Interfaces with Other Disciplines

Assessing productive efficiency of banks using integrated Fuzzy-DEA and bootstrapping: A case of Mozambican banks

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ABSTRACT

Performance analysis has become a vital part of the management practices in the banking industry. There are numerous applications using DEA models to estimate efficiency in banking, and most of them assume that inputs and outputs are known with absolute precision. Here, we propose new Fuzzy-DEA α-level models to assess underlying uncertainty. Further, bootstrap truncated regressions with fixed factors are used to measure the impact of each model on the efficiency scores and to identify the most relevant contextual variables on efficiency. The proposed models have been demonstrated using an application in Mozambican banks to handle the underlying uncertainty. Findings reveal that fuzziness is predominant over randomness in interpreting the results. In addition, fuzziness can be used by decision-makers to identify missing variables to help in interpreting the results. Price of labor, price of capital, and market-share were found to be the significant factors in measuring bank efficiency. Managerial implications are addressed.

1. Introduction

One of the major research areas in banking is the measurement of the relative efficiency of banks by means of popular non-parametric techniques such as Data Envelopment Analysis (Hemmati, Dalghandi, & Nazari, 2013). In recent years, several scholars have developed new Data Envelopment Analysis (DEA) models to handle input and output uncertainty (Hatami-Marbini, Tavana, Saati, & Agrell, 2013). A possible path to handle input/output uncertainty in DEA relies on the use of probability distributions to model their inherent randomness. These distributions are subsequently employed in stochastic DEA models (Morita & Seiford, 1999; Brázdik, 2004; El-Demerdash, El-Khodary, & Tharwat, 2013; Vaninsky, 2013). In such cases, however, these probability distributions require to be somewhat estimable a priori or a posteriori, limiting the use of stochastic DEA models in cases where the event is unique or deterministic. Alternatively, however, uncertainty in input/output may be related to imprecision or vagueness, rather than to randomness. This being the case,

imprecision or vagueness in input/output values can be expressed by membership functions within the ambit of fuzzy logic (Coroianu, Gagolewski, & Grzegorzewski, 2013).

This paper analyses the efficiency of Mozambican banks with three major Fuzzy DEA (FDEA) models based on the α-level approach. Thus far, applications of FDEA to measure bank efficiency have been scarce and focused on ranking DMUs rather than predicting their efficiency levels based on a set of contextual variables (Chen, Chiu, Huang, & Tu, 2013; Puri & Yadav, 2013, 2014; Wang, Lu, & Liu, 2014; Hsiao, Chern, Chiu, & Chiu, 2011; Wu, Yang, & Liang, 2006). This paper innovates first by focusing on Mozambican banks and second by simultaneously adopting three major FDEA models based on the α-level approach in combination with the conditional bootstrapped truncated regression, proposed in this research, in a two-stage approach. The third innovative venue in this research relies on the further testing of the global separability between efficiency scores and contextual variables using Monte Carlo methods, as suggested in Daraio, Simar and Wilson, (2010). In summary, the novelty resides in the practical application of different FDEA models combined with conditional bootstrapped regression in the second stage. This combination of fuzzy and probabilistic approaches also represents a contribution to the emerging literature on possible analytical venues within the ambit of 2-Dimensional Fuzzy Monte Carlo Analysis (2D FMCA).

Specifically, the motivations for the present research are related to the following issues. The first relates to the evaluation of the

relative efficiency of Mozambican banks using FDEA for the first time, using the popular α-level approach. Despite the existence of different types of fuzzy approaches for handling vagueness and uncertainty within the ambit of DEA models—see Emrouznejad and Tavana (2014) for a comprehensive literature review on this subject—the α-level approach was chosen here not only in terms of its popularity among researchers, but also because in this approach an FDEA model is solved by parametric programming using α-levels. Solving this model at a given level of α produces interval efficiency for the decision making unit (DMU) under assessment (Zerafat Angiz, Emrouznejad, & Mustafa, 2010). Although these intervals, when taken in a certain number, can be used to infer the respective fuzzy efficiency, in this paper we are interested in using the crisp values for their lower and upper bounds to assess efficiency drivers in Mozambican banks in the second stage.

The second motivation pertains to expanding the literature by using conditional bootstrapped truncated regression to assess the role of major contextual variables in achieving higher levels of efficiency, considering the impact of three different FDEA models based on the α-level approach as fixed factors. In order to achieve this objective, bootstrapped truncated regressions are reformulated within the context of a two-stage approach, considering different levels of α. The third goal concerns the coverage of a significant time span of a representative sample of Mozambican banking—2003 to 2011—so that uncertainty in its different forms can be assessed. As a matter of fact, the outputs and inputs of banks present different forms of uncertainty within their relationships. For example, credit granting is an output embedded in fuzziness because of the ex-ante risks associated with non-performing loans (Li, 2003). On the other hand, the investment income of a bank, which is not a constant number, changes randomly on account of the market value of the investment target. To evaluate Mozambican bank efficiency more realistically and accurately, this study employs the fuzzy DEA model with data specified in bounded forms to measure the efficiency of banks.

Therefore, this study proposes a predictive model for banking efficiency in Mozambique based on the financial and operational criteria commonly found in the literature and considers uncertainty in the collection of input and output data. The remainder of the paper is organized as follows: Section 2 presents the contextual setting; Section 3 reviews the literature; Section 4 presents the data source and the model; results are discussed and presented in Section 5; and Section 6 sets out the conclusion.

2. Contextual setting

According to KPMG (2014), banking credit is the most important driver of Mozambique economic growth. The average credit interest rate was 20.22% in 2014, which resulted in private sector debt to banks of 28.7%. Adding to this growth, the increase in public debt in 2014 was 16.37%; that is, the Mozambican economy is being pumped up by money to induce growth.

The banks analyzed in the present context account for about 90% of the banking industry and their data are representative of Mozambique banks. BIM-Banco Internacional de Moçambique is the country's largest bank, with a 40% market share. This bank was formed in 2001 through a merger of Millennium BCP and Banco Comercial de Moçambique. BCI – Banco Comercial de Investimentos – is a small investment bank. It is owned sixty percent by the Portuguese public bank Caixa Geral de Depósitos and 40% by small shareholders. Standard Bank is a South African bank and the largest in Africa. Barclays Bank is an English bank with international operations and is active in Mozambique. Barclays Bank Mozambique was founded in 2002 with the acquisition of Banco Austral, a Portuguese owned bank, and is Mozambique's largest bank for personal and commercial banking. BancABC (previously African Banking Corporation) was originally a British Overseas Bank, headquartered in London albeit with all branches overseas; main shareholders currently include the International Finance Corporation, Old Mutual, Botswana insurance Fund Managers and Citi Venture Capital. In 1999, ABC Mozambique was incorporated as BNP NedBank, a joint venture between the Brazilian BNP Paribas and NedBank of South Africa. Mauritius Commercial Bank SA is a subsidiary of The Mauritius Commercial Bank Limited, a Mauritius based bank. Banco ProCredit Mozambique is a Mozambique private bank. SOCREMO – Banco de Microfinanças is a Mozambique microfinance private bank. BMI (Banco Mercantil e de Investimentos) is a private investment bank. ICB – International Commercial Bank is a bank controlled by the ICB Banking Group based in Switzerland and specializes in emerging markets. It focuses on international bank services and foreign trade finance. BOM (Banco Oportunidade de Moçambique) is a microfinance bank. Banco Tchuma provides credit and savings services to emerging Mozambican entrepreneurs, in particular, women.

Competition is high in banking and will continue to be so as long as new competitors seek to enter the Mozambique market. For example, in 2013, Ecobank, a pan-African bank, entered the market by buying Banco ProCredit. Furthermore, Nedbank of South Africa acquired Mozambique's Banco Único.

3. Review of the literature

Charnes, Cooper, and Rhodes (1978) first proposed DEA for the case of constant returns-to-scale, which became known as the CCR (Charnes, Cooper and Rhodes) model. Subsequently, Banker, Charnes, and Cooper (1984) extended the model to the case of varying returns-to-scale; the model came to be known as BCC (Banker, Charnes and Cooper). Both models apply linear programming and allow output/input weighting to compute efficiency scores. Nowadays, several different DEA models are employed in different circumstances, e.g., industries, countries, and organizations involved in efficiency assessment.

