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Time-dependent system reliability of anchored rock slopes considering rock bolt corrosion effect

Shui-Hua Jiang [a], Dian-Qing Li [a,*], Li-Min Zhang [b], Chuang-Bing Zhou [a]

[a] State Key Laboratory of Water Resources and Hydropower Engineering Science, Wuhan University, 8 Donghu South Road, Wuhan 430072, PR China
[b] Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong

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ABSTRACT

This paper aims to investigate the effect of rock bolt corrosion on time-dependent system reliability of anchored rock slopes. First, a corrosion degradation model for reinforcing steel bars in concrete is selected to model the uniform corrosion of rock bolts. Second, two typical failure modes of rock bolts due to corrosion and the resultant slope failure modes are identified. Subsequently, a Monte Carlo simulation-based reliability approach is proposed to perform system reliability analysis of anchored rock slopes. Finally, an example of an anchored rock slope is worked out to investigate the effect of rock bolt corrosion on the time-dependent reliability of the anchored rock slope. The results indicate that the probability of slope failure upon yield failure of rock bolts at the free length only increases slightly with time. On the contrary, the probability of slope failure upon bond failure at the bolt–grout interface increases dramatically with time. The system probability of failure of the anchored rock slope decreases with increasing thickness of bolt cover and increases with increasing water–cement ratio of the grout. During the design and construction of pre-stressed rock bolts, a certain thickness of bolt cover should be guaranteed and the water–cement ratio of the grout should be strictly controlled to enhance the long-term stabilization effect of rock bolts.

1. Introduction

Rock bolts made of different types of steel have been widely used for the reinforcement of jointed rock slopes, nuclear waste repositories, underground mines and tunnels (Siad, 2001; Osgoui and Ünal, 2009; Blanco-Fernandez et al., 2011; Divi et al., 2011). Rock bolts are often exposed to corrosive environments such as chloride and sulfate ions, moisture, fluctuating temperature, and potential seepage of water through the fractures or joints in the rock (Gamboa and Atrens, 2003; Villalba and Atrens, 2009; Karalis et al., 2012; Kang et al., 2013). A survey of rock bolt failures indicated that the life of a rock bolt is mainly controlled by corrosion which largely affects the unbonded free length (Xanthakos, 1991). The causes of failure of an anchored rock slope are often difficult to characterize because imperfections in installation, corrosion protection, and workmanship on rock anchorage system may induce failure either individually or in combination. In this study, however, the corrosion of rock bolt is taken as the sole cause of possible failure of the anchored rock slope. As reported in the literature, some geotechnical engineering accidents were caused by performance degradation or failure of the anchorage system due to the corrosion effects. A committee of FIP (Fédération Internationale de la Précontrainte) collected 35 cases of anchor failures due to corrosion in 1986 (Littlejohn, 1987), in which 19 incidents occurred at or within 1.0 m of the anchor head, 21 incidents occurred at the free length and 2 incidents occurred at the fixed length. A major reason for the 2010 slope failure on Freeway No. 3 in Taiwan was the corrosion of the anchor system according to the post-event investigation (Lee et al., 2013; Wang et al., 2013a). Thus, the corrosion effect of an anchor system on the reliability of anchored geotechnical structures needs to be investigated.

Recently, the reliability-based analysis and design of anchored rock slopes have been extensively investigated in the literature (Li et al., 2009, 2011a,b; Park et al., 2012; Wang et al., 2013a,b). However, time-dependent reliability of anchored rock slopes under the corrosion effects has not been investigated substantially. To our best knowledge, only Cheng (2010) performed a preliminary study on time-dependent reliability of rock slopes considering the corrosion of rock bolts and time-variant shear strength parameters through direct Monte Carlo simulation (MCS). In contrast, time-dependent reliability of other anchored structures such as anchored gravity structures and reinforced earth wall under the corrosion effects has been reported in the literature (Chakravorty et al., 1995; Chau et al., 2012; Xia et al., 2012). For instance, Chakravorty et al. (1995) developed a corrosion deterioration model for rock anchors by considering the analogies between underground corrosion and atmospheric corrosion. The First-Order Reliability Method (FORM) is used to perform time-dependent reliability analysis of an anchored gravity structure considering the corrosion effect of the anchor bars. Chau et al. (2012) conducted finite element analysis of

* Corresponding author. Tel.: +86 27 6877 2496; fax: +86 27 6877 4295.
E-mail address: dianqing@whu.edu.cn (D.-Q. Li).

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the effect of steel strip corrosion on the long-term behavior of reinforced earth walls. Xia et al. (2012) presented a probability-based computational model for predicting the time-dependent deterioration of bond capacity of corroding rock bolts due to chloride attack. Direct MCS is employed to evaluate the bond strength deterioration at the bolt–grout interface in a probabilistic framework.

Although several researchers have attempted to study the time-dependent reliability of anchored structures, the following issues are not well addressed. Firstly, the corrosion degradation model for pre-stressed rock bolts has not been studied extensively. Furthermore, the bond strength deterioration model at the bolt–grout interface needs to be developed. Secondly, for typical geotechnical structures such as rock slopes, their safety depends greatly on the durability of rock-anchored structures. Yet few attempts have been made to study the failure modes of rock bolts considering corrosion effects and the resulting influences on the reliability of geotechnical structures. Finally, the existing studies considered only a single failure mode of anchored structures. The system reliability of anchored structures considering multiple failure modes has not been investigated comprehensively.

