This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2017.2686701, IEEE Transactions on Power Systems

Security Constrained Unit Commitment using Line Outage Distribution Factors

Diego A. Tejada-Arango, *Student Member, IEEE,* Pedro Sánchez-Martın, and Andres Ramos

Abstract**—Security-constrained unit commitment (SCUC) problem is one of the necessary tools for system operators to make operational planning and real-time operation. The internalization of transmission-network and security constraints (e.g. N-1 criterion) could lead to different decisions in the generation dispatch. However, the computational burden of this problem is challenging mainly due to its inherent large problem size. Therefore, this paper proposes an N-1 security constrained formulation based on the Line Outage Distribution Factors (LODF) instead of the one based on Injection Sensitivity Factors (ISF). This formulation is at the same time more compact than analogous formulations for contingency constraints; hence, it presents a lower computational burden. The computational efficiency of the proposed formulation is shown by solving the SCUC of the IEEE 118 bus system with LODF and ISF. Additionally, an iterative methodology for filtering the active N-1 congestion constraints is detailed, and its implementation for large-scale systems is described. The results show that the proposed filter reduces the computational time by approximately 70% in comparison to the complete formulation of N-1 constraints in SCUC.**

Index Terms**— mixed-integer linear programming (MIP), security constrained unit commitment (SCUC), line outage distribution factors (LODF), N-1 criterion.**

NOMENCLATURE

A. Indices and Sets

n	Time period running from 1 to N
t	Thermal unit running from 1 to T
h	Hydro unit running from 1 to H
i	Bus running from 1 to I
gi	Set of generators connected to bus
iic	Set of lines in service in steady state
jjc	Set of lines for the N-1 criterion

B. Parameters

D_{ni}	Demand per bus	MW
DU_n, DD_n	Secondary up and down reserve requirements	MW
IG_{ni}	Intermittent generation per bus	MW
$\overline{P}_t, \underline{P}_t, \overline{P}_h, \underline{P}_h$	Maximum and minimum thermal and hydro output	MW
CF_t	Fixed cost	\$/h
CV_t	Variable cost	\$/MWh
$CRU_t\ , CRD_t$	Secondary reserve, up and down, variable cost	\$/MW
C^{ENS}	Energy not served cost	\$/MWh
$CSU_t\ , CSD_t$	Startup and shutdown cost	\$
RU_t, RD_t	Ramp up and down	MW/h
$TSU_t\ , TSD_t$	Startup and shutdown time	h
TU_t, TD_t	Minimum up and down times	h
$PSU_t\ , PSD_t$	Output of startup and shutdown for thermal unit	MW
$PRU_h\ , PRD_h$	Available up and down reserve for hydro unit	MW
EFF_h	Efficiency pumping-production	p.u.
E_h	Available energy for hydro unit	GWh
$\overline{ER}_h, \underline{ER}_h$	Maximum and minimum reservoir energy	GWh
$\overline{F}_{iic}$	Maximum line capacity	MW
$\overline{F'}_{iic}$	Emergency rating of the line	MW
CF'_{iic}	Emergency rating violation cost	\$/MW
ISF^{i}_{iic}	Sensitivity of the flow on line iic to injection at bus i	p.u.
$ISF'^{i,jjc}_{iic}$	Sensitivity of the flow on line iic to injection at bus i with line jjc in outage	p.u.
$PTDF^{i,i'}_{iic}$	Power Transfer Distribution Factor between injections of bus i and i' and the line iic	p.u.
$LODF^{jjc}_{iic}$	Line Outage Distribution Factor between flow of line jjc and the flow of line iic	p.u.
$X_{i'i}$	Line impedance between buses i' and i	p.u.
$\hat{F}'^{jjc}_{n,iic}$	Estimated line power flow on iic, due to outage in line jjc	MW

C. Variables

uc_{nt}	Commitment decision	{0,1}
su_{nt}, sd_{nt}	Startup and shutdown decisions	{0,1}
p^{T}_{nt}	Total thermal output	MW
p_{nt}	Power output above minimum output	MW
p_{nh}	Hydro output	MW
ru_{nt}, rd_{nt}	Up and down secondary reserve of thermal units	MW
ru_{nh}, rd_{nh}	Up and down secondary reserve of hydro units	MW
ig_{ni}	Intermittent generation per bus (e.g. wind or solar)	MW
ens_{ni}	Energy non-supplied per bus	MW

D.A. Tejada-Arango, P. Sánchez, and A. Ramos are with the Institute for Research in Technology (IIT), School of Engineering (ICAI), Comillas Pontifical University, Madrid, Spain (e-mail: dtejada@comillas.edu, psanchez@comillas.edu, aramos@comillas.edu).

$f_{n,iic}$	Line power flow on iic in steady state	MW
$f'^{jjc}_{n,iic}$	Line power flow on iic, due to outage in line jjc	MW
$\Delta f'^{jjc}_{n,iic}$	Slack variable for line power flow on iic, due to outage in line jjc	MW

I. INTRODUCTION

A. Motivation

ELECTRIC power systems have grown in size and complexity over the last years due to the integration of renewable energies and the difficulties to build new network infrastructures. Furthermore, as demonstrated by recent major blackouts around the world, transmission networks play a vital role in the process of our society [1], [2]. Therefore, the System Operator[1] (SO) must schedule the generation resources necessary to achieve an economical, reliable, and secure energy production in the power systems. To perform this task, the Unit Commitment (UC) problem is solved by the SO on a daily and intradaily basis. The objective of the UC problem is to minimize total system operational costs, considering system operation constraints (e.g. spinning reserve and security criteria), transmission limits and the technical limitation of generation units [3]. The mathematical formulation of UC is a Mixed-Integer Programming (MIP) problem and belongs to the class of NP-hard problems. Therefore, the optimal solution is, in general, difficult to obtain due to problem size and NP-hardness [4]. Although it is a widely studied problem, there is still a research interest for large-scale systems regarding the UC problem such as: security and reserve constraints, and processes to reduce space search [3]. In fact, SOs in Europe, North and South America include the transmission network and the security constraints when it comes to solving SCUC.

B. Literature Review

As mentioned in section I.A, the UC is a widely-studied problem. Recent works emphasize the impact of renewables uncertainty [5]–[9], the improvement of the performance of MIP formulations [5], [10]–[12], and security constraints for the UC [13], [14], [6], [15]–[17].