Despite the numerous studies focusing on banking efficiency and productivity using DEA (Berger & Humphrey, 1992, 1997; Fukuyama & Weber 2009a, 2009b, 2010; Holod & Lewis, 2011; Sufian, 2010), an in-depth analysis of banks in Africa is still missing (O'Donnell & Westhnizen, 2002; Azam, Biais, & Dia, 2004; Figueira, Nellis, & Parker, 2006; Kirkpatrick, Murinde, & Tefula, 2007; Okeahalam, 2008; Ikhide, 2008; Kiyota, 2009; Assaf, Barros, & Ibiwoye, 2010; Kebede & Wassie, 2013), thus indicating a literature gap. The situation contrasts with the extensive research that has been carried out on American banks (Berger, Hanweck, & Humphrey, 1987; Bauer, Berger, & Humphrey, 1993; Berger & Humphrey, 1997), on European banks Asian banks (Assaf et al., 2010; Barros, Peypoch, & Williams, 2010; Barros, Managi, & Matousek, 2012b; Berger et al., 2009; Chen, Skully, & Brown, 2005; Kumbhakar & Wang, 2005), and even South American banks (Staub, Souza, & Tabak, 2010; Wanke & Barros, 2014).

Even though DEA might be sufficient to determine efficiency levels, this method does not per se provide details of the determinants related to inefficiency. In this sense, several studies proposed a combination approach of measuring and explaining bank efficiency scores (Fethi & Pasiouras, 2010) using DEA in the first stage to determine efficiency scores and a regression model in the second stage to explain the respective drivers. For example, Ariff and Can (2008), Casu and Molyneux (2003), and San, Theng, and Heng (2011) employed Tobit regression to explain bank performance in terms of contextual variables after running DEA models.

Traditionally, however, DEA models consider that output and input are crisp numbers. If input and output values were fuzzy, traditional DEA could not be able to assess efficiency levels in a proper manner. This being the case, several researchers (Cooper, Park, & Yu, 1999; Despotis & Smirlis, 2002; Guo & Tanaka, 2001; Jahanshahloo, Soleimani-damaneh, & Nasrabadi, 2004; Kao & Liu, 2000b) started structuring FDEA models, allowing for the measurement of outputs

and inputs as fuzzy numbers. Particularly with respect to FDEA applications on banking, studies to assess efficiency in the financial sector still remain scarce, and their major focus tends to relate to ranking of DMUs based on computed fuzzy efficiencies rather than predicting or explaining efficiency levels in terms of contextual variables (Chen et al., 2013; Puri & Yadav, 2013, 2014; Hsiao et al., 2011 ; Wang et al., 2014; Wu et al., 2006).

Wu et al. (2006) introduced fuzzy logic into DEA formulation in order to deal with the environmental variables and thus assess the performance of bank branches in different regions. The inner-province and inter-province comparison were given based on the fuzzy DEA results. The authors also compared these results with those obtained from traditional DEA analysis. Hsiao et al. (2011) proposed the use of a fuzzy super-efficiency slack-based measure DEA to analyze the operational performance of 24 commercial banks facing problems on loan and investment parameters with vague characteristics. Wang et al. (2014) investigated the association between the performance of bank holding companies and their intellectual capital. The authors applied fuzzy multiple objective programming approaches to calculate efficiency scores. Puri and Yadav (2013) evaluated the fuzzy input mix-efficiency using the α-level based approach for the State Bank of Patiala in the Punjab state of India, with districts as the DMUs. Puri and Yadav (2014) proposed another fuzzy DEA model with undesirable fuzzy outputs, which can be solved as a crisp linear program for each α in (0, 1] using the α-level based approach, which will be further discussed in Section 4.2. The authors applied the model in the public banking sector in India for the period 2009–2011. Chen et al. (2013) applied the Fuzzy Slack-Based Measurement model in the Taiwan banking sector under market risk.

According to Hatami-Marbini, Emrouznejad, and Tavana (2011a), the huge dissemination of different models within a large scope of applications in terms of efficiency measurement demonstrates that FDEA models represent an effective path for handling uncertainty and vagueness when inputs/outputs are imprecise (Kao & Liu, 2000b). However, the authors point out research challenges that should be addressed in future studies. Although these challenges will be further explored in the methodology section, we briefly present their implications in light of the literature review on banking efficiency and provide additional details on how our study differs from previous ones.

One of the challenges is the imperative for a unified FDEA approach to account for the numerous FDEA models and frameworks. In this research, we employed a two-stage approach as an attempt to allow the simultaneous application of different major FDEA models, since it is very often not possible to know what model should be chosen over another. Here, in the first stage, three major FDEA models based on the α-level approach are used to compute crisp values for the efficiency scores; in the second stage, these underlying models (Guo & Tanaka, 2008; Kao & Liu, 2000a; Saati, Memariani, & Jahanshahloo, 2002) and the nature of their efficiency scores (whether lower, upper, or middle values) are used as fixed factors in a proposed approach called conditional bootstrapped truncated regression in order to control for the impact of several contextual variables related to bank efficiency in Mozambique.

Another relevant challenge pointed out by Hatami-Marbini et al. (2011a) is related to the sensitivity analysis issue, since fuzzy data are, by definition, less robust and not fixed. In this paper, given the difficulty of obtaining reliable data sources on Mozambican banking and the fact that the conversion of the values originally expressed in the Mozambican national currency into the US dollar is often subject to financial crisis and/or currency board controls, we decided to treat, as triangular fuzzy numbers – following Puri and Yadav (2013) – all the outputs and the inputs, with their lower and upper values defined by an offset of 20% from their respective mean crisp values. Further, in this research, the sensitivity analysis conducted on fuzzy efficiency scores observes the combined probabilistic-fuzzy approach advocated by Arunraj, Mandal, and Maiti (2013), where both random-

ness and uncertainty are jointly considered as useful properties of probabilistic and fuzzy methods.

One last challenge mentioned by Hatami-Marbini et al. (2011a) concerns real-life applications in a sense, since most published papers have used hypothetical data or simple examples to support their rationale. Here, we use a case study, obtained from the real world, on the banking sector of Mozambique in assessing its efficiency drivers despite all sources of uncertainty – probabilistic (randomness) and possibilistic (fuzziness) – surrounding the data.

4. Methodology

This section presents the major methodological steps adopted in this research. After presenting in Section 4.1 the data collected in terms of inputs, outputs, and contextual variables, the two stage-approach is explained in detail. Section 4.2 is devoted to discussing the application of the three major FDEA models used in this research. Section 4.3 depicts the conditional bootstrapped truncated regression proposed to be used in the second stage, while Section 4.4 sets out how the results were validated and interpreted by means of sensitivity analysis. At last, Section 4.5 addresses the global separability issue between efficiency scores and contextual variables within the ambit of this two-stage approach.

4.1. The data

The data on 13 Mozambican banks was obtained from KPMG's yearly report of Mozambique's top 100 companies and encompassed the period from 2003 to 2011. The inputs and the outputs considered observed not only those commonly found in the literature review but also the availability of data. As regards the lack of differentiation in efficiency scores, one of the most common problems in DEA is caused by an excessive number of input and output variables with respect to the number of DMUs (Adler & Berechman, 2001); this research observes the convention that the minimal number of DMUs should be three times greater than the sum of the number of inputs and outputs (Barros, Gonçalves, & Peypoch, 2012a). In fact, there are 117 observations (13 DMUs ∗ 9 years), which is greater than the total number of inputs and outputs multiplied by three, as detailed next.

The choice of inputs and outputs is perhaps the most important task in employing DEA to measure the relative efficiency of the DMUs. Two approaches are widely used to identify a bank's inputs and outputs: the production approach and the intermediation approach (e.g. Aly, Grabowski, Pasurka, & Rangan, 1990; Barros, Liang, & Peypoch, 2014; Berger & Humphrey, 1992; Favero & Pepi, 1995; Miller & Noulas, 1996; Sherman & Gold, 1985; Yue, 1992; Sealey & Lindley, 1977). Under the production approach, banks are treated as a firm to produce loans, deposits, and other assets by using labor and capital. However, banks are considered as financial intermediaries to transform deposits, purchase funds and labor into loans and other assets under the intermediation approach. More specifically, deposits are treated as an input under the production approach and an output under the intermediation approach.

