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Strategic choice of flexible production technology using game theory approach

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A B S T R A C T

This paper examines the conditions under which a firm would choose a flexible production technology or a dedicated technology in a duopoly environment. We model this technology choice by having two firms simultaneously select from two production technologies in the first stage and subsequently take in a Cournot production quantity subgame. Conditions under which technology equilibriums exist are given. We find that the premium a firm is willing to pay for flexibility increases as the market size increases and the product substitutability decreases. We also find that Prisoner's Dilemma does not necessarily occur in the production technology game, which is different from previous studies.

1. Introduction

The environment faced by manufacturing firms is increasingly uncertain because of fast and dramatic changes in customer expectations, competition, and technology [1]. Many manufacturing firms have faced the decision of whether or not to invest into what is known as flexible manufacturing systems (FMS). Examples of implementations of these new systems abound in the auto-mobile, machine tool, aerospace, heavy machinery, electronics and military equipment industries [2]. FMS brings a firm the ability to accommodate with various internal and external changes, thus promotes the performance and competitiveness of the firm. FMS also provides a firm the ability to produce multiple products simultaneously and enter multiple markets. However, FMS also makes firms compete more fiercely if they produce the same product type and sell them in the same market. In this paper we analyze firms' strategic choices of flexible production technology, and the factors that affect the choices.

Our study is motivated by many practical examples on firms' decisions to invest in FMS. We have observed that in some industries, most firms invest for flexible production technology. For example, in the tri-networks (telecommunications network, the cable TV network and the Internet) industry in China, the "big three" (i.e. China Telecom, China Mobile and China Unicom) have invested heavily for the advanced flexible cable technology. The flexible cable technology enables the firm to enter the three network markets simultaneously. In contrast, in some other industries, most firms invest for dedicated production technology. For example, in the auto industry in China, several main firms almost focus on one auto type. Jiefang trucks, Hongqi cars and Ankai buses are famous brands in China. There are also some industries where both flexible and dedicated technologies coexist in most firms. Upton [3] studied 61 plants in North America in the paper industry with quite comparable products (e.g., letter-size paper) and finds that, some firms have adopted flexible production technology while others have not. As a result, products manufactured by different companies—and hence using different technologies—compete directly in the market.

It is plausible that a firm's decision on technology investment is mainly determined by the cost differential between flexible and dedicated technology. For example, it does not increase much cost to invest for the flexible cable technology than a dedicated cable technology; however, it is much more expensive to build a flexible auto production line, which can produce different auto types than to build a dedicated one. Nevertheless, there may be some other factors that also affect firms' decision in common markets, e.g. correlation between products' demand, competitors' decisions. Then how do the possible factors affect the decisions of firm managers on technology investment? In other words, how to strategically select the technology in a competitive market? This is an important problem facing firm strategic managers.

In this paper, we aim to investigate the impact of possible factors on a firm's strategic choice of technology. Specifically, three main factors are considered, i.e. the cost differential between different technologies, the correlation between products'

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demand, and competitors' decisions. We model the technology choice as a two-stage non-cooperative duopoly game. In the first stage, two firms simultaneously select from two production technologies. If a firm chooses to invest in the flexible technology, it can produce two products and enter both markets. If a firm chooses to invest in a dedicated technology, it can only produce one product and enter only one market. Given a set of technologies chosen by the firms in the first stage, firms take in a subgame—Cournot production quantity game in the second stage. We seek a subgame-perfect equilibrium in such a game. In a similar background, Röller and Tombak [2] addressed that if both firms select the flexible technology (one equilibrium), then the firms will be trapped in a Prisoner's dilemma-like situation, i.e. both firms investing in flexible technology is detrimental to both firms. However, as will be described in our paper, we find that the ''Prisoner's dilemma'' does not necessarily occur in this equilibrium.

To keep the model simple, we in this paper do not take customer behavior into account, such as customers' brand preference. It is assumed that customers do not have brand preference for the same product type. Considering customer behavior, the demand function can be revised to reflect customers' preference and thus the manufacturing planning results should be different. Some authors have made some attempts to model customers' behavior in manufacturing planning. For example, Makris and Chryssolouris [28] developed a model to estimate the probability that a customer actually place an order once he has received a potential delivery date for a product. Using a market simulation model, Pasek et al. [29] investigated customers' behavior in decision under the mass-customization conditions.

The paper is organized as follows. In Section 2, we review the related literature in flexible technology choice. In Section 3, we describe the problem investigated in this paper, and provide the specifications of the basic model. In Section 4, we present technology equilibriums in the duopoly game, where conditions for the equilibriums are given. In Section 5, we analyze the conditions for ''Prisoner's dilemma'' to occur in the technology game. In Section 6, we summarize our findings and give some future research directions.

2. Literature review

In the past half century, flexible manufacturing technology has been widely used in manufacturing industry and extensively studied by scholars. Many scholars have done a lot of work on the concept and measurement of manufacturing flexibility. Sethi and Sethi [19] critically reviewed the literature, classified manufacturing flexibility into eleven types, and summarized the concept, purpose, means and measurement method for each type of flexibility. Kumar [20] proposed a method for measuring manufacturing flexibility using the concept of entropic. Chryssolouris and his group members have published many results regarding the concept and measurement of manufacturing flexibility [21–27]. He et al. [30] proposed a method to guide flexibility investment by quantifying required flexibility level and available flexibility level, and then determining a best-suited flexible configuration. These research results provide some theoretical basis for flexible technology investment.

There is also abundant research on flexible production technology investment, which can be classified into three streams. Papers in the first stream investigate flexibility improvement for a centralized manufacturing system. For example, Jordan and Graves [4] proposed three guidelines to add flexibility in the context of process flexibility for a single-stage manufacturing system, which is called ''chaining principle'' in the literature [5]. This work was extended to multistage manufacturing systems by Graves and Tomlin [6]. Considering Bill of Material (BOM) of products, Hua and He [5] further developed guidelines to improve process flexibility of machine lines and manufacturing system. The main point of these studies is that flexible capacity arranged in a right way will make the resulted manufacturing or service system with high flexibility and well performance. Based on the assumption that there is a central planner responsible for the whole system, most of these studies conclude that flexible technology is always better than dedicated technology for the system to achieve outstanding performance. Different from these papers, this paper concentrates on the flexible technology choice in decentralized manufacturing system. A decentralized manufacturing system is composed of multiple autonomous firms, which produce multiple products and sell them in a whole market. There is no central planner who is responsible for the whole system.