This paper focuses on Security Constrained Unit Commitment (SCUC). The objective of SCUC is to obtain a UC schedule at minimum production cost without compromising the system reliability [18]. The reliability of a system is interpreted as satisfying two functions: adequacy and security. An adequate amount of capacity resources must be available to meet the peak demand (adequacy), and the system must be able to withstand changes or contingencies on a daily and hourly basis (security). The adequacy component has been incorporated into the SCUC by the system spinning reserve [19], [20]. These constraints allow to internalize outages of generation units. On the other hand, the security component has been represented by the N-1 and N-k transmission line outage constraints [1], [14], [18], [21]–[23].

The representation of the network is a key factor in the SCUC. The SCUC becomes a nonlinear, non-convex, mixed-integer problem, when the power flow equations are considered [1], [18], [24], [25]. Nevertheless, the DC power flow model is used, instead of the AC equations, in order to have a MIP problem in the SCUC [4], [12], [18], [20]. For the representation of the DC power flow or network constraints, the Injection Sensitivity Factors (ISF) are commonly used [26], [18], because this kind of formulation avoids the inclusion of more variables such as voltage angle per bus. However, the main drawback in the N-1 contingency representation through the ISF is the large number of coefficients required to calculate the power flow in contingency state that create heavy models which greatly increase the computational burden of SCUC. Therefore, more compact MIP-based network representation models are needed in order to create more computationally efficient SCUC formulations.

The ISF is one of the so called Linear Sensitivity Factors (LSF). Other LSF such as the Line Outage Distribution Factors (LODF) have applications in optimal power flow calculation [27], [28], generation capacity expansion planning [29], and real-time operation [30], due to the straightforward way to calculate post-contingency power flows through a line [30], [26], [31], [32]. Although iterative techniques have been applied in industry for SCUC [33], [34], the authors are not aware of any academic publication which proves why the application of the LODF in the SCUC is more efficient. The LSF, including the LODF, has been analyzed in more detail in the section II.

Another aspect to be considered in the SCUC is the size of the problem (number of variables, constraints, and nonzero elements). Despite the significant improvements in MIP solving, developments in the processing capacity, and increase of available memory in the computers, the time and the size required to solve SCUC problems continues to be a critical feature that limits the size and scope of SCUC models [5]. This situation is more relevant when the N-k outages or contingencies are considered in the SCUC, because in a moderated size system the problem becomes computationally intractable [23]. One possible strategy to solve large-scale SCUC problems considering line outages is Benders decomposition [18], [23], [35]–[37]. Nevertheless, improving the SCUC formulation can dramatically reduce its computational burden and thus allow the implementation of more advanced and more computational demanding problems, such as stochastic formulations in the SCUC [7]–[9], [14]. In addition, even with the consideration of N-1 line outages in the SCUC, not all those constraints are contributing directly to set up the SCUC feasible region; in fact, most of them are superfluous and could be discarded [38], [39]. For example, in [39], [40], in an Optimal Power Flow (OPF) implemented with N-1 constraints on the IEEE 118-bus system [41], it was found that 99.3% of its constraints were superfluous. In order to address this situation, the umbrella constraints concept was introduced in [39], [40]. The umbrella constraints are conditions which are necessary and sufficient to the

---

[1] Depending on the regulation of the country, SO could be an independent organization (Independent System Operator – ISO) or part of the transmission company (Transmission System Operator – TSO).

description of the feasible set of an OPF problem, and can be extended to the SCUC problem. Thus, by identifying and removing the unnecessary or non-umbrella constraints from such problems, these SCUC problems run faster. As it is shown in [39], [40], the umbrella constraint discovery problem formulation is a linear program, which, nonetheless, is not a trivial optimization problem when dealing with practical problems. On the other hand, reference [38] proves that determining inactive constraints in the SCUC is equivalent to solving a series of small-scale and simpler MILP problems than the original one. Instead of solving the discovery problem to determine the umbrella constraints [39] or solving small-scale problems [38], an iterative algorithm based on LODF is proposed in this paper to find the active constraints that are binding the SCUC to implement the N-1 preventive criterion. The proposed algorithm reduces the search space and enhances the computational burden of the problem resolution.

C. Contributions

This paper presents an alternative set of constraints to represent line outages in the SCUC based on the LODF. This formulation describes the same integer problem as [18], and it yields the same optimal results. The main contributions of this paper are:

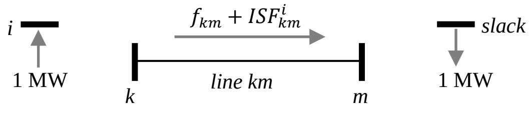
1) A compact formulation for SCUC problems based on a combination of ISF for pre-contingency constraints and LODF for post-contingency power flow due to the N-1 criterion. Thus, the proposed formulation reduces the computational burden of reference SCUC formulations, in terms of core memory and in potential solution speed-up.
2) A methodology for large scale problems based on the use of LODF as filter of the line outages that are binding the solution of SCUC. The methodology allows to solve the SCUC in an iterative way, not only reducing the size of the problem but also its computational time.

D. Paper Structure

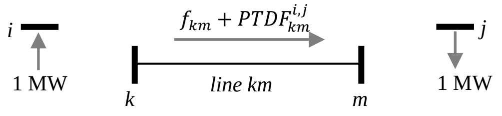
The remainder of this paper is organized as follows. Section II describes LSF conceptually and shows their mathematical formulation. Section III provides the mathematical formulation for the SCUC. This section also conceptually evaluates the advantages of reducing the coefficients of security constraints in the proposed model using LODF. Section IV presents a methodology based on LODF to reduce the search space of the SCUC. Section V shows case studies which validate the performance of the formulation. Finally, Section VI draws conclusions and suggests future works.

II. LINEAR SENSITIVITY FACTORS (LSF)

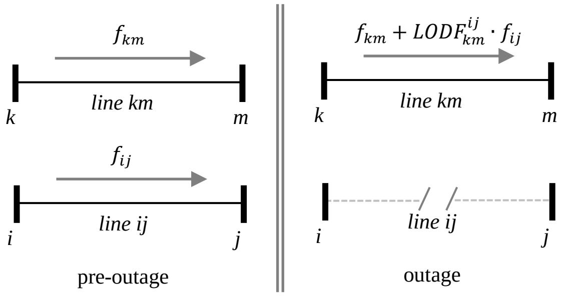
When the linearized approximation approach of the power flow (DC power flow) is used, the sensitivity factors are called LSF. The sensitivity factors are obtained from the analysis of the Jacobian Matrix from the power flow equations [18], [26]. Sensitivities can be easily calculated even for large systems and are dependent upon the network topology. Sensitivities are typically used in the OPF. In this context a common application is to determine overloaded lines due to nodal injections [28]. In this paper the following LSF are analyzed:



(a) ISF



(b) PTDF



(c) LODF

Fig. 1. (a) ISF definition. (b) PTDF definition. (c) LODF definition.

