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Nash equilibrium in competitive insurance

Anastasios Dosis

Department of Economics - ESSEC Business School and THEMA, 3 Avenue Bernard Hirsch, 95021 Cergy-Pontoise Cedex, France

HIGHLIGHTS

- I study an insurance market with any finite number of types as a standard duopoly.
- I formally specify demand functions and profits.
- I provide an easy proof for the (non) existence of (pure strategy) Nash equilibrium.

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ABSTRACT

I formalize a rather stylized insurance market with adverse selection as a standard duopoly. I formally specify demand functions and profits and prove that a Nash equilibrium in pure strategies exists if and only if the well-known Rothschild–Stiglitz allocation is efficient.

1. Motivation

Rothschild and Stiglitz (1976) show that an equilibrium may not exist in competitive insurance markets with adverse selection. Nonetheless, their analysis does not explicitly specify a competition game, and the arguments are, for the most part, diagrammatic and concern only two possible types. Furthermore, each company is allowed to offer only one contract. Riley (1979) and Wilson (1977) extend the result to more than two types. They also propose "reactive" equilibria. Miyazaki (1977) and Spence (1978) allow companies to offer menus of contracts and show that reactive equilibrium exists and is efficient. Notably, neither of these papers explicitly specifies a competition game. Engers and Fernandez (1987) and Hellwig (1987) propose extensive-form games that depart from "Bertrand-type" competition and show that equilibrium exists but highlight the difficulties of explicitly modeling the reactive equilibria of Wilson and Riley. Classic microeconomics textbooks such as Jehle and Reny (2011) and Mas-Colell et al. (1995) examine games in which companies compete by offering menus of contracts but focus on the two-type case. Netzer and Scheuer (2014) analyze an extensive-form game in which companies can become inactive at a cost and show that an equilibrium may also exist or fail to exist in the two-type case. Dasgupta and Maskin (1986a, b) and Rosenthal and Weiss (1984) prove the existence of mixed-strategy equilibria in the two-type case. In this note, I formalize a rather stylized insurance market with any finite number of types as a standard duopoly and provide a step-by-step proof for the (non) existence of (pure strategy) Nash equilibrium.

2. The model

■ **Consumers and companies.** There is a measure one of consumers. Each consumer belongs to one of a finite set of types $\theta = 1, \ldots, N$. For simplicity, I sometimes denote the set of types by Θ. The share of type-θ consumers in the population is λ^θ, with $\sum_\theta \lambda^\theta = 1$. There are two possible (individual) states $\omega = 0, 1$, where $\omega = 1$ represents the state in which a consumer suffers an accident, and $\omega = 0$, the state in which there is no accident. Uncertainty is purely idiosyncratic, and hence, states occur independently across different consumers. Each consumer begins with endowment W and suffers a loss ℓ, where $W > \ell > 0$ if and only if the accident occurs. A consumer of type θ has probability

E-mail address: dosis@essec.com.

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π_ω^θ of being in state ω, with $\sum_\omega \pi_\omega^\theta = 1$ for every θ. Moreover, let $\pi_0^1 < \pi_0^2 < \cdots < \pi_0^N$. An insurance contract is $x = (p, b) \in \mathcal{X}$, where $\mathcal{X} = \{(\alpha, \beta) \in \mathbb{R}_+^2 : \alpha \le W, \alpha - \beta \le W - \ell\}$. In insurance terms, p specifies the insurance premium and b the benefit that the consumer receives if and only if the accident occurs. A consumer of type θ has preferences represented by an expected utility function $U^\theta(x) = \pi_0^\theta u(W-p) + \pi_1^\theta u(W-\ell-p+b)$, where u is continuous, strictly increasing and strictly concave. The status quo utility of type θ is $\underline{U}^\theta = \pi_0^\theta u(W) + \pi_1^\theta u(W-\ell)$. Finally, there exist two symmetric companies in the market $i = 1, 2$. Because I only consider symmetric companies, there is no loss of generality in assuming the existence of only two companies. If type θ buys contract x from company i, then the latter earns an expected profit equal to $\zeta^\theta(x) = p - \pi_1^\theta b$.

□ **Allocations.** An *allocation* is a vector of contracts indexed by the set of types, $(x^\theta)_\theta$.[1] An allocation $(x^\theta)_\theta$ is *incentive compatible* iff $U^\theta(x^\theta) \ge U^\theta(x^{\theta'})$ for every $\theta, \theta' \in \Theta$. Efficient allocations play a key role in studying the existence of an equilibrium. An efficient allocation is formally defined below.