Fortin and Leclerc (2007), however, showed that with an incomplete list of assets and liabilities, the ratio between assets and liabilities included in the model of banking production strongly influences the efficiency score under the intermediation approach. In fact, the authors found that the average score varies significantly according to the definition of inputs and outputs, thus biasing the analysis. Fortin and Leclerc (2007) also advocate either the production approach or the value-added approach. In the production approach, both credit and deposits services are included in the outputs of the banking although the high level of correlation between both types of services may lead to some specification problems. On the other hand, a value-added approach, such as that developed by

Table 1.
Descriptive statistics for the inputs, outputs and the contextual variables.

Variables	Min	Max	Mean	SD	
Inputs and outputs	Total costs (USD/year)	17740.00	6297180.00	821296.07	1157230.34
	Employee costs (USD/year)	4783.00	1380714.00	248910.24	282163.90
	Total deposits (USD)	(78831.34)	34134168.33	4654231.91	7961176.43
	Income before tax (USD/year)	(23099.00)	4056188.00	259347.22	595742.32
	Total credit (USD/year)	(126549.18)	20606226.25	2504382.04	4563029.51
Control variables	Price of labor (employees cost/number of employees)	50.35	4350.35	1174.76	1292.78
	Price of capital (depreciation/total assets)	0.00	0.16	0.03	0.03
	Price of deposits (impairment and provisions/total deposits)	−0.05	1.94	0.07	0.26
	Trend	1.00	9.00	5.00	2.59
	Market-share (%)	0.00	0.43	0.07	0.12
Contextual variables	Foreign ownership (1 = yes/0 = no)	0.00	1.00	0.75	0.43
	Government ownership (1 = yes / 0 = no)	0.00	1.00	0.02	0.13
	Merger and acquisition (1 = yes/0 = no)	0.00	1.00	0.08	0.27
	IFRS accounting principles (1 = yes/0 = no)	0.00	1.00	0.46	0.50
	Active dividend policy (1 = yes/0 = no)	0.00	1.00	0.35	0.48

Fixler and Zieschang (1999), offers an alternative that takes into account the cost of funds to measure the average interest rate spread. Therefore, taking into consideration the risk of biasing the analysis for the Mozambican banks under the intermediation approach and the detailed data requirement under the value-added approach, the production approach in banking is adopted in this research.

The inputs and the outputs considered observed not only those commonly found in the literature review but also the availability of data. The input variables included total costs – excluding employee costs – (USD/year) and employee costs (USD/year). Output variables included total deposits (USD), income before tax (USD/year), and total credit (USD/year). Their descriptive statistics are presented in Table 1.

Besides these inputs and outputs, it should be noted that control variables such as trend, market-share, price of deposits, price of capital, and price of labor were also collected for each bank. The idea is to control for the variations in the market dynamics and in the price paid for these inputs by each bank over the course of time. In addition, five contextual and business-related variables were collected to explain differences in the efficiency levels. These variables are also presented in Table 1 and are related to the ownership/origin of the bank, i.e., (i) whether foreign; (ii) whether Governmental; (iii) whether the bank resulted from a merger and acquisition (M&A) process – the governance structure of the bank; (iv) whether it has an active dividend policy; and (v) whether it adheres to IFRS accounting principles. Especially with respect to M&A processes, there are nine occurrences within the data, encompassing Banco Internacional de Moçambique (in 2005); Banco Comercial de Investimento (in 2003); Banco Pro-Credit (in 2005, 2006, and 2007); and SOCREMO – Banco de Microfinanças (in 2005, 2006, 2007, and 2008).

It is noteworthy that these contextual variables, while neither inputs nor outputs, are deemed to affect the production process. Because these variables identify the sources of efficiency variations, they are also directly linked to policy formulation. Past studies in banking have introduced these contextual variables as exogenous (Assaf, Barros, & Matousek, 2011; Assaf et al., 2010; Assaf, Matousek, & Tsionas, 2013); put differently, an important underlying assumption on contextual variables considered by all these banking authors is that these contextual variables are exogenous, that is, they affect efficiency levels without being affected by them. Here, therefore, these contextual variables represent decision variables based on the banking discretion rather than endogenous variables generated within the ambit of an efficiency model or a production process.

Correlation analyses presented in Tables 2 and 3 indicate significant positive relationships between the input and the output variables, which are, therefore, isotonic, thus justifying their inclusion in the model (Wang, Lu, & Tsai, 2011).

Table 2
Correlations between output variables.

	Total deposits	Income before tax	Total credit
Total deposits	1.000		
Income before tax	0.603	1.000	
Total credit	0.650	0.682	1.000

Table 3
Correlations between input variables.

	Total costs	Employee costs
Total costs	1.000	
Employee costs	0.564	1.000

4.2. Fuzzy DEA

There are two approaches for modeling uncertainty within the ambit of DEA: fuzzy and stochastic. The latter uses probability distributions to model the error process (Sengupta, 1992). The former, however, departs from the fuzzy set algebra (Zadeh, 1965b) and this is the cornerstone that permits fuzziness and vagueness to be treated in uncertain circumstances. FDEA models found in literature are usually classified according to four general approaches (Lertworasirikul, Fang, Joines, & Nutt, 2003; Hatami-Marbini, Saati, & Tavana, 2011b): (i) tolerance, (ii) α-level, (iii) fuzzy ranking, and (iv) possibility. Here we will confine the focus to the major α-level approaches found in literature, as compiled in Hatami-Marbini et al. (2011a).

The α-level approach is possibly the most popular, given the numerous papers produced using its variations, despite the fact that their models are not computationally efficient. This is so because α-level models demand more linear programs to be solved for each α value (Soleimani-damaneh, Jahanshahloo, & Abbasbandy, 2006). Within the α-level approach, the FDEA model is first converted into a pair of parametric programs so that the lower and upper bounds of the efficiency scores can be computed next for a given value of α (Emrouznejad & Tavana, 2014).

The rationale behind the selection of the α-level approach in this study is related to a number of aspects. First, when using this approach, fuzzy inputs and outputs may be expressed as crisp numbers representing the limiting bounds of the intervals for different α-levels (Chen et al., 2013), thus allowing the uncertainty of the data collected from Mozambican banks to be easily modelled as triangular fuzzy numbers. Second, in the situation of various α levels for the inputs and the outputs, FDEA may be translated into traditional DEA (crisp) models in light of the Extension Principle, thus making solving their respective linear programs simpler

(Yager, 1981; Zadeh, 1965a; Zimmerman, 1976). Third, owing to the input and output data being fuzzy numbers, the efficiency scores are also fuzzy numbers (Puri & Yadav, 2013). Moreover, as long as the efficiency values considered here are the upper and lower "crisp" bounds computed for various α levels, the membership functions for the true fuzzy efficiency cannot be reconstructed, which has a number of implications on how fuzzy efficiencies should be ranked (Chen et al., 2013; Puri & Yadav, 2013; Hsiao et al., 2011). These bounds, however, can be treated as crisp values and incorporated into statistical modeling as efficiency scores subjected to certain fixed effects or treatments in order to properly assess the impact of different contextual variables.