1) Injection Shift Factor (ISF_{km}^{i}): represents the fraction of the additional 1MW injection at bus i that goes through line between the bus k and bus m, see Fig. 1(a).
2) Power Transfer Distribution Factor ($PTDF_{k,m}^{i,j}$): represents the change in power transfer on line between the bus k and bus m, with respect to the power transfer of 1MW from an injection node i to a consumption node j, see Fig. 1(b).
3) Line Outage Distribution Factor ($LODF_{km}^{ij}$): represents the portion of the pre-outage real power line flow on the line between the bus i and bus j that is redistributed to the line between the bus k and bus m as a result of the outage of line between the bus i and bus j, see Fig. 1(c).

In the DC power flow context, the computation of the ISF_{km}^{i} is defined by equation (1) [18], [26]. The $PTDF_{km}^{i,j}$ and the $LODF_{km}^{ij}$ can be calculated from the ISF_{km}^{i} as presented in equations (2) and (3) [31], [32]. In this context, PTDF and LODF are slack independent. However, the slack node could change the results in AC transmission models [2], [42].

$$ISF_{km}^{i} = \frac{X_{ki} - X_{mi}}{X_{km}} \tag{1}$$

$$PTDF_{km}^{i,j} = ISF_{km}^{i} - ISF_{km}^{j} \tag{2}$$

$$LODF_{km}^{ij} = \frac{PTDF_{km}^{i,j}}{1 - PTDF_{ij}^{i,j}} = \frac{ISF_{km}^{i} - ISF_{km}^{j}}{1 - \left(ISF_{ij}^{i} - ISF_{ij}^{j}\right)} \tag{3}$$

In section III, the LSF presented in this section will be used to formulate the network constraints in the SCUC, highlighting the advantages of the LODF in large scale problems.

III. MATHEMATICAL FORMULATION FOR SCUC

Several formulations for the SCUC were analyzed in the literature review. Nevertheless, the tight and compact MIP formulation presented in [5], [11], [12], [43] was selected as the reference formulation in this paper for the SCUC. It was selected due to the computational efficiency at solving the network-constrained UC problem. The objective function in (4) minimizes the total variable cost of the system (production cost, startup and shutdown cost, non-served energy cost, etc.). This problem is subject to the following constraints: balance between generation and demand (5), up and down secondary reserve requirements (6) and (7), constraints in the capacity of thermal units (8)-(12) where constraints (8) and (11) describe the first and the last period cases, constraints in the capacity of thermal units (11) and (12), total thermal output including startup and shutdown trajectories (13), up and down ramps of thermal units (14) and (15), commitment, startup and shutdown logic of thermal units (16), minimum up and down times of thermal units (17) and (18), constraints in the hydro energy production (19), (20), and (21), maximum intermittent generation per bus (22), and thermal and hydro production positive condition (23) and (24).

$$\begin{aligned} min \textstyle\sum_{n,t} CSU_t \cdot su_{nt} + \textstyle\sum_{n,t} CSD_t \cdot sd_{nt} \\ + \textstyle\sum_{n,t} CF_t \cdot uc_{nt} + \textstyle\sum_{n,t} CV_t \cdot p_{nt}^T \\ + \textstyle\sum_{n,t} CRU_t \cdot ru_{nt} + \textstyle\sum_{n,t} CRD_t \cdot rd_{nt} \\ + \textstyle\sum_{n,i} C^{ENS} \cdot ens_{ni} + \textstyle\sum_{n,iic,jjc} CF'_{iic} \cdot \Delta f'^{jjc}_{n,iic} \end{aligned} \qquad (4)$$

s.t.

$$\textstyle\sum_t p_{nt}^T + \sum_h p_{nh} + \sum_i ig_{ni} + \sum_i ens_{ni} = \sum_i D_{ni} \qquad \forall n \qquad (5)$$

$$\textstyle\sum_t ru_{nt} + \sum_h ru_{nh} \geq DU_n \qquad \forall n \qquad (6)$$

$$\textstyle\sum_t rd_{nt} + \sum_h rd_{nh} \geq DD_n \qquad \forall n \qquad (7)$$

$$p_{1t} + ru_{1t} \leq uc_{1t}(\overline{P}_t - \underline{P_t}) - sd_{2t}(\overline{P}_t - PSD_t) \qquad \forall t \qquad (8)$$

$$\begin{aligned} p_{nt} + ru_{nt} \leq uc_{nt}(\overline{P}_t - \underline{P_t}) \\ - sd_{n+1,t}(\overline{P}_t - PSD_t) \\ - su_{nt} \cdot max(PSD_t - PSU_t, 0) \end{aligned} \qquad \forall t, n = 2, \dots, N-1 \qquad (9)$$

$$\begin{aligned} p_{nt} + ru_{nt} \leq uc_{nt}(\overline{P}_t - \underline{P_t}) \\ - sd_{n+1,t} \cdot max(PSU_t - PSD_t, 0) \\ - su_{nt}(\overline{P}_t - PSU_t) \end{aligned} \qquad \forall t, n = 2, \dots, N-1 \qquad (10)$$

$$p_{Nt} + ru_{Nt} \leq uc_{Nt}(\overline{P}_t - \underline{P_t}) - su_{Nt}(\overline{P}_t - PSU_t) \qquad \forall t \qquad (11)$$

$$p_{nt} - rd_{nt} \geq 0 \qquad \forall nt \qquad (12)$$

$$\begin{aligned} p_{nt}^T = p_{nt} + \underline{P_t} uc_{nt} \\ + \textstyle\sum_{n'=1}^{TSD_t} PSD_t sd_{n-n'+1,t} \\ + \textstyle\sum_{n'=1}^{TSU_t} PSU_t su_{n-n'+1+TSU_t,t} \end{aligned} \qquad \forall nt \qquad (13)$$

$$p_{nt} - p_{n-1,t} + ru_{nt} \leq RU_t \qquad \forall nt \qquad (14)$$

$$p_{nt} - p_{n-1,t} - rd_{nt} \geq -RD_t \qquad \forall nt \qquad (15)$$

$$uc_{nt} - uc_{n-1,t} = su_{nt} - sd_{nt} \qquad \forall nt \qquad (16)$$

$$\textstyle\sum_{n'=n+1-TU_t}^{n} su_{n't} \leq uc_{nt} \qquad \forall nt \qquad (17)$$

$$\textstyle\sum_{n'=n+1-TD_t}^{n} sd_{n't} \leq 1 - uc_{nt} \qquad \forall nt \qquad (18)$$