The second stream of literature considers investment in flexible versus dedicated technology/capacity for one or more firms. In these papers, technology choice is made given that the firm has decided to produce a certain number of products, i.e. flexible technology is not considered as a necessity to enter a new market. Fine and Freund [7] modeled a firm manufacturing n products within two decision stages. In the first stage, the firm must choose the capacity levels for the n dedicated resources as well as for the one flexible resource that can manufacture all n products. In the second stage (when demand is realized), the firm decides on production quantities given the capacity constraints. Fine and Freund [7] showed that the decision to invest in flexibility is based on the cost differential between the dedicated and flexible technologies. Van Mieghem [8] developed a similar model and finds that flexibility is beneficial even with perfect positive correlation if product margins are different. Bish and Wang [9] studied the optimal resource investment decision faced by a two-product, price-setting firm that operates in a monopolistic setting and employs a postponed pricing scheme. The firm has the option to invest in dedicated resources as well as a more expensive, flexible resource that can satisfy both products. Bish and Wang [9] provided the structure of the firm's optimal resource investment strategy as a function of demand parameters and investment costs, and shows that the flexible resource investment decision follows a threshold policy. Goyal and Netessine [10] explored the impact of competition on a firm's choice of technology (product-flexible or product-dedicated) and capacity investment decisions. They modeled two firms competing with each other in two markets characterized by price-dependent and uncertain demand, and showed that the firms may respond to competition by adopting a technology, which is the same as or different from what the competitor adopts. Our paper differs from these papers in that we regard the flexible technology as a necessity of a firm to enter a new market, not just a pure technology of production.

Papers in the third stream of literature look at the strategic value of flexibility as entering a new market or deterring the market entry of rivals. Röller and Tombak [2] and Kim et al. [11] examined the market conditions under which firms would choose a more flexible production technology to enter their rival's market. They assume that the two products are substitutes. They use a two-stage game in which firms choose between flexible and less flexible production technologies in the first stage and subsequently choose output. For substitutable products, they find that firms are trapped in a Prisoner's dilemma-like situation: while each can choose one market and make a monopoly profit in it, both firms invade both markets by choosing flexible technology, and thus earn less profit. They also find that the mixed technology equilibrium (i.e. one firm chooses flexible technology and the other chooses dedicated technology) does not exist. Röller and Tombak [12] examined the implications of market structure on investment in flexible manufacturing systems of n firms. They analyzed the technological choice of the firms on whether to serve one or two markets at the same time in a two-stage game model. They found that in equilibrium, a large proportion of FMS firms is associated with more concentrated markets (i.e. n is small). Norman and Thisse [13] investigated the strategic implications of flexible manufacturing for market structure and performance. They found that the introduction of FMS may change the market environment so much that a first-mover is able to preempt existing and new markets and exercise monopoly power while being able to deter entry of the last-mover. Tseng [14] also addressed the question of how the strategic choice of flexible manufacturing technologies intensifies and interacts with competition among firms. Focusing on the manufacturing flexibility that allows a firm to produce its outputs at shorter expected delivery time, they found that increasing the number of firms will decrease firms' incentive to acquire more flexible manufacturing technologies. Our paper differs from these papers in that either the assumptions or the problems investigated are different.

While our model is somewhat similar to that of Röller and Tombak [2], we allow the two products to be substitutes or complements. By substitutes we mean two products in which each one can take the place of the other one, and the demands of the two products are negatively correlated. For example, a LCD monitor and a LED monitor are substitutes because they both provide the display service. More people selecting LED monitors means less people selecting LCD monitors. By complements we mean two products in which the increased usage of one product will lead to the increased usage of the other, thus the demands of the two products are positively correlated. For example, a toothbrush and a tube of toothpaste are compliments because each one can be used only with the other one. More people selecting toothbrush means more people selecting toothpaste.

Another difference between our work and Röller and Tombak's [2] is, in our paper, the investment costs of flexible technology are allowed to be different for different firms, which is not the case in Röller and Tombak's [2].

3. The model

The problem we investigate in this paper is how to strategically choose from two production technologies, one flexible and one dedicated, for each firm in the competitive markets. If a firm chooses the flexible technology, it produces two products and enters two markets. If a firm chooses the dedicated technology, it will concentrate on one product market. However, the firm selecting dedicated technology may also face the competition on production quantity derived from the other firm which chooses the flexible technology.

We model the technology–quantity choice as a two-stage sequential duopoly non-cooperative game of complete information. Consider two firms (1 and 2), which intend to produce two products (A and B). In the first stage, firms simultaneously choose from a flexible production technology (F) and a dedicated technology (D). In the second stage a Cournot game in quantities is played and each firm needs to decide its production quantities. Two markets exist, one for product A, and another for product B. The flexible production technology allows a firm to participate in both markets (i.e. the equipment is capable of producing both product A and product B) whereas the dedicated technology limits the firm to producing either product A or B. Without loss of generality, if a firm selects the dedicated technology, we assume that firm 1 produces product A, and firm 2 product B.

There exists a fixed cost of the firm associated with the chosen technology k ($k=F,D$). We use some notations to formulate the model:

f_i^k	the fixed cost of firm i ($i=1,2$) for the investment of technology k
q_i^j	firm i's ($i=1,2$) production quantity of product j ($j=A,B$)
$Q^j = q_1^j + q_2^j$	the total production quantity for product j ($j=A,B$) in the market
P^j	the market price for product j ($j=A,B$)
λ	the product substitutability parameter, $\lambda \in [-1, 1]$

$\lambda > 0$ ($\lambda < 0$) signifies that the two products are substitutes (complements). Specially, $\lambda=0$ indicates that the demands of the two products are independent, and the two products are neither substitutes nor complements. We will show later that the $\lambda=0$ case is actually a degradation of either the $\lambda > 0$ case or the $\lambda < 0$ case, and it seamlessly connects the two opposite cases.

3.1. Assumptions

The assumptions used in this paper are listed as follows:

(i) Assume that the market price of a product is a function of the total production quantity in the market, i.e. $P^A = \alpha - Q^A - \lambda Q^B$, $P^B = \alpha - Q^B - \lambda Q^A$ [10]. In these inverse demand functions, α is the demand curve intercept and represents a measure of market size, which is same for both products.

(ii) Assume that $f_i^F > f_i^D$ ($i=1,2$). For simplicity and without loss of generality, we normalize the fixed cost of the dedicated technology to zero, *viz*, $f_i^D = 0$ ($i=1,2$). Similar assumption is used in Röller and Tombak [2].

(iii) The unit production cost of any product is assumed to be zero, for any technology [10].

(iv) All the parameters are assumed to be common knowledge. This assumption is made to simplify our discussion.

In assumption (ii), f_i^k can be regarded as the cost of necessary organizational changes when adopting new technologies [15]. The fixed cost of the flexible technology includes the computer that controls an FMS, among others, which is often greater than that of dedicated systems [16]. Thus the assumption $f_i^F > f_i^D$ ($i=1,2$) is reasonable. We assume that the fixed cost of flexible technology varies considerably across firms, because it has been observed that the capital costs of acquiring FMS varies substantially across firms [18]. We also assume that $f_i^D = 0$ ($i=1,2$), because we are only interested in the difference between f_i^F and f_i^D. We could have considered nonzero fixed cost of the dedicated technology. Nevertheless, it merely complicates the algebra and does not add additional insights.

In assumption (iii), we assume that the marginal costs are equal for both technologies. This assumption is motivated by empirical observation. See, for example, the cases referred to in [17]. We further assume that the marginal costs are equal to zero. This assumption is equivalent to the assumption that the production cost is linear with production quantity [12], and Fine and Pappu [18] argued that the latter is reasonable since in highly automated manufacturing systems most of the variable costs are material costs.