Definition 2.1. An allocation $(x^\theta)_\theta$ is efficient if and only if: (i) it is incentive compatible, (ii) $\sum_\theta \lambda^\theta \pi^\theta(x^\theta) \ge 0$, and (iii) there exists no other allocation $(\hat{x}^\theta)_\theta$ that satisfies (i), (ii) and $U^\theta(\hat{x}^\theta) \ge U^\theta(x^\theta)$ for every θ, with the inequality being strict for at least one θ.

Efficiency, as is defined here, is standard Pareto efficiency subject to incentive constraints. Note that, as is fairly standard in these environments, efficiency is defined with respect to the payoff of the consumers and the average resource constraint. One can establish the following result regarding the set of efficient allocations.

Lemma 2.2. *If allocation $(x^\theta)_\theta$ is efficient, then $\sum_\theta \lambda^\theta \zeta^\theta(x^\theta) = 0$.*

Proof. I prove the result by contraposition. Suppose that $(x^\theta)_\theta$ is an incentive compatible allocation such that $\sum_\theta \lambda^\theta \zeta^\theta(x^\theta) > 0$. Consider allocation $(\tilde{x}^\theta)_\theta$, where for $\tilde{x}^\theta$,

$$u(W - \tilde{p}^\theta) = \epsilon u(W - p^\theta) + (1-\epsilon)u(W) \tag{2.1}$$

and

$$u(W - \ell - \tilde{p}^\theta + \tilde{b}^\theta) = \epsilon u(W - \ell - p^\theta + b^\theta) + (1-\epsilon)u(W - \ell + \hat{b}) \tag{2.2}$$

for $\hat{b} > 0$. Because $u(\cdot)$ is strictly concave, by Jensen's inequality, for every $\theta \in \Theta$,

$$W - \tilde{p}^\theta < \epsilon(W - p^\theta) + (1-\epsilon)W \tag{2.3}$$

and

$$W - \ell - \tilde{p}^\theta + \tilde{b}^\theta < \epsilon(W - \ell - p^\theta + b^\theta) + (1-\epsilon)(W - \ell + \hat{b}). \tag{2.4}$$

Multiplying Eq. (2.3) by π_0^θ and Eq. (2.4) by π_1^θ and summing them up yields

$$\zeta^\theta(\tilde{x}^\theta) > \epsilon \zeta^\theta(x^\theta) - (1-\epsilon)\pi_1^\theta \hat{b}. \tag{2.5}$$

Multiplying Eq. (2.5) by λ^θ and summing over θ yields

$$\sum_\theta \lambda^\theta \zeta^\theta(\tilde{x}^\theta) > \epsilon \sum_\theta \lambda^\theta \zeta^\theta(x^\theta) - (1-\epsilon)\sum_\theta \lambda^\theta \pi_1^\theta \hat{b}. \tag{2.6}$$

Because $(x^\theta)_\theta$ is incentive compatible by definition and due to Eqs. (2.1) and (2.2), for every $\epsilon \in (0, 1)$ the following are true:

$$U^\theta(x^\theta) \ge U^\theta(x^{\theta'}) \quad \forall \quad \theta, \theta'$$

$$U^\theta(\tilde{x}^\theta) = \epsilon U^\theta(x^\theta) + (1-\epsilon)\big(\pi_0^\theta u(W) + \pi_1^\theta u(W - \ell + \hat{b})\big) \quad \forall \quad \theta$$

$$U^\theta(\tilde{x}^{\theta'}) = \epsilon U^\theta(x^{\theta'}) + (1-\epsilon)\big(\pi_0^\theta u(W) + \pi_1^\theta u(W - \ell + \hat{b})\big) \quad \forall \quad \theta, \theta'.$$

Therefore, $(\tilde{x}^\theta)_\theta$ is incentive compatible. Evidently, there exist ϵ and $\hat{b}$ such that $U^\theta(\tilde{x}^\theta) > U^\theta(x^\theta)$ for every $\theta \in \Theta$ and $\sum_\theta \lambda^\theta \zeta^\theta(\tilde{x}^\theta) > 0$. Hence, $(x^\theta)_\theta$ is not efficient. □

An allocation that plays a significant role in insurance markets with adverse selection is what is usually called the *Rothschild–Stiglitz Allocation* (RSA). This is identified in nearly all studies mentioned in the introduction. It maximizes the payoff of every type within the set of incentive compatible allocations that make positive profits type-by-type. A formal definition of a RSA follows.