$$\textstyle\sum_n p_{nh} \leq E_h \qquad \forall h \qquad (19)$$

$$0 \leq ru_{nh} \leq PRU_h \qquad \forall nh \qquad (20)$$

$$0 \leq rd_{nh} \leq PRD_h \qquad \forall nh \qquad (21)$$

$$0 \leq ig_{ni} \leq IG_{ni} \qquad \forall ni \qquad (22)$$

$$p_{nt} \geq 0 \qquad \forall nt \qquad (23)$$

$$p_{nh} \geq 0 \qquad \forall nh \qquad (24)$$

A. Reference formulation for N-1 security constraint

As presented in the literature review in section I.B, the ISF is widely used for the representation of the electrical network in optimization problems [18], [26]. Furthermore, in section II, the mathematical expression to calculate the ISF was shown. Therefore, the power flow through a line can be computed as the sum of all the net power injections in the nodes (generation production minus the demand) as presented in equation (25). The ISF can be re-calculated for each topology obtained from the line outage in the system [18], and then the N-1 security constraint can be formulated as presented in equation (26). In fact, equation (26) is a general formulation, and the ISF can be obtained for multiple line outages in the network.

$$\begin{aligned} f_{n,iic} = \textstyle\sum_i ISF_{iic}^{i} \cdot \big(\textstyle\sum_{t \in gi} p_{nt}^T \\ + \textstyle\sum_{h \in gi} p_{nh} + ig_{ni} + ens_{ni} - D_{ni}\big) \end{aligned} \qquad \forall n, iic \qquad (25)$$

$$\begin{aligned} f'^{jjc}_{n,iic} = \textstyle\sum_i ISF'^{i,jjc}_{iic} \cdot \big(\textstyle\sum_{t \in gi} p_{nt}^T \\ + \textstyle\sum_{h \in gi} p_{nh} + ig_{ni} + ens_{ni} - D_{ni}\big) \end{aligned} \qquad \forall n, iic, jjc \qquad (26)$$

$$\left|f_{n,iic}\right| \leq \overline{F}_{iic} \qquad \forall n, iic \qquad (27)$$

$$\left|f'^{jjc}_{n,iic}\right| \leq \overline{F'}_{iic} + \Delta f'^{jjc}_{n,iic} \qquad \forall n, iic, jjc \qquad (28)$$

TABLE I
COMPARISON OF NUMBER OF NONZERO COEFFICIENTS

N-1 security Constraint	Non-zero Coefficients[(a)]
Ref	$NL \cdot NO \cdot NB \cdot NG$
$LODF$	$NL \cdot NO$

[(a)] NL = number of lines, NO = number of outages, NB = number of buses, and NG = number of generators.

One of the drawbacks of equation (26) is that the number of coefficients in the constraint increases with the number of lines, buses, and outages, see Table I. The number of non-zero coefficients is calculated based on the elements which are not equal to zero in the constraint matrix of the optimization problem. A larger number of non-zero coefficients is related to a denser constraint matrix and therefore it is harder to solve for the optimization software. The number of non-zero coefficients has a significant impact on solution times as it is shown in Section V.A (see also [11] and references therein). The variable $f_{n,iic}$ is limited by the maximum line capacity $\overline{F}_{iic}$, equation (27). The variable $f'^{jjc}_{n,iic}$ is constrained by the emergency rating of the line $\overline{F'}_{iic}$ plus the slack variable associated to violations on emergency rating of the line, equation (28). For the following sections this formulation will be referred to as Ref.

B. LODF formulation

In view of the LODF definition presented in section II, the real power flows on transmission lines caused by lines on outage with a pre-contingency power flow can be obtained from equation (29), [31], [32].

$$f'^{jjc}_{n,iic} = f_{n,iic} + LODF^{jjc}_{iic} \cdot f_{n,jjc} \qquad \forall n, iic, jjc \qquad (29)$$

First, equation (29) does not reduce the number of constraints or variables, compared to the Ref formulation. However, the number of non-zero coefficients in the security constraints is reduced by a factor equal to the number of buses in the network multiplied by the number of generators (see Table I). This advantage allows to reduce the non-zero elements of the problem, which decreases the computational burden of the SCUC problem. In section V this reduction has been quantified in the IEEE 118 bus test system. It is important to highlight that equation (25) is also needed to calculate the line's power flow in steady state or pre-contingency. In addition, equations (27) and (28) are included in the proposed formulation with LODF. Finally, the equation (26) is replaced by equation (29).

IV. LODF POST-CONTINGENCY FILTER

One of the drawbacks described in section I.B for large scale problems, is that most of the N-1 contingency constraints (relationship between the failed line and the post-contingency flow of another line), are superfluous and do not set up the feasibility space of the SCUC problem. Therefore, an iterative filtering process is proposed using the LODF to set up the minimum set of contingency constraints that limits the feasible region of the problem (see Fig. 2), which corresponds to the umbrella constraints associated with the contingency constraints. The proposed filter is based on equation (29). Therefore, at the end of each iteration, the estimation of the post-contingency power flow $\widehat{F}'^{jjc}_{n,iic}$ is determined using equation (30).

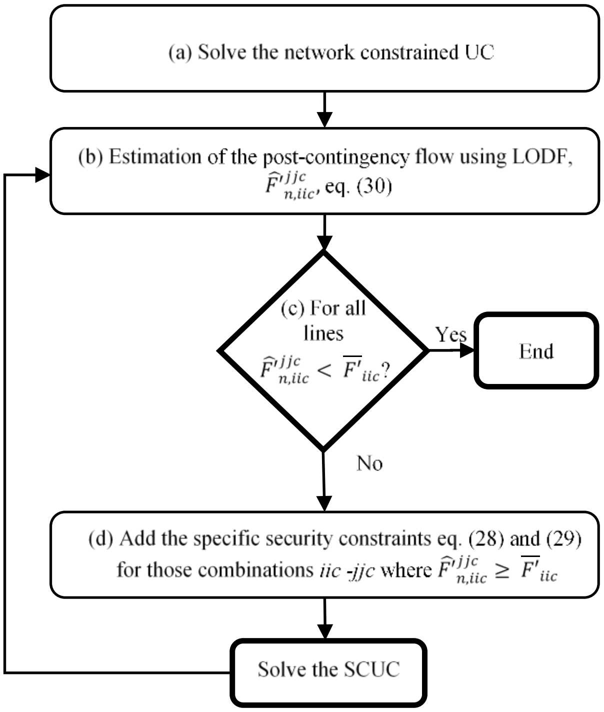


Fig. 2. Flowchart of the LODF Post-Contingency Filter for the SCUC problem

$$\widehat{F}'^{jjc}_{n,iic} = f_{n,iic} + LODF^{jjc}_{iic} \cdot f_{n,jjc} \qquad \forall n, iic, jjc \qquad (30)$$