The objective of this paper is to investigate the effect of rock bolt corrosion on time-dependent system reliability of anchored rock slopes. Based on experimental data for uniform corrosion of rock bolts, a corrosion rate model for reinforcing steels in concrete structures is selected to model the uniform corrosion of rock bolts. Furthermore, the bond strength deterioration model at the bolt–grout interface is investigated. Thereafter, two typical failure modes of rock bolts due to corrosion and the resultant failure models of anchored rock slopes are identified. Then, a MCS-based reliability approach is presented to perform time-dependent system reliability analysis of anchored rock slope considering the corrosion of rock bolts. Parametric studies are carried out to investigate the effect of the thickness of bolt cover and the water–cement ratio of grout on the time-dependent system reliability of an anchored rock slope.

2. Corrosion rate model for rock bolts

Rock slopes reinforced by pre-stressed rock bolts may be subjected to various types of corrosion. This study will only focus on the corrosion of rock bolts by assuming that the rock slope itself does not have appreciable deterioration. In the literature, a corrosion deterioration model for rock bolts in anchored rock slopes is not available. To the best of our knowledge, to date neither theoretical nor empirical data can adequately predict the corrosion rate of rock bolts in anchored rock slopes. However, corrosion rate models for reinforcing steels in concrete structures have been studied extensively. Since there exist some analogies between the corrosion of the rock bolts in anchored rock slopes and that of reinforcing steels in concrete structures (Chakravorty et al., 1995), the corrosion rate model for reinforcing steels in concrete structures can be adopted for modeling the corrosion rate of rock bolts.

An improved corrosion rate model for reinforcing steels in concrete structures proposed by Vu and Stewart (2000) is selected to describe the corrosion deterioration process of rock bolts. This corrosion rate model is selected because it is relatively simple and accounts for the effect of thickness of steel cover and water–cement ratio of grout which are assumed to be two major factors affecting the corrosion rate. The heterogeneous profile of chloride concentration at the surface of grout cover for different cross-sections will lead to a variation of time to corrosion initiation over the anchor length. Theoretically, non-uniform corrosion of steel–bolts over the anchor length will occur. However, each element of a rock bolt is assumed to be subject to uniform corrosion in this study for simplicity. Some outdoor experimental data on uniform corrosion of rock bolts is collected to validate the applicability of this corrosion rate model in this section. In this model, for the typical environmental condition of an ambient relative humidity of 75% and a temperature of 20 °C, the influence of the thickness of steel cover and water–cement ratio is expressed empirically as

$$\begin{aligned} i_{\mathrm{corr}}(t) &= \frac{37.8\times10^{-3}(1-w/c)^{-1.64}}{d_c}0.85t^{-0.29}\times11.6\times10^{-6} \\ &= 3.727\times10^{-7}\frac{(1-w/c)^{-1.64}t^{-0.29}}{d_c} \end{aligned} \tag{1}$$

where $i_{\mathrm{corr}}(t)$ is the corrosion rate (m/year) at time t (years) since the initiation of corrosion; d_c is the thickness of the steel cover (m); and w/c is the water–cement ratio of grout. The corrosion penetration depth, Δd, over a service period of t years can be derived as

$$\Delta d(t) = \int_0^t i_{\mathrm{corr}}(\tau)d\tau = 5.249\times10^{-7}\frac{(1-w/c)^{-1.64}t^{0.71}}{d_c}. \tag{2}$$

Zeng et al. (2002) presented 180-day corrosion experimental data on exposed rock bolts in five different environmental conditions as illustrated in Fig. 1. Liu (1996) and Liu and Weyers (1998) conducted a five-year outdoor experiment on the dynamic process of corrosion rate of steel in concrete and developed a theoretical corrosion rate model incorporating the effect of chloride ingress and temperature fluctuation. The corrosion experimental data reported by Zeng et al. (2002) and Liu (1996) provides a good basis for validating the applicability of the corrosion rate model proposed by Vu and Stewart (2000). Fig. 1 shows the variation of corrosion rate of a rock bolt steel with a diameter of 28 mm and a thickness of bolt cover of 46 mm with time. In Fig. 1, pH is a measure of the acidity or the basicity of an aqueous solution in geotechnical media. For comparison, the results produced by the Vu and Stewart (2000) model for various water–cement ratios of grout are also provided in Fig. 1. Note that the corrosion rate model proposed by Vu and Stewart (2000) with a water–cement ratio w/c of 0.4 matches well with that proposed by Liu and Weyers (1998) with a surface chloride concentration of 6.0 kg/m^3. Additionally, the corrosion rate model proposed by Vu and Stewart (2000) is suitable for describing the corrosion deterioration process of rock bolts subjected to the environmental conditions of the indoor exposure with humidity and alternate wetting and drying.

Fig. 2 shows the variations of the corrosion penetration depth of the rock bolt with time on log scales within an assumed service period of 100 years. The corrosion penetration depths of rock bolts associated with the models proposed by Vu and Stewart (2000) and Liu and Weyers (1998) are obtained through the integration of corrosion rate over the service period. Note that the corrosion penetration depth of

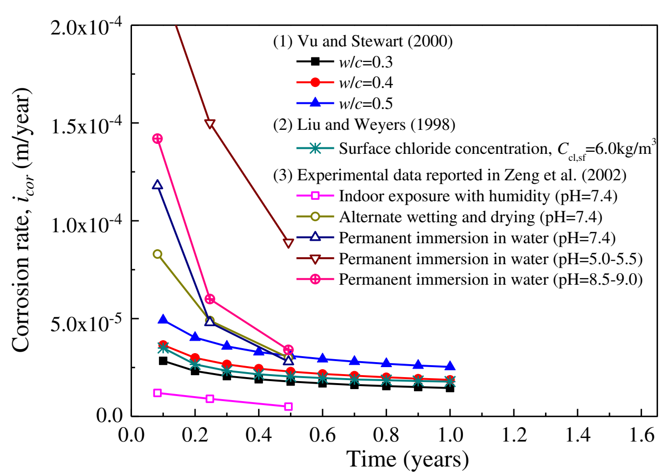

Fig. 1. Variation of the corrosion rate of rock bolt with time.

