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Structural mechanics of knitted fabrics for apparel and composite materials

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Abstract *The importance of fabric biaxial extension, in-plane compression, shear and bending properties, have been widely recognised by textile scientists and engineers for the evaluation of the three-dimensional formability and drape of textile materials in apparel products and three-dimensional preforms. In contrast to woven fabrics where bending and shear properties determine the fabric formability, knitted fabrics have very high formability as a direct result of their easy biaxial extension properties. This ability to form three-dimensional shapes using the biaxial extensibility of knitted structures enables these knitted textile materials to be utilised for a wide variety of close fitting apparel garments and shaped composite preforms. Some representative biaxial extension curves for the plain knitted structure are described in this paper. These curves illustrate an unusual shape for the load-extension curve of a textile material arising from the pre-tension or pre-stress. The pre-stress yields an initial high tensile modulus for the structure in contrast to the very low initial modulus characteristic of apparel textiles. Accordingly, for knitted textile materials, it is shown how biaxial extension of the fabric introduces a fabric pre-stress to maximise the three-dimensional fabric formability especially when subjected to transverse compression by the resin or matrix in a composite material. Typical uniaxial and biaxial tensile stress–strain curves for knitted fabrics are compared.*

1. Introduction

Concentrating on the plain weft-knitted structure, we investigate uniaxial and biaxial extension in this section and transverse compression in a subsequent section. The mechanisms of deformation are studied for each of these mechanical properties.

The load-extension properties of knitted fabrics have been investigated theoretically by several workers (Shanahan and Postle, 1974a; Popper, 1966; Whitney and Epting, 1966; Kawabata *et al.*, 1973; MacRory *et al.*, 1975; Shanahan and Postle, 1974b), acknowledging the importance of this subject as far as the end use of knitted fabrics is concerned. Popper (1966) and Whitney and Epting (1966) analysed the biaxial deformation of plain-knitted fabrics after a sufficient load had already been applied to straighten the threads in the fabrics. Kawabata *et al.* (1973) used as their model a fabric built up from several bent and loaded yarn segments and assumed that a yarn segment extends so that the slope of the yarn remains constant at the extremity of the segment. Further geometrical assumptions were made in applying this technique to an

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actual fabric. MacRory *et al.* (1975) used a similar technique but, instead of the two-dimensional model of Kawabata *et al.* (1973), they used a three-dimensional model and assumed that the shape of the yarn in the contacting region of the plain-knitted fabric could be described by a helix. This technique ignores the deformation of certain segments within the structure.

Theoretical load-extension curves for plain-knitted fabric have been derived by three force methods (MacRory *et al.*, 1975; Shanahan and Postle, 1974b; Hepworth, 1978, 1980). Except for the analysis of walewise extension by Shanahan and Postle (1974a, b), all the predicted tensile moduli were higher than the experimentally observed values.

Only two methods of analysis have been capable of including both length and width jamming in the determination of the tensile properties of plain-knitted fabric. Using the Hepworth and Leaf force model, Hepworth (1980) predicted the load-extension behaviour of plain-knitted fabrics under uniaxial and biaxial tension. de Jong and Postle (1977) determined the effects of very small uniaxial loads ($PL^2/B = 2$) [1] on the plain-knitted loop, using the energy model. The energy analysis includes yarn compression, which is omitted from the Hepworth and Leaf force model. The energy analysis is used here to predict fabric tensile behaviour at uniaxial and biaxial (dimensionless) loads up to $PL^2/B = 4.0$. A wide range of biaxial load ratios is used.

2. Uniaxial extension

Uniaxial load-extension curves

Load-extension curves calculated from the energy considerations for plain-knitted fabrics are shown in Figure 1 for uniaxial walewise loading and in Figure 2 for uniaxial coursewise loading. The figures also provide a comparison with the results of the force analysis of Hepworth (1980). The sharp changes in the modulus, characteristic of the Hepworth and Leaf force model, are absent from the results of the energy equations because the onset and release of jamming are gradual processes when yarn compressibility is included. Jamming forces cause a pre-stress when they are parallel to the direction of extension (Hepworth, 1980) and a restriction on fabric extensibility when they are normal to the direction of fabric extension.