The proposed methodology starts by calculating the network constrained UC, which means only equations (25) and (27) will be included in the SCUC (see Fig. 2(a)). The network constrained UC obtains the pre-contingency line flows. Then, the post-contingency filter uses equation (30) based on LODF to compute all the post-contingency line iic power flow ($\widehat{F}'^{jjc}_{n,iic}$) when the line jjc fails (see Fig. 2(b)). Then, if all post-contingency power flows are under the maximum value ($\widehat{F}'^{jjc}_{n,iic} < \overline{F'}_{iic}$) (all combinations are under loaded) then it is not necessary to include more pre- and post-contingency combinations and the final solution is obtained, which corresponds to the End block in Fig. 2(c). If the above condition is not fulfilled, then every combination between failed line iic and overloaded flow at line jjc ($\widehat{F}'^{jjc}_{n,iic} \geq \overline{F'}_{iic}$) is computed and stored per time period, and afterwards added to the SCUC formulation using the corresponding constraints (28) and (29) (see Fig. 2(d)). After solving the SCUC with the binding combinations of equations (28) and (29), the new pre-contingency flows are analyzed again to verify if additional combinations should be added using the proposed filter again (30). This iterative approach checks all the contingencies at each iteration. Then if the estimated power flow on line iic due to outage in line jjc is higher than maximum line iic capacity, those contingencies are added to the SCUC in the

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPWRS.2017.2686701, IEEE Transactions on Power Systems

next iteration. This set of constraints (29) represents the umbrella constraints associated to the security constraints. In real power systems, it is expected, from the experience of the ISO, that only a short list of N-1 line outages, compared to all the available lines in the system, is required to establish a secure operation.

The proposed filtering methodology provides an efficient way to find out the active contingencies to include in the SCUC. Other iteration techniques applied to SCUC such as Benders decomposition and Lagrangian relaxation need to solve subproblems in order to perform the security analysis taking into account corrective actions at each contingency [33], but increasing the total solution time. However, the proposed filtering methodology does not require solving a mathematical programming subproblem for each contingency. Another advantage of this methodology is that it is not necessary to solve the ad-hoc discovery problem to determine the umbrella constraints, which could be a larger problem than the SCUC [39], [40]. Numerical results are shown for a test system in section V.B.

V.NUMERICAL RESULTS

The performance of the proposed formulation and methodology have been evaluated using the modified IEEE 118-bus test system, available online at [44], for a time span of 24 hours and also for the 4 times interconnected IEEE 118-bus systems. The system consist of 118 buses; 186 transmission lines; 54 thermal units; 91 loads, with average and maximum aggregated levels of 3991MW and 5592MW, respectively; and three wind units, with aggregated average and maximum production for the nominal wind case of 867MW and 1333MW, respectively. The number of considered lines outages is 177. This number is less than the number of lines in IEEE 118-bus system, because the radial transmission lines have been disregarded.

The models are solved using the commercial solver GUROBI 6.0 [45]. All instances were solved using three different relative optimality criteria or integrality gaps (IntGap), 1E-2, 5E-3, and 1E-3. The IntGap is defined as $|Z_{BP} - Z_{BF}| / |Z_{BP}|$, where Z_{BF} is the objective function value of the current best integer solution while Z_{BP} is the best possible integer solution. The problems are solved until they hit the time limit of 2 hours or until they reach the given IntGap. The solver defaults settings were used for all the experiments, which were run on an Intel-i7 CPU @3.4-GHz computer with 16GB of RAM memory and four cores.

A. Comparison between formulations without filter

For this case study, in the LODF formulation, the nonzero elements are only 5.4% of the *Ref* formulation. On the other hand, the RAM usage is reduced from 11128MB in *Ref* formulation to 616MB in LODF formulation. Therefore, the LODF formulation is almost 20 times more compact[2] than the *Ref* formulation. This is a consequence of the reduction on the number of non-zero coefficients shown in Table I.

TABLE II
COMPUTATIONAL BURDEN

	IntGap	CPU Time [s]	Nodes explored
Ref	1 E-02	399	62
	5 E-03	468	129
	1 E-03	5731	1650
LODF	1 E-02	38	54
	5 E-03	46	136
	1 E-03	994	4859
	0	3452	122816

TABLE III
COMPUTATIONAL BURDEN AT EACH ITERATION - INTGAP = 1E-3

Iteration	N-1 added	CPU Time [s]	Nodes explored	Obj. function [M$]
1	-	5	0	0.809868
2	215	33	923	0.831672
3	13	72	1437	0.834148
4	12	82	1625	0.835073
5	5	79	1572	0.835080
Total	245	271	5557	-

TABLE IV
COMPUTATIONAL BURDEN AT EACH ITERATION - INTGAP = 0

Iteration	N-1 added	CPU Time [s]	Nodes explored	Obj. function [M$]
1	-	12	388	0.809809
2	207	65	2137	0.831522
3	13	93	4405	0.834118
4	12	469	42170	0.835008
5	5	356	35426	0.835040
Total	237	995	84526	-

The tightness of the formulations can be measured using IntGap as a reference at the first iteration. For instance, if formulation A has a lower IntGap at the first iteration than formulation B, it means that the formulation A is tighter or stronger than B. The tightness of the formulations is important, because a lower IntGap at the first iteration implies that we are closer to the optimal solution and therefore, a lower CPU time is expected. *Ref* formulation has an IntGap equal to 57.24% at iteration one while LODF formulation has an IntGap equal to 7.44% at iteration one. Thus, LODF formulation is tighter than *Ref* formulation.

Both tightness and compactness of LODF formulation have impact on CPU time. Actually, Table II presents the results for computational burden obtained for each formulation using GUROBI. The results have shown a significant reduction of CPU time for all the IntGap. In fact, the LODF formulation allows to solve the SCUC for all IntGap in CPU times under 1 hour, situation that is not possible with the *Ref* formulation, see Table II.

B. Results using filtering methodology

The SCUC for 177 line outages in the IEEE 118-bus system was solved in the previous section. Therefore, 790,128 constraints are included in the complete formulation of SCUC. However, from the results analysis, only 94 N-1 equations (0.012%) are binding constraints. This case shows the relevance of the methodology presented in section IV. Firstly,

---

[2] Small density of the constraint matrix.