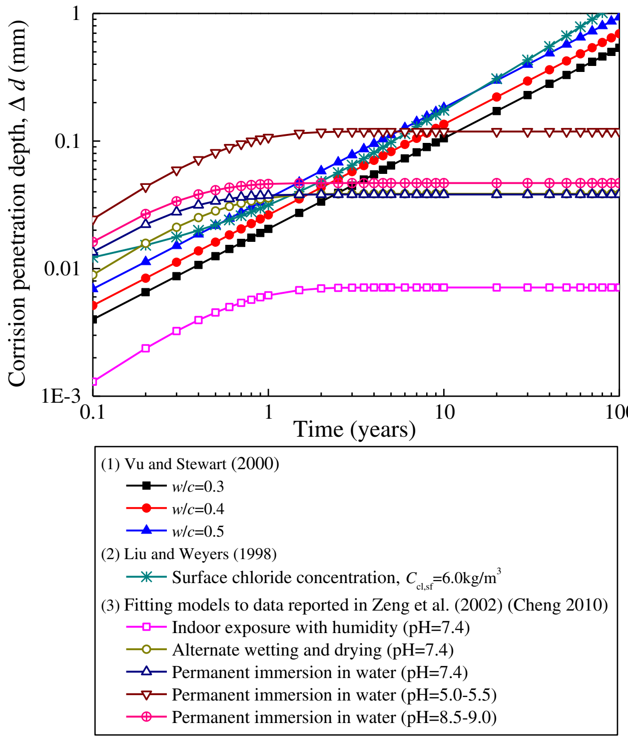

Fig. 2. Variation of the corrosion penetration depth of rock bolt with time.

the rock bolt produced by the Vu and Stewart (2000) model also agrees well with that produced by the Liu and Weyers (1998) model with a surface chloride concentration of 6.0 kg/m^3. Cheng (2010) also studied the variation of corrosion penetration depth by adopting exponential function fitting models based on the 180-day experimental data under five different environmental conditions as reported in Zeng et al. (2002). The results obtained from the exponential fitting models (Cheng, 2010) are consistent with those obtained from the Vu and Stewart (2000) model only within the first year. The corrosion penetration depth obtained from Cheng (2010) almost remains the same with time after one year, which may be unrealistic from an intuitive way. These results imply that the results produced by the fitting models based on the short-term corrosion experimental data cannot accurately predict the long-term rock bolt corrosion. Thus, the corrosion rate model proposed by Vu and Stewart (2000) is finally adopted to model the corrosion rate of rock bolts and conduct time-dependent system reliability analysis of anchored rock slopes.

3. Failure modes of rock bolt due to corrosion

To conduct a system reliability analysis of an anchored rock slope, the first step is to identify the relevant failure modes based on information of slope stability and forces acting on the rock slope. For the stability model of anchored rock slope shown in Fig. 4, if no reinforcement measures are taken, the factor of safety is calculated as 0.947 when the mean values of the involved random variables in Table 1 are used. It is evident that some reinforcement measures are needed to improve the slope stability. In this study, multi-row rock bolts are adopted to stabilize the rock slope, which will be illustrated in Section 4.1. Failure modes of reinforced rock slopes include not only tensile, shear and bending failures of the reinforcements, but also pullout of the reinforcements from the rock mass (e.g., Siad, 2001). As reported in FHWA (1999), there are four failure modes associated with anchorage systems as follows: failure of the reinforcement, failure of the rock mass, failure of the grout-reinforcement bond, and failure of the rock–grout bond.

In this study, the failure of reinforcements only refers to external axial tensile forces exceeding tensile strengths supplied by rock bolts since the limit equilibrium method (LEM) is used to calculate the factor of safety against sliding of the anchored rock slope. If the axial tensile forces provided by the rock bolts due to corrosion decrease dramatically, the anchored rock slope may fail. Therefore, we will focus on the potential tensile failure modes of the rock bolts. For illustrative purposes, there are two major tensile failure modes of rock bolts due to corrosion: the yield failure mode at the free length of rock bolt and the bond failure mode at the bolt–grout interface of the fixed length of rock bolt. The failure modes of the anchored rock slope corresponding to these two failure modes of rock bolts are investigated.

3.1. Yield failure at free length of rock bolt

The corrosion of rock bolt is taken into account through the sacrificial thickness, namely, the metal thickness that will be lost by corrosion during the service life (Chau et al., 2012). During the corrosion process, the cross-sectional area of rock bolt decreases with increasing sacrificial thickness, which results in the loss of yield strength of the rock bolt. When the corrosion penetration depth reaches a threshold value, the yield failure of the free length of rock bolt will occur due to a large pre-stress. The time-dependent tension force at the free length is expressed as (Cheng, 2010)

$$T_{i1}(t) = \frac{\pi}{4} {d_b}^2 (1-\eta_s(t)) f_{t0} \alpha_{st}(t) \tag{3}$$

where d_b is the diameter of rock bolt; f_{t0} is the initial yield strength of rock bolt; $\eta_s(t)$ is the percentage reduction of cross-sectional area of rock bolt at the tth service year; and $\alpha_{st}(t)$ is the reduction of yield strength in percentage of a rock bolt at the tth service year. $\eta_s(t)$ and $\alpha_{st}(t)$ can be derived as (Cheng, 2010)

$$\eta_s(t) = \frac{{d_b}^2 - [d_b - 2\Delta d(t)]^2}{{d_b}^2} \tag{4}$$

and

$$\alpha_{st}(t) = \frac{0.985 - 1.028\eta_s(t)}{1-\eta_s(t)}. \tag{5}$$

Table 1
Statistical parameters of input random variables.