3.2. Model formulation

We schematically represent the technology game as a 2×2 matrix in Fig. 1, which is typical for strategic-form games. Each

		Firm 2	
		F	D
Firm 1	F	$\pi_1^{(F,F)}, \pi_2^{(F,F)}$	$\pi_1^{(F,D)}, \pi_2^{(F,D)}$
	D	$\pi_1^{(D,F)}, \pi_2^{(D,F)}$	$\pi_1^{(D,D)}, \pi_2^{(D,D)}$

Fig. 1. Payoff matrix of the technology game.

firm is endowed with two strategies (F and D) and each row–column intersection signifies a subgame in the technology game, while the matrix entries signify payoffs (i.e. profits of firms) in the second-stage production quantity game. For example, $\pi_1^{(F,F)}$ signifies the profit of firm 1 if both firms select flexible technology in the first stage of the game. We seek the conditions that guarantee each of the four possible pure-strategy Nash equilibriums of this 2×2 non-cooperative game, i.e. (F,F), (D,D), (F,D) and (D,F), where the first entry signifies the technology choice of firm 1, and the second firm 2.

Depending on the technology choices in the first stage, a variety of situations have to be analyzed in the second stage. The optimization problem of a firm should be built for each specific technology combination.

The optimization problems of two firms if both of them invest in flexible technology are formulated as follows:

$$\max_{q_i^A, q_i^B} \pi_i^{(F,F)} = [\alpha-(q_i^A+q_j^A)-\lambda(q_i^B+q_j^B)]q_i^A + [\alpha-(q_i^B+q_j^B)-\lambda(q_i^A+q_j^A)]q_i^B - f_i^F \quad (1)$$

where $i=1,2$.

The optimization problems of two firms if both of them invest in dedicated technology are formulated as follows:

$$\max_{q_1^A} \pi_1^{(D,D)} = (\alpha-q_1^A-\lambda q_2^B)q_1^A \quad (2)$$

$$\max_{q_2^B} \pi_2^{(D,D)} = (\alpha-q_2^B-\lambda q_1^A)q_2^B \quad (3)$$

The optimization problems of two firms if firm 1 invests in flexible technology while firm 2 invests in dedicated technology are formulated as follows:

$$\max_{q_1^A, q_1^B} \pi_1^{(F,D)} = [\alpha-q_1^A-\lambda(q_1^B+q_2^B)]q_1^A + [\alpha-(q_1^B+q_2^B)-\lambda q_1^A]q_1^B - f_1^F \quad (4)$$

$$\max_{q_2^B} \pi_2^{(F,D)} = [\alpha-(q_1^B+q_2^B)-\lambda q_1^A]q_2^B \quad (5)$$

The optimization problems of two firms if firm 1 invests in dedicated technology while firm 2 invests in flexible technology are formulated as follows:

$$\max_{q_i^A} \pi_1^{(D,F)} = [\alpha-(q_1^A+q_2^A)-\lambda q_2^B]q_1^A \quad (6)$$

$$\max_{q_2^A, q_2^B} \pi_2^{(D,F)} = [\alpha-(q_1^A+q_2^A)-\lambda q_2^B]q_2^A + [a-q_2^B-\lambda(q_1^A+q_2^A)]q_2^B - f_2^F \quad (7)$$

For the objective functions (1)–(7), we have the following lemma:

Lemma 1. *π_i is jointly concave in its decision variables, q_i^A and/or q_i^B, $i=1,2$.*

The proof of Lemma 1 is given in Appendix A.

Based on Lemma 1, the optimal values of decision variables in the second stage of the game can be obtained from the first-order conditions.

Table 1
Optimal value of decision variables and maximized profits.

	Technology combinations			
	(F,F)	(D,D)	(F,D)	(D,F)
$q_1^{A^*}$	$\frac{\alpha}{3(1+\lambda)}$	$\frac{\alpha}{2+\lambda}$	$\frac{\alpha}{2(1+\lambda)}$	$\frac{\alpha}{3}$
$q_1^{B^*}$	$\frac{\alpha}{3(1+\lambda)}$	N/A	$\frac{\alpha(2-\lambda)}{6(1+\lambda)}$	N/A
$q_2^{A^*}$	$\frac{\alpha}{3(1+\lambda)}$	N/A	N/A	$\frac{\alpha(2-\lambda)}{6(1+\lambda)}$
$q_2^{B^*}$	$\frac{\alpha}{3(1+\lambda)}$	$\frac{\alpha}{2+\lambda}$	$\frac{\alpha}{3}$	$\frac{\alpha}{2(1+\lambda)}$
π_1^*	$\frac{2\alpha^2}{9(1+\lambda)}-f_1^F$	$\frac{\alpha^2}{(2+\lambda)^2}$	$\frac{\alpha^2(13-5\lambda)}{36(1+\lambda)}-f_1^F$	$\frac{\alpha^2}{9}$
π_2^*	$\frac{2\alpha^2}{9(1+\lambda)}-f_2^F$	$\frac{\alpha^2}{(2+\lambda)^2}$	$\frac{\alpha^2}{9}$	$\frac{\alpha^2(13-5\lambda)}{36(1+\lambda)}-f_2^F$

4. The technology game

For the two-stage duopoly game, we should firstly solve for the subgame-perfect Nash equilibriums in the second stage game. By the first-order conditions, we give the maximized profits of the two firms and the corresponding value of the decision variables in each of the four possible subgames in Table 1 (the details of the computation are given in Appendix B). In Table 1, the asterisk in a superscript indicates the optimal value (of decision variables) and maximized value (of profits).

4.1. Best response functions

As is typical for such games, the solution is obtained by considering the best response functions of each firm given the technology choice of the other firm [10]. Without loss of generality, we assume that firm 1 selects technology firstly, and firm 2 reacts for firm 1's selection.

From Fig. 1, it is clear that if firm 1 has chosen the flexible production technology, then firm 2 will also choose the flexibility technology if $\pi_2^{(F,F)^*} > \pi_2^{(F,D)^*}$, otherwise it will choose the dedicated technology. If firm 1 has chosen the dedicated manufacturing technology, then firm 2 will also choose the dedicated technology if $\pi_2^{(D,F)^*} < \pi_2^{(D,D)^*}$, otherwise it will choose the flexible technology.

Proposition 1. *(i) When firm 1 invests in flexible technology, the best response of firm 2 is to invest in flexible technology whenever*

$$f_2^F < \bar{f}_1 = \frac{\alpha^2(1-\lambda)}{9(1+\lambda)}$$

Otherwise, firm 2 should invest in dedicated technology.

(ii) When firm 1 invests in dedicated technology, the best response of firm 2 is to invest in dedicated technology whenever

$$f_2^F > \bar{f}_2 = \frac{\alpha^2(13-5\lambda)}{36(1+\lambda)} - \frac{\alpha^2}{(2+\lambda)^2}$$

Otherwise, firm 2 should invest in flexible technology.

(iii) Above conclusions are symmetric for firms 1 and 2.

The proof of Proposition 1 is given in Appendix C.

Proposition 1 implies that, firms are more willing to invest in flexible production technology if the investment cost is small, and not willing to invest if the investment cost is large. $\bar{f}_1$ and $\bar{f}_2$ are two thresholds of the fixed cost for firms to invest in flexible technology. Then we are interested in analyzing the effects of two parameters, α and λ, on the thresholds $\bar{f}_1$ and $\bar{f}_2$. The results are shown in Proposition 2.

