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Hesitant fuzzy linguistic projection model to multi-criteria decision making for hospital decision support systems

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ABSTRACT

To improve the ability and efficiency of the hospital management, it is needed for us to handle the decision making problems so as to assist the hospital decision support systems. Considering the complexity and urgency of the hospital management affairs, this paper proposes a projection model with hesitant fuzzy linguistic term sets to solve the decision making problems under consideration. The proposed model not only can describe the uncertainties of the problems and the hesitancy of the decision makers, but also can decrease subjective and increase objectives of the decision making results. Then, the error analysis method is provided to obtain the weights of the criteria with hesitant fuzzy linguistic information. Furthermore, we make comparisons between the proposed model and other decision making methods, and present its advantages and drawbacks. Finally, a case study on hospital decision support systems is made to illustrate the validity and applicability of the proposed model.

1. Introduction

With the rapid development of information technology, the competition becomes rigorous in medical service market, which prompts the hospitals to improve their working efficiency and quality. Hospital information system (HIS) (Suzuki, Omori, Akiyama, & Fukuhara, 2005), which obtains the medical management policies and measurements by data analysis, was introduced to support the hospital management by inputting/outputting the medical activities data on a mobile terminal. By providing some successful examples and the experiences of computer applications, Kuperman and Gardner (1991) presented the guidance on how to design a HIS with few mistakes. Afterwards, from the aspects of hospital stays and hospitalization cost, Evans, Classen, Stevens, Pestotnik, and Gardner (1993) used the HIS to assess the effects of adverse drug affairs.

Nevertheless, the limitations of the HISs gradually emerge in practice with the accumulation of the clinical and administrative data. In such a case, the hospital decision support system (HDSS) (Zhang, Shu-Tao, Zhang, & Jian-Bo, 2005), which possesses various data analysis and data mining techniques, was provided to deal with the extensive data of medical management. By the HDSS, some new findings and laws of the clinical medicine and hospital management can be obtained. Since the HDSS is effective to assist the hospital management strategies, many researchers have made efforts to investigate the problems related to the HDSS. Zhang et al. (2005) applied the online analytical processing (OLAP) into the HDSS to solve the medical decision-making problems. With data summary and visual tools, Barrett, Mondick, Narayan, Vijayakumar, and Vijayakumar (2008) established the nonlinear models to design the decision support systems for pediatric pharmacotherapy. In addition, Shklovskiy-Kordi, Shakin, Ptashko, and Surin (2005) proposed a method to process the clinical data.

To enhance the practicability of the HDSS, Kawamoto, Houlihan, Balas, and Lobach (2005) collected and analyzed the relevant literature to identify the features of clinical decision support systems. Nanni, Brahnam, Lumini, and Barrier (2010) studied the artificial intelligence techniques, which can be utilized in the decision support systems in healthcare. To address the clinical decision support process, Aleksovska-Stojkovska and Loskovska (2013) used the data mining methods to extract and collect the individual asthma patients' information.

According to the needs of hospital administrators in each department, several topics in the HDSS have been constructed including work efficiency, work quality, outpatient service, hospitalization and so on. To improve the medical service quality, the HDSS needs to convert the existing data into the decision making assistant information in view of different requirements, and provide the DMs with effective solutions. For example, the rational use of drug is a management demand for the doctors to prescribe medications, which is put forward by health

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authorities and hospital management department. It requires the HDSS to review the prescriptions from each doctor, department, ward and even the whole hospital according to the monthly, quarterly and annual periods. Moreover, the detailed information of each drug, including variety, specification and amount, is also needed to be statistically analyzed. On this basis, when faced with a variety of diagnostic conditions, the HDSS can provide some reasonable medication plans by using the data warehouse, OLAP, and data mining.

However, it is hard to provide decision support only relying on data statistics and analysis, since we usually obtain several available alternatives by the HDSS according to the historical data. Subsequently, an important problem, which is how to select the optimal alternative among the available alternatives in the medical management problems, should be noticed and solved. To address this problem, the relevant knowledge base and expert system are necessary for the final decision result, which provide experience and wisdom from the DMs. In the traditional medical decision-making processes, the DMs usually hold a meeting to discuss their opinions and find the most satisfactory solution, which effectively avoid the individual bias. Nevertheless, it is inconvenient to reconcile each DM's time and the discussion always takes a long time so that the decision efficiency will be reduced. More seriously, there exists 'group think' (Janis, 1982) in practical decision-making processes, which suppresses viewpoints from the minority and often results in an irrational outcome. Therefore, in order to improve the operability and feasibility of the HDSS, it is necessary to establish the methods to determine the most desirable alternative which performs best from all the available alternatives with some criteria, such as clinical diagnosis analysis, drug procurement, sector assessment, etc. After the system automatically generates several feasible alternatives, the DMs can enter the human-computer interaction interface at their convenience, which signifies that they can input their own evaluations without external influences. Hence, adding the multi-criteria decision making (MCDM) processes into the HDSS certainly not only improves the efficiency and quality of hospital service, but also provides the accurate information to hospital administrators.

Actually, the decision-making problems exist some complexities and uncertainties, which are hard to be predicted and disposed. Fuzzy set (FS) (Zadeh, 1965), which is a descriptive tool, was defined to depict the vagueness in the decision-making process. With the evolution of the practical things, the FSs, which are expressed by the numbers between 0 and 1, are insufficient to be used by the DMs in the actual problems. In this case, linguistic terms (Zadeh, 1975) were proposed to represent the decision-making information. As the linguistic terms are in accordance with people's descriptive conventions, the theory based on linguistic terms has been deeply investigated. Herrera and Herrera-Viedma (2000) presented the steps of decision analysis to solve the MCDM problems with linguistic information. Wang and Chu (2004) established a group decision making (GDM) method with linguistic terms to evaluate the flexibility of a manufacturing system. Later on, Chen, Chan, and Shiu (2006) provided a novel GDM method to measure performance under linguistic circumstance. Furthermore, the extensions of linguistic theories have been studied to handle the problems with the unbalanced linguistic information (Herrera, Herrera-Viedma, & Martínez, 2008), incomplete linguistic information (Xu, 2006), etc.

However, due to the fuzzy thoughts of people, when the DMs face the decision-making problems with many complicating factors, they are willing to give several linguistic terms to present their assessments, instead of a single linguistic term. For this reason, Rodriguez et al. (2011, 2012) proposed the concept of hesitant fuzzy linguistic term sets (HFLTSs), which allow the DMs to assess the objects by consecutive linguistic terms. Afterwards, a lot of decision making methods with hesitant fuzzy linguistic information have been investigated. Lin, Zhao, Wang, and Wei (2014) studied the desired properties of the linguistic aggregation operators and applied them to solve the MCDM problems. Subsequently, Wei, Zhao, and Tang (2015) extended the HFLTS, and then established a novel decision-making model with the extended form. Furthermore, Liao and Xu (2015) defined the distance and similarity measures for HFLTSs and applied them into the MCDM problems.

As stated before, based on several available alternatives obtained by the data mining techniques in the HDSS, it is necessary to add the decision-making methods to the HDSS, which can help the DMs to get the optimal alternative. On the one hand, there exist many emergency events in the HDSS, which are related to life rescuing. On the other hand, the medical management consists a majority of complex factors, such as the satisfied levels of patients and relatives, the speed of medical logistics and so on, which are difficult to be evaluated. Thus, there is an immediate need to find out the efficient tools to let the DMs express their opinions flexibly and conveniently. Fortunately, hesitant fuzzy linguistic information can be well used to address the problem, which not merely have the strong ability to depict the uncertainties of the decision-making problems, but also conform to people's descriptive conventions. Even though the existing literature has made significant contributions to the decision-making methods of the MCDM problems with hesitant fuzzy linguistic information, they are difficult to be programmed into the HDSS for their relatively complex logics.