Kao and Liu (2000a) developed a procedure to measure the efficiencies when inputs and outputs are fuzzy, starting out with a modified BCC model. This model is solved first at a given level of α-level and leads to an interval efficiency – lower and upper bounds – for each DMU. Let $(w_p)_\alpha^L$ be the lower bound and $(w_p)_\alpha^U$ be the upper bound of the fuzzy efficiency score for a specific α-level. Furthermore, let $\tilde{x}_{ij}$ and $\tilde{y}_{rj}$ denote, respectively, the input and output values for the DMU$_p$. The pair of mathematical models proposed in Kao and Liu (2000a) is given as follows:

$$(w_p)_\alpha^L = max \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^L + u_0$$

$$s.t. \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ip})_\alpha^U + u_0 \le 0,$$

$$\sum_{r=1}^{s} u_r (Y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^L + u_0 \le 0, \; j, \; \forall \, j \neq p$$

$$\sum_{i=1}^{m} v_i (X_{ip})_\alpha^U = 1, \; u_r, \; v_i \ge 0, \; \forall \, r, i \tag{1}$$

$$(w_p)_\alpha^U = max \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^U + u_0$$

$$s.t. \sum_{r=1}^{s} u_r (Y_{rp})_\alpha^U - \sum_{i=1}^{m} v_i (X_{ip})_\alpha^L + u_0 \le 0,$$

$$\sum_{r=1}^{s} u_r (Y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i (X_{ij})_\alpha^U + u_0 \le 0, \; \forall \, j, \; j \neq p$$

$$\sum_{i=1}^{m} v_i (X_{ip})_\alpha^L = 1, \; u_r, \; v_i \ge 0, \; \forall \, r, i \tag{2}$$

where $\left[(X_{ij})_\alpha^L, (X_{ij})_\alpha^U\right]$ and $\left[(Y_{rj})_\alpha^L, (Y_{rj})_\alpha^U\right]$ are α-level specifications for the respective inputs/outputs in their fuzzy form.

In turn, Saati et al. (2002) presented a fuzzy CCR model in its possibilistic form, transforming this model into an interval programming by means of α-levels. The transformed model could be solved, for a given α, as a crisp linear program. More precisely, model (3) proposed by Saati et al. (2002) is derived for a particular case where the inputs and outputs are triangular fuzzy numbers:

$$max \; w_p = \sum_{r=1}^{s} y'_{rp}$$

$$s.t. \sum_{r=1}^{s} y'_{rj} - \sum_{i=1}^{m} x'_{ij} \le 0, \; \forall \, j,$$

$$v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^l \right) \le x'_{ij} \le v_i \left(\alpha x_{ij}^m + (1 - \alpha) x_{ij}^u \right), \; \forall \, i, j,$$

$$u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^l \right) \le y'_{rj} \le u_r \left(\alpha y_{rj}^m + (1 - \alpha) y_{rj}^u \right), \; \forall \, i, j,$$

$$\sum_{i=1}^{m} x'_{ip} = 1, \; u_r, \; v_i \ge 0, \; \forall \, r, j \tag{3}$$

where $\tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u)$ and $\tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u)$ are the inputs and outputs expressed in terms of triangular fuzzy numbers, and x'_{ij} and y'_{ij} are decision variables used to convert the original fuzzy model into a linear program with $\alpha \in [0, 1]$.

More recently, created a FDEA model to compute efficiency within the assurance region concept. The author applied the α level approach and Zadeh's extension principle (Zadeh, 1978; Zimmermann, 1996) to convert this model into a pair of parametric mathematical programs. Therefore, the relationship importance of the inputs and outputs is given as $\frac{L_{I\delta}}{U_{Iq}} \le \frac{v_\delta}{v_q} \le \frac{U_{I\delta}}{L_{Iq}}, \delta < q = 1, \dots, m$; and $\frac{L_{O\delta}}{U_{Oq}} \le \frac{v_\delta}{v_q} \le \frac{U_{O\delta}}{L_{Oq}}, \delta < q = 2, \dots, s$; respectively.

The two parametric models proposed are as follows:

$$(W_p)_\alpha^L = max \sum_{r=1}^{s} u_r (y_{rp})_\alpha^L$$

$$s.t. \sum_{r=1}^{s} u_r (y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i (x_{ij})_\alpha^L \le 0 \; \forall \, j, \; j \neq p,$$

$$-v_\delta + I_{\delta q}^L v_q \le 0, \; v_\delta - I_{\delta q}^U v_q \le 0, \; \forall \, \delta < q,$$

$$-u_\delta + O_{\delta q}^L u_q \le 0, \; u_\delta - O_{\delta q}^U u_q \le 0, \; \forall \, \delta < q,$$

$$\sum_{i=1}^{m} v_i (x_{ip})_\alpha^U = 1, \; u_r, \; v_i \ge 0, \; \forall \, r, j. \tag{4}$$

$$(W_p)_\alpha^U = max \sum_{r=1}^{s} u_r (y_{rp})_\alpha^U$$

$$s.t. \sum_{r=1}^{s} u_r (y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i (x_{ij})_\alpha^U \le 0, \; \forall \, j, \; j \neq p,$$

$$-v_\delta + I_{\delta q}^L v_q \le 0, \; v_\delta - I_{\delta q}^U v_q \le 0, \; \forall \, \delta < q,$$

$$-u_\delta + O_{\delta q}^L u_q \le 0, \; u_\delta - O_{\delta q}^U u_q \le 0, \; \forall \, \delta < q,$$

$$\sum_{i=1}^{m} v_i (x_{ip})_\alpha^L = 1, \; u_r, \; v_i \ge 0, \; \forall \, r, j. \tag{5}$$

where $I_{\delta q}^L = \frac{L_{I\delta}}{U_{Iq}}$, $I_{\delta q}^U = \frac{U_{I\delta}}{L_{Iq}}$, $O_{\delta q}^L = \frac{L_{O\delta}}{U_{Oq}}$ and $O_{\delta q}^U = \frac{U_{O\delta}}{L_{Oq}}$.

4.3. Conditional bootstrapped truncated regression on the α-level efficiency bounds

This section presents the grounds for regressing the upper and lower bounds of the efficiency scores (obtained for each α-level) against the set of contextual variables and fixed factors. Here we consider these lower and upper efficiency bounds as crisp values obtained when solving the converted FDEA model, for different α values, in terms of the pair of parametric linear programs (Guo & Tanaka, 2008; Kao & Liu, 2000a). The same rationale presented here applies to the mean crisp efficiency values found when solving Saati et al. (2002) and Guo and Tanaka (2008) FDEA models.

First, the current state of research on fuzzy linear regression is discussed, not only in terms of addressing its major differences from statistical linear models, but also in terms of identifying its major limitations/criticisms for more widespread use among academics and practitioners. Keeping these limitations in mind, we depart from the bootstrapped truncated regression model frequently used in two-stage DEA analysis and propose a modified version: the conditional bootstrapped truncated model in terms of α so that the different FDEA models and their respective types of scores can be handled as fixed factors and the contextual variables as covariates.

4.3.1. Fuzzy linear regression

Fuzzy linear regression was introduced by Tanaka, Uejima, and Asai (1982) to model situations in which the practitioner cannot accurately measure the dependent variable. As long as traditional statistical regression models can only fit crisp data, fuzzy linear regression models can be used to fit both fuzzy and crisp data (Chang & Ayyub, 2001). For such data, fuzzy set theory provides a means for modeling linguistic variables utilizing membership functions. In contrast to the traditional statistical regression models which are based on probability theory, fuzzy regression is based simultaneously on possibility theory (Dubois, 1988) and fuzzy set theory (Zadeh, 1965b).

Since the introduction of fuzzy linear regression, the literature on the subject has grown rapidly (Pasha, Razzaghnia, Allahviranloo, Yari, & Mostafaei, 2007). In general, there are two approaches in fuzzy regression analysis: the linear programming-based method (Hojati, Bector, & Smimou, 2005; Nasrabadi & Nasrabadi, 2004; Peters, 1994; Sakawa, 1992) and the fuzzy least squares method (Dubois & Prade, 1980). The first method is based on minimizing fuzziness as an optimal criterion. Its major advantage is simplicity in programming and computation. The second method uses a fuzzy least-squares approach to minimize the errors between the observed and predicted values.

In statistical regression analysis, the errors derived from the adjustment of a regression model into the observed data are assumed to be observational errors caused by a random variable following some statistical distribution (e.g., normal, with constant variance and zero mean). However, fuzzy regression analysis views these errors as the underlying uncertainty or fuzziness that exists within the model structure, as proposed by Tanaka et al. (1982, 1988, 1989). This being the case, according to Chang and Ayyub (2001), statistical regressions are meant for handling random errors determined from crisp estimated and observed data. These errors are different in nature from fuzziness or uncertainty. On the other hand, fuzzy regression analyses are meant to model observed fuzzy data.

