Available online at www.sciencedirect.com

SCIENCE DIRECT®

Agricultural Water Management 82 (2006) 210–222

Agricultural water management

www.elsevier.com/locate/agwat

Falling water tables in a sloping/nonsloping aquifer under various initial water table profiles

R.M. Singh [a], K.K. Singh [a,1], S.R. Singh [b,*]

[a] *Department of Irrigation and Drainage Engineering, G.B. Pant University of Agriculture and Technology, Pantnagar 263145, Dist. Udham Singh Nagar, India*

[b] *Department of Mathematics Statistics and Computer Science, G.B. Pant University of Agriculture and Technology, Pantnagar 263145, Dist. Udham Singh Nagar, India*

Accepted 4 June 2005

Abstract

An analytical solution for the Boussinesq equation using Werner method of linearization have been obtained to describe falling water table between two parallel drains installed at sloping/nonsloping aquifer for flat, parabola and elliptical initial water table profiles. Midpoints of falling water table between parallel drains obtained from proposed analytical solutions for these initial conditions were compared with both the laboratory and field data. Midpoint water tables obtained from various solutions for nonsloping aquifer were also compared with the result obtained from Boussinesq exact solution using parameters of a drainage experimental site. Tchebycheff norm was used to rank the performance of the proposed solutions. It was observed that the proposed analytical solution for elliptical initial water table profile provides a better result as compared to other solutions and found to be more realistic in modeling the falling water profile.

Keywords: Watertable; Sloping; Nonsloping; Aquifer; Boussinesq equation

* Corresponding author. Tel.: +91 5944 233343; fax: +91 5944 233473.
E-mail addresses: singhkeke@yahoo.com (K.K. Singh), singh_shivaraj@rediffmail.com (S.R. Singh).

1 Tel.: +91 5944 233675 (R)/233307 (O)/233413 (O), fax: +91 5944 233473.

0378-3774/$ – see front matter © 2006 Published by Elsevier B.V.
doi:10.1016/j.agwat.2005.06.003

1. Introduction

Most of the subsurface drainage theories related to sloping or nonsloping land have been developed using Boussinesq equation (1904), based on the principle of continuity and Dupuit-Forchheimer assumptions. Further in most of the transient flow studies, the water table between drains was assumed to be a flat surface at the start of each drainage cycle between two drains except at the drains where the water table drops suddenly to zero, which occurs in the situation when the drains are put into operation for the first time or after a very long period of time.

Dumm (1964) obtained an analytical solution of linearized Boussinesq equation assuming the initial water table described by a fourth degree parabola. Moody (1966) obtained a numerical solution of the nonlinear Boussinesq equation considering drains to be lying some distance above the horizontal impervious layer with an initial water table profile that was described by fourth degree parabola. Dass and Morel-Seytoux (1974) obtained the solutions of one dimensional nonlinear Boussinesq equation by Galerkin finite element technique for three initial conditions: flat, and two types of parabola. Skaggs (1975) obtained numerical solution of Boussinesq equation for initially parabolic and elliptical conditions. Uzaik and Chieng (1989) presented a solution of linearized Boussinesq equation with initial condition in the form of an ellipse (approximated by the two negative exponential functions). Upadhyaya and Chauhan (2001) mentioned that initial shape of water table may be assumed flat, parabola or elliptical depending on soil characteristics. However, they obtained analytical solutions of the Boussinesq equation linearized by Baumann and Werner (1953, 1957) methods and numerical solutions for nonlinear form of the Boussinesq equation using finite difference, finite element and hybrid finite analytic methods, only for flat initial water table profile.

In real situation, a flat water table does not occur after the installation of parallel drains and thus a solution for initially parabolic or elliptical profile should be used for drain spacing. This is probably the case for drainage of irrigated lands and soils with higher hydraulic conductivity. Therefore, various initial water table profiles were being implemented for obtaining analytical solution of Boussinesq equation with Werner linearization for falling water table between drains in sloping/nonsloping aquifer. The objective of this study was to obtain analytical solutions of Boussinesq equation linearized by Werner method for different initial water table profiles to describe falling water tables between two drains lying on a sloping/nonsloping impermeable barrier. The midpoints of falling water tables for the above flow conditions obtained from various solutions were compared with laboratory and field data and the result obtained from Boussinesq exact solution.