Although the fabric as a whole is not loaded at zero tension, there is a pressure exerted between yarns where jamming occurs in the relaxed structure. This pressure is counterbalanced by a yarn tension existing elsewhere in the loop. This pre-stressed jammed structure was quoted by Hepworth (1980) as the reason for the discontinuity in the uniaxial tensile load-extension curves predicted from the model incorporating incompressible yarns.

Typical computed shapes of the plain-knitted fabric in its relaxed, walewise extended state and coursewise extended state are shown in Figure 3.

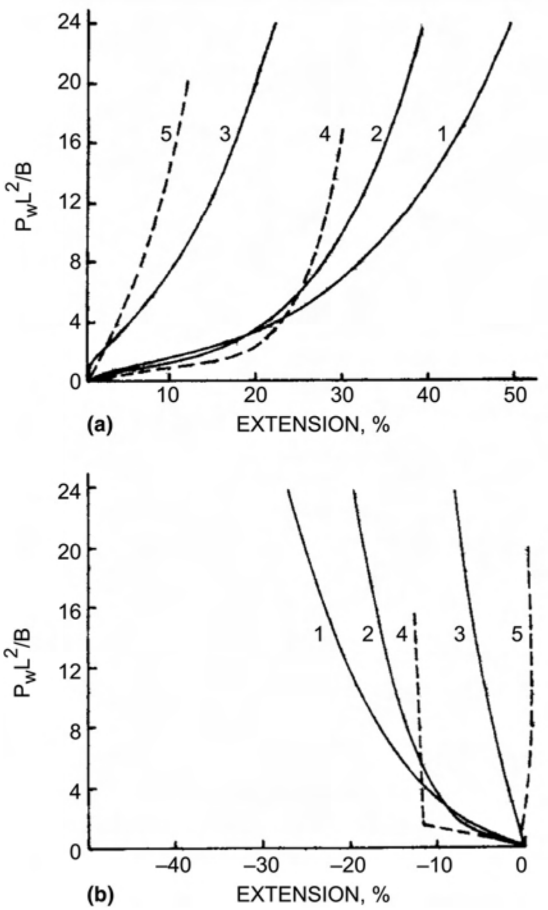


Figure 1. Uniaxial load-extension curves for the plain-knitted structure in walewise loading, showing the effects of yarn compression index a and fabric tightness d_{min}/L, for (a) walewise extension and (b) coursewise extension (in walewise loading): curve 1, $a = 10$, $d_{min}/L = 0.22$; curve 2, $a = 20$, $d_{min}/L = 0.22$; curve 3, $a = 20$, $d_{min}/L = 0.26$; curve 4, results of Hepworth, $d/L = 0.22$; curve 5, results of Hepworth, $d/L = 0.26$.

Frictional resistance to slippage

The significance of the inter-yarn forces calculated by the energy method in relation to frictional slippage in the fabric is of interest. Yarn tension in the interlocking regions is approximately equal to the fabric tension per loop and the direction of the inter-yarn forces at the point of closest contact is almost normal to the yarn axis. Therefore, slippage of the yarns past each other occurs in the contact region when

$$\frac{\text{fabric tension per loop}}{\text{total inter-yarn force}} > \mu \qquad (1)$$

where μ is the coefficient of friction between the yarns.

Thus, once μ is known, equation (1) can be used to determine whether inter-yarn slippage occurs. The range of μ for yarn-to-yarn friction is approximately