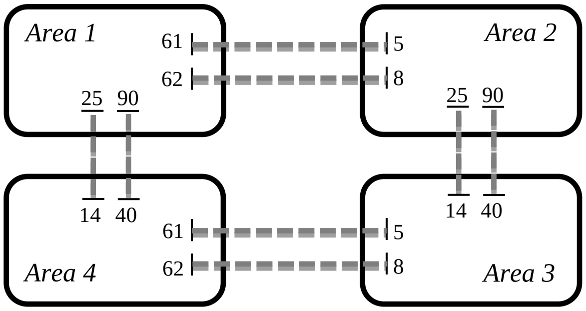


Fig. 3. Modified IEEE 118-bus test system

the IntGap = 1E-3 is selected for the test[3]. As result, the algorithm solves the SCUC in five iterations (see Table III). In the first iteration, the network constrained UC was solved. Then, 215 line outages were identified as candidates to be included in the N-1 security constraint. Afterwards, the SCUC is solved till the 5th iteration, until no more line outages are candidates to be included in the SCUC, and the algorithm ends. The total number of N-1 constraints added is 245 (0.03%), higher than the 94 that are relevant in the complete problem. However, it is still a very low percentage of the total N-1 constraints of the complete problem. Finally, the total CPU time is 271s, 73% less than the 994s necessary to solve SCUC with all the N-1 security constraints (see Table II). The methodology proposed is also tested with IntGap = 0. The results are presented in Table IV. In this case, one additional iteration is necessary compared to the case with IntGap = 1E-3. The total number of N-1 added constraints is 237 (0.029%). On the other hand, the total CPU time is around the 995s, 71% less than the 3452s necessary to solve SCUC with all the N-1 security constraints (see Table II). It is important to highlight that at each iteration the value of the objective function is increasing until it achieves an optimal value which coincides with the one obtained with all N-1 security constraints.

C. Results for large scale test system

The proposed methodology is tested in a large-scale system. We have created a test system based on IEEE 118-bus test system. This large-scale system has 4 interconnected areas, each area corresponding to one IEEE 118-bus test system. Therefore, the size of the large-scale system is four times the original IEEE 118-bus test system. Each area is interconnected with other two areas through two transmission lines (see Fig. 3). The interconnection lines have a reactance of 0.0493p.u. and a maximum capacity of 750MW. Consequently, the built-up system is composed of 472 buses, 752 transmission lines, 364 demand sides, and 216 thermal units. The complete set of possible N-1 contingencies is 12,286,560 for an optimization period of 24 hours, and disregarding the radial transmission lines.

The size of the complete problem (including all N-1 contingencies) using $LODF$ formulation is 12,911MB. The SCUC problem using $LODF$ formulation is solved in 28,083 s with an IntGap = 1E-3. The SCUC problem cannot be solved as a complete problem using the *Ref* formulation and therefore a decomposition technique, such as Benders decomposition, is required. The iterative approach is used in this large-scale problem to solve the SCUC problem in less CPU time. A Relaxed Mixed-Integer Linear Programming (RMIP)[4] version of the model is used for the first five iterations, and then the Mixed-Integer Linear Programming (MIP) version of the model is used for further iterations until convergence is achieved as it is shown in Fig. 2. This strategy helps to reduce the CPU time at the beginning of the iterative approach and enhances the overall performance in large-scale systems. Table V shows the results of the iterative approach using filtering methodology. Eight iterations are needed to solve the SCUC and the total CPU time is 1011s, 96% faster that the solution using the complete set of contingencies (28,083 s). The size of the problem at the last iteration is 78% lower than SCUC problem with all contingencies.

TABLE V
LARGE SCALE SYSTEM COMPUTATIONAL BURDEN AT EACH ITERATION

Iteration	N-1 added	CPU Time [s]	Size [MB]	Obj. function [M$]
1	-	83	2631	2.220797
2	186	94	2817	2.275138
3	28	142	2817	2.278236
4	7	121	2817	2.278253
5	35	117	2817	2.278760
6	16	155	2817	3.361450
7	309	148	2817	3.409381
8	2	150	2817	3.409381
Total	583	1011	-	-

TABLE VI
RESULTS SUMMARY FOR EACH REGIONAL GROUP

Regional Group	Continental Central East (CCE)	Continental South West (CSW)	North Sea (NS)	Continental Central South (CCS)
Countries	AT, CZ, DE, PL	ES, FR, PT	BE, DE, DK, FR, GB, NL	AT, CH, DE, FR, IT, SI
Nodes	298	367	631	673
Thermal gen.	455	446	596	804
Lines	639	821	1337	1371
Initial size [MB]	2524	4124	9988	11476
Final size [MB]	2612	4248	10268	11784
Iterations	9	8	8	7
N-1 added	335	766	1403	1256
% of whole set of N-1	0.0197%	0.0136%	0.0113%	0.0144%
CPU Time [s]	324	698	4723	4526

D. Results for European cases

The European Network of Transmission System Operators for Electricity (ENTSO-E) is structured into six regional groups for grid planning and other system development tasks.

---

[3] For the iterative algorithm, the option *"LazyConstraints"* in GUROBI is used for equation (30).

[4] The RMIP problem is the same as the MIP problem in all respects except all the integer variables are relaxed.

The countries belonging to each regional group are shown in [46]. We have selected four simplified versions of these regional groups to test our proposal in medium- and large-scale systems. Table VI shows the regional group and countries that we have selected. These European cases are used for planning studies and they are not used in day-ahead dispatch.

Table VI shows the number of thermal generators, nodes, and transmission lines for each regional group. We have applied the iterative approach proposed in Section IV to solve the SCUC for each regional group. The results show that the iterative approach ends after 7 to 9 iterations, and the amount of N-1 constraints added to the problem is small in comparison to the set of all possible N-1 constraints. The final size of the problem also indicates that the number of N-1 constraints added to the SCUC is increasing by less than 5% of the initial size of the problem. Therefore, we are fulfilling the security constraints in the UC without solving the full SCUC.

VI. CONCLUSION

This paper has presented a formulation for the N-1 security constraint for the SCUC using the Line Outage Distribution Factors (LODF). It was shown, conceptually and numerically, that the proposed formulation reduces the nonzero elements, in comparison to similar formulations that are available in the literature. Consequently, the computational burden for the SCUC problem is reduced, and optimal solutions can be obtained significantly faster. Furthermore, with the methodology proposed for large-scale systems is possible to reduce even more the total CPU time. The results have shown that the proposed methodology reduces computational time by 71% to 73% in comparison to the complete formulation of N-1 constraints in SCUC for the analyzed case studies. The proposed formulation and methodology can support the SOs evaluation of their day ahead planning in a faster way, and the real-time re-dispatch. The applicability of this methodology does not depend on the size of the system and always provides a computational advantage, even better for larger scale problems. We have shown that for a large-scale system, the compactness of the LODF formulation allows to solve the SCUC for the complete set of contingencies in the system. This situation is not possible using the Ref formulation without the use of a decomposition technique such as Benders decomposition. We also have shown the applicability of the LODF post-contingency filter in medium and large scale European cases.