2. Mathematical formulation of the problem

The physical problem of subsurface drainage considered for the present study is illustrated in Fig. 1. While formulating the boundary value problem for a falling water table between two parallel drains, it is assumed that due to previously recharge the water table has reached the land surface at the midpoint and it starts falling with drainage of the

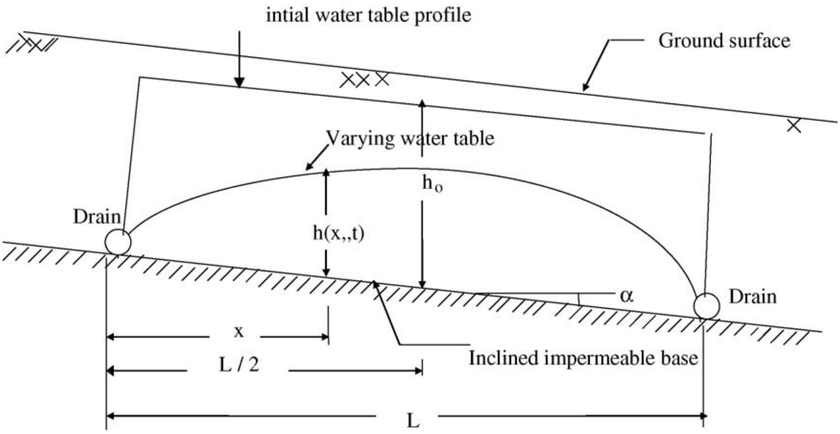

Fig. 1. Definition sketch for falling water table between drains in a sloping aquifer.

aquifer. The aquifer is homogeneous, isotropic and resting on a sloping impermeable base. The Boussinesq equation has been commonly used for mathematical modeling of subsurface drainage problems. The nonlinear second order partial differential equation (Boussinesq, 1904) for describing falling water tables between two drains may be produced as

$$\left(\frac{\partial h}{\partial x}\right)^2 + h\left(\frac{\partial^2 h}{\partial x^2}\right) - \alpha\left(\frac{\partial h}{\partial x}\right) = \frac{f}{K}\frac{\partial h}{\partial t} \tag{1}$$

where h is the height of water table above the impermeable layer [L] at a distance x and time t; α the slope of the impermeable barrier having a small value such that $\alpha = \sin \alpha = -\tan \alpha$; K the hydraulic conductivity [L T^{-1}]; and f the drainable porosity of the aquifer. Werner transformation has been used for linearization of equation (1); on applying the Werner transformation, $z = h^2$, and by setting $(1/\sqrt{z})(\partial z/\partial t) = (1/D)(\partial z/\partial t)$. Where characteristics depth D is the average depth of flow. After applying Werner linearization technique the Eq. (1) can be expressed as

$$\frac{\partial^2 z}{\partial x^2} - \frac{\alpha}{D}\frac{\partial z}{\partial x} = \frac{f}{KD}\frac{\partial z}{\partial t} \tag{2}$$

where characteristic depth D is the average depth of flow.

The initial and boundary conditions for flow problem may be written as

$$z(x,0) = z_0 = h_0^2(x) \quad \text{at}\, t = 0 \qquad \text{for}\, 0 < x < L \tag{3}$$

$$z(0,t) = 0 \qquad \text{at}\, t > 0 \qquad \text{for}\, x = 0 \tag{4}$$

$$z(L,t) = 0 \qquad \text{at}\, t > 0 \qquad \text{for}\, x = L. \tag{5}$$

Different studies have indicated that the initial condition seems to be more complicated for which the choice depends on drainage situations. Four types of initial water table shapes that we considered are given as

Case 1. A constant water table height h_0, exists everywhere between the drains except at the drains where water table suddenly drops to zero

$$\begin{array}{lll} h(x,0) = h_0 & \text{for} & 0 < x < L \\ h(x,0) = h_0 & \text{for} & t \geq 0 \text{ at } x = 0 \text{ and } x = L. \end{array} \tag{6}$$