The projection model (Villemagne & Skelton, 1987), which has been studied in different fields (Jin, Zhang, & Liu, 2010; Wei, Alsaadi, Hayat, & Alsaedi, 2016; Xu & Cai, 2012; Xu, Xu, & Cheng, 2009), consists of two parts: (1) the cosine of the included angle of each alternative and the ideal alternative; (2) the module of each alternative and the ideal alternative. The projection model is valid to reduce the loss of information by pairwise comparison of each alternative and the ideal alternative. What's more, the process of the projection model is simple and convenient for operation, which fits the characteristics of the HDSS's programming system.

Consequently, this paper aims to construct a projection model with hesitant fuzzy linguistic information to assist the HDSS. The main contributions of this paper can be briefly summarized as:

(1) The paper proposes a MCDM approach for the HDSS to overcome the defects in the traditional decision-making processes, which not only saves a lot of time for the DMs, but also enhances the objectivity of the decision results.
(2) Considering the complexity and uncertainty of the hospital management affairs, the paper utilizes the HFLTS to depict the linguistic evaluation information in decision-making processes.
(3) The paper establishes a hesitant fuzzy linguistic projection model (HFLPM), which cannot merely capture the uncertainty of the objects, but also portray the decision-making information from the viewpoints of quality, to assist the HDSS to improve the quality and efficiency of the hospital management.
(4) The paper introduces a projection model to handle the decision-making problems in the HDSS, which is convenient and straightforward to obtain the results, especially in emergency medical management.
(5) The paper proposes an error analysis method to determine the criteria weights with hesitant fuzzy linguistic information, which uses a possibility degree formula to obtain the ranking of criteria. The result shows that the error analysis method is well suited for hesitant fuzzy linguistic environment.

In addition, the superiority of the proposed model is verified by comparing it with other MCDM methods; Also, the paper applies the proposed model and other MCDM methods to handle the selection problem of the medicine purchase projects for tuberculosis in the HDSS. The contrastive conclusions demonstrate that the HFLPM is effective and appropriate to support the HDSS.

The reminder of this paper is organized as follows: In Section 2, we review some fundamental knowledge of the hesitant fuzzy linguistic information. Section 3 proposes a projection model to handle the MCDM problems with hesitant fuzzy linguistic information, and then provides the error analysis method to obtain the criteria weights.

Section 4 makes comparisons between the proposed model and other decision-making methods with HFTLSs. Section 5 provides a practical example, which involves selecting the optimal medicine purchase project in the HDSS, to illustrate the applicability and effectiveness of the proposed model. This paper ends with some conclusions in Section 6.

2. Preliminaries

Firstly, we recall the linguistic term set (LTS) $S=\{s_t\ |t=-\tau,...,-1,0,1,...,\tau\}$ (Xu, 2005b), which satisfies the following characteristics: (1) The set is ordered: if $\alpha>\beta$, then $s_\alpha>s_\beta$; (2) There exists a negation operator: $neg(s_\alpha)=s_{g-\alpha}$; (3) If $s_\alpha>s_\beta$, then max $\{s_\alpha,s_\beta\}=s_\alpha$ and min $\{s_\alpha,s_\beta\}=s_\beta$. For any two linguistic terms $s_\alpha,s_\beta\in S$, there are three basic operations: $s_\alpha\oplus s_\beta=s_{\alpha+\beta}$, $s_\alpha\ominus s_\beta=s_{\alpha-\beta}$ and $\lambda s_\alpha=s_{\lambda\alpha}$. Based on the LTS, Rodriguez, Martinez, and Herrera (2012) defined a hesitant fuzzy linguistic term set (HFLTS) as an ordered finite subset of the consecutive linguistic terms in S.

For example, let

$$S=\left\{\begin{matrix} s_{-4}\text{: extremely poor, } s_{-3}\text{: very poor, } s_{-2}\text{:poor,}\\ s_{-1}\text{: slightly poor, } s_0\text{: middling, } s_1\text{:slightly good,}\\ s_2\text{: good, } s_3\text{: very good, } s_4\text{:extremely good}\end{matrix}\right\}$$

be a LTS, then a HFLTS on S can be presented as Liao and Xu (2015): $\widehat{S}=\{h_i|h_i\subset S\}$, where $h_1=\{s_1\text{: slightly good, } s_2\text{: good}\}$, $h_2=\{s_3\text{: very good}\}$, $h_3=\{s_{-2}\text{: poor, } s_{-1}\text{: slightly poor, } s_0\text{: middling}\}$ and h_i $(i=1,2,3)$ are called the hesitant fuzzy linguistic elements (HFLEs).

For a HFLE h, the following operations (Rodriguez et al., 2012) can be used to determine its lower bound and upper bound:

(1) Lower bound: $h^-=\min(s_t)=s_k,s_t\in h$ and $s_t\geqslant s_k$.
(2) Upper bound: $h^+=\max(s_t)=s_k,s_t\in h$ and $s_t\leqslant s_k$.

Considering that we usually need to rank the HFLEs in the decision-making process, Liao, Xu, and Zeng (2015) proposed the score function and the variance function of a HFLE $h=\{s_l|l=1,...,L\}$ as $\rho(h)=s_{\frac{1}{L}\sum_1^L l}$ and $\sigma(h)=s_{\frac{1}{L}\sqrt{\sum_{s_l,s_k\in h}(l-k)^2}}$, respectively, where L is the number of linguistic terms in h.

Later on, the hesitant fuzzy linguistic preference relation (HFLPR) (Zhu & Xu, 2014) was provided to compare the objects in pair. For a collection of objects $X=\{x_i\ |i=1,2,...,n\}$, a HFLPR of the objects is presented by $R=(r_{jk})_{m\times m}\subset X\times X$, where $r_{jk}=\{r_{jk}^l\ |l=1,2,...,L_{jk}\}$ (L_{jk} is the number of linguistic terms in r_{jk}) is a HFLE, indicating hesitant degrees to which x_j is preferred to x_k. For all $j,k=1,2,...,m$, r_{jk} $(j<k)$ should satisfy the following conditions:

$$\begin{aligned}&r_{jk}^l\oplus r_{kj}^l=s_0,\quad r_{jj}=\{s_0\},\quad L_{jk}=L_{kj},\\ &r_{jk}^l<r_{jk}^{l+1},\quad r_{kj}^{l+1}<r_{kj}^l\end{aligned}\tag{2.1}$$

where r_{jk}^l and r_{kj}^l are l th elements in r_{jk} and r_{kj}, respectively.

3. Hesitant fuzzy linguistic projection model to the MCDM problems

In this section, we demonstrate the whole process of the HFLPM, and then, the error analysis method with hesitant fuzzy linguistic information is proposed to obtain the criteria weights.

3.1. Hesitant fuzzy linguistic projection model

Motivated by Zhang and Wu (2014), we firstly define an inverse function of a symmetrical LTS as follows:

Definition 3.1. Let I: $S\to[-\tau,\tau]$ be a function from S to $[-\tau,\tau]$, such that $I(s_t)=t$ for any $s_t\in S$. Obviously, there exists an inverse function I^{-1}: $[-\tau,\tau]\to S$, such that $I^{-1}(t)=s_t$ for any $t\in[-\tau,\tau]$.