Regarding further improvements, the proposed SCUC formulation could be extended by introducing transmission losses, outages in generation, and evaluation of N-k line outages. It is possible to use the Generalized Line Outage Distribution Factors presented in [32] for the N-k line outages.

REFERENCES

[1] A. Tavakoli, M. J. Sanjari, H. Karami, S. H. Hosseinian, and G. B. Gharehpetian, “Imperialistic Competitive Algorithm Based Unit Commitment Considering Risk of Cascading Blackout,” *Electr. Power Compon. Syst.*, vol. 43, no. 4, pp. 374–383, Feb. 2015.

[2] A. Gomez-Exposito, A. J. Conejo, and C. Canizares, *Electric Energy Systems: Analysis and Operation*. CRC Press, 2008.

[3] B. F. Hobbs, W. R. S. Jr, R. E. Bixby, M. H. Rothkopf, R. P. O'Neill, and H. Chao, “Why this Book? New Capabilities and New Needs for Unit Commitment Modeling,” in *The Next Generation of Electric Power Unit Commitment Models*, B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. Chao, Eds. Springer US, 2002, pp. 1–14.

[4] B. F. Hobbs, M. H. Rothkopf, R. P. O'Neill, and H. Chao, *The Next Generation of Electric Power Unit Commitment Models*. Springer Science & Business Media, 2006.

[5] G. Morales-España, “Unit Commitment: Computational Performance, System Representation and Wind Uncertainty Management,” Doctoral thesis, Pontifical Comillas University, KTH Royal Institute of Technology, and Delft University of Technology, 2014.

[6] A. Kalantari, J. F. Restrepo, and F. D. Galiana, “Security-Constrained Unit Commitment With Uncertain Wind Generation: The Loadability Set Approach,” *Power Syst. IEEE Trans. On*, vol. 28, no. 2, pp. 1787–1796, May 2013.

[7] Ruiwei Jiang, Jianhui Wang, and Yongpei Guan, “Robust Unit Commitment With Wind Power and Pumped Storage Hydro,” *Power Syst. IEEE Trans. On*, vol. 27, no. 2, pp. 800–810, May 2012.

[8] H. Quan, D. Srinivasan, A. M. Khambadkone, and A. Khosravi, “A computational framework for uncertainty integration in stochastic unit commitment with intermittent renewable energy sources,” *Appl. Energy*, vol. 152, pp. 71–82, Aug. 2015.

[9] B. Vieira, A. Viana, M. Matos, and J. P. Pedroso, “A multiple criteria utility-based approach for unit commitment with wind power and pumped storage hydro,” *Electr. Power Syst. Res.*, vol. 131, pp. 244–254, Feb. 2016.

[10] C. Liu, M. Shahidehpour, Z. Li, and M. Fotuhi-Firuzabad, “Component and Mode Models for the Short-Term Scheduling of Combined-Cycle Units,” *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 976–990, May 2009.

[11] G. Morales-Espana, J. M. Latorre, and A. Ramos, “Tight and Compact MILP Formulation for the Thermal Unit Commitment Problem,” *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4897–4908, Nov. 2013.

[12] G. Morales-Espana, C. M. Correa-Posada, and A. Ramos, “Tight and Compact MIP Formulation of Configuration-Based Combined-Cycle Units,” *Power Syst. IEEE Trans. On*, vol. PP, no. 99, pp. 1–10, 2015.

[13] R. Chhetri, *Security Constrained Unit Commitment Considering Multiple Regions*. University of New Brunswick (Canada), 2006.

[14] M. Shahidehpour, “Security-Constrained Unit Commitment with Stochastic Constraints,” in *IEEE Power Engineering Society General Meeting, 2007*, 2007, pp. 1–3.

[15] Yong Fu, Zuyi Li, and Lei Wu, “Modeling and Solution of the Large-Scale Security-Constrained Unit Commitment,” *Power Syst. IEEE Trans. On*, vol. 28, no. 4, pp. 3524–3533, Nov. 2013.

[16] M. M. R. Sahebi and S. H. Hosseini, “Stochastic security constrained unit commitment incorporating demand side reserve,” *Int. J. Electr. Power Energy Syst.*, vol. 56, pp. 175–184, Mar. 2014.

[17] E. Nasrolahpour and H. Ghasemi, “A stochastic security constrained unit commitment model for reconfigurable networks with high wind power penetration,” *Electr. Power Syst. Res.*, vol. 121, pp. 341–350, Apr. 2015.

[18] M. Shahidehpour, H. Yamin, and Z. Li, *Market operations in electric power systems: forecasting, scheduling, and risk management*. New York: Institute of Electrical and Electronics Engineers, Wiley-Interscience, 2002.

[19] G. Morales-Espana, A. Ramos, and J. Garcia-Gonzalez, “An MIP Formulation for Joint Market-Clearing of Energy and Reserves Based on Ramp Scheduling,” *Power Syst. IEEE Trans. On*, vol. 29, no. 1, pp. 476–488, Jan. 2014.

[20] G. Morales-Espana, R. Baldick, J. Garcia-Gonzalez, and A. Ramos, “Power-Capacity and Ramp-Capability Reserves for Wind Integration in Power-Based UC,” *Sustain. Energy IEEE Trans. On*, vol. PP, no. 99, pp. 1–11, 2015.

[21] C. Sahin, M. Shahidehpour, and I. Erkmen, “Allocation of Hourly Reserve Versus Demand Response for Security-Constrained Scheduling of Stochastic Wind Energy,” *IEEE Trans. Sustain. Energy*, vol. 4, no. 1, pp. 219–228, Jan. 2013.

[22] D. Bertsimas, E. Litvinov, X. A. Sun, J. Zhao, and T. Zheng, “Adaptive Robust Optimization for the Security Constrained Unit Commitment

Problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52–63, Feb. 2013.

[23] R. L.-Y. Chen, N. Fan, A. Pinar, and J.-P. Watson, "Contingency-constrained unit commitment with post-contingency corrective recourse," *Ann. Oper. Res.*, pp. 1–27, Dec. 2014.