Case 2. The water table shape is a fourth degree parabola (parabola type1) of the following form,

$$h(x,\ 0) = 8h_0\left(\frac{x}{L} - 3\frac{x^2}{L^2} + 4\frac{x^3}{L^3} - 2\frac{x^4}{L^4}\right). \tag{7}$$

Case 3. The water table shape is another type of parabola (parabola type2) expressed as

$$h(x,0) = \frac{16h_0}{L^4}x^2(L-x)^2. \tag{8}$$

Case 4. The water table shape is an ellipse

$$\frac{(x-(L/2))^2}{L^2(h_0+d)^2/4\{(h_0+d)^2-d^2\}} + \frac{(h+d)^2}{(h_0+d)^2} = 1 \tag{9}$$

where h_0 is the midpoint initial water table height above the drains and d the depth above the impervious layer, where drains are installed.

3. Analytical solutions

3.1. Analytical solution for falling water table in a sloping aquifer

An analytical Solution of linearized Boussinesq equation $(\partial^2 z/\partial x^2) - (\alpha/D)(\partial z/\partial x) = (f/KD)(\partial z/\partial t)$ is obtained by using a transformation that converts it into a heat flow equation. The transformation be given as

$$z(x,t) = V(x,t)e^{sx - s^2 at} \tag{10}$$

where $a = KD/f$, and $s = \alpha/2D$. Applying the transformation (10), the governing Eq. (2) and the initial and boundary conditions, Eqs. (3)–(5) become

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{a}\frac{\partial V}{\partial t} \tag{11}$$

$$V(x,0) = h_0^2(x)e^{-sx} = f(x) \tag{12}$$

$$V(0,t)=0 \tag{13}$$

$$V(L,t)=0 \tag{14}$$

$h_0(x)$ will vary for different initial water table profiles, as illustrated for different cases 1–4 as mentioned in Eqs. (6)–(9). Solution of the above boundary value problem given by Eqs. (11)–(14) is obtained from Ozisik (1980), expressed as

$$V(x,t)=\frac{2}{L}\sum_{m=1}^{\infty}e^{-a\beta_m^2 t}\sin\beta_m x\left[\int_0^L f(x)\sin\beta_m x\mathrm{d}x\right]. \tag{15}$$

Thus in above Eq. (15) incorporating Eq. (12), we get

$$V(x,t)=\frac{2}{L}\sum_{m=1}^{\infty}e^{-a\beta_m^2 t}\sin\beta_m x\left[\int_0^L (h_0^2(x))e^{-sx}\sin\beta_m x\mathrm{d}x\right]. \tag{16}$$

This on simplification gives

$$V(x,0)=\frac{2h_0^2}{L}\sum_{m=1}^{\infty}C_m e^{-a\beta_m^2 t}\sin\beta_m x \tag{17}$$

where C_m is a coefficient and its values for different initial water table profiles will be different. Substituting Eq. (17) in equation (10), we get

$$h^2(x,t)=\frac{2}{L}(h_0^2)e^{sx-s^2at}\sum_{m=1}^{\infty}C_m e^{-a\beta_m^2 t}\sin\beta_m x. \tag{18}$$

For different cases of initial water table profiles the values of C_m are as fallows:

Case 1. Flat initial water table profile

$$C_m=\left[\frac{\beta_m}{\beta_m^2+s^2}\left[1-(-1)^m e^{-sl}\right]\right].$$

Case 2. Parabola type1 initial water table profile

$$\begin{aligned}C_m=\Bigg[\Bigg\{\Bigg\{&\frac{(3s^2\beta_m-\beta_m^3)}{L^2(s^2+\beta_m^2)^3}+\frac{204(5s^4\beta_m-10s^2\beta_m^3+\beta^5)}{L^4(s^2+\beta_m^2)^5}\\&+\frac{10080(7s^6\beta_m-35s^4\beta_m^3+21s^2\beta^5-\beta_m^7)}{L^6(s^2+\beta_m^2)^7}\\&+\frac{80640(9s^8\beta_m-84s^6\beta_m^3+126s^4\beta^5-36s^2\beta_m^7+\beta_m^9)}{L^8(s^2+\beta_m^2)^9}\Bigg\}\left[1-(-1)^m e^{-sl}\right]\Bigg\}\end{aligned}$$