The MCDM problem with hesitant fuzzy linguistic information can be described as follows:

Let $A=\{A_i\ |i=1,2,...,n\}$ be an alternative set and $C=\{C_j\ |j=1,2,...,m\}$ be a criterion set of the MCDM problem. $\omega=(\omega_1,\omega_2,...,\omega_m)^T$ is the weight vector of the m criteria with $\omega_j\in[0,1]$ for $j\in\{1,2,...,m\}$ and $\sum_{j=1}^m\omega_j=1$. The DMs give the HFLEs to evaluate the alternatives with respect to the criteria. All the opinions of the DMs construct the hesitant fuzzy linguistic decision matrix (HFLDM), denoted as $H=(h_{ij})_{n\times m}$, where $h_{ij}=\{h_{ij}^l\ |l=1,2,...,L_{ij}\}$ is a HFLE (L_{ij} is the number of the HFLE h_{ij}), which indicates the possible degree that the alternative A_i satisfies the criterion C_j.

Based on Definition 3.1, suppose that $I(h_{ij}^l)$ $(l=1,2,...,L_{ij})$ are the subscripts of the HFLEs h_{ij}^l $(l=1,2,...,L_{ij})$, then all the HFLEs can be converted to the numbers between 0 and 1 by the following formula:

$$u(h_{ij}^l)=\frac{|I(h_{ij}^l)|}{2\tau}\tag{3.1}$$

Afterwards, the module of A_i, which represents the distance from A_i to the origin point, can be calculated by

$$|A_i|=\sqrt{\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}[u(h_{ij}^l)]^2\right]}\tag{3.2}$$

According to the HFLDM, the hesitant fuzzy linguistic positive ideal solution (HFL-PIS) $A^+=\{h_1^+,h_2^+,...,h_m^+\}$ and the hesitant fuzzy linguistic negative ideal solution (HFL-NIS) $A^-=\{h_1^-,h_2^-,...,h_m^-\}$ (Liao & Xu, 2015) can be obtained:

$$h_j^+=\begin{cases}\max\limits_{i=1,2,...,n} h_{ij}^+, & \text{for the benefit criterion } C_j\\ \min\limits_{i=1,2,...,n} h_{ij}^-, & \text{for the cost criterion } C_j\end{cases},\quad \text{for } j=1,2,...,m\tag{3.3}$$

$$h_j^-=\begin{cases}\max\limits_{i=1,2,...,n} h_{ij}^-, & \text{for the benefit criterion } C_j\\ \min\limits_{i=1,2,...,n} h_{ij}^+, & \text{for the cost criterion } C_j\end{cases},\quad \text{for } j=1,2,...,m\tag{3.4}$$

Based on the above equations, the cosine of the included angle between the alternative A_i and the HFL-PIS A^+ is

$$\begin{aligned}\cos(A_i,A^+)&=\frac{\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}(u(h_{ij}^l)\cdot u((h_j^l)^+))\right]}{|A_i||A^+|}\\ &=\frac{\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}\left(\frac{|I(h_{ij}^l)|}{2\tau}\cdot\frac{|I((h_j^l)^+)|}{2\tau}\right)\right]}{\left[\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}\left[\frac{I(h_{ij}^l)}{2\tau}\right]^2\right]\cdot\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}\left[\frac{I((h_j^l)^+)}{2\tau}\right]^2\right]\right]^{1/2}}\\ &=\frac{\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}(|I(h_{ij}^l)|\cdot|I((h_j^l)^+)|)\right]}{\left[\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}[I(h_{ij}^l)]^2\right]\cdot\sum_{j=1}^m\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}[I((h_j^l)^+)]^2\right]\right]^{1/2}}\end{aligned}\tag{3.5}$$

where $|A^+|=\sqrt{\sum_{j=1}^m\left[\frac{\omega_j}{L_j^+}\sum_{l=1}^{L_j^+}[u((h_j^l)^+)]^2\right]}$. Correspondingly, we can get the cosine of the included angle between the alternative A_i and the HFL-NIS $A^-=\{h_1^-,h_2^-,...,h_m^-\}$.

Remark 1. L_{ij} is the maximum number of the HFLE h_{ij} and h_j^+, which means that the shorter one should be extended to the equal length of the longer one with the optimistic method (Wei, Zhao, & Tang, 2014).

As we all know, a vector contains the direction and the module. $\cos(A_i,A^+)$ only reflects the similarity measure of the direction of A_i and A^+, but ignores the length of the vector. To comprehensively reflect the similarity degree between A_i and A^+, we introduce the projection of A_i on A^+ as follows:

$$\mathrm{Pr}j_{A^+}A_i = |A_i|\cos(A_i,A^+) = |A_i|\frac{\sum_{j=1}^{m}\left[\frac{\omega_j}{L_{ij}}\sum_{l=1}^{L_{ij}}(u(h_{ij}^l)\cdot u((h_j^l)^+))\right]}{|A_i||A^+|} \tag{3.6}$$

Remark 2. The projection formula measures the similarity between two alternatives with simultaneously considering the numerical distance and expressive direction of the two alternatives.

The greater value of $\mathrm{Pr}j_{A^+}A_i$ represents the closer the distance of A_i to A^+, which indicates that the alternative A_i performs better. In order to make the results more accurate and more reasonable, it is needed to combine the projections of A_i on A^+ and A^-.

Definition 3.2. Let V_i be the integrated value of the alternative A_i, then it can be obtained by

$$V_i = \alpha\cdot\mathrm{Pr}j_{A^+}A_i-(1-\alpha)\cdot\mathrm{Pr}j_{A^-}A_i \tag{3.7}$$

where α $(0\leqslant\alpha\leqslant 1)$ is up to the preference of the DMs on the ideal solutions. Obviously, the greater value of V_i, the better the alternative A_i, which means that A_i is closer to the positive ideal solution and keeps away from the negative ideal solution.

3.2. Criteria weights derived by the error analysis method

In practice, the criteria weights are usually unknown. To handle such a situation, the HFLPR (Zhu & Xu, 2014) was proposed as an effective tool to obtain the criteria weights by pairwise comparisons of the criteria. Apparently, for the decision-making problems of medical management, the DMs are hard to determine the criteria weights directly since the factors of the HDSS are polytropic. Thus, it needs to provide some methods to get the criteria weights efficiently. Some researches have addressed this kind of problems, a majority of them have focused on establishing some goal programming models so as to derive the criteria weights from the HFLPRs (Meng & Chen, 2015; Rodriguez, Martinez, & Herrera, 2013; Wei, 2016; Zhang, Xu, & Wang, 2015). Nevertheless, these optimization models are complex in the calculating process, which are unsuitable to deal with the medical management affairs.

Error analysis method, firstly introduced by Xu (2012), is simple and convenient to obtain the criteria weights without considering the consistency of the HFLPR and constructing the programming models for the HFLPR. Hence, we propose an error analysis process to handle the preference relation with the hesitant fuzzy linguistic information. Firstly, we provide a novel expression of the HFLE as follows:

Definition 3.3. Let $r = \{r^l|l = 1,...,L\}$ be a HFLE, and $r^+ = \max(r^l)$ for $l\in\{1,...,L\}$ and $r^- = \min(r^l)$ for $l\in\{1,...,L\}$, then r can be presented by the form of the error distribution:

$$r = \bar{r}\oplus\Delta r \quad \text{or} \quad r = \bar{r}\ominus\Delta r \tag{3.8}$$

where $\bar{r} = \frac{1}{L}\underset{r_l\in r}{\oplus} r^l$ and $\Delta r = \frac{1}{2}(r^+\ominus r^-)$.