Proposition 2. *(1)* $(\partial \bar{f}_1/\partial \alpha) \geq 0$, $(\partial \bar{f}_2/\partial \alpha) \geq 0$, $(\partial \bar{f}_1/\partial \lambda) < 0$, $(\partial \bar{f}_2/\partial \lambda) < 0$;

(2) $\bar{f}_1 = \bar{f}_2$ *if* $\lambda = 0$;

(3) $\bar{f}_1 = \bar{f}_2 = 0$ *if* $\lambda = 1$.

The proof of Proposition 2 is given in Appendix D.

Proposition 2 shows that both the thresholds, $\bar{f}_1$ and $\bar{f}_2$, are non-decreasing in the market size, α, and decreasing in the product substitutability parameter, λ.

Proposition 2 implies that as the market size increases, the premium a firm is willing to pay for flexibility increases. This is because the larger market increases the opportunity of flexible firms to profit from selling two products, which encourages more active participation in both markets.

Proposition 2 also implies that as the product substitutability increases, the premium a firm is willing to pay for flexibility decreases. We explain this for two cases respectively. For the case that has two products as complements ($-1 \leq \lambda < 0$), Proposition 2 implies that as the two products becomes more complementary (λ decreases), the thresholds increases, and firms are more willing to pay for flexible technology investment. This is because as λ decreases, the demands of complements are more positively correlated, which benefits a firm more if the two products are simultaneously produced. This encourages more active participation in both markets, and thus leads to higher thresholds. For the case that the two products as substitutes ($0 < \lambda \leq 1$), Proposition 2 implies that as the two products becomes more substitutable (λ increases), the thresholds decreases, and firms are less willing to pay for flexible technology investment. As the extreme point, if $\lambda = 1$, then $\bar{f}_1 = \bar{f}_2 = 0$, which means both the two firms will not invest in flexible technology unless the fixed cost of investing in flexible technology is zero. This is because as λ increases, the demands of substitutes are more negatively correlated. A firm can earn nearly the same profit if it produces one product or two substitutable products. This discourages firms from investing in flexible technology to participate in both markets, and thus leads to lower thresholds.

4.2. Nash equilibrium of the technology game

Based on the best response functions above, we have the following proposition about the pure strategy equilibriums of the technology game:

Proposition 3. *In the technology game, if* $-1 \leq \lambda < 1$, *then*

(i) *if* $f_1^F < \bar{f}_1$ *and* $f_2^F < \bar{f}_1$, *then (F,F) is an equilibrium;*
(ii) *if* $f_1^F > \bar{f}_2$ *and* $f_2^F > \bar{f}_2$, *then (D,D) is an equilibrium;*
(iii) *if* $f_1^F < \bar{f}_2$ *and* $f_2^F > \bar{f}_1$, *then (F,D) is an equilibrium;*
(iv) *if* $f_1^F > \bar{f}_1$ *and* $f_2^F < \bar{f}_2$, *then (D,F) is an equilibrium;*

where $\bar{f}_1 = (\alpha^2(1-\lambda)/9(1+\lambda))$, $\bar{f}_2 = (\alpha^2(13-5\lambda)/36(1+\lambda)) - (\alpha^2/(2+\lambda)^2)$.

If $\lambda = 1$, *then (D,D) is the sole equilibrium.*

The proof of Proposition 3 is given in Appendix E.

The relative magnitude of $\bar{f}_1$ and $\bar{f}_2$ depends on whether λ is positive or negative (please refer to Appendix E for the proof). Therefore, we illustrate the equilibriums and their corresponding conditions in Figs. 2 and 3, for $-1 \leq \lambda < 0$ and $0 < \lambda < 1$, respectively.

It can be observed from Figs. 2 and 3 that, there are some differences for the equilibriums of the technology game when the two products are complements ($-1 \leq \lambda < 0$) or substitutes ($0 < \lambda < 1$). These difference occurs only when $\lambda \neq 0$ and $\lambda \neq 1$. If $\lambda = 0$, we have $\bar{f}_1 = \bar{f}_2$. Figs. 2 and 3 will become the same, and there are only four blocks. If $\lambda = 1$, we have $\bar{f}_1 = \bar{f}_2 = 0$. Both Figs. 2 and 3 have only one block. In other words, $\lambda = 0$ is a special case of either $-1 \leq \lambda < 0$ or $0 < \lambda < 1$; $\lambda = 1$ is a special case of $0 < \lambda < 1$.

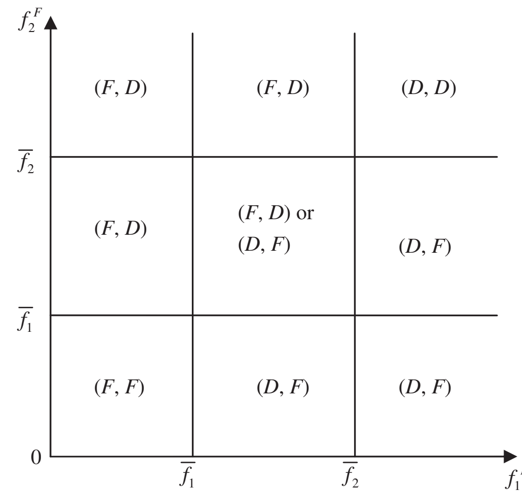

Fig. 2. Technology equilibriums on the distribution of investment costs for $-1 \leq \lambda < 0$.

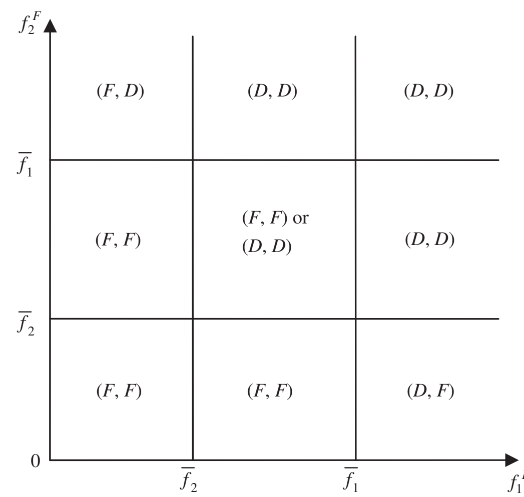

Fig. 3. Technology equilibriums on the distribution of investment costs for $0 < \lambda \leq 1$.