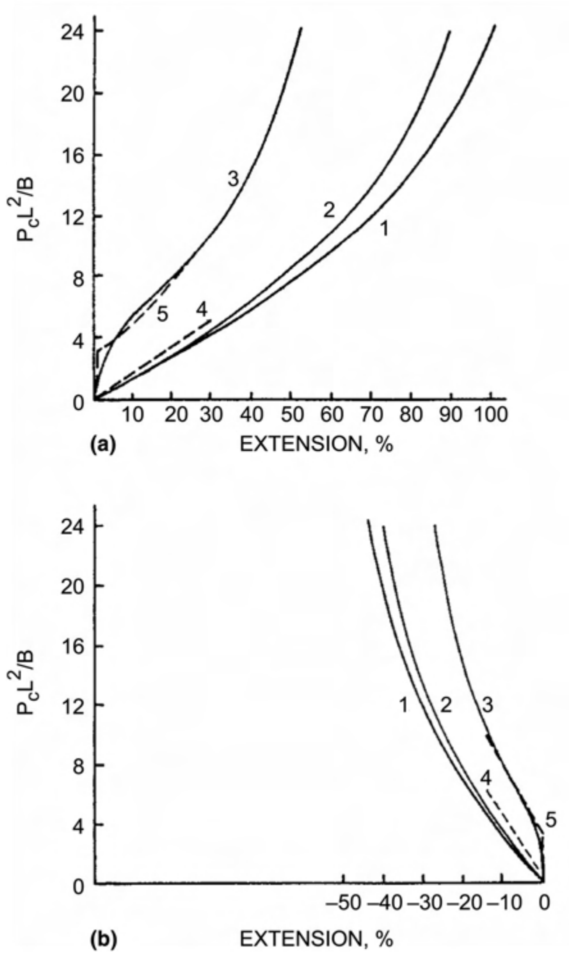


Figure 2.
As Figure 1, but coursewise loading: (a) coursewise extension and (b) walewise extension (in coursewise loading)

$0.1 < \mu < 0.4$ (Yanagawa *et al.*, 1970). If the ratio on the left-hand side of equation (1) is high relative to μ, the interlacing yarns are likely to slip during fabric extension. In this way, it has been calculated (Hart, 1981) that the interlacing yarns are more likely to slip when the plain-knitted fabric is extended in the wale direction. Setting increases the ratio on the left-hand side, thereby facilitating yarn slippage during fabric extension.

The jamming forces

As the fabric is extended uniaxially, the jamming forces between neighbouring loops increase in the direction normal to extension. In walewise extension the width jamming yarns come closer together (see Figure 3(b)), thereby increasing the force between the interlacing yarns (Hart, 1981). This would reduce the

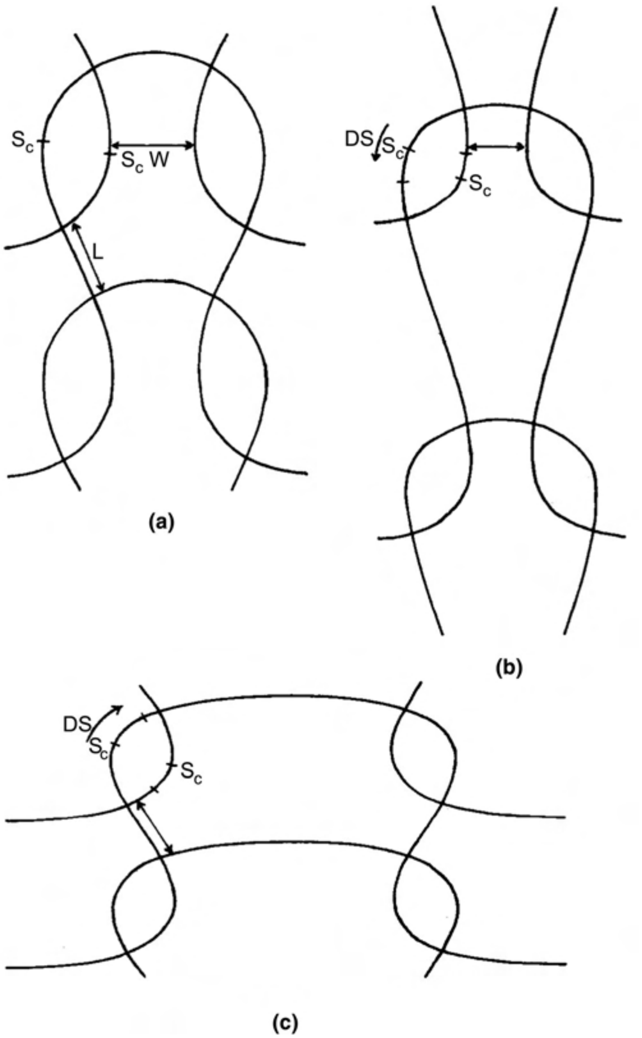


Figure 3.
The effect of yarn slippage on jamming for the plain-knitted structure: (a) untensioned fabric (W and L indicate where width and length jamming will occur); (b) walewise tension (the direction of slippage, labelled DS, puts more yarn in the stem of the loop, bringing the width-jamming yarns closer together); (c) coursewise tension (the direction of slippage puts more yarn in the arc, bringing the length-jamming yarns closer together)

tendency for the interlacing yarns to slip. Similarly, in coursewise extension, Figure 3(c) shows that the length-jamming yarns come closer together again, increasing the force between interlacing yarns. This would also reduce the tendency for further yarn slippage to occur.