Random variable	Distribution	Mean	COV	Other parameters	Reference
c (kPa)	Normal	20	0.3	–	Low (2007), Li et al. (2011a)
ϕ (°)	Normal	32	0.2	–	Low (2007), Li et al. (2011a)
f_{t0} (kPa)	Lognormal	400,000	0.05	–	Cheng (2010)
τ_{b0} (kPa)	Lognormal	1000	0.15	–	Cheng (2010)
r_w	Truncated exponential	0.5	–	[0, 1.0]	Low (2007), Li et al. (2011a, b)
k_h	Truncated exponential	0.08	–	[0, 0.16]	Low (2007), Li et al. (2011a)

Note: The symbol "–" denotes that the parameter is not available.

3.2. Bond failure at the bolt–grout interface of the fixed length of rock bolt

The stabilization mechanism of a rock slope with rock bolts also includes the transfer of the tensile force provided by the rock bolt to the ground through the shear resistance mobilized at the bolt–grout interface at the fixed length of rock bolt. The rock bolt corrosion can lead to the reduction of bond strength at the bolt–grout interface due to material loss. Note that corrosion products such as rust will be generated around the rock bolt steel in the early corrosion stage, which will increase the bond strength at the bolt–grout interface to some extent (Xia et al., 2012). However, when the rust concentration over steel surface accumulates gradually to exceed a threshold value, the grout may break. Consequently, the moisture, oxygen, chloride and sulfate ions in the ambient environment surrounding the rock bolt will concentrate on the rock bolt surface rapidly, which will further increase the corrosion rate. Then, the bond strength at the bolt–grout interface of the fixed length will be reduced substantially. Following Xia et al. (2012), the moment of grout spalling is approximated as the initiation time for analyzing the deterioration process of bond strength at the bolt–grout interface. The time-dependent shear resistance at the bolt–grout interface of fixed length is given by (Xia et al., 2012)

$$T_{i2}(t) = \pi d_b L_{ai} \tau_{b0} R(t) \tag{6}$$

in which L_{ai} is the fixed length of the ith row rock bolts; τ_{b0} is the initial bond strength at the bolt–grout interface; and $R(t)$ is a normalized bond strength ratio, which is also defined as the ratio of the bond strength at the bolt–grout interface of a rock bolt at the tth service year to the original bond strength of un-corroded specimens.

Based on the experimental data, Bhargava et al. (2007) proposed empirical models to evaluate the progressive degradation of the bond strength between the reinforcing steel and the concrete for specimens without stirrups. These models are validated through data from pullout tests and flexural tests. The empirical model based on pullout test experimental data is employed herein to represent the bond strength values at the bolt–grout interface varying with time. The normalized bond strength ratio $R(t)$ is expressed as

$$R(t) = \begin{cases} 1 & \text{for } X_\mathrm{p}(t) \le 1.5\% \\ 1.192\exp\left(-0.117 X_\mathrm{p}(t)\right) & \text{for } X_\mathrm{p}(t) > 1.5\% \end{cases} \tag{7}$$

where $X_p(t)$ is the corrosion level which is measured by the percentage mass loss of the corroded rock bolt steel after the removal of rust to the original mass of rock bolt, which is equal to the percentage reduction in bar cross-section area in the case of uniform steel corrosion. Under the assumption that the density of rock bolt steel remains unchanged during the process of uniform corrosion, $X_p(t)$ is expressed as

$$X_\mathrm{p}(t) = \frac{{d_b}^2 - [d_b - 2\Delta d(t)]^2}{{d_b}^2}. \tag{8}$$

In the literature, Val et al. (1998) indicated that the corrosion of reinforcing bars would cause cracking and ultimately spalling or delamination of the concrete cover, with subsequent reduction of the bond strength. Xia et al. (2012) investigated the relationship between the bond strength at the bolt–grout interface and the uniform corrosion penetration depth. The degradation models of bond strength proposed by Val et al. (1998) and Xia et al. (2012) are used to further validate the capacity of the bond strength degradation model shown in Eq. (7). Fig. 3 shows the variations of normalized bond strength ratio obtained from three different degradation models with corrosion penetration depth. Note that the degradation model proposed by Xia et al. (2012) is consistent with that proposed by Bhargava et al. (2007). The degradation model proposed by Val et al. (1998) agrees well with that proposed by Bhargava et al. (2007) only for the corrosion penetration depth below 1.25 mm. Generally, the deterioration model proposed by Bhargava et al. (2007) has a good applicability for describing the degradation process of bond strength due to rock bolt corrosion, which is adopted in this study for illustration.

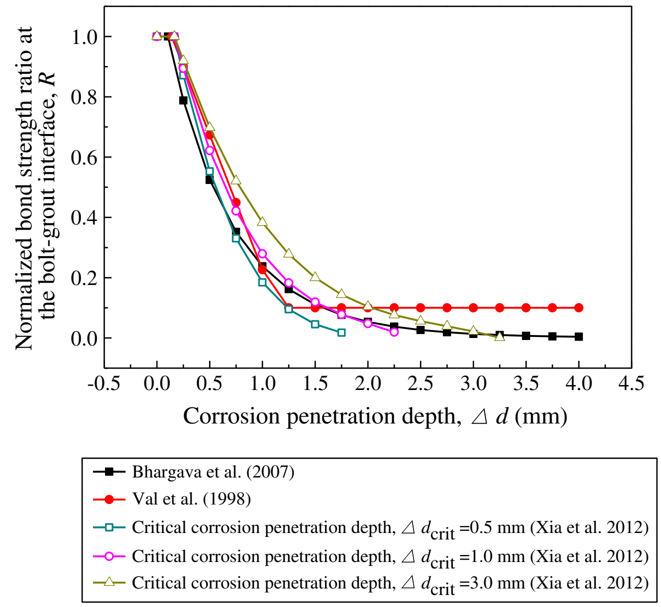

Fig. 3. Variation of normalized bond strength ratio at the bolt–grout interface with corrosion penetration depth.