Definition 2.3. An allocation $(x^\theta)_\theta$ is an RSA if and only if: (i) it is incentive compatible, (ii) $\zeta^\theta(x^\theta) \ge 0$ for every $\theta \in \Theta$, and (iii) there exists no other allocation $(\tilde{x}^\theta)_\theta$ that satisfies (i), (ii) and $U^\theta(\tilde{x}^\theta) \ge U^\theta(\tilde{x}^\theta)$ for every θ, with the inequality being strict for at least one θ.

Remarks. It is well known that with only two possible types, the RSA is efficient when the share of type-1 consumers (i.e., the high-risk consumers) in the population is sufficiently large. A similar result applies here. Note first that in the RSA, type 1's contract is $(\pi_1^1\ell, \ell)$ (i.e., the full-coverage contract that makes zero profits if taken by type 1 only) and all incentive constraints are binding. Therefore, every contract that is preferred by a group of types higher in the rank than type 1 over the RSA allocation is also preferred by type 1. Evidently, every such contract is loss-making if taken only by type 1, given that $(\pi_1^1\ell, \ell)$ is the payoff-maximizing contract for type 1 that makes zero profits. If the share of type-1 consumers is sufficiently large, then every menu of contracts that is preferred by a subset of types (e.g., $\{1, \ldots, n\}$) necessarily makes negative profits. Hence, the RSA satisfies Definition 2.1.

□ **Menus, demands and profits.** Each of the two companies selects a menu of contracts. The set of possible menus for each company is $\mathcal{X}^N$. Let m_i denote a menu for company i and $\boldsymbol{m} = (m_1, m_2)$ a profile of menus. Based on all contracts that are available in the market, each consumer purchases a contract from one of the two companies. Let $(q_1^\theta(\mathbf{m}), q_2^\theta(\mathbf{m}))$, where $q_i^\theta(\mathbf{m}) : \{x : x \in m_i\} \to [0, \lambda^\theta]$, denote a pair of measures for every $\mathbf{m} \in \mathcal{X}^{2N}$. Each of these measures represents the demand function from type-θ consumers to company i when the menus of contracts are $\mathbf{m} = (m_1, m_2)$. For every θ, $\mathbf{m}$ and i, the following sequential rationality conditions must be satisfied:

$$q_i^\theta(x|\mathbf{m}) = 0 \text{ if } \quad U^\theta(x) < \max_{y \in m_1 \cup m_2 \cup \{(0,0)\}} U^\theta(y) \tag{2.7}$$

$$q_0^\theta(\mathbf{m}) + \sum_i \sum_{x \in m_i} q_i^\theta(x|\mathbf{m}) = \lambda^\theta,$$

$$\text{where } q_0^\theta(\mathbf{m}) = \begin{cases} >0, & \text{if } U^\theta(0,0) > \max_{y \in m_1 \cup m_2} U^\theta(y) \\ 0, & \text{otherwise.} \end{cases} \tag{2.8}$$

Eq. (2.7) states that the demand for a contract is zero when this contract does not belong to the set of contracts that maximize the utility of type θ among all the contracts that are offered in the market (i.e., $m_1 \cup m_2 \cup \{(0, 0)\}$). Eq. (2.8) states that the measures sum to λ^θ; the ex ante share of type θ. q_0^θ represents the share of types that does not buy any insurance. This is strictly positive if and

[1] An allocation defines a mapping from the type space to the set of contracts. In mechanism design jargon, an allocation is a direct revelation mechanism.

only if no contract offered by the two companies provides a strictly higher payoff than the null contract (i.e., $(0,0)$). Based on $q_i(\mathbf{m})$, the expected profit of firm i is written as

$$\Pi_i(m_i, m_{-i}|q_i(\mathbf{m})) = \sum_\theta \sum_{x\in m_i} q_i^\theta(x|\mathbf{m})\zeta^\theta(x).$$

3. Existence of an equilibrium

A formal definition of Nash equilibrium follows:

Definition 3.1. A Nash equilibrium consists of a profile of actions $\bar{\mathbf{m}} = (\bar{m}_1, \bar{m}_2)$ such that for every i $\bar{m}_i \in \arg\max_{m_i} \Pi_i(m_i, \bar{m}_{-i}|q_i(\mathbf{m}))$ for some $(q_1^\theta(\mathbf{m}), q_2^\theta(\mathbf{m}))$ satisfying Eqs. (2.7) and (2.8) for every θ and $\mathbf{m}$.