In contrast, jamming in the direction of extension decreases very rapidly with increasing applied tensile load, and the magnitude of the jamming force is comparable with the difference observed between the initial modulus and the modulus after the jamming has diminished. Thus the jamming pre-stress effect explains the initially high tensile modulus of the theoretical curves.

Yarn compression

The principal effect of yarn compression is to increase fabric extensibility. The area of contact is larger between interlocking yarns for compressible yarns than for relatively incompressible yarns, resulting in decreasing freedom of movement in the fabric. For compressible yarns, these effects give the high pre-stresses at very low loads and the high fabric rigidity up to loads of $PL^2/B = 4$, which are shown most clearly in Figure 1(a).

As the tensile load increases further, fabric extension by the mechanism of yarn compression becomes significant, not only because of compression in the interlocking region, but also because of compression between jamming yarns at right-angles. In walewise extension, for example, yarn compression between width-jamming yarns allows more reduction in the length of the arc of the loop by yarn slippage, as shown in Figure 3(b).

3. Biaxial extension

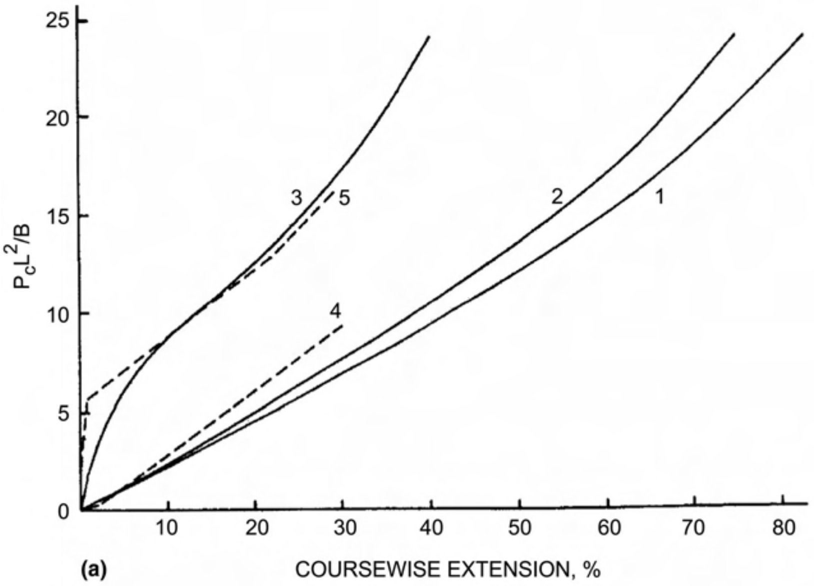
The tensile properties of plain-knitted fabrics have been calculated (Hart, 1981) from the energy equations under many combinations of walewise and coursewise biaxial loads. The biaxial condition, $P_cL^2/B = 2P_wL^2/B$, was used for the biaxial load-extension curves shown in Figure 4, where P_c and P_w are the loads applied in the course and wale directions, respectively.

In Figure 5 a family of biaxial load-extension curves has been developed using a fixed walewise load P_wL^2/B and varying the coursewise load P_cL^2/B. The stress values were based on relaxed fabric dimensions. Computed shapes are shown in Figure 6 for the plain-knitted structure in the relaxed state and the biaxially extended state for equal biaxial stress, $PL^2/B = 20$.

At high fabric tensions and when the coursewise load is greater or equal to the walewise load ($P_cL^2/B \geq P_wL^2/B$), the fabric is usually in compression in the walewise direction (negative walewise extension in Figures 4 and 5) and the length-jamming forces are large, as in uniaxial coursewise extension.

Comparison with the analysis of Hepworth (1980) in Figure 4 shows good agreement, particularly at high loads. The main difference is that, in Hepworth's results, fabric extensibility varies less with fabric tightness. This effect is related to yarn compressibility. For compressible yarns used in the energy model, the area of contact increases with increasing fabric tightness; this causes a reduction in the freedom of movement in the fabric. Even without yarn compression, freedom of movement is reduced by increasing tightness but yarn compressibility magnifies the effect.