Motivated by the form of error distribution, we can develop a method for determining the median weight vector based on the absolute deviation of the criteria values:

$$R = \overline{R}\oplus\Delta R \quad \text{or} \quad R = \overline{R}\ominus\Delta R \tag{3.9}$$

where $\overline{R} = (\bar{r}_{jk})_{m\times m}$, $\Delta R = (r_{jk})_{m\times m}$, $\bar{r}_{jk} = \frac{1}{L_{jk}}\underset{r_{jk}^l\in r_{jk}}{\oplus} r_{jk}^l$, and $\Delta r_{jk} = \frac{1}{2}(r_{jk}^+\ominus r_{jk}^-)$.

To eliminate the impacts of the dimension on the decision results, we propose a method to normalize the average matrix $\overline{R} = (\bar{r}_{jk})_{m\times m}$ into the matrix $\overline{B} = (\bar{b}_{jk})_{m\times m}$, which is

$$\bar{b}_{jk} = s_\gamma \tag{3.10}$$

where $\gamma = I(\bar{r}_{jk})/\sum_{k=1}^{m}I(\bar{r}_{jk})$.

After that, the average assessment value of each criterion, which indicates the average superiority of each criterion to other criteria, can be obtained by the following formula:

$$b_j = \frac{1}{m}\left(\overset{m}{\underset{k=1}{\oplus}}\bar{b}_{jk}\right),\quad \text{for } j = 1,2,...,m \tag{3.11}$$

According to Eq. (3.10), we know that $b_j = s_{1/m}$. Then, the absolute difference between each element of $\overline{B} = (\bar{b}_{jk})_{m\times m}$ and the average assessment value of the corresponding criterion (b_j) is calculated, which represents the distance between the preference of this criterion over another criterion and the average assessment value of this criterion. Subsequently, we can construct the absolute difference matrix $\dot{B} = (\dot{b}_{jk})_{m\times m}$, where

$$\dot{b}_{jk} = |\bar{b}_{jk}\ominus b_j|,\quad \text{for all } j,k = 1,2,...,m \tag{3.12}$$

By aggregating the sum of the absolute difference of each row, we can determine the importance of each criterion. Obviously, the greater the sum of the absolute difference is, the higher the superiority of this criterion is, this is to say, the more important the criterion is. On the contrary, the lower the sum of the absolute difference is, the less important the criterion is.

Furthermore, we can get the median weight vector of the criteria $\bar{\varpi}_j = (\bar{\varpi}_1,\bar{\varpi}_2,...,\bar{\varpi}_m)^T$ by

$$\bar{\varpi}_j = \sum_{k=1}^{m}I(\dot{b}_{jk})\bigg/\sum_{j=1}^{m}\sum_{k=1}^{m}I(\dot{b}_{jk})\quad \text{for } j = 1,2,...,m \tag{3.13}$$

where $I(\dot{b}_{jk})$ is the subscript of $\dot{b}_{jk}$.

Remark 3. This method for deriving the median weight vector takes each element in the HELEs into account, which represent all the information of the HELEs. It can preserve more information from the original HFLPR and make the obtained results more accurate.

Afterwards, we can utilize the error propagation formula (Yoon, 1989) to derive the calculation formula for the propagation errors of weights, which is denoted as:

$$\sigma_z^2 = \sum_{i=1}^{m}\left(\frac{\partial f}{\partial y_i}\right)^2\sigma_{y_i}^2 \tag{3.14}$$

where $z = f(y_1,y_2,...,y_m),y_i\in Y$ is a random function and $\sigma_{y_i}^2$ is the random error of the variable y_i.

In practical problem, the limiting error, which is commonly used to represent the random error, is deduced from the general formula as:

$$\sigma_{z\lim}^2 = \sum_{i=1}^{m}\left(\frac{\partial f}{\partial y_i}\right)^2\sigma_{y_i\lim}^2 \tag{3.15}$$

According to Eq. (3.10) and Eq. (3.15), the random error of $\bar{b}_{jk}$ can be got as:

$$\Delta(I(\bar{b}_{jk}))^2 = \frac{1}{(\sum_{q=1}^{m}I(\bar{r}_{qk}))^4}\left[I(\Delta r_{jk})^2\left(\sum_{\substack{q=1\\q\neq i}}^{m}I(\bar{r}_{qk})\right)^2 + (I(\bar{r}_{jk}))^2\left(\sum_{\substack{q=1\\q\neq i}}^{m}I(\Delta r_{qk})^2\right)\right] \tag{3.16}$$

Then, based on the above equations, we can derive the propagation errors of weights in hesitant fuzzy linguistic environment as:

$$\Delta\varpi_j^2 = \frac{1}{\left[\sum_{q=1}^{m}\sum_{k=1}^{m}\left|I(\bar{b}_{qk})-\frac{1}{m}\right|\right]^4}\left\{\left[\sum_{\substack{q=1\\q\neq j}}^{m}\sum_{k=1}^{m}\left|I(\bar{b}_{qk})-\frac{1}{m}\right|\right]^2 \sum_{k=1}^{m}\Delta(I(\bar{b}_{jk}))^2 + \left[\sum_{k=1}^{m}\left|I(\bar{b}_{jk})-\frac{1}{m}\right|\right]^2 \sum_{\substack{q=1\\q\neq j}}^{m}\sum_{k=1}^{m}\Delta(I(\bar{b}_{qk}))^2\right\} \tag{3.17}$$

The interval weight vector of the HFLPR can be obtained by

$$\tilde{\varpi}_j = [\tilde{\varpi}_j^-,\tilde{\varpi}_j^+] = [\max\{\varpi_j-\Delta\varpi_j,0\},\min\{\varpi_j+\Delta\varpi_j,1\}] \quad \text{for } j = 1,2,...,m \tag{3.18}$$

which reflects the ranges of the relative importance of the criteria C_j ($j = 1,2,...,m$).

Remark 4. The interval weight vector derived by this method, which is developed by combing the median weights and the propagation errors of weights, has high accuracy of weight errors.

Before obtaining the criteria weights, a possibility degree formula (Xu & Da, 2002) is needed to handle the interval weight vector $\tilde{\varpi}_j$ for $j = 1,2,...,m$:

$$p_{jk}(\tilde{\varpi}_j \geqslant \tilde{\varpi}_k) = \max\left\{1-\max\left(\frac{\tilde{\varpi}_k^+-\tilde{\varpi}_j^-}{\tilde{\varpi}_j^+-\tilde{\varpi}_j^- + \tilde{\varpi}_k^+-\tilde{\varpi}_k^-},0\right),0\right\} \tag{3.19}$$

All the possibility degrees got by Eq. (3.19) can construct a possibility degree matrix $P = (p_{jk})_{m\times m}$, which satisfies

$$p_{jk} \geqslant 0,\ p_{jk} + p_{kj} = 1,\ p_{jj} = 0.5, \quad \text{for all } j,k = 1,2,...,m \tag{3.20}$$

Then the criteria weights can be derived by the following formula (Xu, 2001):

$$\omega_j = \frac{1}{m(m-1)}\left(\sum_{k=1}^{m} p_{jk} + \frac{m}{2}-1\right), \quad \text{for } j = 1,2,...,m \tag{3.21}$$

To this end, a graph is provided to clearly demonstrate the process of the error analysis method (see Fig. 1).

As we have introduced above, the error analysis method can effectively handle the hesitant fuzzy linguistic information in the decision-making processes. When faced with emergency and complex situations in the HDSS, the error analysis method can not only save amount of time, but also simplify the procedure of calculation with straightforward logic. In addition, the error analysis method can preserve original information to the greatest extent, which makes it well suited for the high requirements on information completeness of the HDSS in the decision-making processes.