Next we briefly explain the existence of equilibriums. Firstly, consider the case that the two products are complements ($-1 \leq \lambda < 0$). In this case, we have $\bar{f}_1 < \bar{f}_2$ (see the details in Appendix E). If the fixed cost of flexible technology of two firms are less than the smaller threshold $\bar{f}_1$ (i.e. $f_1^F < \bar{f}_1$, $f_2^F < \bar{f}_1$), then (F, F) is the technology equilibrium, because investing in flexible technology is always the best strategy of a firm whatever the other firm chooses. Each firm will bear a loss if it deviates from this equilibrium alone. For example, if firm 1 changes its choice to be a dedicated technology, then its profit will decrease since $(\alpha^2/9) < (2\alpha^2/9(1+\lambda)) - f_1^F$, which is also similar for firm 2 since $(\alpha^2/9) < (2\alpha^2/9(1+\lambda)) - f_2^F$ (see Table 1 for the details). If the fixed cost of flexible technology of two firms are both more than the larger threshold $\bar{f}_2$ (i.e. $f_1^F > \bar{f}_2$, $f_2^F > \bar{f}_2$), then (D,D) is the equilibrium. Each firm will bear a loss if it deviates from this equilibrium alone. For example, if firm 1 changes its choice to be dedicated technology, then its profit will decrease since $(\alpha^2(13-5\lambda)/36(1+\lambda)) - f_1^F < (2\alpha^2/(2+\lambda)^2)$, similar for firm 2 since $(\alpha^2(13-5\lambda)/36(1+\lambda)) - f_2^F < (2\alpha^2/(2+\lambda)^2))$. Other equilibriums can be similarly explained. Note that if $\bar{f}_1 < f_1^F < \bar{f}_2$ and $\bar{f}_1 < f_2^F < \bar{f}_2$, both (*F*,*D*) and (*D*,*F*) are equilibriums. Which one actually occurs depends on the action of the first mover.

Then we consider the case that the two products are substitutes ($0<\lambda<1$). In this case, we have $\bar{f}_2<\bar{f}_1$. If the fixed cost of flexible technology of two firms are both less than the smaller threshold $\bar{f}_2$ (i.e. $f_1^F<\bar{f}_2, f_2^F<\bar{f}_2$), then (*F*,*F*) is the equilibrium. Each firm will bear a loss if it deviates from this equilibrium alone. If the fixed cost of flexible technology of both firms are larger than the larger threshold $\bar{f}_1$ (i.e. $f_1^F>\bar{f}_1, f_2^F>\bar{f}_1$), then (*D*,*D*) is the equilibrium. Each firm will bear a loss if it deviates from this equilibrium alone. Similarly, note that if $\bar{f}_2<f_1^F<\bar{f}_1$ and $\bar{f}_2<f_2^F<\bar{f}_1$, both (*F*,*F*) and (*D*,*D*) are equilibriums. Which one actually occurs depends on the action of the first mover.

Specially, it can be observed from Figs. 2 and 3 that mixed equilibriums (*F*,*D*) and (*D*,*F*) can exist under some conditions. However, in Röller and Tombak [2] and Kim et al. [11], which focus on similar background for $0\le\lambda\le1$, it is concluded that there are no parameter values for which mixed equilibriums could exist. This is because in their papers, they assume that the fixed cost of flexible technology is the same for different firms. In this paper, we assume that the fixed costs of flexible technology may be different for different firms, and find the parameter values for which mixed equilibriums could exist.

5. Prisoner's dilemma

It can be learned from Section 4 that each of the four technology equilibriums may exist under certain conditions. Then it is necessary to analyze whether firms are better off if one or two firms adopt flexible technology.

For the case that the two products are substitutes, Röller and Tombak [2] have addressed that if (*F*,*F*) is the equilibrium, firms are trapped in a Prisoner's dilemma-like situation: while each can choose one market and make a monopoly profit in it, both firms invade both markets by choosing flexible technology, and hence intensify competition. As a result, Röller and Tombak [2] show that flexible technology is detrimental to both firms. However, the case that the two products are complements is not discussed in that paper. In this section, we will find out whether Prisoner's dilemma occurs in the technology game by examining two firms' profits in equilibriums.

Note that in (*F*,*F*) and (*D*,*D*) equilibriums, the profits of the two firms are symmetric except for the difference in f_i^F ($i=1,2$). Then whether Prisoner's dilemma occurs is determined by the relative magnitude of total profit of two firms for (*F*,*F*) and (*D*,*D*) equilibriums. For completeness of analysis, we summarize the total profit of the two firms in each equilibrium (denoted by Π^*, which is equal to the sum of π_1^* and π_2^* in Table 1) in Table 2.

Proposition 4. *The total profit of two firms for (F,F) equilibrium is always less than that for (F,D) or (D,F) equilibriums.*

Table 2
Total profit of the two firms.

Technology equilibriums	Π^*
(*F*,*F*)	$\frac{4\alpha^2}{9(1+\lambda)}-f_1^F-f_2^F$
(*D*,*D*)	$\frac{2\alpha^2}{(2+\lambda)^2}$
(*F*,*D*)	$\frac{\alpha^2(17-\lambda)}{36(1+\lambda)}-f_1^F$
(*D*,*F*)	$\frac{\alpha^2(17-\lambda)}{36(1+\lambda)}-f_2^F$

Proposition 4 is easy to prove since

$$\Pi^{(F,F)^*}-\Pi^{(F,D)^*}=\frac{4\alpha^2}{9(1+\lambda)}-\frac{\alpha^2(17-\lambda)}{36(1+\lambda)}-f_2^F=\frac{\alpha^2(\lambda-1)}{36(1+\lambda)}-f_2^F<0, \quad (8)$$

and

$$\Pi^{(F,F)^*}-\Pi^{(D,F)^*}=\frac{4\alpha^2}{9(1+\lambda)}-\frac{\alpha^2(17-\lambda)}{36(1+\lambda)}-f_1^F=\frac{\alpha^2(\lambda-1)}{36(1+\lambda)}-f_1^F<0. \quad (9)$$

Proposition 4 implies that once a firm has invested in flexible technology, it will decrease the total profit of the two firms if the other firm changes from dedicated technology to flexible technology. Thus it is interesting that whether the (*F*,*F*) equilibrium leads to Prisoner's dilemma-like situation to the two firms.

Recall that in the technology game, (*F*,*F*) is an equilibrium for the two firms if $f_1^F<\bar{f}_1$ and $f_2^F<\bar{f}_1$ (see the technology equilibriums of the two firms shown in Figs. 2 and 3). As argued in Röller and Tombak [2], if $f_1^F<\bar{f}_1$ and $f_2^F<\bar{f}_1$, the firms are trapped in a Prisoner's dilemma-like situation, i.e. $\Pi^{(F,F)^*}<\Pi^{(D,D)^*}$. However, in their paper, it is assumed that the two products are substitutes (i.e. $0\le\lambda<1$). If the two products are complements (i.e. $-1\le\lambda<0$), we show that this conclusion is not necessarily valid:

$$\Pi^{(F,F)^*}-\Pi^{(D,D)^*}=\frac{4\alpha^2}{9(1+\lambda)}-\frac{2\alpha^2}{(2+\lambda)^2}-f_1^F-f_2^F=\frac{2\alpha^2(\lambda-1)(2\lambda+1)}{9(1+\lambda)(2+\lambda)^2}-f_1^F-f_2^F. \quad (10)$$

It can be observed from (10) that if $-0.5<\lambda<1$, then $\Pi^{(F,F)^*}-\Pi^{(D,D)^*}<0$; if $-1<\lambda<-0.5$, then total profit for (*F*,*F*) equilibrium may be larger than that for (*D*,*D*) equilibrium if f_1^F and f_2^F are rather small. Note that in the right hand of (10)

$$\frac{2\alpha^2(\lambda-1)(2\lambda+1)}{9(1+\lambda)(2+\lambda)^2}=\frac{2\alpha^2(1-\lambda)}{9(1+\lambda)}\cdot\frac{(-1-2\lambda)}{(2+\lambda)^2}=2\bar{f}_1\cdot\frac{(-1-2\lambda)}{(2+\lambda)^2}. \quad (11)$$