4. Illustrative example

4.1. Case geometry and material properties

An anchored rock slope studied by Shukla and Hossain (2011) and Low (2007) is employed to investigate the effect of rock bolt corrosion on the system reliability of an anchored slope. The geometry of the anchored rock slope is illustrated in Fig. 4. Based on Cheng (2010) and Shukla and Hossain (2011), the following parameters are adopted. The overall slope height is $H = 12$ m. The angles of inclination of the slope face and potential failure plane to the horizontal are $\psi_f = 60°$ and $\psi_p = 35°$, respectively. The distance between the crest of slope and the tension crack (top width of the slope) is $B = 4$ m. The unit weights of rock and water are $\gamma = 26$ kN/m^3 and $\gamma_\mathrm{w} = 10$ kN/m^3, respectively. The tension crack with a depth of $z = 4.35$ m is filled with water to a depth z_w. Following Low (2007), the extent of tension crack filled with water is often characterized by a ratio $r_w = z_w/z$. The top of the slope is subjected to a surcharge load q. Additionally, horizontal seismic loads k_hW and k_hqB act on the slope, where k_h is the horizontal seismic coefficient and W is the weight of rock block. The vertical seismic loads are not considered.

To ensure the safety of the rock slope, the slope is reinforced with four rows of rock bolts in Fig. 4 and these rock bolts are assumed to fail, if any, at the same time. The resultant anchor stabilizing forces T_i ($i = 1$ ~ 4) are inclined to the normal direction of the failure plane at the same angle of α_i ($i = 1$ ~ 4) $= 35°$. The diameter of the rock bolt is $d_b = 28$ mm and the fixed length of rock bolt is L_{ai} ($i = 1$ ~ 4) $=$ 4 m. Both horizontal and vertical spacings between rock bolts are equal to 2.5 m. The material of rock bolt is made of HRB400 hot-rolled ribbed steel bars with an elastic modulus of $E_b = 2.0 \times 10^8$ kPa. A water–cement ratio of the cement grout $w/c = 0.4$ was used in most cases (Hyett et al., 1992). The diameter of cement grout is $d_g = 120$ mm, and the corresponding thickness of bolt cover is $d_c = 46$ mm. Fig. 5 illustrates the interactions of a typical rock bolt

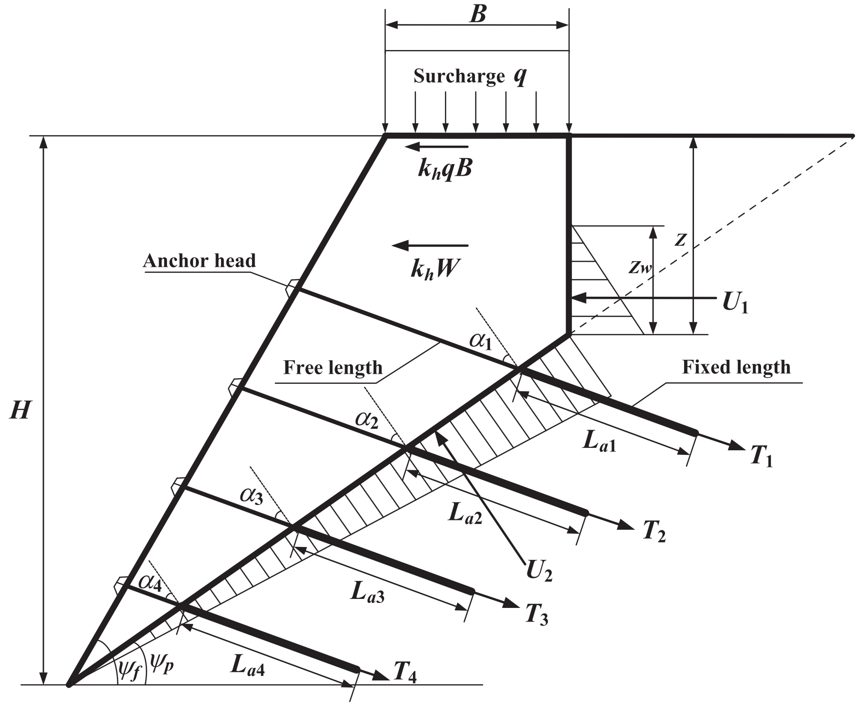

Fig. 4. An anchored rock slope with a single slip surface.

in the rock slope system through its components: the anchor head, the free length and the fixed length.

According to engineering practice, the case where a tension crack has a lower water level is more common than the one that has a higher water level. The minimum value of r_w being 0 corresponds to the case in which the tension crack is dry, whereas the maximum value of r_w is 1.0 when the tension crack is completely filled with water. Thus, a truncated exponential distribution is used for the distribution of r_w. The original

