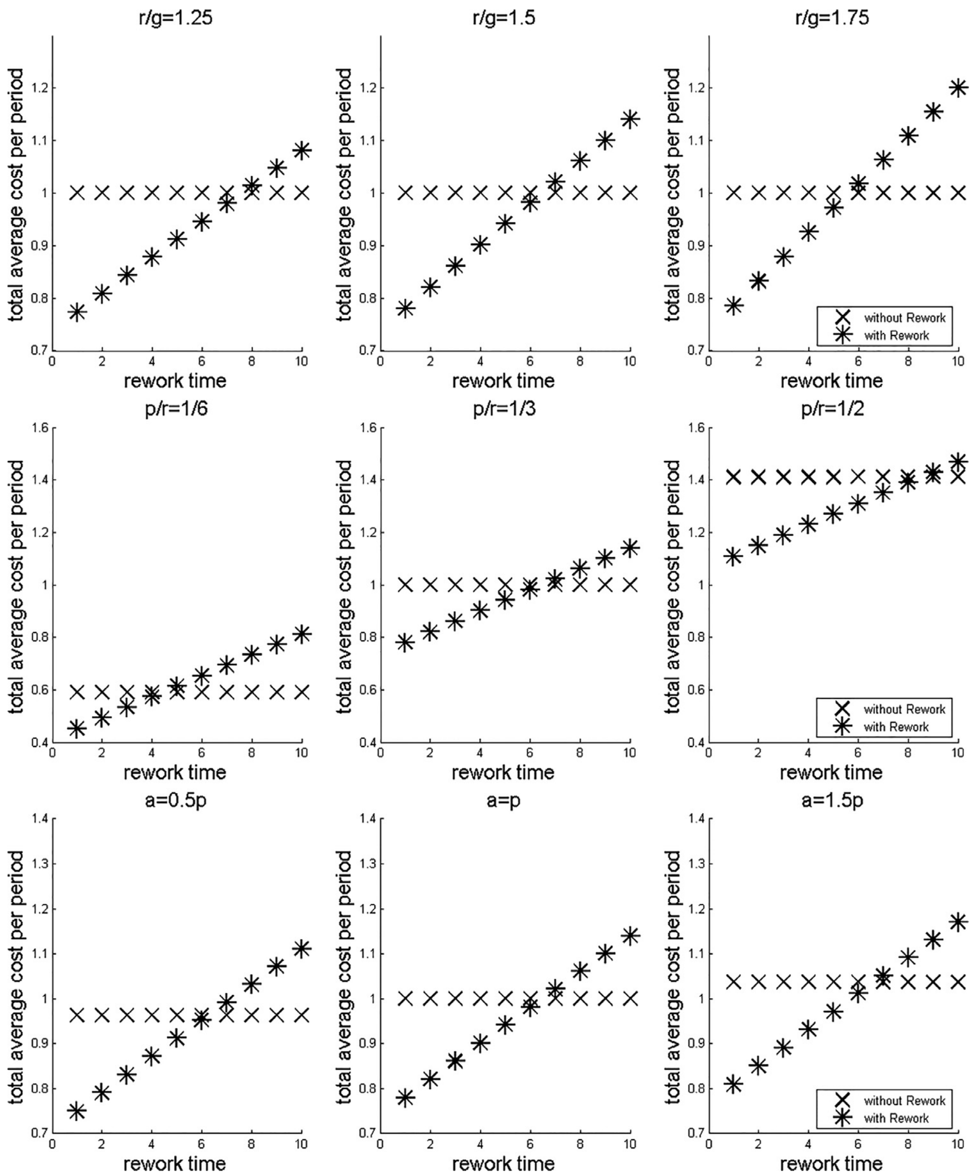

Fig. 4. Effect of cost parameter variations on total cost.

if all defective items are scrapped, these products have to be produced again, which becomes more expensive with increasing production costs. If production becomes expensive, the length of the rework time becomes more and more negligible to a point where reworking is always cheaper than producing defective items again.