When $-1<\lambda<-0.5$, we have $0<((-1-2\lambda)/(2+\lambda)^2)<1$, thus if $f_1^F+f_2^F\le2\bar{f}_1\cdot((-1-2\lambda)/(2+\lambda)^2)$, $\Pi^{(F,F)^*}-\Pi^{(D,D)^*}\ge0$. This implies that if $f_1^F+f_2^F\le2\bar{f}_1\cdot((-1-2\lambda)/(2+\lambda)^2)$, Prisoner's dilemma-like situation as described in Röller and Tombak [2] does not occur. However, if $f_1^F<\bar{f}_1$, $f_2^F<\bar{f}_1$, but $f_1^F+f_2^F>2\bar{f}_1\cdot((-1-2\lambda)/(2+\lambda)^2)$, then Prisoner's dilemma-like situation does occur. Thus we have the following proposition:

Proposition 5. *Prisoner's dilemma-like situation occurs if and only if* $f_1^F<\bar{f}_1, f_2^F<\bar{f}_1$, and $f_1^F+f_2^F>2\bar{f}_1\cdot((-1-2\lambda)/(2+\lambda)^2)$.

Proposition 5 implies that, when the two products are sufficiently complementary ($-1<\lambda<-0.5$), and the investment cost for flexible technology is not too large, then both firms investing flexible technology will benefit themselves simultaneously: prisoner's dilemma-like situation does not occur.

6. Conclusions

In this paper, we consider the technology–quantity choice of two autonomous duopoly firms in two competitive markets. We model the technology–quantity choice as a two-stage sequential duopoly non-cooperative game of complete information. In the first stage, firms simultaneously choose from a flexible production technology (*F*) and a dedicated technology (*D*). The flexible technology enables the firm to simultaneously produce two products and thus enter two markets; whereas the dedicated technology limits the firm to producing only one product and thus entering one market. In the second stage a Cournot game in quantities is played conditioning on the first-stage choice of technologies. Compared with previous studies, this paper takes a look at the strategic choice of flexible production technology from a more general viewpoint. In this paper, the products can be substitutes or complements, and the investment costs of flexible technology are allowed to be firm-specific. These relaxed assumptions make the results different from previous studies.

Considering both the cases that the two products are substitutes or complements, we have identified the conditions under which each of the four subgame perfect equilibriums (i.e. (*F*, *F*), (*D*, *D*), (*F*, *D*) and (*D*, *F*)) exists. This is different from Röller and Tombak [2] and Kim et al. [11], which consider only the case that the two products are substitutes and concluded that the mixed equilibriums (i.e. (*F*, *D*) and (*D*, *F*)) do not exist.

We find that as the market size increases, the premium a firm is willing to pay for flexibility increases. This is because the larger market encourages more active participation in both markets. We also find that as the product substitutability increases, the premium a firm is willing to pay for flexibility decreases. This is because as the product substitutability increases, the demands of substitutes are more negatively correlated. A firm can earn nearly the same if it produces one product or two substitutable products. However, as the product substitutability decreases, the demands of complements are more positively correlated, which benefits a firm more if the two products are simultaneously produced by flexible technology.

To make the results more visible, we illustrate the main conclusions by a numerical example. As an example, we set $\alpha = 1000$ and assume that $f_1^F = f_2^F$, which means the fixed cost for the investment of flexible technology are the same for both firm 1 and firm 2. We draw the technology equilibriums distribution with different values of $f_1^F(f_2^F)$ and λ in Figs. 4 and 5. These two figures show that as the product substitution parameter increases (i.e. the two products become more substitutable), both firms are less likely to invest in flexible technology but more likely to concentrate on their own and separate markets. We also draw the technology equilibriums distribution with different values of $f_1^F(f_2^F)$ and α in Figs. 6 and 7. These two figures show that as the market size increases, both firms are more likely to invest in flexible technology.

If the two products are complements, we also find that the Prisoner's dilemma-like situation as described in Röller and Tombak [2] does not necessarily occur, i.e. both firms investing in flexible technology may benefit themselves simultaneously.

The model proposed in this paper can be extended in many ways. First, in this paper, the cost of a technology is assumed as a fixed cost, which is irrelevant to the capacity of the equipment. In future research, capacity-related cost of different technologies can be considered and capacity constraints can be included in the production game. Second, demand uncertainty is not taken into

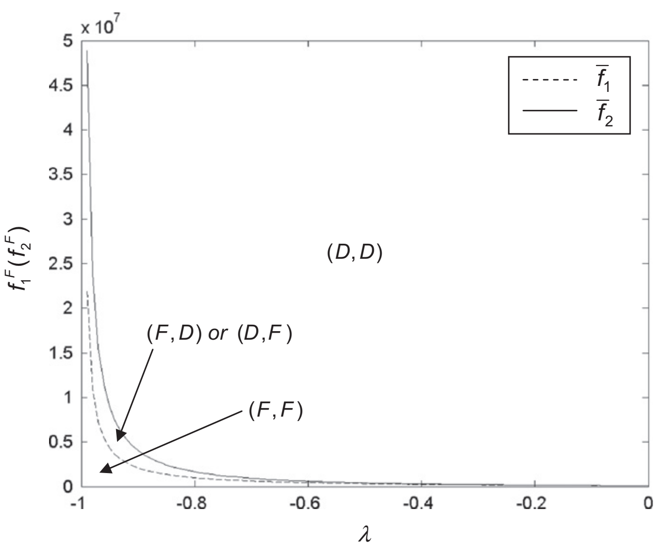

Fig. 4. Technology equilibriums with $\alpha = 1000$ for complementary products ($-1 \leq \lambda \leq 0$).

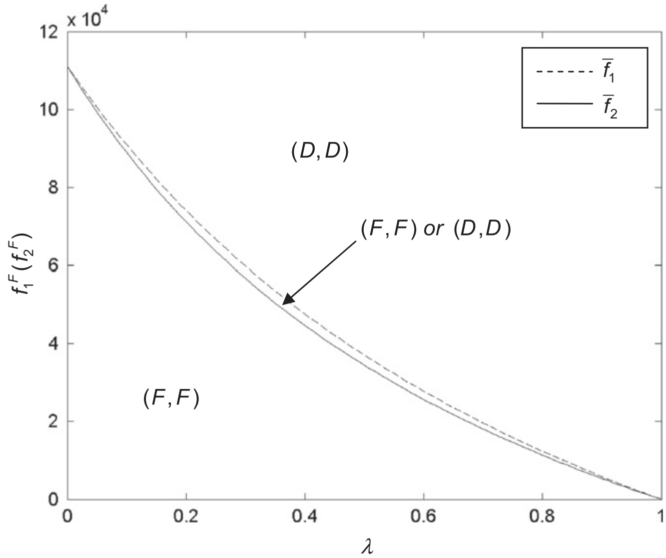

Fig. 5. Technology equilibriums with $\alpha = 1000$ for substitutable products ($0 \leq \lambda \leq 1$).