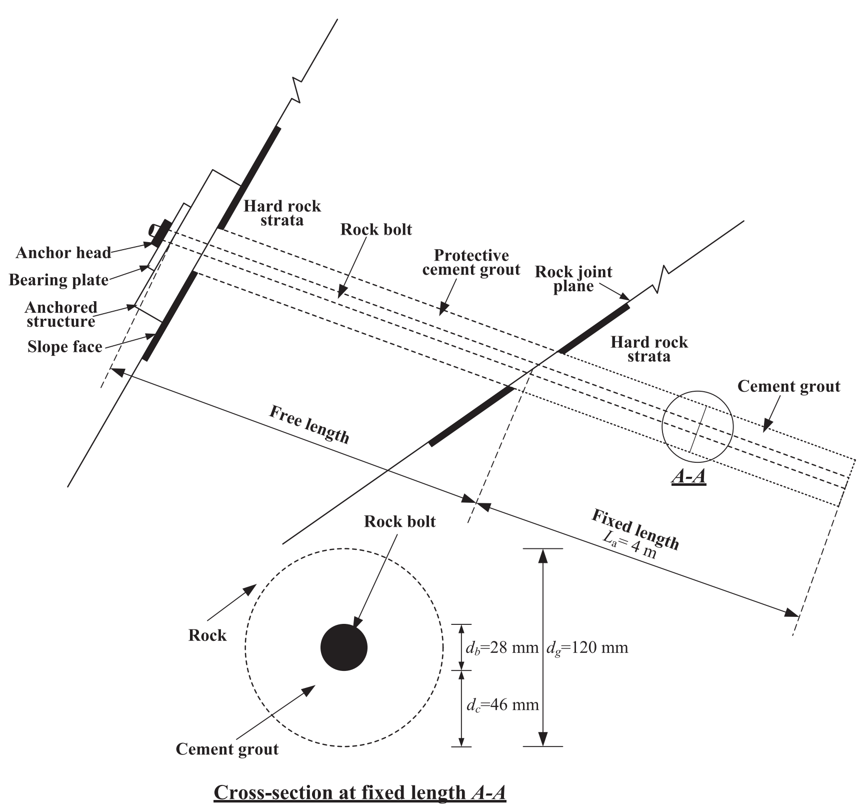

Fig. 5. Illustration of components of a typical rock bolt.

exponential distribution of r_w has a mean of 0.5, which is truncated within the interval [0, 1.0]. The corresponding probability density function and cumulative distribution function are given by (Li et al., 2011a)

$$f(r_w) = \frac{2}{1-e^{-2}} e^{-2r_w} \quad 0 \le r_w \le 1.0 \tag{9}$$

and

$$F(r_w) = \frac{1-e^{-2r_w}}{1-e^{-2}} \quad 0 \le r_w \le 1.0. \tag{10}$$

Based on Eqs. (9) and (10), the mean value of the truncated exponential distribution r_w is obtained as 0.343, while the original mean value is 0.5. Similarly, the horizontal seismic coefficient k_h follows an exponential distribution with a mean of 0.08, which is truncated within the interval [0, 0.16]. The mean of k_h is obtained as 0.055 after truncation, which is significantly smaller than the original mean value of 0.08. Additionally, the initial yield strength of the rock bolt steel is denoted by f_{t0}. The initial bond strength at the bolt–grout interface is denoted by τ_{b0}. The cohesion and internal friction angle of the potential failure plane are represented by c and ϕ. These parameters are treated as independent random variables for simplicity and their statistical values are summarized in Table 1.

4.2. Factor of safety of the anchored rock slope

Shukla and Hossain (2011) analyzed the stability of the rock block using the LEM and presented a general expression for the factor of safety for a slice of unit thickness through the slope. Only the force equilibrium is considered without any resistance to sliding at the lateral boundaries of the sliding block. However, when the rock slope is reinforced with multi-row rock bolts and the horizontal spacing between rock bolts is not equal to unit thickness through the rock slope, the expression for the factor of safety for a slice of unit thickness proposed by Shukla and Hossain (2011) is not valid. For this reason, a modified expression of the factor of safety for a slice with a thickness through the slope equal to the horizontal spacing, s, between rock bolts is derived as

$$FS = \frac{2c^*Ps + \left[(Q+2q^*R)\dfrac{\cos\left(\theta+\psi_p\right)}{\cos\theta}s - \dfrac{z_w^{*\,2}}{\gamma^*}\sin\psi_p s - \dfrac{z_w^*}{\gamma^*}Ps + 2\sum_{i=1}^{n} T_i^*\cos\alpha_i \right]\tan\phi}{(Q+2q^*R)\dfrac{\sin\left(\theta+\psi_p\right)}{\cos\theta}s + \dfrac{z_w^{*\,2}}{\gamma^*}\cos\psi_p s - 2\sum_{i=1}^{n} T_i^*\sin\alpha_i} \tag{11}$$

where q is the surcharge at the top of the slope and $q = 0$ is used for simplicity; n is the number of rows of rock bolts, and $n = 4$ is used in this example; T_i is the stabilizing force provided by the ith row rock bolt; and α_i is the angle of the ith row rock bolt inclined to the normal direction of the failure plane. The non-dimensional parameters are expressed as follows: $c^* = c / \gamma H, q^* = q / \gamma H, z^* = z / H, z_w{}^* = z_w / H, \gamma^* = \gamma / \gamma_w$, and $T_i{}^* = T_i / \gamma H^2$. The other parameters are given by

$$\theta = \tan^{-1}(k_h) \tag{12}$$

$$P = (1-z^*)\csc\psi_p \tag{13}$$

$$Q = \left(1-z^{*2}\right)\cot\psi_p - \cot\psi_f \tag{14}$$

$$R = (1-z^*)\cot\psi_p - \cot\psi_f. \tag{15}$$

Note that when $T_i = T_{i1}$, FS corresponds to the yield failure mode at the free length of the rock bolt. When $T_i = T_{i2}$, FS corresponds to the bond failure mode at the bolt–grout surface. T_{i1} and T_{i2} are calculated using Eqs. (3) and (6), respectively. It should be mentioned that numerical methods need to be used to evaluate the slope stability if the stress–strain information of the slope is of concern or the rock slope involves complicated discontinuities such as joints in rock mass. The most widely used numerical methods in rock mechanics are the finite element method (FEM), the finite difference method (FDM) and the newly developed numerical manifold method (NMM) (Zhu et al., 2011; Zhuang et al., 2012; Cai et al., 2013).