The jamming forces fall very rapidly in biaxial extension. If it were not for the increase in yarn compression during biaxial extension, the initial reduction in bending energy for slack fabrics would cause a negative modulus in part of the load-extension curve, as illustrated in the results of Hepworth in Figure 4. In contrast, the compression energy decreases with increasing load initially for tight fabrics, while the bending energy increases. The jamming forces are so



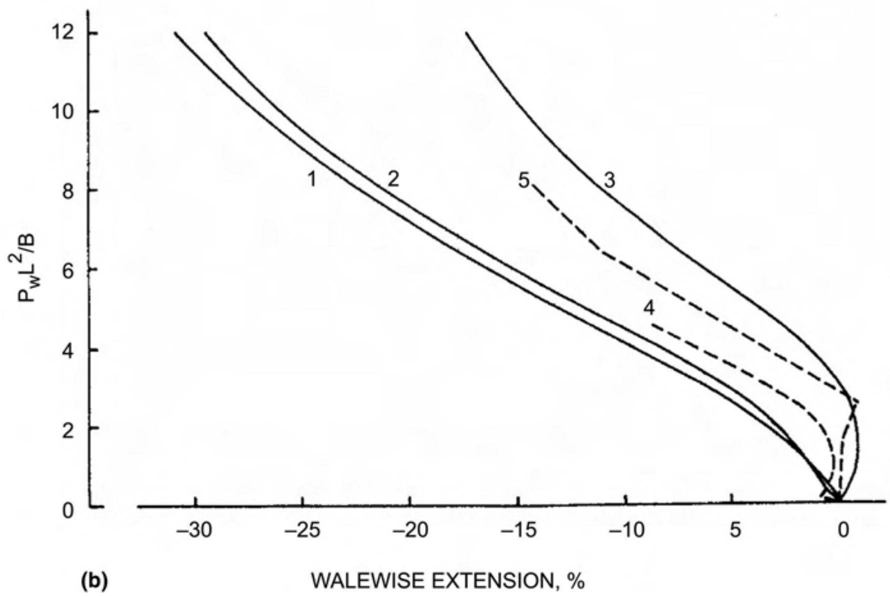


Figure 4.
Biaxial tensile load-extension curves for the plain-knitted structure, where $P_cL^2/B = 2P_wL^2/B$, showing the effect of fabric tightness d_{min}/L, for (a) the coursewise direction and (b) the walewise direction: curves as in Figure 1