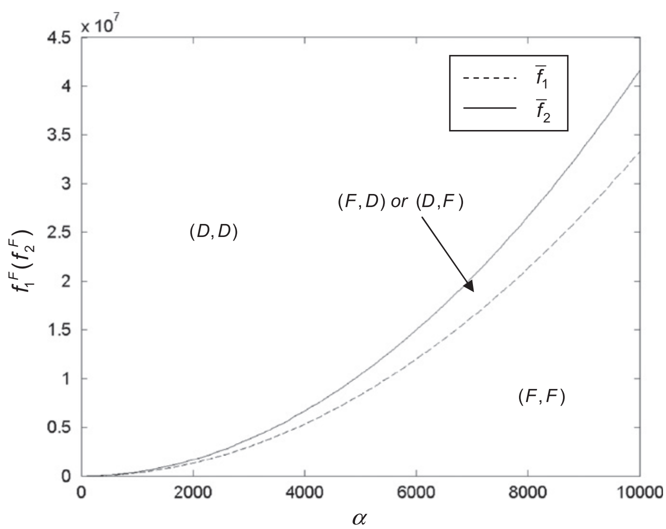

Fig. 6. Technology equilibriums with $0 \leq \alpha \leq 10{,}000$ for complementary products ($\lambda = -0.5$).

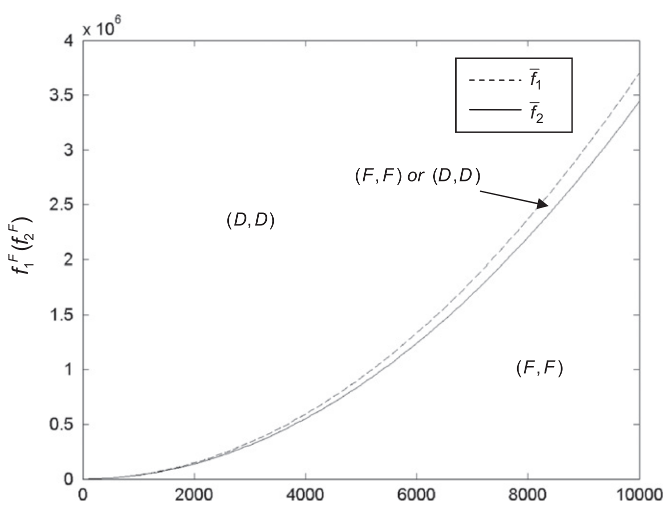

Fig. 7. Technology equilibriums with $0 \leq \alpha \leq 10{,}000$ for substitutable products ($\lambda = 0.5$).

account since we focus on the strategic value of flexibility as a weapon of competition and entering a new market. Demand uncertainty could be included in the model development in future research. Third, we assume that all the parameters are common knowledge in this paper. Information asymmetry can be considered in the future. Also, customer behavior can be further taken into account into the model as described in the introduction part.

Acknowledgements

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Appendix A

Proof of Lemma 1. For π_i to be concave in q_i^A and/or q_i^B $(i=1,2)$, π_i should satisfy the following conditions: (i) $(\partial^2\pi_i/\partial(q_i^A)^2)<0$; (ii) $(\partial^2\pi_i/\partial(q_i^B)^2)<0$; and (iii) $(\partial^2\pi_i/\partial(q_i^A)^2)(\partial^2\pi_i/\partial(q_i^B)^2)\geq(\partial^2\pi_i/\partial q_i^A q_i^B)^2$. Since the proofs for the four technology combinations are analogous, we give the proof of Lemma 1 for (*F*, *F*) technology combination and omit others.

For the objective function (1), we have

$$\frac{\partial\pi_1^{(F,F)}}{\partial q_1^A}=\alpha-2q_1^A-q_2^A-2\lambda q_1^B-\lambda q_2^B,\quad \frac{\partial\pi_1^{(F,F)}}{\partial q_1^B}=\alpha-2q_1^B-q_2^B-2\lambda q_1^A-\lambda q_2^A,$$
$$\frac{\partial\pi_2^{(F,F)}}{\partial q_2^A}=\alpha-2q_2^A-q_1^A-2\lambda q_2^B-\lambda q_1^B,\quad \frac{\partial\pi_2^{(F,F)}}{\partial q_2^B}=\alpha-2q_2^B-q_1^B-2\lambda q_2^A-\lambda q_1^A,$$
$$\frac{\partial^2\pi_i^{(F,F)}}{\partial(q_i^A)^2}=\frac{\partial^2\pi_i^{(F,F)}}{\partial(q_i^B)^2}=-2<0\quad i=1,2.$$

Thus conditions (i) and (ii) hold. Furthermore, $(\partial^2\pi_i^{(F,F)}/\partial(q_i^A)^2)(\partial^2\pi_i^{(F,F)}/\partial(q_i^B)^2)-(\partial^2\pi_i^{(F,F)}/\partial q_i^A q_i^B)^2=4-4\lambda^2\geq 0, i=1,2$. The inequality exists since $\lambda\in[-1,1]$. Thus condition (iii) holds.

Appendix B. Details of the computation in Table 1

We need to compute the maximized profit of each firm in each of the four possible technology combinations.

(1) If both firms choose flexible production technology, then the profits of firm 1 and firm 2 are respectively as follows:

$$\pi_1^{(F,F)}=[\alpha-(q_1^A+q_2^A)-\lambda(q_1^B+q_2^B)]q_1^A+[\alpha-(q_1^B+q_2^B)-\lambda(q_1^A+q_2^A)]q_1^B-f_1^F,$$
$$\pi_2^{(F,F)}=[\alpha-(q_1^A+q_2^A)-\lambda(q_1^B+q_2^B)]q_2^A+[\alpha-(q_1^B+q_2^B)-\lambda(q_1^A+q_2^A)]q_2^B-f_2^F.$$

For firm 1, only q_1^A and q_1^B are decision variables. For firm 2, only q_2^A and q_2^B are decision variables. From the first-order conditions:

$$\frac{\partial\pi_1^{(F,F)}}{\partial q_1^A}=\alpha-2q_1^A-q_2^A-2\lambda q_1^B-\lambda q_2^B=0,$$
$$\frac{\partial\pi_1^{(F,F)}}{\partial q_1^B}=\alpha-2q_1^B-q_2^B-2\lambda q_1^A-\lambda q_2^A=0,$$
$$\frac{\partial\pi_2^{(F,F)}}{\partial q_2^A}=\alpha-2q_2^A-q_1^A-2\lambda q_2^B-\lambda q_1^B=0,$$
$$\frac{\partial\pi_2^{(F,F)}}{\partial q_2^B}=\alpha-2q_2^B-q_1^B-2\lambda q_2^A-\lambda q_1^A=0,$$

we get the optimal production quantities of each firm as $q_1^{A^*}=q_2^{A^*}=q_1^{B^*}=q_2^{B^*}=(\alpha/3(1+\lambda))$, and the profit of each firm as $\pi_1^{(F,F)^*}=(2\alpha^2/9(1+\lambda))-f_1^F$, $\pi_2^{(F,F)^*}=(2\alpha^2/9(1+\lambda))-f_2^F\ldots$

(2) If both firms choose the dedicated manufacturing technology, then the profits of firm 1 and firm 2 are respectively as follows:

$$\pi_1^{(D,D)}=(\alpha-q_1^A-\lambda q_2^B)q_1^A,$$
$$\pi_2^{(D,D)}=(\alpha-q_2^B-\lambda q_1^A)q_2^B.$$

The optimal production quantities of the firms can be obtained by the first-order conditions. Thus we have $q_1^{A^*}=q_2^{B^*}=(\alpha/(2+\lambda))$, and $\pi_1^{(D,D)^*}=\pi_2^{(D,D)^*}=(\alpha^2/(2+\lambda)^2)$.