4.3. System reliability of the rock slope

As mentioned earlier, it is assumed that a rock bolt only supplies an increased normal force and a decreased sliding force to stabilize a slope when the LEM is employed to calculate the factor of safety. In this study, the minimum value between the yield force and the bond force is regarded as the stabilizing force provided by each rock bolt for deterministic stability analysis of the rock slope. In contrast, for system reliability analysis of the rock slope, the yield failure mode and the bond failure mode related to the tensile failure of the rock bolts are considered separately to investigate the system reliability of the anchored rock slope. According to the failure mechanism of the anchored rock slope, the failure of the anchored rock slope occurs when the failure of any one of the failure modes occurs. In addition, the occurrence chances of the two failure modes vary with time due to the corrosion effect. Such reliability is a typical time-dependent series system reliability problem. The performance functions for the above two failure modes associated with the considered anchored rock slope are expressed as

$$\begin{cases} G_1(t) = FS_1(T_{i1}(t, f_{t0}), c, \phi, r_w, k_h) - 1 \\ G_2(t) = FS_2(T_{i2}(t, \tau_{b0}), c, \phi, r_w, k_h) - 1 \end{cases} \tag{16}$$

where $G_1(t)$ and $G_2(t)$ are the performance functions associated with the yield failure mode and the bond failure mode, respectively. For the first performance function in Eq. (16), only the yield failure of rock bolt is considered and the bond failure at the bolt–grout interface is not likely to occur. Similarly, only the bond failure at the bolt–grout interface is taken into account in the second performance function.

Based on Eq. (16), the performance function of the series system reliability of the anchored rock slope, $G(t)$, is expressed as

$$G(t) = \min(G_1(t), G_2(t)). \tag{17}$$

Note that the case that the slope may fail when the two failure modes are both reached can be incorporated in the system reliability analysis through Eqs. (16) and (17).

The reliability analysis with explicit performance functions can be carried out readily using the direct MCS or a non-intrusive stochastic finite element method (Zhang et al., 2011; Jiang et al., 2013). To produce sufficiently accurate results, the direct MCS with a sample size of 10^6 is used to calculate the system probability of slope failure. Such a sample size can accurately calculate a probability of failure exceeding 10^{-4}. For a probability of failure below 10^{-4}, more samples are needed. The system probability of failure of the anchored rock slope can be obtained from a MATLAB function: SYSpf = sum(min(G1(X(imod,:),tcor), G2(X(imod,:), tcor)) < 0)/Nsample, in which tcor, imod and Nsample denote the service time t (years) of rock bolts, the ith set of Monte-Carlo samples and the total number of samples, respectively.

4.4. Reliability analysis results

For the considered rock slope shown in Fig. 4, the factor of safety is calculated as 1.64 at the initiation of corrosion ($t = 0$) when the mean values of the involved random variables in Table 1 are adopted. The

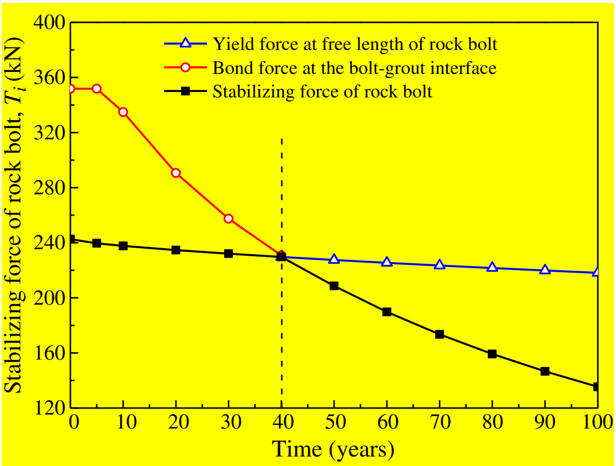

Fig. 6. Variation of the stabilizing force of each rock bolt with the service time.

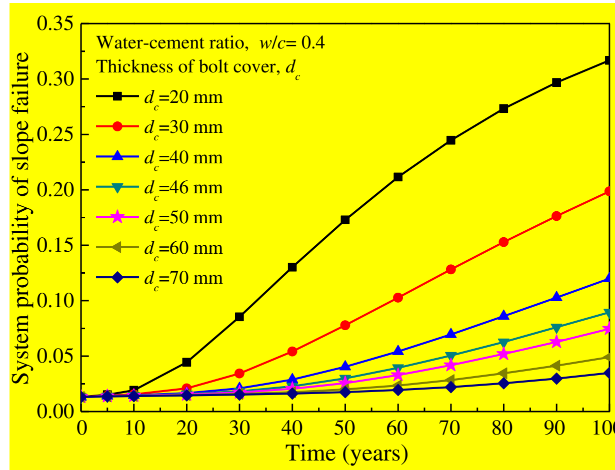

Fig. 8. Effect of the thickness of bolt cover on system probability of slope failure.

stabilizing force of rock bolt will degrade due to the corrosion of rock bolts. Accordingly, the factor of safety of the anchored rock slope will decrease. Fig. 6 shows the variations of the yield force at free length of rock bolt and the bond force at the bolt–grout interface of the fixed length of rock bolt with the service time. The minimum value between the yield force and the bond force is also plotted in Fig. 6. It is evident that the bond force at the bolt–grout interface degrades significantly with the service time. In comparison with the bond force, the yield force almost remains the same with the service time. For a service period $t \leq 40$ years, the yield force at the free length of rock bolt is smaller than the bond force at the bolt–grout interface. Thus, the corrosion at the free length of rock bolt has a relatively significant influence on the stability of the anchored rock slope. On the contrary, for $t > 40$ years, the bond force at the bolt–grout interface is smaller than the yield force. Thus, the stability of the anchored rock slope is mainly influenced by the corrosion at the fixed length of rock bolt.