Next, a change in quality control cost is analyzed. It is obvious that this parameter has only small effects. Only the total costs increase because of the higher cost parameter. The small effect arises due to small changes in the cost parameter. For larger variations an effect is recognizable in such a way that reworking becomes more and more favorable with an increasing quality control cost per item. This effect results from the assumption that reworked products are of perfect quality without any further inspection.

With these results in mind, a planner should focus mainly on production and rework costs when he has to decide whether to scrap or rework defective items. The higher the production cost relative to rework cost, the more profitable is rework.

5.5. Managerial insights

Whether to dispose or rework defective items is a difficult decision for a planner due to lots of different influencing parameters. It is obvious that reworking becomes more favorable for shorter rework times and lower corresponding costs compared to production and disposal. Nevertheless, a planner should not only focus on these effects but should be aware that the decision between rework and disposal has wide-ranging consequences especially regarding the required safety stock. Reducing the safety stock is important especially in situations where limited storage capacity is available. Although rework is in some situations more costly than disposal, the safety stock level under rework is always below the safety stock level under disposal. The reason for this is that under rework former defective items enter the warehouse after rework at a certain time wherefore these quantities – different from disposal – do not have to be reordered.

Even though the safety stock level is lower under rework, the difference compared to disposal of defectives highly depends on the parameter setting. As an example consider the demand variability, which has no effect on the question whether disposal or rework is favorable, but a high effect on the safety stock level. The difference in safety stock between rework and disposal decreases with increasing demand variability, which means rework is even more favorable for low demand variability. This is not intuitive at the first moment, because one would expect that rework becomes more favorable for increasing uncertainty in the system. The effect can be explained in two ways: First, under low demand variability the uncertainty due to random yield is dominant. This uncertainty is lower if defective items are reworked and therefore do not leave the system and do not have to be reordered. Second, under low demand variability less safety stock is available to hedge against demand uncertainty. Thus, there is less stock available which can also be used to hedge against yield uncertainty. Indeed, there is a pooling effect which reduces the safety stock level under high demand and yield uncertainty.

6. Summary and Outlook

We studied a single-stage production system with stochastic proportional yield, which results in random yield losses in each period. Since only items of perfect quality are stored in the warehouse to satisfy stochastic customer demand, defective products are either disposed of or reworked. We assumed that the reworking process converts all defective items into products that satisfy the required quality standards.

Our contribution was (1) to show how to determine the base-stock level minimizing average inventory holding and backorder cost in a production environment where defective products are not disposed of but reworked, and (2) to develop a mathematical model to be used as a decision-making support for the planner when it comes to the question of whether defective items should be disposed of or reworked.

The results are as follows: (1) the adaptation of the steady-state approach to a situation where defective products are not disposed of but reworked is not easy. The reworking process results in covariances between orders, which are difficult to calculate exactly because the joint distribution is unknown. We presented an approximation of the covariances depending on the reworking times. A numerical study confirmed that the approximation works very well. In 319 of 528 instances the approximation leads to the optimal solution as determined by the simulation. For all other 209 instances the average deviation equals approximately 0.10%. Over all 528 instances we get an average deviation from optimum of 0.10% with a standard deviation of 0.23%, which is excellent.

(2) We introduced a mathematical model addressing production, quality control, rework and inventory holding, and backorder cost. Based on this model, we analyzed the effect of varying parameter settings. The results show that the demand variation as well as the cost ratio $b/(b+h)$ has nearly no effect on cost and on the decision whether to dispose of or rework defective items. Thus, the model is robust for these parameters. On the other hand, the mean and the coefficient of variation of the yield have an enormous effect. The higher the mean yield, the less valuable is reworking because only a few items are of imperfect quality. The higher the coefficient of variation of the yield, the more valuable is a rework. Concerning a change in the cost parameters, the ratio of production and rework cost is the main determiner of whether to dispose of or rework a defective item.

Future research should focus on production systems where the planner can decide in each period if he or she wants to rework defective items or dispose of them. A mixture of both strategies is conceivable, where some products are scrapped and some are reworked. Furthermore, the reworking process like the production process might be imperfect, which means that either good products would be classified as defective or imperfect products would stay in the system by mistake. In such situations it might be necessary to place several inspection stations in tandem.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2017.11.019

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