As one would expect, when fuzzy data approach their crisp state in fuzzy regression (e.g. $\alpha = 1$), the results should approach those obtained from the statistical regression analysis (Chang & Ayyub, 2001). This property, however, still does not exist in actual fuzzy regression models. The basic reason is that fuzzy regression takes the fuzziness assumption as a substitute for the randomness assumption in statistical analysis. In other words, fuzziness is treated as a replacement to randomness, rather than being modeled in a complementary fashion to the underlying randomness. Chang and Ayyub (2001) called this aspect as the "limiting behavior" of fuzzy regression methods. This behavior has unfortunately segregated the used of fuzzy regression from the well-received ordinary least-squares regression. For the same reason, the use of fuzzy regression methods has drawn some criticism from statisticians, for example, Redden and Woodall (1994).

4.3.2. Bootstrapped truncated regression

Methods for treating DEA scores obtained in the first stage using regression or statistical models in the second stage have evolved over the years (e.g., Banker, 1993; Cooper, Seiford, & Tone, 2007; Simar & Wilson, 2007). As a matter of fact, the impact of contextual variables on efficiency scores has been acknowledged by the use of two-stage approaches (Fried, Lovell, Schmidt, & Yaisawarng, 2002). Although some early two-stage studies employ Tobit regression in a second stage or other non-parametric tests (e.g. Turner, Windle, & Dressner, 2004), Simar and Wilson (2007) showed that truncated regression combined with bootstrapping as a resampling technique best overcomes the unknown serial correlation complicating the two-stage analysis. These issues are detailed next.

4.3.3. The proposed approach

In this research – and also putting into perspective the current limitations on fuzzy regression methods – we depart from the approach of Simar and Wilson (2007) and propose conditional bootstrapped truncated regression to analyze the crisp values derived from FDEA models based on α-levels for the upper, lower, and middle efficiency values. Here, the following conditional modeling is tested in Mozambican banks:

$$\theta_j | \alpha = k + Z_j \delta + F_j \gamma + \varepsilon_j, \, j = 1, \ldots, n \tag{6}$$

The modeling can be understood as the first-order approximation of the unknown true relationship. In Eq. (6), α is a real value bounded between 0 and 1 and represents the α-level of the membership function for the efficiency score, k is the constant term, ε_j is statistical noise, F_j is vector of dummy variables that represent the fixed effects for the type of the FDEA model used (Guo & Tanaka, 2008; Kao & Liu, 2000a; Saati et al., 2002) and the type of score derived (whether lower, upper, or middle), and Z_j is a vector of the control and the contextual variables for DMU j that is expected to be related to the DMU's efficiency score, θ_j, taken as a crisp value. As suggested by Croissant and Millo (2012), the Hausman test was performed to assess the suitability of treating the underlying model and the type of the score as fixed rather than random effects.

Specifically, noting that the distribution of ε_j is restricted by the condition $\varepsilon_j \geq 1 - k - Z_j \delta - F_j \gamma$ (since both sides of (6) are bounded by unit), the steps proposed in Simar and Wilson (2007) are followed here, and it is assumed that this distribution is truncated normal with zero mean (before truncation), unknown variance, and (left) truncation point determined by this very condition. Furthermore, replacing the true but unobserved regressand in (6), SE_j, by the respective FDEA estimate, $\overline{\theta}_j$, the conditional econometric model formally becomes:

$$\overline{\theta}_j | \alpha \approx k + Z_j \delta + F_j \gamma + \varepsilon_j, \, j = 1, \ldots, n, \tag{7}$$

where

$$\varepsilon_j \sim N(0, \sigma_\varepsilon^2), \text{ so that } \varepsilon_j \geq 1 - k - Z_j \delta - F_j \gamma, \, j = 1, \ldots, n, \tag{8}$$

which is evaluated via maximal likelihood estimation as regards $(\delta, \sigma_\varepsilon^2)$ obtained from the data. The respective computations for the parametric bootstrap for this conditional regression were carried out with R codes developed by the authors. It uses information both on the distributional assumption and on the parametric structure obtained from the data.

4.4. On the combination of probabilistic and fuzzy approaches

Putting into perspective the issues raised in Section 4.3, within the ambit of combined probabilistic-fuzzy approaches, randomness and uncertainty should have their useful properties jointly considered whenever possible (Arunaj et al., 2013). A growing number of studies in the literature employ variants of combined probabilistic-fuzzy approaches in several aspects of decision-making. More specifically, 2-Dimentional Fuzzy Monte Carlo Analysis (2D FMCA) uses a combination of probability and possibility theory to include probabilistic and imprecise information in the same analytical model. Arunaj et al. (2013) presented a comprehensive literature review on these issues and some of their key aspects are addressed next.

Guyonnet et al. (2003), for example, combined MCA with fuzzy calculations to assess uncertainty in risk management. In turn, Kentel and Aral (2004) used a similar approach in order to generate the resulting combinations between probability density functions of random variables and membership functions of fuzzy variables. These resulting combinations were used for determining the fuzzy uncertainty estimates at certain percentiles of risk for given individuals or groups. Early works on 2D FMCA can be also traced back to Zonouz and Miremadi (2006), who developed a fuzzy-MCA for fault tree analysis. In their approach, the variability in the random variables of the risk is treated using probability density functions, and the uncertainty associated with them is treated by using fuzzy numbers for the parameters of these random variables.

In this research, a specific application of the 2D FMCA approach is developed to assess the efficiency levels and their determinants in the Mozambican banking industry. More precisely, the approach used here starts off from the α-level FDEA based models – where production inputs and outputs are treated as triangular fuzzy numbers (as in Puri & Yadav, 2013) with a 20% offset from their central values – and culminates with the proposed conditional bootstrapped truncated regression, thereby allowing the treatment of these FDEA models and their respective bounds as fixed factors.

More precisely, the conditional bootstrapped truncated regressions – discussed in Section 4.3.3 – are performed each time for a given α-level (say 0; 0.1; 0.2;...; 1, as in Hsiao et al., 2011) including the respective crisp values for the lower and upper bounds as the efficiency measures obtained within each treatment. Readers should be aware, however, that the α-level values within this set are primarily used in the three major FDEA models – presented in Section 4.2 – so as to determine crisp values for the input and the output bounds, thus allowing the computation of the respective efficiency levels in Kao and Liu (2000a), Saati et al. (2002).

4.5. On global separability and the adequacy of the two-stage approach

Simar and Wilson (2011) examined the wide-spread practice where efficiency estimates are regressed on some environmental variables in what is commonly known as a two-stage analysis. In a broader sense, the authors argue that this is done without specifying a statistical model in which such structures would follow from the first stage where the initial DEA estimates are obtained. As such, these two-stage approaches are not structural, but rather ad hoc. The most important underlying assumption regarding two-stage analysis concerns global separability. The next paragraphs detail this assumption.

In general lines, the vector of environmental factors or contextual variables, Z, may either affect the range of attainable values of the inputs and the outputs (X,Y), including the shape of the production set, or it may only affect the distribution of inefficiencies inside a set with boundaries not depending on Z (meaning that only the probability of being less or more far from the efficient frontier may depend on Z) or both. Under separability, the environmental factors have no influence whatsoever on the support of XY, and the only potential remaining impact of the environmental factors on the production process may be on the distribution of the efficiencies.

To understand the importance of the "separability" condition, let $X \in R_p^+$ denote a vector of p input quantities, and let $Y \in R_q^+$ denote a vector of q output quantities. In addition, let $Z \in Z \subseteq R_r$ denote a vector of r environmental variables with domain Z. Let $S_n = \{(X_i, Y_i, Z_i)\}n_i = 1$ denote a set of observations. The separability assumption in Simar and Wilson (2011) implies that the sample observations (X_i, Y_i, Z_i) in S_n are realizations of identically, independently distributed random variables (X, Y, Z) with probability density function $f(x, y, z)$ which has support over a compact set $P \subset R_{p+q}^+ \times R_r$ with level sets $P(z)$ defined by $P(z) = \{(X, Y) \mid Z = z, X \text{ can produce } Y\}$. Now let $F = UP(z) \subset R_{p+q}$, $z \in Z$. Under the "separability" condition, $P(z) = F \ \forall \ z \in Z$ and hence $P = F \times Z$. If this condition is violated, then $P(z)$ is different than F for some $z \in Z$. Whether this is the case or not is ultimately an empirical question to be assessed within the ambit of each study.