3.3. The procedure of hesitant fuzzy linguistic projection model

Based on the above results, the procedure of the projection model for the MCDM problem with HFLEs can be concluded as:

Step 1. Determine the alternatives $A = \{A_i\ |i = 1,2,...,n\}$ and the criteria $C = \{C_j\ |j = 1,2,...,m\}$ of the MCDM problem;
Step 2. Invite the DMs to compare the criteria in pair and give the corresponding HFLPR. Moreover, let the DMs provide the HFLDM, which contains the performance assessments of the alternatives with respect to the criteria;
Step 3. Drive the criteria weights by the error analysis method, and determine the HFL-PIS $A^+ = \{h_1^+,h_2^+,...,h_m^+\}$ and the HFL-NIS $A^- = \{h_1^-,h_2^-,...,h_m^-\}$;
Step 4. Calculate the cosine of the included angle between each alternative and the HFL-PIS with respect each criterion, in the same way, we calculate the cosine of the included angle between each alternative and the HFL-NIS. Then, we construct the positive ideal separation matrix F^+ and the negative ideal separation matrix F^-, respectively:

$$F^+ = \begin{pmatrix}\cos(A_1,A^+)\\ \cos(A_2,A^+)\\ \vdots\\ \cos(A_n,A^+)\end{pmatrix} \quad \text{and} \quad F^- = \begin{pmatrix}\cos(A_1,A^-)\\ \cos(A_2,A^-)\\ \vdots\\ \cos(A_n,A^-)\end{pmatrix} \tag{3.22}$$

Step 5. Obtain the projection of the alternative A_i on the HFL-PIS A^+ and the projection of the alternative A_i on the HFL-NIS A^-;
Step 6. Aggregate the integrated values of the alternatives by Eq. (3.7), and rank the alternatives.

The procedure of the projection model can be shown in Fig. 2.

4. Comparative analyses

There exist some methods to handle the MCDM problems with hesitant fuzzy linguistic information, such as the method based on Euclidean distance (Singh, 2015; Xu, Xu, Merigo, & Wang, 2015), the method based on the Hausdorff distance (Wang, Wu, Wang, Zhang, & Chen, 2016a), the likelihood-based TODIM (an acronym in Portuguese for Iterative Multi-criteria Decision Making) method (Wang, Wang, &

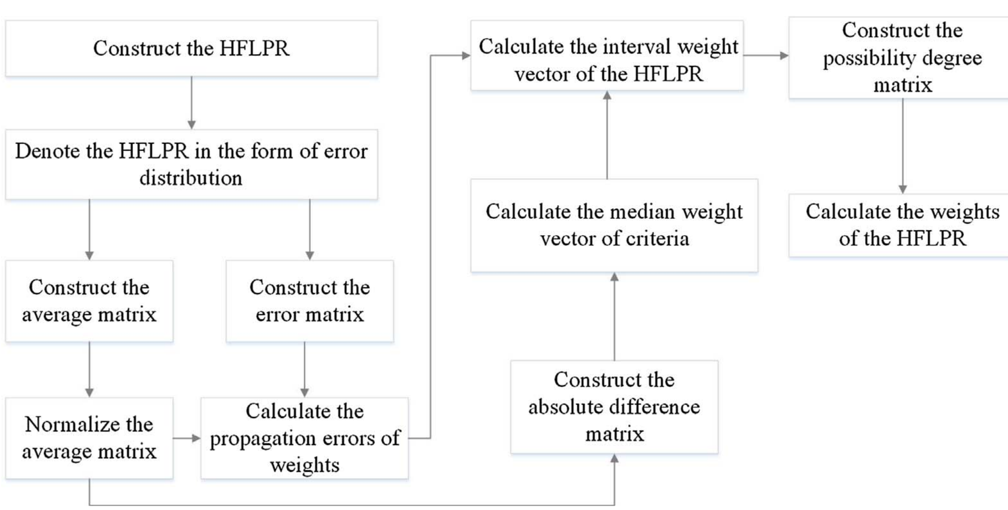

Fig. 1. The process of the error analysis method to derive the criteria weights from the HFLPR.

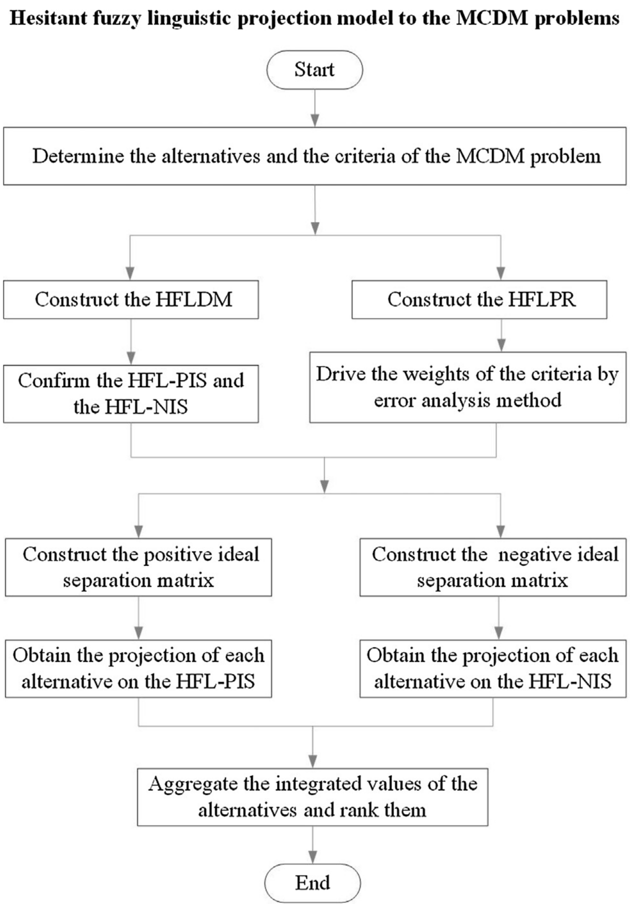

Fig. 2. The procedure of HFLPM.

Zhang, 2016b) and so on. In order to verify the reasonability and superiority of the proposed model, here we make comparisons between the proposed model and several hesitant fuzzy linguistic decision-making methods.

From the above discussion, we can draw some conclusions as follows:

(1) The projection model uses cosine similarity to measure the projection of the weight vector of the alternatives on the ideal solution. Compared with the Euclidean distance, the projection value focuses on not only the difference of the two vectors but also their direction. As for the Euclidean distance, when the relative position of the two alternatives remains the same, the distance doesn't change if the location of the origin point is moved, which indicates that the Euclidean distance cannot depict the direction of the two alternatives.
(2) In short distance measurement, the sensitivity of Hausdorff distance is not as good as the projection model since it is not easily affected by the distance between two close alternatives. In addition, Hausdorff distance is more vulnerable to the interference error.
(3) The thought of the likelihood-based TODIM method is similar to the projection model except that the TODIM method, which calculates the overall advantage degree of each alternative for other alternatives and rank the alternatives according to the overall degrees, is more complex than the projection model. It indicates that the likelihood-based TODIM is inefficient to be utilized in the decision-making problems of medical management.