$$
-\left\{\left\{\frac{18\left(4s^3\beta_m-4s\beta_m^3\right)}{L^3\left(s^2+\beta_m^2\right)^4}+\frac{1680\left(6s^5\beta_m-20s^3\beta_m^3+6s\beta^5\right)}{L^5\left(s^2+\beta_m^2\right)^6}+\frac{40320\left(8s^7\beta_m-56s^5\beta_m^3+56s^3\beta^5-s\beta_m^7\right)}{L^6\left(s^2+\beta_m^2\right)^7}\right\}\left[1+(-1)^m e^{-sl}\right]\right\}\Bigg].
$$

Case 3. Parabola type2 initial water table profile

$$
C_m=\left[\left\{\frac{24\left(5s^4\beta_m-10s^2\beta_m^3+\beta^5\right)}{L^4\left(s^2+\beta_m^2\right)^5}+\frac{4320\left(7s^6\beta_m-35s^4\beta_m^3+21s^2\beta^5-\beta_m^7\right)}{L^6\left(s^2+\beta_m^2\right)^7}+\frac{40320\left(9s^8\beta_m-84s^6\beta_m^3+126s^4\beta^5-36s^2\beta_m^7+\beta_m^9\right)}{L^8\left(s^2+\beta_m^2\right)^9}\right\}\left[1-(-1)^m e^{-sl}\right]-\left\{\frac{480\left(6s^5\beta_m-20s^3\beta_m^3+6s\beta^5\right)}{L^5\left(s^2+\beta_m^2\right)^6}+\frac{20160\left(8s^7\beta_m-56s^5\beta_m^3+56s^3\beta^5-8s\beta_m^7\right)}{L^7\left(s^2+\beta_m^2\right)^8}\right\}\left[1+(-1)^m e^{-sl}\right]\right].
$$

Case 4. Elliptical initial water table profile

$$
C_m=\left[\left\{\frac{2s\beta_m}{L(s^2+\beta_m^2)^2}\right\}\left[1+(-1)^m e^{-sL}\right]-\left\{\frac{2(3s^2\beta_m-\beta_m^3)}{L^2(s^2+\beta_m^2)^3}\right\}\left[1-(-1)^m e^{-sL}\right]\right].
$$

3.2. Analytical solution for falling water table in a nonsloping aquifer

A simplified form of Boussinesq equation to describe water table fluctuation in a horizontal aquifer can be obtained by putting $\alpha = 0$ in Eq. (2). The solution for such flow condition can be obtained independently by setting an appropriate boundary value problem. Here analytical solution for falling water table in a nonsloping aquifer obtained as special case of the solution obtained for the sloping aquifers by putting $s = 0$.

$$
h^2(x,t)=\frac{2}{L}h_0^2\sum_{m=1}^{\infty}C_m e^{-a\beta_m^2 t}\sin\beta_m x \tag{19}
$$

where for different type of initial water table profiles the values of C_m are as fallows:

Case 1. Flat initial water table profile

$$
C_m=\frac{1}{\beta_m}\left[1-(-1)^m\right].
$$

Case 2. Parabola type1 initial water table profile

$$C_m = 128\left[\left\{ -\frac{3}{L^2\beta_m^3} + \frac{204}{L^4\beta^5} - \frac{10080}{L^6\beta_m^7} + \frac{80640}{L^8\beta_m^9} \right\}[1-(-1)^m]\right].$$

Case 3. Parabola type2 initial water table profile

$$C_m = 256\left[\left\{ \frac{24}{L^4\beta^5} - \frac{4320}{L^6\beta_m^7} + \frac{40320}{L^8\beta_m^9} \right\}[1-(-1)^m]\right].$$

Case 4. Elliptical initial water table profile

$$C_m = 4\left[\left\{ \frac{2}{L^2\beta_m^3} \right\}[1-(-1)^m]\right].$$

4. Result and discussion

The analytical solution of the linearized Boussinesq equation using Werner method of linearization for four cases of initial water table profiles were validated both with laboratory and field data as well as with the Boussinesq exact solution for the falling water table condition. Water table profiles computed for various times using the proposed analytical solution for elliptical initial water table profile in sloping aquifer (8% slopes) were compared with water table profile for nonsloping aquifer.