Daraio et al. (2010) provided a method for testing H0: $P(z) = F \ \forall \ z \in Z$ versus H1: $P(z)$ is different than F for some $z \in Z$. In order to test these null hypothesis consider the test statistics $\tau_{Frontier,n}(S_n) = n^{-1} \sum_{i=1}^n \hat{D}_{Frontier,i} \hat{D}'_{Frontier,i} \geq 0$ where $\hat{D}_{Frontier,i} = (Y_i \hat{\lambda}_{Frontier}(X_i, Y_i/S_n) - Y_i \hat{\lambda}_{Frontier}(X_i, Y_i/Z_i, S_n))$ and its complementary $\hat{D}'_{Frontier,i}$ are $(q \times 1)$ vectors. This statistics give estimates of the mean integrated square difference between P and $F \times Z$. If separability assumption holds, we should expect these statistics to be "close" to zero; otherwise, we should expect them to lie "far" from zero. The authors indicate how to conduct a series of Monte Carlo experiments

for various sample sizes m in order to choose the value of m that minimizes the volatility of the object of interest and, therefore, obtain critical values for the rejection rates for the separability test. Readers should refer to their work for further details.

In this research, an R code was structured upon the packages np (Hayfield & Racine, 2008) and FNN (Beygelzimer, Kakadet, & Langford, 2015) to compute the statistics of this test. In situations where the "separability" condition is satisfied, it would be straightforward to perform the second stage analysis. For instance, one might estimate the model regression model by the maximum-likelihood method using standard software (Simar & Wilson, 2011). Besides, readers should pay attention to the fact that, under standard assumptions, where properties of traditional DEA estimators have been derived, the mass of estimates equal to one may negatively affect this test statistic, leading to values far from zero.

5. Results and discussion

The distributions of the fuzzy estimates calculated for 13 selected Mozambican banks from 2003 to 2011, using a meta-frontier (O'Donnell, Rao, & Battese, 2007) and the previously discussed FDEA models based on the α-level approach, are given in Fig. 1. As regards the contextual variables and test of global separability (Daraio et al., 2010), the empirical values of the test statistic for the FDEA scores was found to be close to zero for each one of the alpha cuts and to be statistically significant at 0.05, except for the alpha-cuts of 0.3 and 1.0 – this last one for a sample fraction of 50% –, as indicated in the Appendix for the simulation results obtained for the critical values. Global separability, therefore, appears to be consistent with the use of FDEA on the sample data to the detriment of DEA models. This suggests that the contextual and control variables considered here affect only the distribution of efficiencies and not the attainable input/output combinations (or the shape of the underlying production set).

In general terms, the fuzzy estimates mostly ranged from 0.05 to 0.50. As expected, the lower and upper fuzzy efficiency estimates derived from the models of Guo and Tanaka (2008) and Kao and Liu (2000a) present higher uncertainty for lower values of α-level. This uncertainty is reflected in the differences between the median values of their respective distributions with each model. As long as α-levels increase towards one, the median values for the fuzzy estimates obtained within these two models converge to the same value. Additionally, a quick visual inspection of the outliers and interquartile range of these distributions reveals that randomness also decreases with higher values of α-levels.

However, as regards the model of Saati et al. (2002), it is interesting to note an upward movement in its central tendency towards higher values of α-levels, and an increase in randomness, as evidenced by outliers and interquartile range dispersion. In general terms, while in the models of Guo and Tanaka (2008) and Kao and Liu (2000a) there is a trade-off between uncertainty and randomness – that is, both models behave similarly in terms of uncertainty and randomness towards higher values of α – in the model of Saati et al. (2002) there is a trade-off between uncertainty and randomness: a higher value of α lowers uncertainty to the detriment of randomness.

These different features observed in the simultaneous comparison of major FDEA models based on the α-level approach suggest (i) that further research is necessary to investigate their behavior in other datasets and (ii) corroborate the need to treat these models and the estimates obtained within them as fixed factors that should be controlled for when assessing the impact of contextual variables in Mozambican banking efficiency.

Results for the conditional bootstrapped truncated regression performed on different α-levels reflect the impact of different types of estimates (lower, mean, and upper) and FDEA models. Fig. 2 presents their mean fixed impacts measured in terms of the
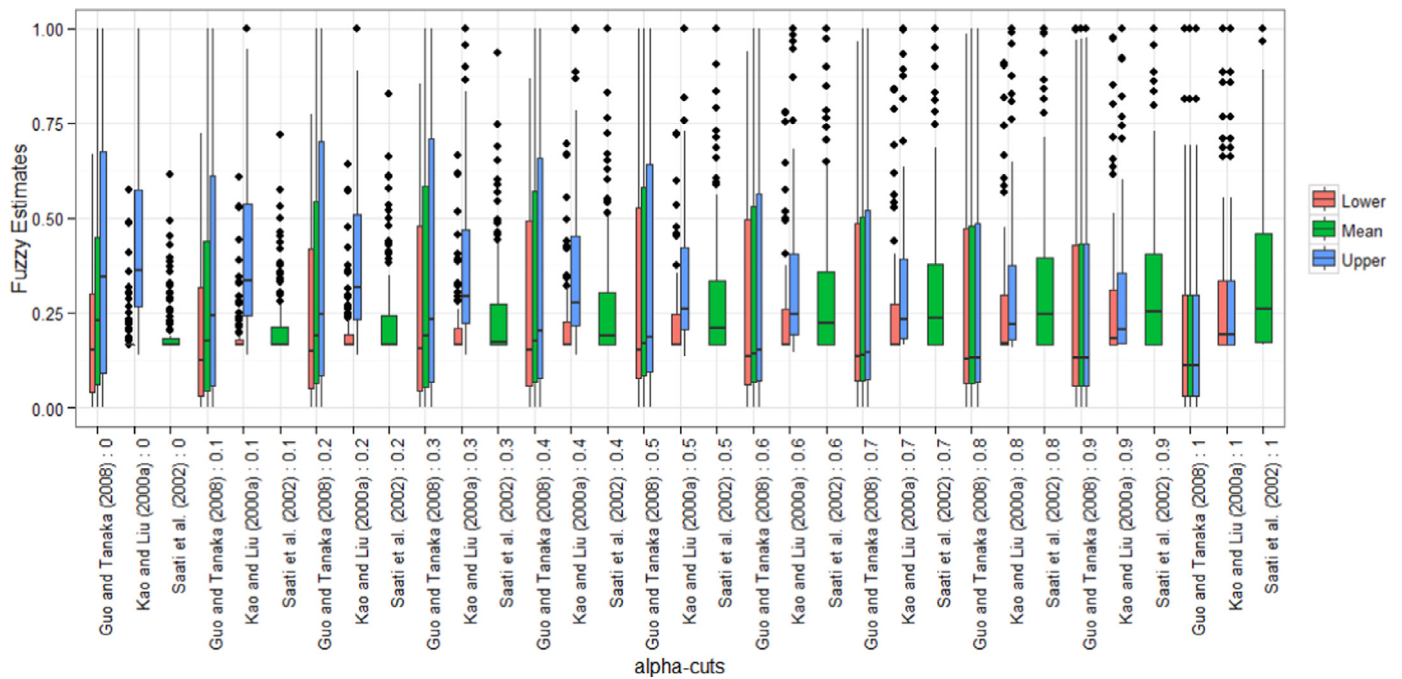
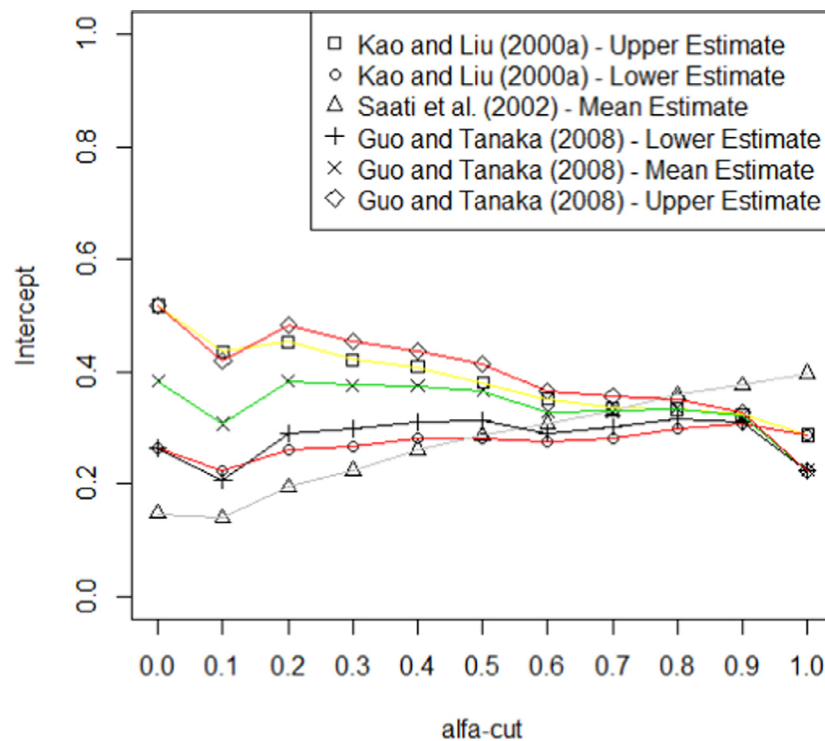Fig. 1. Fuzzy estimates of Mozambican banking efficiency levels.