Furthermore, the advantages and drawbacks of the projection model can be summarized:

Advantages:

(1) Considering the uncertainties, complexities and urgency of the decision-making problems in the HDSS, the proposed model introduces the hesitant fuzzy linguistic information, which can successfully portray the fuzziness of the problems, to express the decision-making information.
(2) The projection model can eliminate the errors caused by the different assessment dimensions of the DMs, which means that the model is objective to handle the MCDM problems. Furthermore, the projection model is effective to be applied in hesitant fuzzy linguistic environment since it is sensitive to measure the short distance.
(3) In the proposed model, all the elements of two sets are compared in pairs. It simultaneously utilizes the module and the cosine of the included angle of two alternatives to measure their closeness to the ideal solution, which performs well in qualitative analysis.
(4) By comparing the alternatives in pair, the projection model reserves all DMs' original assessment information in the decision-making processes, which is considered as a perfect and reasonable decision-making method.
(5) As stated before, the projection model is relatively straightforward and convenient, which determines that it is appropriate to solve the decision-making problems in the HDSS, especially in the emergency medical management affairs.

Drawbacks:

(1) The projection model, which accents on the qualitative analysis, is insufficient to handle the quantitative information.
(2) This paper only introduces the projection model into hesitant fuzzy linguistic environment, it would be ideal to discuss the model with other decision-making information, for example, probabilistic linguistic information.

5. The selection of the medicine purchase project in the HDSS with hesitant fuzzy linguistic projection model

5.1. Background of the HDSS

The HDSS has become a new development direction of hospital digitalization construction. At the technical level, a high-level integration framework of the HDSS based on hospital data warehouse consists of four main parts:

(1) Data acquiring layer. Data sources include patient information, drug information, clinic information, hospitalization information and other external data. To guarantee the consistency of the information, the source data must be cleaned, extracted and converted into a unified type of data.
(2) Data storage layer. Store and manage the comprehensive data of the decision-oriented subject, and reorganize the data to support the decision-making processes in the HDSS according to the actual needs of the problems.
(3) Data process layer. Obtain the useful information from the data warehouse by online analysis.
(4) Data access layer. The information and knowledge obtained from the data process layer are displayed to the users with providing the permission of the HDSS to the users.

These four parts can be detailly illustrated by Fig. 3.

In practice, several functional modules in the HDSS must be confirmed, which represent different types of affairs that hospital administrators need to deal with, such as clinical diagnosis analysis, drug procurement, sector assessment, etc. With this prerequisite, the HDSS can obtain the relevant data involved in each module and determine the meaningful indicators for analysis and decision support. Later on, the data models of the HDSS, which contain indicators of different

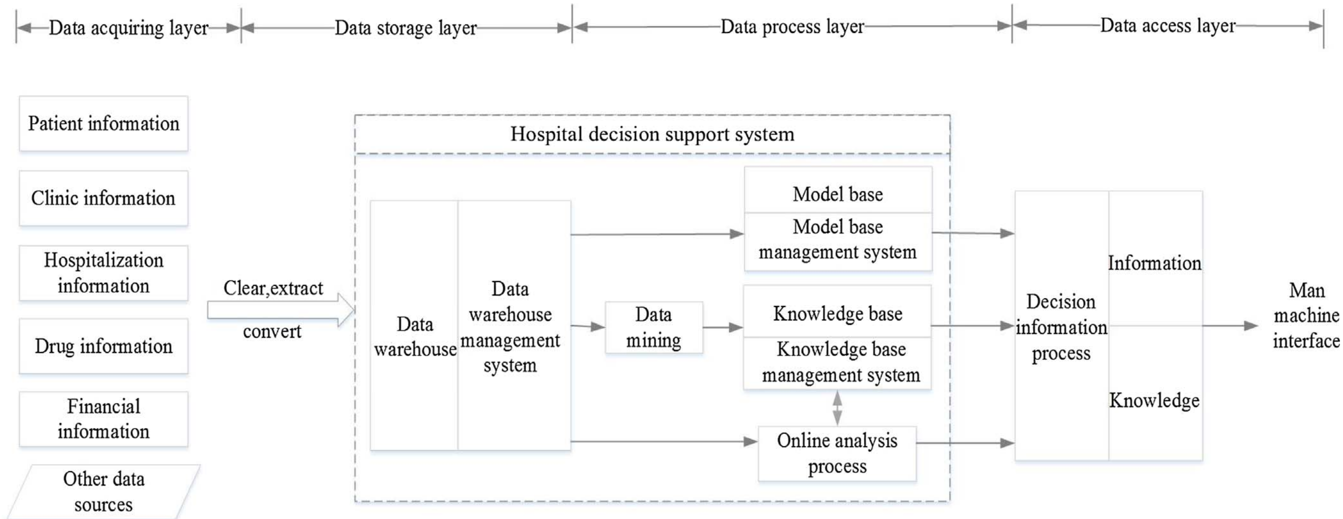

Fig. 3. The high-level integration framework of the HDSS.

dimensions, are established separately. After extracting and converting the original data, the HDSS analyzes the correlation between those data and draw some significant conclusions by data mining. For example, therapeutic regimens for a severe disease can be got in the module for clinical diagnosis, and the concurrent relationship between various diseases can be discovered in the module for pathological analysis.

In face of important hospital management affairs, to avoid mistakes in decision making, it requires wisdom and experience of the DMs. We apply the MCDM method to modules that need to pick up the optimal solution from several alternatives derived by data mining. Subsequently, the HDSS with the decision-making method makes it convenient for the DMs to enter the human-computer interaction interface, and input their assessments into the system at any time. When the opinions of each DM have been collected, the HDSS will automatically generate the most satisfactory solution by means of the proposed method.

Considering that there exist many emergency events in the HDSS, which are related to life rescuing, it needs to get a reasonable solution as soon as possible. Moreover, medical management affairs involve numerous complicated factors, such as the description of patients' condition, risk of treatment, and technical level of physicians, which are hard to be evaluated. The HFLPM firstly provides a natural way for the DMs to give their assessments, which is effective to depict the uncertainties of the medical management affairs. Afterwards, it generates the HFLDMs and the HFLPRs by synthesizing the views of each DM, and determine the HFL-PIS and the HFL-NIS simultaneously. The proposed model can reduce the information loss from the initial decision matrix, which improves the accuracy of decision result. In addition, the straightforward calculation logic of the proposed model can greatly improve the efficiency of the decision-making processes.

During the past few years, medical and health conditions, a new indicator to evaluate a hospital, have been greatly concerned by the society. Drug procurement, as the first bridge of drug production and drug consumption, should be handled by the hospitals in responsible ways. In recent years, the price of drugs gets higher and a lot of counterfeited medicines emerge, which increase the challenge for the hospital management in the drug procurement process. Thus, it is needed to provide the decision support to the hospitals on the drug procurement aspect to improve the hospital management and control expenses (see Table 1).

5.2. The proposed model for the MCDM problems of medicine purchase project selection

For the MCDM problems of medicine purchase project in HDSS, the DMs firstly extract the useful information and use the HDSS to automatically generate several possible alternatives. Then, After the DMs input their assessments on those candidates into the HDSS, the HFLPM is used to select the optimal alternative from the possible alternatives.

As mentioned in the introduction section, the hesitant fuzzy linguistic information depicts the uncertainties and complexities of the medical management effectively, and the projection model obtains the results conveniently. Therefore, here we apply the proposed model to deal with the problems of medicine purchase project selecting.

There are five criteria that we should consider in the drug procurement process: (1) therapeutic effects; (2) drug profits; (3) medical cost; (4) treatment cycle; (5) drug qualities. We denote the five criteria as C_j (j = 1,2,3,4,5). To treat the tuberculosis, four drug procurement plans have been obtained (as shown in Table 2) by the HDSS. Based on which, it is inevitable to further select the most desirable drug procurement plan for the hospital.