4.1. Comparison with experimental model

Chauhan conducted an experimental investigation on a Hele-Shaw viscous model as reported by Upadhyaya and Chauhan (2001). The experiment for the nonsloping case was conducted at 68 °F using Shell Tellus oil, with a model permeability of 5.91 cm/min. The sloping case experiments were conducted at 72 °F with a model permeability of 7.34 cm/min. The initial oil profile was parallel to the impervious layer in experiments. Midpoint fall of oil for several time intervals were recorded visually above the impermeable layer and were reported for the cases of 0, 4, 6 and 8% slopes of the impermeable barrier.

The values of midpoint decline of water tables obtained from analytical solutions for different initial water table profiles were compared with the experimental results of the Hele-Shaw model for nonsloping and sloping aquifers reported by Upadhyaya and Chauhan (2001). Since the initial oil profile of Hele-Shaw model was parallel to the impervious layer in experiments, the comparisons were made by considering initial water table heights at 25 min as the starting point because it is assumed that during this period of time the water table profile will take its actual shape. For comparison, the following values of different parameters have been used as: the initial water table at midpoint above the drains or impermeable barrier $h_0 = 25.4$, 24.77, 24.49 and 24.23 cm for 0, 4, 6, and 8% slope, respectively, specific yield $f = 1.0$, spacing between two drains $L = 254$ cm, and hydraulic conductivity for non-sloping and sloping cases $K = 5.91$ and 7.34 cm/min, respectively.

It may be observed that in case of nonsloping (Table 1)and sloping (Table 2) aquifers the values of midpoint water table heights computed by analytical solution for elliptical initial water table profile are the closest to the experimental results. Tchebycheff norm was used to rank the performance of the proposed analytical solutions. Tchebycheff norm as reported by Prenter (1975) is employed to measure the goodness and accuracy of various theoretical solutions. This norm describes the maximum difference between the theoretical solution and the experimental model. It is evident from Tables 1 and 2 that in nonsloping and sloping aquifers, the least value of Tchebycheff norm is obtained for the elliptical initial water table profile. The performance of various solutions are compared with the laboratory data. Thus, for both sloping and nonsloping cases, the analytical solution for the elliptical initial water table profile may be considered an excellent one.

4.2. Comparison with measured water tables in the drainage field

This example is based on data from Basin No. 3 in the Canadian work reported by Dumm (1964), in which the weighted average permeability during specific time period was derived from point permeability measured at various heights above the barrier by piezometer method. These data represent the case where the drain is located immediately above an impermeable barrier. Specific yield during each time period was derived from the measured water table height and shape, and discharge rate at beginning and end of each time period.

Comparison of water table heights computed from the obtained the analytical solutions was made with the measured from Basin No. 3 as reported by Dumm (1964). The observed and computed water table heights are presented in Table 3. It may be observed from Table 3 that the values of midpoint fall of water table with time obtained from developed analytical solution for elliptical initial water table profile are in close accordance with the observed water table heights.

Table 1
Experimental and computed midpoint transient falling water table (cm) for nonsloping aquifer

Time (min)	Exp results	Computed water table height (m)						
		Dumm (1954)	Dumm (1964)	Uzaik and Chieng (1989)	Proposed analytical solution			
					Case1	Case2	Case3	Case4
0	30.48							
25	25.40							
50	19.81	19.38	17.88	19.38	22.20	20.65	16.71	19.99
100	13.59	11.84	10.92	11.84	15.32	14.25	11.53	13.79
150	10.29	10.08	9.30	10.08	11.71	10.90	8.81	10.54
200	8.26	8.61	7.93	8.61	9.42	8.76	7.09	8.48
250	7.11	7.42	6.83	7.42	7.82	7.30	5.89	7.06
300	6.15	6.71	6.17	6.71	6.91	6.43	5.21	6.22
Value of Tcheby-cheff norm		1.75	2.67	1.75	2.39	0.84	3.10	0.22