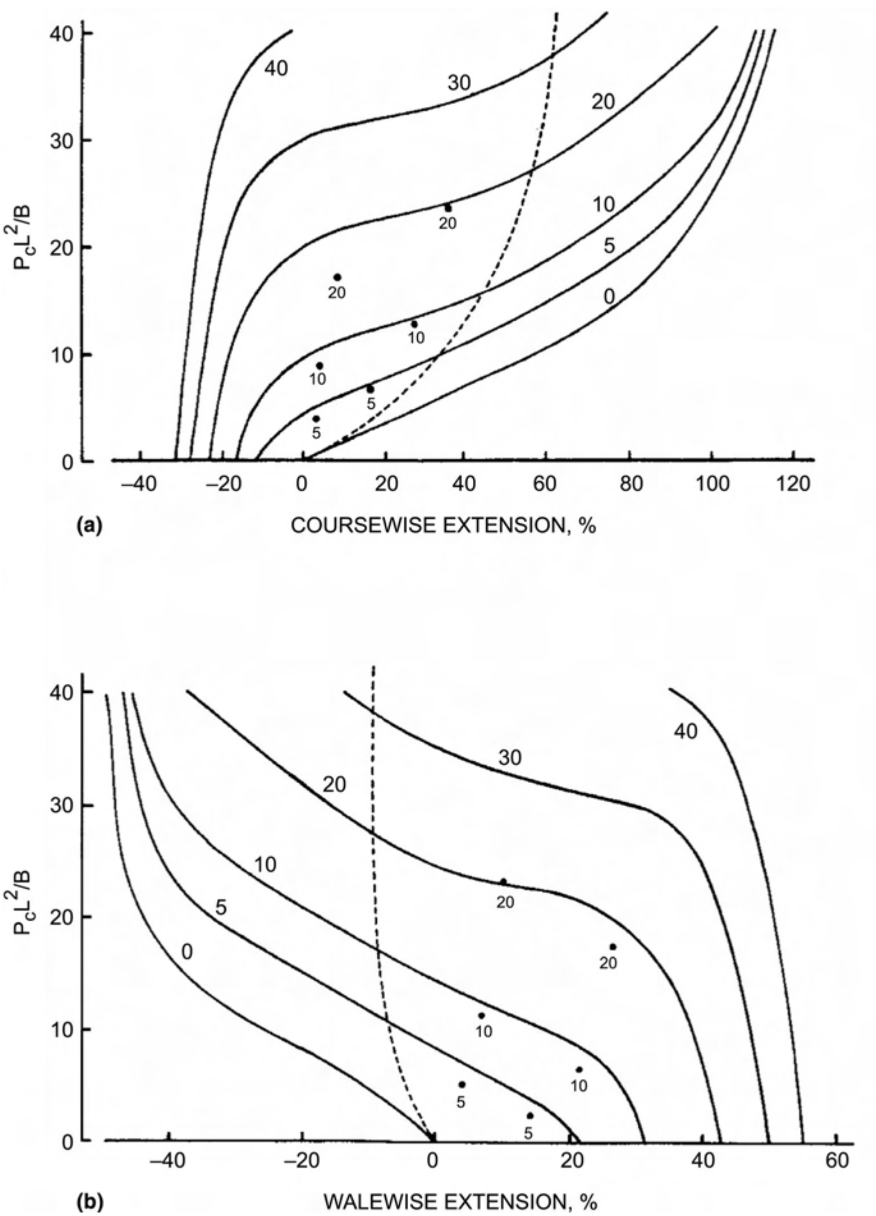


Figure 5.
Biaxial tensile load-extension curves for fixed walewise loads P_wL^2/B (values indicated on the curves) and varying coursewise loads P_cL^2/B for (a) coursewise extension and (b) walewise extension (fabric compression index $a = 10$; fabric tightness $d_{min}/L = 0.22$; highly set fabrics $\phi = 0.99$); •, experimental results of MacRory *et al.*; - - - - -, load combinations that cause no inter-yarn slippage

large that their reduction is responsible for most of the changes in compression energy.

A large variation in the ratio of the walewise load to the coursewise load is shown in Figure 5. Coursewise extensibility is generally greater than walewise extensibility because of the ease of yarn straightening in the arc of the loop. Walewise contraction is larger than coursewise contraction, but it only occurs when the coursewise load per yarn is greater than the walewise load per yarn. Slippage is the main cause of contraction but Figure 5 shows that a small walewise contraction still occurs for the particular load ratio where there is no slippage.

Figure 5 is most useful for experimental comparison because experimental testers cannot easily apply a constant load ratio. The results of MacRory *et al.* (1977) agree very closely with the predicted deformations in both fabric directions. The $d_{\min}$ used in Figure 5 is less than the yarn diameter measured by MacRory *et al.*. Experimental methods of determining yarn diameter appear to overestimate $d_{\min}$ because of yarn flattening in the contacting region. It is this effect far more that the yarn compression during fabric extension that explains the large discrepancy between the model of MacRory *et al.* and their experimental results. Results computed from the energy equations show that an extreme range of compression indices does not cause more than 30 percent variation in fabric extensibility (Figure 6).

4. Transverse compression of the plain-knitted structure

The transverse compression force was applied at the end point of the loop (Hart, 1981). An alternative method would be to simulate compression between parallel plates. This latter method would allow more accurate specification of fabric thickness, but would differ only slightly from the present analysis at the low pressures used. At high pressures such as those which exist in a composite material, the usefulness of theoretical results is limited by the accuracy of the compression energy function.