The decision-making process to solve the MCDM problem with selecting the optimal drug procurement plan for the tuberculosis treatment can be made as follows:

Step 1. Invite three experts from West China Hospital, who have extensive experience in the treatment of tuberculosis, to compare the five criteria in pair and obtain the corresponding HFLPR. Moreover, let them provide the HFLDM to present the assessment values of the four plans with respect to the five criteria. For each expert, he/she provided his/her judgements on the linguistic evaluation scale $S=\{s_{-4},\cdots,s_{-1},s_0,s_1,\cdots,s_4\}$ where $s_{-4}=$ *very poor*, $s_{-3}=$ *poor*, $s_{-2}=$ *a little poor*, $s_{-1}=$ *slightly poor*, $s_0=$ *middling*, $s_1=$ *slightly good*, $s_2=$ *a little good*, $s_3=$ *good*, $s_4=$ *very good*. Then, we collected all judgements by interviewing the experts one by one. For example, when the experts gave their assessments on the therapeutic effects C_1 in the plan A_2, two experts thought that the therapeutic effects of A_2 was "*a little good* (s_2)", the other expert thought that the therapeutic effect of A_2 was "*good* (s_3)". Hence, the evaluation value of the criterion C_1 in the alternative A_2 was obtained, denoted as $\{s_2,s_3\}$. In this way, all the assessment values from the experts were got to construct the HFLPR (as shown in Table 3) and the HFLDM (as shown in Table 4).

Step 2. By the error analysis method, the weights of the criteria can be obtained as:

Table 1
The decision-making methods with hesitant fuzzy linguistic information.

Methods	Descriptions
Projection model	The projection model is established based on the cosine similarity measure that refers to the ratio of the inner product of two columns and the product of their length. It selects the desirable alternative with the nearest distance from the ideal solution
Method based on Euclidean distance (Xu et al., 2015)	On the basis of the Euclidean distance, the method first measures the distance between each alternative and the ideal solution, and then selects the optimal alternative with the nearest distance from the ideal solution
Method based on Hausdorff distance (Wang et al., 2016a)	The method based on the classical Hausdorff distance, which can measure the semblable extent of two non-empty sets in a metric space, contains the orientation information of the alternatives
Likelihood-based TODIM method (Wang et al., 2016b)	Likelihood-based TODIM method introduces a likelihood function, which is based on a generalized function of the possibility degree of real numbers, into the TODIM so as to address the decision-making problems, where the DMs are of bounded rationality

Table 2
The drug procurement plans.

Plans	Drug combination	Specifications	Producer
A_1	Rifampicin Capsules Pyrazinamide Ethambutol Hydrochloride Isoniazid	300 mg × 2 500 mg × 3 250 mg × 3 300 mg × 1	HongQi pharmaceutical factory
A_2	Sodium Aluminate Isoniazid Streptomycin	200 mg × 2 300 mg × 1 150 mg × 2	YiKang pharmaceutical factory
A_3	Rifampicin Capsules Isoniazid	300 mg × 2 300 mg × 2	XieLi pharmaceutical factory
A_4	Rifampicin Capsules Ethambutol Hydrochloride Isoniazid	300 mg × 2 400 mg × 3 300 mg × 1	HongQi pharmaceutical factory

Table 3
The HFLPR of the five criteria.

	C_1	C_2	C_3	C_4	C_5
C_1	$\{s_0\}$	$\{s_{-1},s_0\}$	$\{s_1,s_2,s_3\}$	$\{s_2,s_3\}$	$\{s_3\}$
C_2	$\{s_1,s_0\}$	$\{s_0\}$	$\{s_2\}$	$\{s_2,s_3\}$	$\{s_1,s_2\}$
C_2	$\{s_{-1},s_{-2},s_{-3}\}$	$\{s_{-2}\}$	$\{s_0\}$	$\{s_0,s_1\}$	$\{s_{-1},s_0\}$
C_4	$\{s_{-2},s_{-3}\}$	$\{s_{-2},s_{-3}\}$	$\{s_0,s_{-1}\}$	$\{s_0\}$	$\{s_{-3},s_{-2},s_{-1}\}$
C_5	$\{s_{-3}\}$	$\{s_{-1},s_{-2}\}$	$\{s_1,s_0\}$	$\{s_3,s_2,s_1\}$	$\{s_0\}$

Table 4
The HFLDM of the four plans with respect to the five criteria.

	C_1	C_2	C_3	C_4	C_5
A_1	$\{s_1,s_2,s_3\}$	$\{s_0,s_1,s_2\}$	$\{s_3\}$	$\{s_{-3}\}$	$\{s_3\}$
A_2	$\{s_2,s_3\}$	$\{s_1,s_2,s_3\}$	$\{s_2,s_3\}$	$\{s_{-3},s_{-2}\}$	$\{s_1,s_2,s_3\}$
A_3	$\{s_0,s_1\}$	$\{s_{-1},s_0\}$	$\{s_1,s_2\}$	$\{s_{-1},s_0\}$	$\{s_2,s_3\}$
A_4	$\{s_0\}$	$\{s_{-3}\}$	$\{s_{-3},s_{-2}\}$	$\{s_1,s_2\}$	$\{s_1,s_2\}$

$\omega = (0.20,0.13,0.23,0.14,0.30)^T$

For the HFLDM of the DMs, the HFL-PIS $A^+ = \{h_1^+,h_2^+,h_3^+,h_4^+,h_5^+\}$ and the HFL-NIS $A^- = \{h_1^-,h_2^-,h_3^-,h_4^-,h_5^-\}$ can be respectively got as:

$$A^+ = (\{s_2,s_3\},\{s_1,s_2,s_3\},\{s_{-3},s_{-2}\},\{s_{-3}\},\{s_3\})^T$$
$$A^- = (\{s_0\},\{s_{-3}\},\{s_3\},\{s_1,s_2\},\{s_1,s_2\})^T$$

Step 3. The positive ideal separation matrix F^+ and the negative ideal separation matrix F^- can be constructed as:

$$F^+ = \begin{bmatrix} 0.859 \\ 0.946 \\ 0.813 \\ 0.684 \end{bmatrix} \text{ and } F^- = \begin{bmatrix} 0.795 \\ 0.809 \\ 0.722 \\ 0.988 \end{bmatrix}$$

Step 4. Calculate the projections of $A_i (i = 1,2,3,4)$ on A^+ and the projections of $A_i (i = 1,2,3,4)$ on A^-, denoted as $\mathrm{Prj}_{A^+}A_i$ and $\mathrm{Prj}_{A^-}A_i$ $(i = 1,2,3,4)$:

$$\mathrm{Prj}_{A^+}A_1 = 0.37 \quad \mathrm{Prj}_{A^-}A_1 = 0.35$$
$$\mathrm{Prj}_{A^+}A_2 = 0.38 \quad \mathrm{Prj}_{A^-}A_2 = 0.32$$
$$\mathrm{Prj}_{A^+}A_3 = 0.21 \quad \mathrm{Prj}_{A^-}A_3 = 0.20$$
$$\mathrm{Prj}_{A^+}A_4 = 0.22 \quad \mathrm{Prj}_{A^-}A_4 = 0.32$$

Step 5. Obtain the integrated values of the alternatives $V_i (i = 1,2,3,4)$, and rank the alternatives. Without loss of generality, let $\alpha = 0.5$, then

$$V_1 = 0.5 \times 0.37 - 0.5 \times 0.35 = 0.01 \quad V_2 = 0.5 \times 0.38 - 0.5 \times 0.32 = 0.03$$
$$V_3 = 0.5 \times 0.21 - 0.5 \times 0.20 = 0.005 \quad V_4 = 0.5 \times 0.22 - 0.5 \times 0.32 = -0.05$$

The greater the value V_i, the better the alternative A_i. Thus, the ranking of the four drug procurement plans is $A_2 \succ A_1 \succ A_3 \succ A_4$, which indicates that A_2 performs best. Although the price of the plan A_2 is stiff, the excellent quality and the short treatment cycle of plan A_2 determine that it is the optimal choice for tuberculosis treatment. The obtained result was approved by the hospital experts as the best prescription in drug procurement plans of this paper (due to the different condition of each patient, the HDSS provides different alternatives according to specific circumstance).