Table 2
Experimental and computed midpoint transient falling water table (cm) for different slopes of impermeable layer

Time (min)	4% slope					6% slope					8% slope				
	Exp results	Analytical solutions				Exp results	Analytica solutions				Exp results	Analytical solutions			
		Case1	Case2	Case3	Case4		Case1	Case2	Case3	Case4		Case1	Case2	Case3	Case4
0	29.85					29.82					29.77				
25	24.77					24.49					24.23				
50	18.75	20.57	19.18	15.52	18.52	18.29	20.42	19.03	15.39	18.36	18.03	20.24	18.85	15.24	18.21
100	12.34	13.61	11.43	10.31	12.27	11.81	13.41	12.50	10.11	12.07	11.61	13.23	12.32	9.96	11.89
150	9.27	10.24	9.55	7.72	9.22	9.02	9.86	9.17	7.42	8.86	8.26	9.75	9.07	7.32	8.74
200	7.26	8.26	7.70	6.22	7.42	6.65	8.10	7.52	6.07	7.26	6.25	7.47	6.93	5.56	6.68
250	6.27	6.76	6.27	5.08	6.07	5.36	6.30	5.84	4.70	5.64	4.85	5.94	5.49	4.39	5.31
300	4.95	6.02	5.61	4.52	5.41	4.37	5.23	4.85	3.91	4.70	3.76	4.83	4.42	3.51	4.27
Value of Tchebycheff norm		1.82	0.91	3.23	0.46	–	2.13	0.87	2.90	0.61	–	2.21	0.82	2.79	0.51

Table 3
Measured and computed midpoint transient falling water table (cm) when drains installed at the impervious layer

Date (Oct.)	t (day)	K (cm/day)	f (%)	Water table height (cm)					
				Measured	Computed form analytical solutions				
					Dumm (1964)	Uzaik and Chieng (1989)	Case2	Case3	Csae4
3			85.34						
4	1	42.37	23.2	82.30	78.94	97.10	81.99	66.50	79.25
5	1	40.84	23.8	76.20	73.76	95.45	79.86	64.92	77.11
8	3	35.05	6.5	45.72	45.11	53.79	50.29	33.53	45.72
9	1	28.96	4.0	39.62	38.41	43.62	40.23	32.61	38.71
15	6	23.77	2.5	18.29	17.68	25.91	21.34	8.23	18.29
Value of Tchebycheff norm					3.36	14.8	4.57	15.8	3.05

4.3. Comparison of computed water tables with result of Boussinesq exact solution

The numerical results obtained from developed analytical solutions were compared with water tables computed from exact solution by using the value of drainage design parameters obtained at a drainage experimental site in Sampla, Haryana, India. The Sampla experimental site is situated at $28^\circ 47' 12''$N latitude and $76^\circ 45' 20''$E longitudes. The annual average rainfall at this site is 650 mm and the annual average pan evaporation is 2000 mm. More than 80% of the total rainfall occurs during a 90-day period that starts from 15th June. During the monsoon season, the water table fluctuates from the soil surface to a depth of about 1.5 m in summer. The hydraulic conductivity of the soil was measured to be 3.0 m/day, and the average drainable porosity estimated as 14%. The drains were placed at a depth of 1.8 m with a spacing of 50 m (Verma et al., 1998). The computed water table heights are given in Table 4. It may be observed from Table 4 that values of midpoint fall of water table with time obtained by developed analytical solution for elliptical initial water table profile are close to the water table heights predicted from Boussinesq exact solution.

4.4. Physical example for characterizing falling water table

Analytical solution for elliptical initial water table profile was applied for a physical situation (Sampla site mentioned above in Section 4.3). The initial midpoint water table was assumed to be located at the ground surface (Fig. 2).