Fig. 2. Conditional bootstrapped truncated regression results for the intercept.

intercept. One thousand bootstrap replications were computed for each one of the α-levels; confidence intervals were omitted for the sake of readability.

From a quick inspection of Fig. 2, several conclusions can be drawn. As expected, the fixed effects on the intercept converge when $\alpha = 1$, particularly within the models of Guo and Tanaka (2008) and Kao and Liu (2000a). Furthermore, the upper estimates for these models are fairly close within $0 < \alpha < 1$. They are equal when $\alpha = 0$

but present maximal difference when $\alpha = 1$. The same happens with their lower estimates. In contrast, the fixed effect of the model of Saati et al. (2002) systematically increases with higher values of α and crosses the mean estimate derived from Guo and Tanaka (2008) when $\alpha = 0.7$.

As regards the contextual variables presented in Table 1, Fig. 3 presents the results for their coefficient estimates with the respective 95% confidence intervals, considering different values of α. A
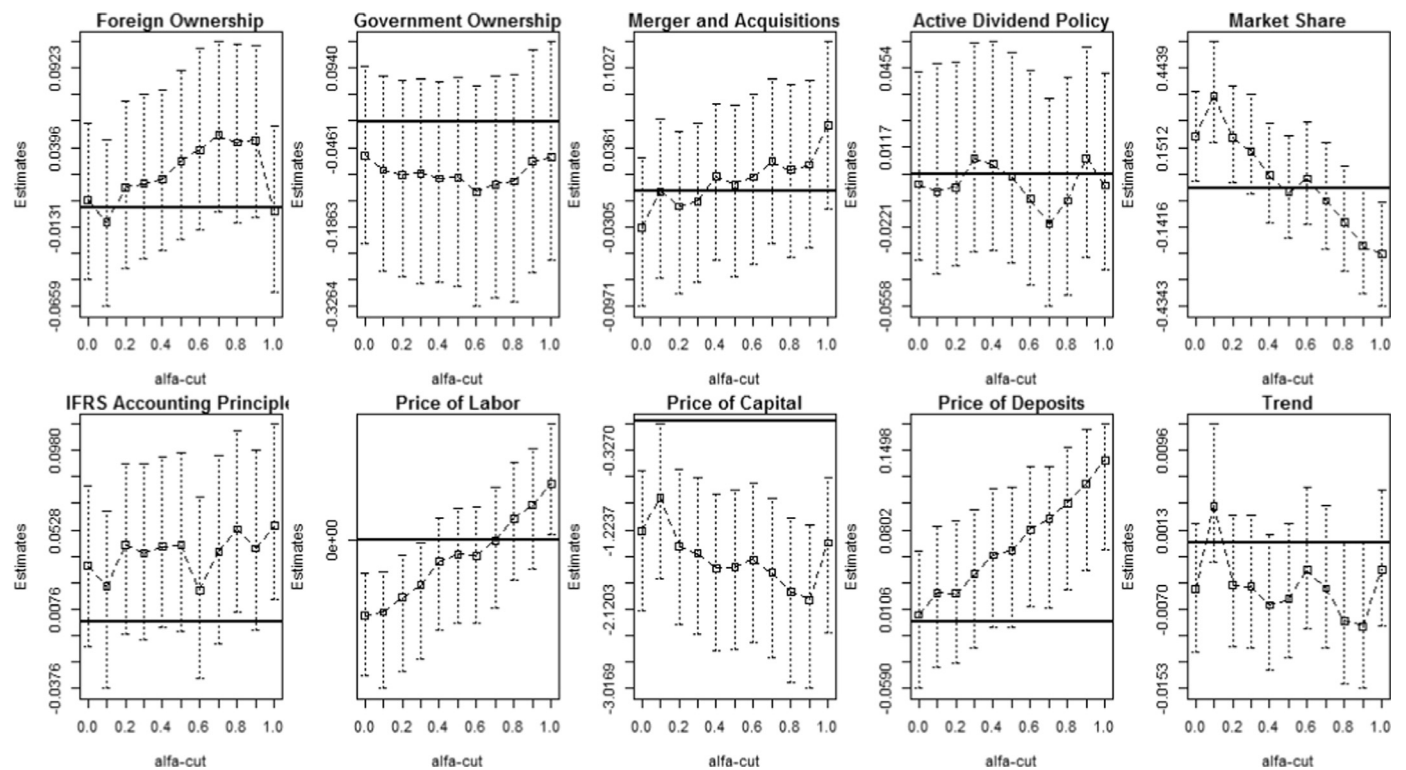
Fig. 3. Coefficient estimates for the conditional bootstrapped truncated regression.

solid line marks the zero in each graph, thus indicating whether a contextual variable is significant or not for a given value of α. Several contextual variables are not statistically significant, regardless of the uncertainty level in inputs and outputs: foreign ownership, government ownership, merger and acquisitions, active dividend policy, and trend.

However, several contextual variables are significant to some α-values: price of deposits, price of labor, IFRS accounting principles,
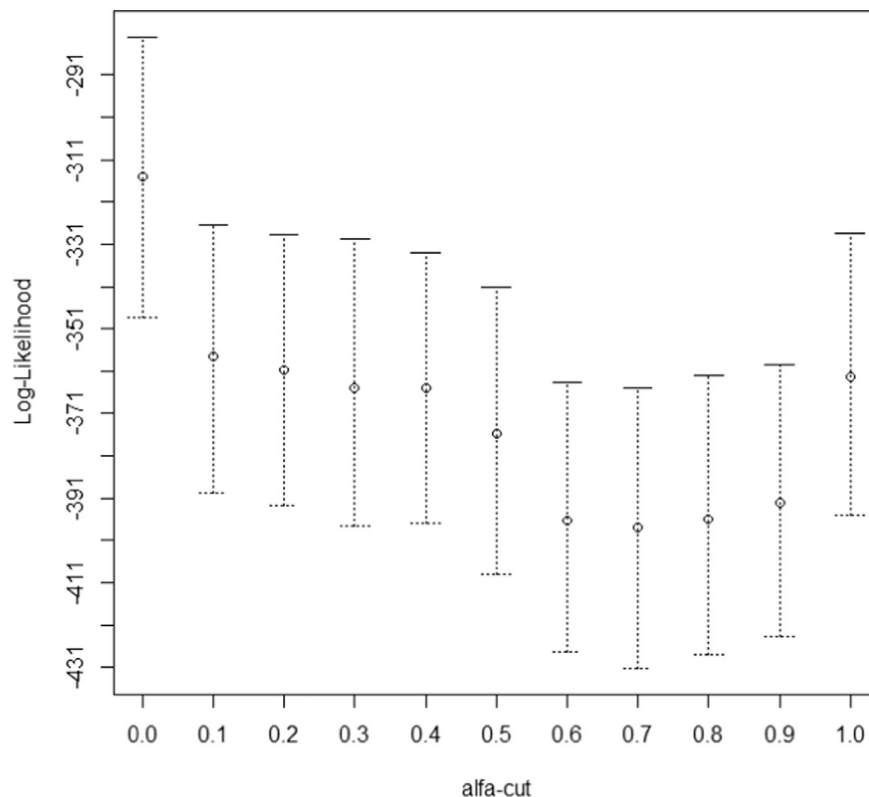

Fig. 4. Log-likelihood confidence intervals for the conditional bootstrap truncated regressions.

and market-share. It is interesting to note that, in such cases, the sign of the significant impact may depend upon the uncertainty level, or the degree of fuzziness, that the input and output data are subjected to. This suggests that (i) uncertainty and randomness interact in the input/output level and (ii) uncertainty or fuzziness is the predominant effect for interpreting the results, provided it is capable of changing the sign of the relationship between the contextual variable and the efficiency scores (controlling, however, for the type of model and type of estimate). This ambiguity, which is intrinsic to fuzzy systems, opens the room for researchers and practitioners to investigate further the actual sources of efficiency and possibly shed light on other obscure contextual variables that may constitute valuable sources for interpreting the results.