A maximum compressive load of $PL^2/B = 5$ was used to derive, from energy considerations, the series of transverse load-compression curves shown

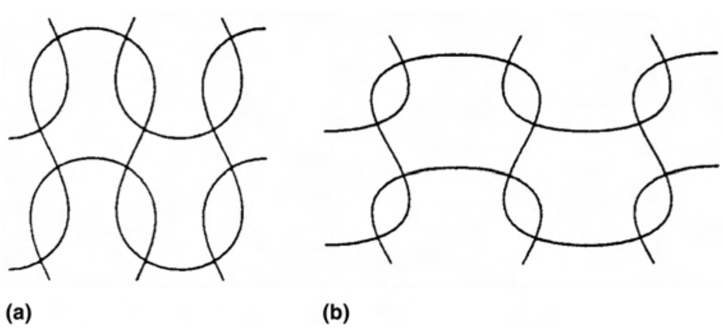


Figure 6. Fabric plane projections of the plain-knitted structure for (a) relaxed fabric and (b) fabric under equal biaxial stressed ($PL^2/B = 20$; $a = 20$; $d_{\min}/L = 0.22$).

in Figure 7. For a typical plain-knitted fabric used by Postle (1971), this load is equivalent to a pressure of 84 kPa, slightly larger than the maximum pressure used experimentally.

Figure 8 shows that the transverse compression modulus for the plain-knitted structure decreases and therefore the fabric compressibility increases with increasing fabric slackness and yarn compressibility. The main mechanisms are the rotation of the arc of the loop into the fabric plane and yarn compression. The former causes an increase in the course spacing and a build-up of the length-jamming forces. This rotation also reduces the fabric width. This effect is most easily understood by considering the arcs of the loop as rotating links of a chain. A counteracting effect is caused by the increase in width-jamming forces. The jamming forces are so large for very tight fabrics that the fabric extends coursewise during transverse compression.

Figure 8 shows that the following equation for the fabric thickness t of the form derived by Williams and Leaf (1974) from experimental results fits the theoretical curve within 5 percent:

$$t = a + b\ \exp\left(-k\frac{\mathrm{PL}^2}{B}\right) \tag{2}$$

where $a = 0.417$, $b = 0.089$ and $k = 0.256$. The constants used in this equation have been chosen to give the correct moduli at 0 and $PL^2/B = 5$. All three constants are dependent on fabric parameters, as concluded by Williams and Leaf (1974).

The measurement of fabric thickness t/L at almost constant $d_{\min}$, as performed experimentally, is equivalent to finding $t/d_{\min}$ at a constant

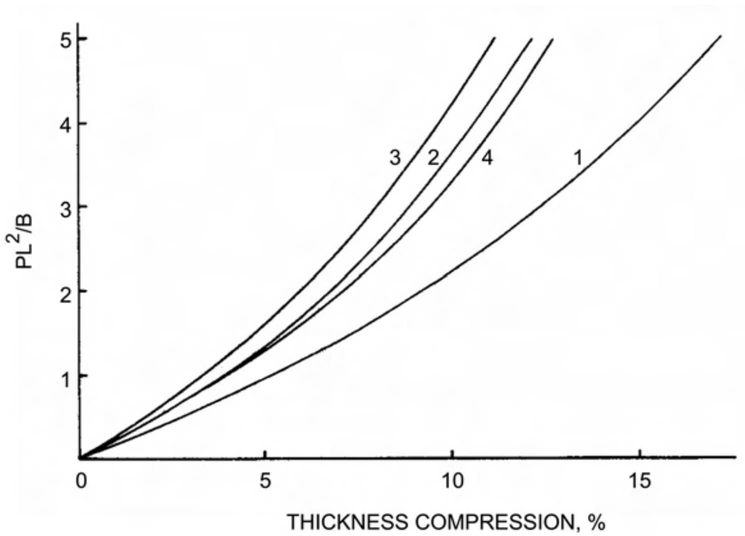


Figure 7.
Transverse load–compression curves for the plain-knitted structure calculated from the energy equations, showing the effects of the yarn compression index a and the fabric tightness $d_{\min}/L$ for set fabrics with form factor $\phi =$ 0.99 : curve 1, $a = 10$, $d_{\min}/L = 0.22$; curve 2, $a = 20$, $d_{\min}/L = 0.22$; curve 3, $a = 20$, $d_{\min}/L = 0.235$; curve 4, from the formula derived by Williams and Leaf, i.e. $t = a + b \exp[-k(PL^2/B)]$, where $a = 0.417$, $b = 0.089$ and $k = 0.256$

quarter-loop length L. There is only a small variation (less than 3 percent) in t/d_{min} with changing fabric tightness d_{min}/L (Hart, 1981). This small variation may not be detectable experimentally. The thickness increases with the fabric tightness for the plain-knitted structure, but not linearly.