4.5. Temporal variation of midpoint water table height under different initial conditions

The developed analytical solution has been applied to Sampla site. The mid point water table heights computed from analytical solution for different initial water table profiles for horizontal aquifer have been compared with the t Boussinesq exact solution and are presented in Fig. 3. A large variation in predicted mid point water table heights under different initial water table profiles are evident. Water table height predicted under elliptical initial water table profile were similar to the water table height computed with the

Table 4
Measured and computed midpoint transient falling water table (m) when drains installed at the impervious layer

Time (day)	Water table height computed form analytical solutions (m)					
	Boussinesq exact solution	Dumm (1964)	Uzaik and Chieng (1989)	Case2	Case3	Case4
2	1.58	1.58	1.7	1.63	1.34	1.59
4	1.41	1.44	1.63	1.50	1.03	1.42
6	1.27	1.34	1.58	1.40	0.81	1.29
8	1.16	1.26	1.54	1.31	0.65	1.18
10	1.06	1.20	1.49	1.24	0.53	1.09
12	0.98	1.16	1.47	1.18	0.43	0.99
Value of Tchebycheff norm		0.18	0.49	0.20	0.55	0.03

Boussinesq exact solution. Water table heights predicted from analytical solution for flat and parabola type1 initial water table profiles were higher than those computed using the Boussinesq exact solution. Water table heights predicted from analytical solution for parabola type2 initial water table profile were lower than predicted by the Boussinesq exact solution.

4.6. Field application

The initial shape of the water table may be flat, parabola or elliptical depending on soil characteristics and recharge pattern. Flat water tables occur frequently after a period of heavy rainfall for both open ditch and pipe drainage systems where subirrigation or controlled drainage is practiced for water management. When rainfall occurs during a period of controlled drainage, the water table will often rise up to the surface and assumes a horizontal profile prior to drawdown. Whereas in the case of drainage for irrigated land and soils with higher hydraulic conductivities, it has been observed that a flat initial water table does not exist after the installation of parallel drains and thus solutions for another types of

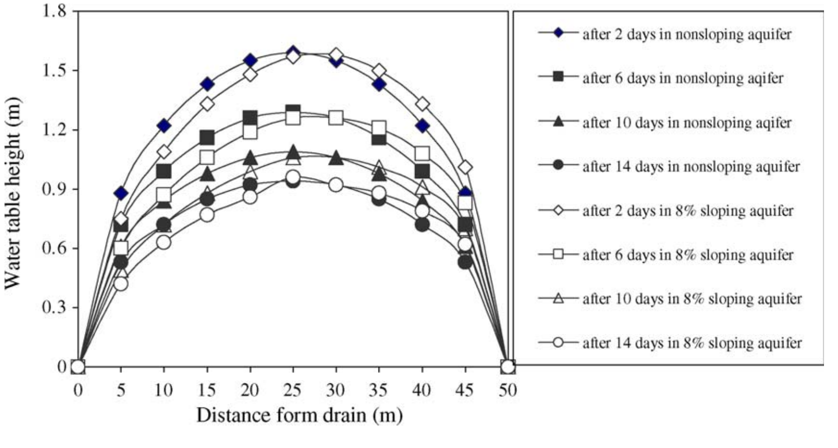

Fig. 2. Water table fall in case of elliptical initial water table profile for nonsloping and sloping aquifers.

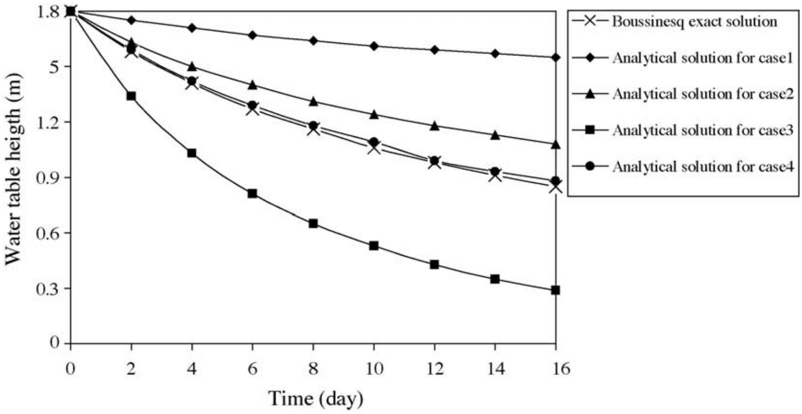

Fig. 3. Temporal variation of midpoint water table height (m).