The price of deposits presents a significant positive effect on efficiency levels for α-values higher than 0.6, indicating that – under low uncertainty – the higher the prices, the larger the inflow of funds to the Mozambican bank, thus contributing to the formation of assets and net profits. The impact of the price of deposits is not significant for higher levels of uncertainty in outputs and inputs; additionally, compliance with IFRS accounting principles have a positive impact on efficiency levels, given uncertainty is low, when α is equal to 0.8 and 1. This result indicates that when measurement uncertainty is high the statistical method adopted here is not capable of capturing its beneficial impacts on efficiency in Mozambican banks.

The price of labor, on the other hand, shows a twofold significant impact on efficiency levels, depending on the level of uncertainty in measuring inputs and outputs. When fuzziness or uncertainty in inputs and outputs is high, the price of labor is negatively related to efficiency, thus indicating that a higher salary mass may contribute to diminishing net profits among other outputs. However, when fuzziness is low, the price of labor may contribute positively to efficiency levels provided the mass of salaries serves as a driver for continuous operational growth. A similar interpretation occurs with the market-share. Its positive (negative) contribution to efficiency levels is captured when measurement fuzziness is high (low). Market-share positive (negative) impacts on efficiency may be related to increasing (diminishing) returns to scale and uncertainty in measuring inputs and outputs, which may hinder the assessment of the most productive scale size.

The last two paragraphs indicate that fuzziness in measuring inputs and outputs sheds some light in other contextual variables that may help in interpreting the issue of banking efficiency in Mozambique with respect to assessment of capital inflow/outflow (price of deposits), average annual growth of outputs (price of labor), and most productive scale size (market-share). There is, however, one variable with an unambiguous interpretation. The price of capital is the only contextual variable that showed sufficient robustness in terms of statistical significance and to uncertainty in input/output measurement. As expected, the higher the price of capital, the lower the efficiency levels in the Mozambican banking industry.

Bootstrapping was also used to build confidence intervals for the log-likelihood measurements for the conditional regression models considering different α-values. These results are presented in Fig. 4. Although one cannot claim that these log-likelihoods are significantly different, since their error bars overlap, it is interesting to note how the regression's performance is affected by the extreme values of uncertainty in the measurement of inputs and outputs. The best fit to the data was verified under $\alpha = 0$, that is, when fuzziness was maximal. Using the best log-likelihood as a criterion to pick up a model, then the contextual variables should be interpreted accordingly. This being the case, the price of deposits and IFRS should not be considered as a significant variable, the price of labor should be analyzed considering its negative impact on efficiency levels, and market-share should be considered as having a positive impact on efficiency levels.

6. Conclusion

This paper presents an analysis of the efficiency of Mozambican banks using major FDEA models based on the α-level approach. FDEA enables the treatment of uncertainties involved in the process of measuring or collecting data regarding inputs and outputs. Here, these variables were modeled as triangular fuzzy numbers with maximal and minimal values determined by an offset of 20% from their mean values. In a second stage, the fuzzy estimates for the efficiency scores for each bank were considered as the dependent variables in conditional bootstrapped truncated regressions, where the random covariates consisted of the contextual variables and the fixed factors were the FDEA models and the type of their scores (whether lower, upper, or middle values). Additional testing on global separability was also conducted to assess the adequacy of this two-stage approach.

Results suggest that the efficiency of the Mozambican banking system can be globally separated from the contextual variables. Based on the conditional bootstrapped truncated regression results, it is possible to explain the efficiency drivers in Mozambican banks. The significant contextual variables are related to the cost structure (price of labor and price of capital) and to the market share of the bank. Therefore, high costs explain the low efficiency of Mozambican banks, although the positive impact of market share on efficiency, when considered in an isolated fashion, suggests increasing returns to scale as efficiency increases towards a higher output level. The ruling economic implication of these findings, in light of the production approach adopted here, is the following. As long as a higher market-share appears to be the only significant contextual driver for higher banking outputs (total deposits, income before tax, and total credit), the price of labor and the price of capital appear to be two significant contextual drivers related to lower banking inputs (total costs and employee costs). Mozambican banks should, therefore, adopt employee downsizing and capital leveraging initiatives, while, expanding their operations in order to move towards higher efficiency standards.

Besides the practical aspect offered to decision-makers, the contributions of this research to the current body of knowledge in the FDEA-banking efficiency literature are fourfold: First, this research addresses a gap in the FDEA literature by proposing a way for handling simultaneously several models based on the α-level approach by incorporating them into the second-stage of the conditional bootstrapped truncated regression approach as fixed effects; secondly, a real case is investigated in which data collection in terms of inputs and outputs was subject to uncertainty or fuzziness because reliable sources of information on the banking industry in Mozambique are scarce; thirdly, the framing of the two-stage FDEA-bootstrapped truncated regression adopted here in terms of the Fuzzy Monte Carlo Analysis, where uncertainty and randomness are supplementary parts of the analytical process, is innovative; and fourthly, a contribution to the nascent literature on the applications of FDEA and banking efficiency has been made.

Limitations of this research are related to the type of FDEA approach chosen (α-level) and to the dataset used. Further studies should be conducted in banking, both incorporating additional FDEA approaches and replicating the two-stage analytics here developed in other datasets, with the goal of corroborating the external validity and robustness of guidelines when analyzing and interpreting the results in light of uncertainty and randomness.

Appendix. Critical values for the separability test.

Alpha-cut	Critical-value	Sample fraction	50 Bootstrap replications			100 Bootstrap replications		
			Sig. = 0.01	Sig. = 0.05	Sig. = 0.10	Sig. = 0.01	Sig. = 0.05	Sig. = 0.10
0	0.030	10%	0.076	0.095	0.098	0.064	0.085	0.098
		50%	0.019	0.026	0.034	0.041	0.054	0.073
0.1	0.022	10%	0.052	0.064	0.074	0.062	0.071	0.096
		50%	0.041	0.055	0.068	0.045	0.058	0.075
0.2	0.017	10%	0.083	0.110	0.126	0.072	0.091	0.123
		50%	0.020	0.034	0.054	0.019	0.030	0.044
0.3	0.171	10%	0.075	0.080	0.104	0.065	0.078	0.095
		50%	0.026	0.032	0.041	0.029	0.035	0.045
0.4	0.022	10%	0.079	0.111	0.123	0.072	0.090	0.123
		50%	0.041	0.055	0.058	0.031	0.041	0.057
0.5	0.015	10%	0.100	0.126	0.138	0.075	0.107	0.140
		50%	0.057	0.064	0.071	0.059	0.062	0.105
0.6	0.015	10%	0.081	0.100	0.138	0.090	0.108	0.134
		50%	0.065	0.076	0.098	0.065	0.087	0.102
0.7	0.077	10%	0.099	0.120	0.130	0.096	0.122	0.160
		50%	0.095	0.107	0.132	0.090	0.104	0.137
0.8	0.078	10%	0.102	0.109	0.135	0.101	0.112	0.152
		50%	0.089	0.092	0.122	0.090	0.094	0.149
0.9	0.034	10%	0.102	0.114	0.136	0.104	0.140	0.151
		50%	0.071	0.077	0.109	0.079	0.100	0.114
1	0.097	10%	0.130	0.160	0.189	0.109	0.131	0.169
		50%	0.035	0.070	0.078	0.053	0.067	0.079

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