To study the existence of an equilibrium, I proceed in steps. The following trivial lemma facilitates the proofs.

Lemma 3.2. *Let $\bar{\mathbf{m}} = (\bar{m}_1, \bar{m}_2)$ be an equilibrium in pure strategies. Then,*

(i) *$\Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) \geq 0$ for every i,*
(ii) *There exists j such that $\Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}})) \leq \sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}| q_i(\bar{\mathbf{m}}))$, with the inequality being strict when $\sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}| q_i(\bar{\mathbf{m}})) > 0$.*

Step 1. The first step is to show that in equilibrium all consumers of the same type purchase a contract with the same terms. This is formally stated in the following lemma:

Lemma 3.3. *Suppose that $\bar{\mathbf{m}} = (\bar{m}_1, \bar{m}_2)$ is an equilibrium in pure strategies. Then, for every $x_1, x_2 \in \arg\max_{x\in\bar{m}_1\cup\bar{m}_2\cup\{(0,0)\}} U^\theta(x)$, $x_1 = x_2$.*

Proof. Suppose that $\bar{\mathbf{m}} = (\bar{m}_1, \bar{m}_2)$ is an equilibrium profile in pure strategies and there exists η and $x_1, x_2 \in \arg\max_{x\in\bar{m}_1\cup\bar{m}_2\cup\{(0,0)\}} U^\eta(x)$ such that $x_1 \neq x_2$. Let $x^\theta(\bar{\mathbf{m}})$ denote the contract chosen by type $\theta \neq \eta$ when the profile of menus is $\bar{\mathbf{m}}$. This contract is unique by definition. Consider contract $\bar{x}^\eta = (\bar{p}^\eta, \bar{b}^\eta)$ such that

$$U^\eta(\bar{x}^\eta) = \frac{\sum_{i\in\{j:x_1\in\bar{m}_j\}} q_i(x_1|\bar{\mathbf{m}})}{\lambda^\eta} U^\eta(x_1) + \frac{\sum_{i\in\{j:x_2\in\bar{m}_j\}} q_i(x_2|\bar{\mathbf{m}})}{\lambda^\eta} U^\eta(x_2). \quad (3.1)$$

As in Lemma 2.2, $U^\theta(x^\theta(\bar{\mathbf{m}})) \geq U^\theta(\bar{x}^\eta)$ for every $\theta \neq \eta$. The profit of contract $\bar{x}^\eta$ is

$$\zeta^\eta(\bar{x}^\eta) > \frac{\sum_{i\in\{j:x_1\in\bar{m}_j\}} q_i(x_1|\bar{\mathbf{m}})}{\lambda^\eta}\zeta^\eta(x_1) + \frac{\sum_{i\in\{j:x_2\in\bar{m}_j\}} q_i(x_2|\bar{\mathbf{m}})}{\lambda^\eta}\zeta^\eta(x_2).$$

The aggregate profit of allocation $\left((x^\theta(\bar{\mathbf{m}}))_{\theta\neq\eta}, \bar{x}^\eta\right)$ is

$$\lambda^\eta\zeta^\eta(\bar{x}^\eta) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(x^\theta(\bar{\mathbf{m}})) > \sum_{i\in\{j:x_1\in\bar{m}_j\}} q_i(x_1|\bar{\mathbf{m}})\zeta^\eta(x_1) + \sum_{i\in\{j:x_2\in\bar{m}_j\}} q_i(x_2|\bar{\mathbf{m}})\zeta^\eta(x_2) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(x^\theta(\bar{\mathbf{m}})) = \sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) \geq 0$$

where the last inequality follows from Lemma 3.2. As in Lemma 2.2, there exists an allocation $(\tilde{x}^\theta)_\theta$ such that $U^\theta(\tilde{x}^\theta) > U^\theta(x^\theta(\bar{\boldsymbol{m}}))$ for every θ and

$$\sum_\theta \lambda^\theta\zeta(\tilde{x}^\theta) > \epsilon(\lambda^\eta\zeta^\eta(\bar{x}^\eta) + \sum_{\theta\neq\eta}\lambda^\theta\zeta^\theta(x^\theta(\bar{\mathbf{m}}))) - (1-\epsilon)\sum_\theta \lambda^\theta\pi_1^\theta\hat{b}$$

for $\epsilon \in (0,1)$ and $\hat{b} > 0$. For ϵ and $\hat{b}$ appropriately chosen, and due to Lemma 3.2, there exists j such that

$$\Pi_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j}|q_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j})) > \sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) \geq \Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}})). \quad (3.2)$$