Fig. 7 shows the variations of the probability of slope failure with the service time. The probabilities of failure for the two considered failure modes and the system probability of slope failure are investigated separately. The probability of slope failure for the yield failure mode increases slightly with the service time. For the assumed service period of 100 years, the probability of failure only increases from 1.3% to 2.1%. In contrast, the probability of slope failure for the bond failure mode increases significantly from 0.22% to 8.9%. The latter is about 40 times the former. In addition, the system probability of slope failure changes significantly with the service time. It is 1.3% at $t = 0$ and increases to 9.0% at $t = 100$ years. Generally, the effect of rock bolt corrosion on the bond failure at the bolt–grout interface is more significant than the yield failure of the free length.

The thickness of bolt cover and the water–cement ratio of grout can affect the corrosion rate of rock bolt because the availability of water, oxygen, chloride and sulfate ions at the rock bolt surface depends primarily on the bolt cover, grout quality (w/c ratio) and environmental conditions. Therefore, it is necessary to investigate the effect of the thickness of bolt cover and the water–cement ratio of grout on the system reliability of the anchored rock slope. Fig. 8 presents the system probabilities of slope failure for various thicknesses of bolt cover at $w/c = 0.4$. Note that the system probability of failure decreases with increasing thickness of bolt cover. In order to enhance the stabilization effect of rock bolts, a certain thickness of bolt cover should be guaranteed. The system probability of slope failure increases slightly with the service time when the thickness of bolt cover reaches a certain value. Taking $d_c = 50$ mm as an example, the system probability of failure only increases from initial value of 1.3% to 7.5% at $t = 100$ years. This result indicates that increasing the thickness of bolt cover can improve the slope safety effectively. Similarly, Fig. 9 shows the system probabilities of slope failure for various grout water–cement ratios at a given bolt cover thickness of $d_c = 46$ mm. It can be observed that the system probability of failure increases as the water–cement ratio of grout increases, especially for water–cement ratios larger than 0.4. To ensure the slope stability, the water–cement ratio of grout should be strictly

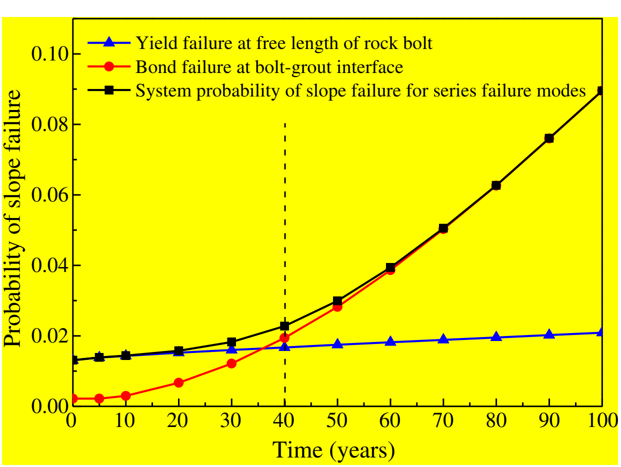

Fig. 7. Variation of the probability of slope failure with the service time.

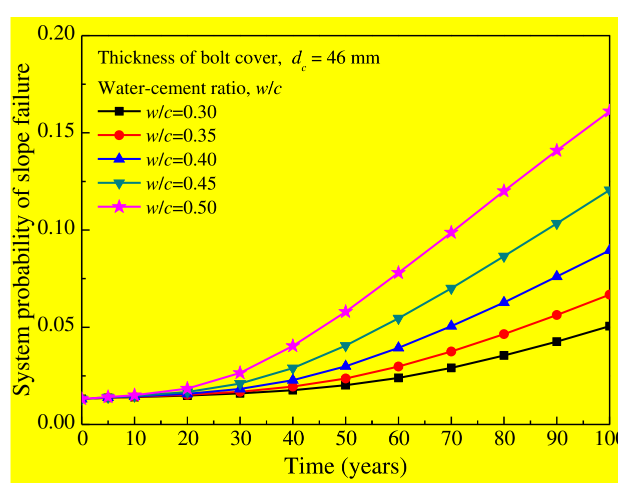

Fig. 9. Effect of water–cement ratio of the grout on system probability of slope failure.

controlled. Additionally, for a small water–cement ratio of grout, the system probability of slope failure changes slightly.

5. Conclusions

This paper has investigated the time-dependent system reliability of rock slopes considering rock bolt corrosion. Two typical failure modes of rock bolts due to corrosion and the resultant failure modes of the anchored rock slope are identified. The time-dependent system reliability analysis is performed using the direct MCS. Several conclusions can be drawn from this study:

(1) The corrosion rate model proposed by Vu and Stewart (2000) can account for the effect of thickness of bolt cover and water–cement ratio of grout. It is suitable for describing the corrosion process of rock bolt under indoor exposure with humidity and alternate wetting and drying. The bond deterioration model proposed by Bhargava et al. (2007) has a good applicability for describing the degradation process of bond strength at the bolt–grout interface due to corrosion.
(2) In the early service period of the anchored rock slope, the corrosion at the free length of rock bolt has a relatively significant influence on the slope stability. In the later service period, the effect of corrosion at the fixed length of rock bolt on the slope stability becomes more significant.
(3) The probability of slope failure in the yield failure mode at the free length of rock bolt increases slightly with time. On the contrary, the probability of slope failure in the bond failure mode at the bolt–grout interface changes dramatically. As expected, the system probability of slope failure increases significantly with time.
(4) The system probability of failure of the anchored rock slope decreases with increasing thickness of bolt cover and decreasing water–cement ratio of the grout. During the design and construction of pre-stressed anchors, a certain thickness of bolt cover should be guaranteed and the water–cement ratio of grout should be strictly controlled to enhance the long-term stabilization effect of rock bolts.
(5) The stress corrosion cracking of rock bolts is not taken into consideration in this paper. The corrosion mechanism of rock bolts subjected to corrosive underground environments is still a challenging problem.

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