5. Conclusion

Several modes of tensile deformation of the plain-knitted structure have been studied. Using the forces, couples, energies and loop shapes that can be evaluated from energy considerations, the mechanisms of knitted fabric extension have been analysed. Previous studies have concentrated on isolated elements of the deformation, but in this paper, the interplay of the major mechanisms – yarn bending, slippage, yarn compression and jamming – has been described in order to explain the overall fabric tensile behaviour.

A redistribution of yarn curvature has the largest single influence on fabric extension and results in greater extensibility in the coursewise direction than in the walewise direction. Structural jamming gives an initial pre-stress in the direction of extension and, at high loads, jamming normal to the direction of extension increases the fabric modulus. Jamming is less significant in biaxial extension than in uniaxial extension for plain-knitted fabrics. Slippage is large in uniaxial extension but would be greatly reduced by friction if it were included in the theoretical analysis. Fabric setting would reduce the frictional resistance.

Yarn compression can increase knitted fabric extensibility by up to 30 percent in the load range used (maximum $PL^2/B = 40$). Yarn compressibility reduces the amount of curvature in regions of loop interlocking and weakens the effect of jamming at high loads.

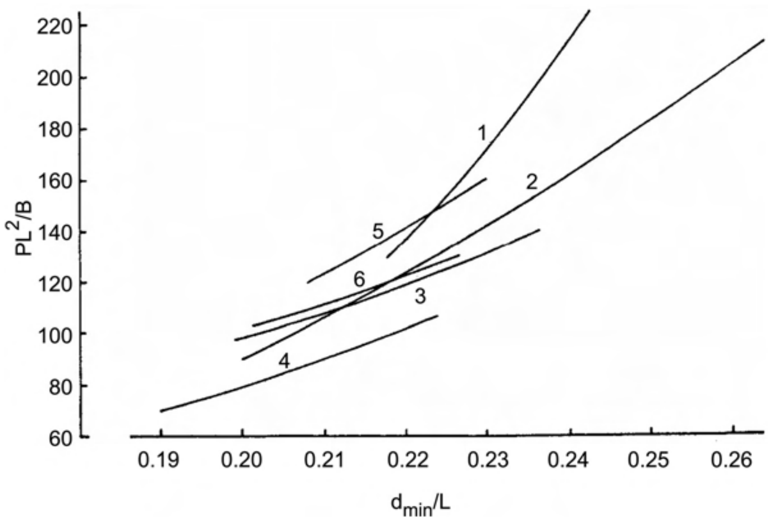


Figure 8. Initial modulus of fabric transverse compression for the plain-knitted structure plotted as a function of fabric tightness d_{min}/L for different values of the yarn compression index a and of the degree ϕ of set: curve 1, $\phi = 0.99$, $a = 30$; curve 2, $\phi = 0.99$, $a = 20$; curve 3, $\phi = 0.99$, $a = 10$; curve 4, $\phi = 0.99$, $a = 5$; curve 5, $\phi = 0$, $a = 20$; curve 6, $\phi = 0$, $a = 10$.

The relaxed or unstressed shape of the knitted structure encountered in knitted apparel yields the characteristic very low initial tensile modulus. On the other hand, the shape of the biaxially stressed or pre-tensioned knitted structure yields the high initial tensile modulus evident from the almost linear yarn segments comprising the pre-stressed knitted loop shape. This latter pre-stressed structure enables the superior formability of knitted textile materials to be adapted to the production of the three-dimensional composite material preforms.

It is clear that we need to interact strongly with the mathematical disciplines of differential geometry and nonlinear dynamical systems in order to advance this challenging field of study. The rewards are potentially very great because success would yield dynamic analytic mathematical solutions to very practical problems in textile, clothing and composite materials science and technology. Such solutions would, for example, facilitate the dynamic display in real time of three-dimensionally deformed textiles or fashion garments on a computer screen since no time consuming numerical procedures for solving complex fabric deformation problems would be required.

Note

1. P is the externally applied force; L is the knitted quarter-loop length; and B is the yarn bending rigidity.

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