Therefore, from Eq. (3.2),

$$\Pi_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j}|q_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j})) > \Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}}))$$

and, from Definition 3.1,

$$\Pi_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j}|q_j((\tilde{x}^\theta)_\theta, \bar{m}_{-j})) \leq \Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}})).$$

Hence, we have a contradiction. □

Thanks to Lemma 3.3, we can uniquely define by $(x^\theta(\mathbf{m}))_\theta$ an allocation associated with equilibrium $\bar{\mathbf{m}} = (\bar{m}_1, \bar{m}_2)$ such that:

$$x^\theta(\mathbf{m}) \in \arg\max_{x\in m_1\cup m_2\cup\{(0,0)\}} U^\theta(x)$$

$(x^\theta(\mathbf{m}))_\theta$ will henceforth be called an *equilibrium allocation*.

Step 2. The second step is to examine the efficiency properties of equilibrium allocations. We can establish the following result:

Proposition 3.4. *If $(x^\theta(\bar{\mathbf{m}}))_\theta$ is an equilibrium allocation, then it is efficient.*

Proof. I prove the result by contradiction. Suppose that $(x^\theta(\bar{\mathbf{m}}))_\theta$ is not efficient. With a straightforward extension of the argument given in the proof of Lemma 2.2, one can show that for every $\epsilon \in (0,1)$, there exists an incentive compatible allocation $(x_\epsilon^\theta)_\theta$ such that $U^\theta(x_\epsilon^\theta) > U^\theta(x^\theta(\bar{\boldsymbol{m}}))$ for every θ and

$$\sum_\theta \lambda^\theta\zeta^\theta(x_\epsilon^\theta) > \epsilon\sum_\theta\lambda^\theta\zeta^\theta(x^\theta(\bar{\mathbf{m}})) = \epsilon\sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})).$$

Therefore, consider $\tilde{m}_j = (x_\epsilon^\theta)_\theta$. The profit of company j from this menu is

$$\Pi_j(\tilde{m}_j, \bar{m}_{-j}|q_j(\tilde{m}_j, \bar{m}_{-j})) = \sum_\theta \lambda^\theta\zeta^\theta(x_\epsilon^\theta) > \epsilon\sum_i \Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) \quad (3.3)$$

because company j attracts all types when the other company offers menu $\bar{m}_{-j}$. For a sufficiently large ϵ:

$$\Pi_j(\tilde{m}_j, \bar{m}_{-j}|q_j(\tilde{m}_j, \bar{m}_{-j})) > \Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}}))$$

which follows from Lemma 3.2, and

$$\Pi_j(\tilde{m}_j, \bar{m}_{-j}|q_j(\tilde{m}_j, \bar{m}_{-j})) \leq \Pi_j(\bar{m}_j, \bar{m}_{-j}|q_j(\bar{\mathbf{m}}))$$

which follows from Definition 3.1. Therefore, we have a contradiction. □

Step 3. The third step is to show that in equilibrium, cross-subsidization is not possible. This is the underlying idea behind the non-existence of equilibrium in Rothschild and Stiglitz (1976).

Lemma 3.5. *If $\bar{\boldsymbol{m}} = (\bar{m}_1, \bar{m}_2)$ is a pure strategy equilibrium, then $\zeta^\theta(x^\theta(\bar{\boldsymbol{m}})) = 0$ for every θ.*

Proof. I prove the result by contradiction. Suppose that $\bar{\boldsymbol{m}} = (\bar{m}_1, \bar{m}_2)$ is an equilibrium such that, for some θ, $\zeta^\theta(x^\theta(\bar{\boldsymbol{m}})) > 0$. Due to Lemmas 2.2 and 3.5, $\Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) = 0$ for every i. Because of the single-crossing property, there exists x' such that

$U^{\eta}(x') > U^{\eta}(x^{\theta}(\bar{\boldsymbol{m}}))$ for every $\eta \geq \theta$ and $U^{\eta}(x') < U^{\eta}(x^{\eta}(\bar{\boldsymbol{m}}))$ for every $\eta < \theta$. Consider contract $\tilde{x}$ such that

$$U^{\theta}(\tilde{x}) = \epsilon U^{\theta}(x^{\theta}(\bar{\boldsymbol{m}})) + (1-\epsilon)U^{\theta}(x') \tag{3.4}$$