initial profiles should be used for designing drain spacing. Under such conditions, the initial condition may be approximated by introducing a parabolic or elliptical profile. Further, a steady state equation based on elliptical initial condition has been widely used in subsurface drainage design. Thus, solutions for initially flat, parabolic and elliptical profiles have wide application in drainage design. The proposed analytical solution is more generalized which can be used for water management in such situations to design subsurface drainage system for sloping and nonsloping aquifers. Development of effective water management practices needs the knowledge of initial water table profile depending on soil characteristics, recharge pattern through rainfall or irrigation.

5. Conclusions

Water table, falling between two parallel drains installed at a sloping/nonsloping, homogeneous, isotropic and unconfined aquifer, has been characterized by Boussinesq equation. Modeling of falling water table has been considered by obtaining the analytical solutions of the Boussinesq equation linearized by the Werner method for different initial water table profiles. Midpoints of falling water tables between two drains as obtained from various solutions were compared with experimental and field data and results obtained. Tchebycheff norm was used to rank the performance of the proposed solutions. Result revealed that the solution for elliptical initial water table profile predicts best values of the midpoint falling water tables between two drains in a horizontal/sloping aquifer where drains are installed on the top of the impervious layer.

Acknowledgements

The first author is highly thankful to the Head, Irrigation & Drainage Engineering Division and the Director, Central Institute of Agricultural Engineering along with the

Indian Council of Agricultural Research, India for granting study leave and sponsoring him to carry out studies to Doctoral degree at G.B. Pant University of Agriculture & Technology, Pantnagar (Uttaranchal), India. The authors are also thankful to Dr. H.S. Chauhan, and Dr. Sewa Ram for their suggestions.

References

Boussinesq, J., 1904. Recherches theoretiques sur l'ecoulement des nappes d'eau infiltree's dans le sol et sur le debit des sources. J. de Math. Pures et Appl., Series 5, Tome X (in French) 5–78.

Dass, P., Morel-Seytoux, H.J., 1974. Subsurface drainage solutions by Galerkin's Method. J. Irrig. Drain. Div., ASCE 100 (IR1), 1–15.

Dumm, L.D., 1954. Drain spacing formula: New formula for determining depth and spacing of subsurface drains in irrigated lands. Agric. Eng. 35 (10), 726–730.

Dumm, L.D., 1964. Transient-flow concept in subsurface drainage: Its validity and use. Trans. ASAE 142–145.

Moody, W.T., 1966. Nonlinear differential equation of drain spacing. J. Irrig. Drain. Div., ASCE 92 (IR2), 1–9.

Ozisik, M.N., 1980. Heat Conduction. Wiley, New York, pp. 71–72, 201–203.

Prenter, P.M., 1975. Spline and Variational Methods. Wiley, New York, pp. 6–11.

Skaggs, R.W., 1975. Drawdown solution for simultaneous drainage and evapotranspiration. J. Irrig. Drain. Div., ASCE 101 (IR4), 1279–1291.

Upadhyaya, A., Chauhan, H.S., 2001. Falling water tables in nonsloping/sloping aquifer. J. Irrig. Drain. Eng. 127 (6), 378–384.

Uzaik, J., Chieng, S., 1989. Drain spacing formula for transient state flow with ellipse as an initial condition. Can. Agric. Eng. 31 (2), 101–105.

Verma, A.K., Gupta, S.K., Singh, K.K., Chauhan, H.S., 1998. An analytical solution for design of bi-level drainage systems. Agril. Water Mgmt. 37, 75–92.

Werner, P.W., 1953. On non-artesian ground water flow. Geofis Pura Appl. 25, 37–43.

Werner, P.W., 1957. Some problems in non-artesian ground water flow. Trans. Am. Geophys. Union 38 (4), 511–518.