[24] A. Castillo, C. Laird, C. A. Silva-Monroy, J.-P. Watson, and R. P. O'Neill, "The Unit Commitment Problem With AC Optimal Power Flow Constraints," *Power Syst. IEEE Trans. On*, vol. PP, no. 99, pp. 1–14, 2016.

[25] A. Lotfjou, M. Shahidehpour, Yong Fu, and Zuyi Li, "Security-Constrained Unit Commitment With AC/DC Transmission Systems," *Power Syst. IEEE Trans. On*, vol. 25, no. 1, pp. 531–542, Feb. 2010.

[26] A. J. Wood, B. F. Wollenberg, and G. B. Sheblé, *Power Generation, Operation and Control*, 3 edition. Wiley-Interscience, 2013.

[27] F. Gonzalez-Longatt and J. L. Rueda, *PowerFactory Applications for Power System Analysis*. Springer, 2014.

[28] Hao Zhu, "ECE530," *ECE530 - Analysis Techniques for Large-Scale Electrical Systems*. [Online]. Available: https://courses.engr.illinois.edu/ece530/. [Accessed: 18-Jan-2016].

[29] V. H. Hinojosa and J. Velásquez, "Improving the mathematical formulation of security-constrained generation capacity expansion planning using power transmission distribution factors and line outage distribution factors," *Electr. Power Syst. Res.*, vol. 140, pp. 391–400, 2016.

[30] J. Zhu, *Optimization of Power System Operation*. John Wiley & Sons, 2015.

[31] J. Guo, Y. Fu, Z. Li, and M. Shahidehpour, "Direct Calculation of Line Outage Distribution Factors," *IEEE Trans. Power Syst.*, vol. 24, no. 3, pp. 1633–1634, Aug. 2009.

[32] T. Güler, G. Gross, and M. Liu, "Generalized Line Outage Distribution Factors," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 879–881, May 2007.

[33] H. Pinto, F. Magnago, S. Brignone, O. Alsac, and B. Stott, "Security Constrained Unit Commitment: Network Modeling and Solution Issues," in *2006 IEEE PES Power Systems Conference and Exposition*, 2006, pp. 1759–1766.

[34] G. A. Dourbois and P. N. Biskas, "A nodal-based security-constrained day-ahead market clearing model incorporating multi-period products," *Electr. Power Syst. Res.*, vol. 141, pp. 124–136, Diciembre 2016.

[35] J. Alemany and F. Magnago, "Benders decomposition applied to security constrained unit commitment: Initialization of the algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 66, pp. 53–66, Mar. 2015.

[36] J. Alemany, D. Moitre, and F. Magnago, "Benders Decomposition applied to Security Constrained Unit Commitment," *Lat. Am. Trans. IEEE Rev. IEEE Am. Lat.*, vol. 11, no. 1, pp. 421–425, Feb. 2013.

[37] M. Shahidehopour and Y. Fu, "Benders decomposition: applying Benders decomposition to power systems," *IEEE Power Energy Mag.*, vol. 3, no. 2, pp. 20–21, Mar. 2005.

[38] Q. Zhai, X. Guan, J. Cheng, and H. Wu, "Fast Identification of Inactive Security Constraints in SCUC Problems," *IEEE Trans. Power Syst.*, vol. 25, no. 4, pp. 1946–1954, Nov. 2010.

[39] A. J. Ardakani and F. Bouffard, "Identification of Umbrella Constraints in DC-Based Security-Constrained Optimal Power Flow," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3924–3934, Nov. 2013.

[40] A. J. Ardakani and F. Bouffard, "Acceleration of Umbrella Constraint Discovery in Generation Scheduling Problems," *IEEE Trans. Power Syst.*, vol. 30, no. 4, pp. 2100–2109, Jul. 2015.

[41] "118 Bus Power Flow Test Case," *Power Systems Test Case Archive*. [Online]. Available: https://www.ee.washington.edu/research/pstca/pf118/pg_tca118bus.htm. [Accessed: 04-Jan-2016].

[42] A. G. Exposito, J. L. M. Ramos, and J. R. Santos, "Slack bus selection to minimize the system power imbalance in load-flow studies," *IEEE Trans. Power Syst.*, vol. 19, no. 2, pp. 987–995, May 2004.

[43] C. Gentile, G. Morales-España, and A. Ramos, "A tight MIP formulation of the unit commitment problem with start-up and shut-down constraints," *EURO J. Comput. Optim.*, pp. 1–25, Apr. 2016.

[44] "Modified IEEE 118-bus test system," *Andres Ramos' Home Page*. [Online]. Available: http://www.iit.upcomillas.es/aramos/IEEE118_SUSD-Ramps.xls. [Accessed: 04-Jan-2016].

[45] "GAMS Documentation Center," *Gurobi 6*. [Online]. Available: https://gams.com/help/index.jsp?topic=%2Fgams.doc%2Fsolvers%2Fgurobi%2Findex.html. [Accessed: 10-Jul-2016].

[46] European Network of Transmission System Operators for Electricity ENTSO-E, "Regional Investment Plan 2015," Oct. 2015.

Diego A. Tejada-Arango (M'11-S'15) received his BSc degree in 2006, from the "Universidad Nacional", in Medellin, Colombia, in electrical engineering, in 2013 he obtained a MSc degree in electrical engineering at Universidad de Antioquia, in Medellin, Colombia with the GIMEL Group; he is working towards a MSc in Research in Engineering Systems Modeling at Universidad Pontificia Comillas, in Madrid, Spain with the IIT. His areas of research are transmission expansion planning, and planning and operation of electric energy systems.

Pedro Sánchez-Martın received the B.S. degree and Ph.D. degree in electrical engineering from the Comillas Pontifical University, Madrid, Spain, in 1993 and 1998, respectively. Currently, he is a Research Fellow with the Technological Research Institute (IIT) and Assistant Professor at ICAI School of Engineering, Comillas Pontifical University. Currently, he is the Head of the Department of Industrial Organization. His areas of interest are planning and operation of power and natural gas systems and management science applied to manufacturing processes.

Andres Ramos received the degree in electrical engineering from Universidad

Pontificia Comillas, Madrid, Spain, in 1982 and the Ph.D. degree in electrical engineering from Universidad Politécnica de Madrid, Madrid, Spain, in 1990. He is a research fellow at Instituto de Investigación Tecnológica, Madrid, Spain, and a full professor at Comillas' School of Engineering, Madrid, Spain, where he has been the Head of the Department of Industrial Organization. His areas of interest include the operation, planning, and economy of power systems and the application of operations research to industrial organization.