$$\zeta^{\theta}(\tilde{x}) > \epsilon \zeta^{\eta}(x^{\theta}(\bar{\boldsymbol{m}})) + (1-\epsilon)\zeta^{\eta}(x') \tag{3.5}$$

$$\epsilon \in [0, 1]. \tag{3.6}$$

Consider now company j and action $\tilde{m}_j$, where $\tilde{m}_j = (\tilde{x}, \dots, \tilde{x})$. For any ϵ satisfying (3.4), (3.5) and (3.6), at least type θ buys contract $\tilde{x}$. For a sufficiently small ϵ, $\zeta^{\eta}(\tilde{x}) > 0$. The profit of company j from $\tilde{m}_j$ when company $-j$ offers $\bar{m}_{-j}$ is

$$0 < \Pi_i(\tilde{m}_j, \bar{m}_{-j}|q_j(\tilde{m}_j, \bar{m}_{-j})) \leq \sum_{\eta \geq \theta}^{N} \lambda^{\eta} \zeta^{\eta}(\tilde{x}) \tag{3.7}$$

where the bounds in (3.7) follow from the fact that for every x, $\zeta^1(x) < \zeta^2(x) < \cdots < \zeta^{\Theta}(x)$. From Definition 3.1, it is true that

$$\Pi_i(\tilde{m}_j, \bar{m}_{-j}|q_j(\tilde{m}_j, \bar{m}_{-j})) \leq \Pi_i(\bar{m}_i, \bar{m}_{-i}|q_i(\bar{\mathbf{m}})) = 0$$

and therefore, we have a contradiction. □

Proposition 3.6. *A Nash equilibrium in pure strategies exists if and only if the RSA is efficient.*

Proof. For convenience, I denote by $(x^{\theta}_{RS})_{\theta}$ the RSA. For the "if" part, suppose that $(x^{\theta}_{RS})_{\theta}$ is efficient. I prove by contradiction that $m^{RS}_1 = m^{RS}_2 = (x^{\theta}_{RS})_{\theta}$ satisfies Definition 3.1. Suppose that it does not. There exist j and $\tilde{m}_j \neq (x^{\theta}_{RS})_{\theta}$ such that for some $\Phi \subseteq \Theta$,

(i) $\max_{x \in \tilde{m}_j} U^{\theta}(x) > U^{\theta}(x^{\theta}_{RS}) \, \forall \, \theta \in \Phi$
(ii) $\max_{x \in \tilde{m}_j} U^{\theta}(x) < U^{\theta}(x^{\theta}_{RS}) \, \forall \, \theta \in \Theta - \Phi$
(iii) $\Pi_j(\tilde{m}_j, m^{RS}_{-j}|q_j(\tilde{m}_j, m^{RS}_{-i})) > \Pi_i(m^{RS}_j, m^{RS}_{-j}|q_j(m^{RS}_j, m^{RS}_{-j}))$

Consider now allocation $\left((x^{\theta}_{RS})_{\theta \in \Theta - \Phi}, (\tilde{x}^{\theta})_{\theta \in \Phi}\right)$, where

$$\tilde{x}^{\theta} \in \{\arg\max_{x \in \tilde{m}_j} U^{\theta}(x)\} \cap \{\arg\max_{x \in \tilde{m}_j} \zeta^{\theta}(x)\}.$$

The profit of this allocation is

$$\sum_{\theta \in \Phi} \lambda^{\theta} \zeta^{\theta}(x^{\theta}_{RS}) + \sum_{\theta \in \Theta - \Phi} \lambda^{\theta} \zeta^{\theta}(\tilde{x}^{\theta}) > 0$$

which follows from (iii) and Definition 2.3. Note, however, that because of (i) and (ii), $\left((x^{\theta}_{RS})_{\theta \in \Theta - \Phi}, (\tilde{x}^{\theta})_{\theta \in \Phi}\right)$ dominates $(x^{\theta}_{RS})_{\theta}$, which contradicts that $(x^{\theta}_{RS})_{\theta}$ is efficient.

For the "only if" part, suppose that $(x^{\theta}_{RS})_{\theta}$ is not efficient. Then, for every efficient allocation $(\hat{x}^{\theta})_{\theta}$, there exists θ such that $\zeta^{\theta}(\hat{x}^{\theta}) > 0$. Suppose that an equilibrium exists. From Lemma 3.5, the equilibrium allocation is efficient. This immediately contradicts Lemma 3.5. □

Remarks. Note that for the existence part (i.e., the first part of Proposition 3.6) no structural assumptions are necessary. The argument straightforwardly extends to considerably more general environments.

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