5.3. *Comparison with some existing methods*

As for the same problem concerning drug procurement, we compare our approach with some existing methods.

5.3.1. *Compared with Ref. (Xu et al., 2015)*

The hesitant fuzzy linguistic ordered weighted Euclidean distance (HFLOWED) operator, which is very effective for the treatment of data in the form of HFLTSs, is provided to solve this problem.

Firstly, we determine the HFLDM and the HFL-PIS $A^+ = \{h_1^+,h_2^+,h_3^+,h_4^+,h_5^+\}$, and then calculate the priority vector $\omega = (\omega_1,\omega_2,\omega_3,\omega_4,\omega_5)^T$ of criteria by the normal distribution-based method (Xu et al., 2015).

After that, the distance between each alternative and the ideal solution can be obtained by the HFLOWED operator as:

$$HFLOWED(A_i,A^+) = \sqrt{\sum_{j=1}^{n} \omega_j (d(h_{ij},h_j^+))^2} \tag{5.1}$$

where $d(h_{ij},h_j^+) = \frac{S_s(I^l(h_{ij}) - I^l(h_j^+))}{(\# b)T}$. In this formula, S_s is a function that indicates a summation of all values in a set, T is the number of linguistic term in S and $\#b$ is the number of each HFLE.

Finally, with the decision information in Section 5.2 and the procedure of the HFLNED operator, the result derived by this method can be listed in Table 5.

Obviously, the closer the distance of A_i to A^+ indicates that the alternative A_i performs better. From Table 5, we can see that A_3, which performs not good or bad with respect to each criterion, is selected as

Table 5
The result obtained from the HFLOWED operator.

Weight vectors	$(0.12,0.21,0.34,0.21,0.12)^T$
Distance between A_i and A^+	$(0.36,0.33,0.32,0.37)^T$
Decision ranking	$A_3 > A_2 > A_1 > A_4$

Table 6
Weight vectors of the HFLPR with different models.

Methods	Weight vectors
The maximum deviation model	$(0.23,0.15,0.17,0,17,0.28)^T$
The proposed model	$(0.20,0.13,0.23,0.14,0.30)^T$

Table 7
Decision rankings with different methods.

Methods	Decision rankings
The maximum deviation model	$A_1 > A_2 > A_3 > A_4$
The proposed model	$A_2 > A_1 > A_3 > A_4$

the optimal solution for this medicine purchase problem. In consequence, different ranking results are obtained from the HFLOWED operator and the proposed model, which is caused by the different essences of the two decision methods. The HFLOWED operator only considers the distance between each alternative and the HFL-PIS and ignores the negative part. However, in practical situations, the DMs need to focus on not only the difference of the two vectors but also the direction of them. Thus, as for the medical management affairs, the results based on the proposed model are more reasonable and reliable than the results derived by the HFLOWED operator.

5.3.2. Compared with the maximum deviation model

The maximum deviation model (Xu, 2005a) is a kind of way to obtain the decision results of the MCDM problems with hesitant fuzzy linguistic information when the criteria weights are completely unknown. With the same decision-making information in the last subsection, here we utilize the maximum deviation model to solve the MCDM problem of selecting the optimal drug procurement plan, and make comparisons between the proposed model and the maximum deviation model. The process of the maximum deviation model is briefly listed as follows (Xu, 2005a):

Firstly, a deviation function is defined to represent the deviation between two alternatives:

$$d(\omega) = \omega_j \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k \neq i} d(r_{ij}, r_{kj}) \tag{5.2}$$

where r_{ij} and r_{kj} are the elements of the HFLDM, and $d(r_{ij},r_{kj}) = \frac{|I(r_{ij}^L) - I(r_{ij}^1)| + |I(r_{kj}^L) - I(r_{kj}^1)|}{4\tau}$.

After that, a non-linear programming mode is constructed to select the weight vector $\omega = (\omega_1,\omega_2,\omega_3,\omega_4,\omega_5)^T$ by:

$$\begin{cases} \max d(\omega) = \omega_j \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k \neq i} d(r_{ij}, r_{kj}) \\ s.\ t. \quad \omega_j \geqslant 0,\ j = 1,2,\ldots,n\ ,\ \sum_{j=1}^{n} \omega_j^2 = 1 \end{cases} \tag{5.3}$$

By solving this model, the weighting vector of criteria can be obtained as:

$$\omega_j = \frac{\sum_{i=1}^{n} \sum_{k \neq i} d(r_{ij}, r_{kj})}{\sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k \neq i} d(r_{ij}, r_{kj})} \tag{5.4}$$

Then, the integrated value of the alternative A_i, denoted as Z_i, is calculated by the following formula:

$$Z_i = \omega_1 \rho(r_{i1}) \oplus \omega_2 \rho(r_{i2}) \oplus \cdots \oplus \omega_m \rho(r_{im}) \tag{5.5}$$

where $\rho(r_{ij})$ is the score function of r_{ij}.

Obviously, the greater value of Z_i, the better the alternative A_i. The decision results are listed in Tables 6 and 7.

According to the maximum deviation model, A_1 is ahead of A_2 with little advantage. The two rankings derived by the different models are both available. The maximum deviation model only uses the difference between the maximum value and the minimum value to measure the distance of two HELEs, which loses a lot of original information from the HEFLDM. Moreover, this method only measures the similarity of two plans through the numerical aspect, but ignores the directional aspect, which lacks comprehensiveness to some degree.

By extending the projection model into the hesitant fuzzy linguistic information, we can fully utilize the original data in the decision-making process. In addition, the proposed model is convenient and effective to handle the MCDM problems when the criteria weights are completely unknown, which possesses the strong practicability and dependability to be used in the MCDM problems in HDSS.

6. Conclusions

To improve the hospital management efficiency, this paper has proposed a hesitant fuzzy linguistic fuzzy projection model to handle the decision-making problems in the HDSS, which considers the characteristics of the medical management, such as uncertainty, complexity, urgency, and so on. Since the decision-making problems in HDSS are various and the factors are polytropic, the paper has proposed the error analysis method to determine the priority vector of the factors. Later on, some discussions have been provided for the proposed model, including comparing it with other decision-making methods, assessing its advantages and drawbacks. Last but not least, the paper has applied the proposed model to select the optimal medicine purchase project. The obtained results have been approved by the hospital experts as the best prescription in drug procurement plans, which manifests that the proposed model has strong applicability and effectiveness to deal with the problems in HDSS. In general, the proposed method not only can process the decision-making problems with all original hesitant fuzzy linguistic information, but also can obtain the decision results straightforwardly and conveniently. In addition, according to the comparative analyses with other MCDM methods, we can easily find that if we only consider the distance of two vectors and ignore their direction, we will lose some important decision information. From this point of view, the proposed model gives a valid way to handle the decision matrix.

Due to the complexity and variability of the HDSS, other MCDM methods in different decision- making environment are worthy paying attention for further research. Furthermore, in addition to the decision modules that require human-computer interactions, there exist a lot of automation modules that need to generate the optimal solution. Subsequently, an important problem, which are about how to predict the assessments of DMs based on their past behavior patterns and preferences, should be thoroughly studied.

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