(3) If firm 1 chooses the flexible production technology, but firm 2 chooses the dedicated manufacturing technology, then the profits of firm 1 and firm 2 are respectively as follows:

$$\pi_1^{(F,D)}=[\alpha-q_1^A-\lambda(q_1^B+q_2^B)]q_1^A+[\alpha-(q_1^B+q_2^B)-\lambda q_1^A]q_1^B-f_1^F,$$
$$\pi_2^{(F,D)}=[\alpha-(q_1^B+q_2^B)-\lambda q_1^A]q_2^B.$$

The optimal production quantities and the profits of the firms are $q_1^{A^*}=(\alpha/2(1+\lambda))$, $q_1^{B^*}=(\alpha(2-\lambda)/6(1+\lambda))$, $q_2^{B^*}=(\alpha/3)$, $\pi_1^{(F,D)^*}=(\alpha^2(13-5\lambda)/36(1+\lambda))-f_1^F$, and $\pi_2^{(F,D)^*}=(\alpha^2/9)$.

(4) If firm 1 chooses the dedicated manufacturing technology, but firm 2 chooses the flexible production technology, then the profits of firm 1 and firm 2 are respectively as follows:

$$\pi_1^{(D,F)}=[\alpha-(q_1^A+q_2^A)-\lambda q_2^B]q_1^A,$$
$$\pi_2^{(D,F)}=[\alpha-(q_1^A+q_2^A)-\lambda q_2^B]q_2^A+[a-q_2^B-\lambda(q_1^A+q_2^A)]q_2^B-f_2^F.$$

The optimal production quantities and the profits of the firms are $q_1^{A^*}=(\alpha/3)$, $q_2^{A^*}=(\alpha(2-\lambda)/6(1+\lambda))$, $q_2^{B^*}=(\alpha/2(1+\lambda))$, $\pi_1^{(D,F)^*}=(\alpha^2/9)$, and $\pi_2^{(D,F)^*}=(\alpha^2(13-5\lambda)/36(1+\lambda))-f_2^F$.

Appendix C

Proof of Proposition 1.

(i) According to the maximized profit values of two firms in Table 1, if firm 1 has chosen the flexible production technology, then firm 2 will also choose the flexibility technology if $\pi_2^{(F,F)^*}>\pi_2^{(F,D)^*}$, i.e. $(2\alpha^2/9(1+\lambda))-f_2^F\ z>(\alpha^2/9)$, or $f_2^F<(\alpha^2(1-\lambda)/\ 9(1+\lambda))$. Let $\overline{f}_1=(\alpha^2(1-\lambda)/9(1+\lambda))$, thus firm 2 will also choose the flexibility technology whenever $f_2^F<\overline{f}_1=\ (\alpha^2(1-\lambda)/9(1+\lambda))$; otherwise, firm 2 should invest in dedicated technology.

(ii) If firm 1 has chosen the dedicated manufacturing technology, then firm 2 will also choose the dedicated technology if $\pi_2^{(D,F)^*}<\pi_2^{(D,D)^*}$, i.e. $(\alpha^2(13-5\lambda)/36(1+\lambda))-f_2^F<(\alpha^2/(2+\lambda)^2)$ or $f_2^F>(\alpha^2(13-5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)$. Let $\overline{f}_2=(\alpha^2(13-\ 5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)$, thus firm 2 will also choose the dedicated technology whenever $f_2^F>\overline{f}_2=(\alpha^2(13-5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)$; otherwise, firm 2 should invest in flexible technology.

(iii) Because firm 1 and firm 2 are symmetric except for their fixed cost of flexible technology, above conclusions are also valid if firm 1 and firm 2 are exchanged.

Appendix D

Proof of Proposition 2.

(1) Recall that $\bar{f}_1=(\alpha^2(1-\lambda)/9(1+\lambda))$, $\bar{f}_2=(\alpha^2(13-5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)$, $-1\le\lambda\le 1$. Take the derivatives of $\bar{f}_1$ and $\bar{f}_2$ to α and λ, respectively, as follows:

$$\frac{\partial\bar{f}_1}{\partial\alpha}=\frac{2\alpha(1-\lambda)}{9(1+\lambda)}\ge 0,\quad \frac{\partial\bar{f}_1}{\partial\lambda}=\frac{-2\alpha^2}{9(1+\lambda)^2}<0.$$

$$\frac{\partial\bar{f}_2}{\partial\alpha}=\frac{\alpha(13-5\lambda)}{18(1+\lambda)}-\frac{2\alpha}{(2+\lambda)^2}=\frac{\alpha[5(1-\lambda^3)+7(1-\lambda^2)+4(1-\lambda)]}{18(1+\lambda)(2+\lambda)^2}\ge 0.$$

$$\frac{\partial\bar{f}_2}{\partial\lambda}=\frac{-\alpha^2}{2(1+\lambda)^2}+\frac{2\alpha^2}{(2+\lambda)^3}=-\frac{\alpha^2[\lambda^2(\lambda+2)+4(1+\lambda)]}{2(2+\lambda)^3(1+\lambda)^2}<0.,$$

(2) If $\lambda=0$, then $\bar{f}_1=(\alpha^2/9)$, and $\bar{f}_2=(13\alpha^2/36)-(\alpha^2/4)=(\alpha^2/9)$. Thus we have $\bar{f}_1=\bar{f}_2$.

(3) If $\lambda=1$, then $\bar{f}_1=0$, and $\bar{f}_2=(\alpha^2/9)-(\alpha^2/9)=0$. Thus we have $\bar{f}_1=\bar{f}_2=0$.

Appendix E

Proof of Proposition 3. To prove Proposition 3, we need to compare the profits to find the conditions for the equilibriums.

From Fig. 1, it is clear that (*F*, *F*) is an equilibrium when the following two conditions are satisfied: (i)$\pi_1^{(F,F)^*}>\pi_1^{(D,F)^*}$, and (ii)$\pi_2^{(F,F)^*}>\pi_2^{(F,D)^*}$. According to Table 1, the two conditions are respectively equal to $(2\alpha^2/9(1+\lambda))-f_1^F>(\alpha^2/9)$ and $(2\alpha^2/9(1+\lambda))-f_2^F>(\alpha^2/9)$, i.e. $f_1^F<(\alpha^2(1-\lambda)/9(1+\lambda))=\bar{f}_1$, and $f_2^F<(\alpha^2(1-\lambda)/9(1+\lambda))=\bar{f}_1$.

(*D*, *D*) is an equilibrium when the following two conditions are satisfied: (i) $\pi_1^{(D,D)^*}>\pi_1^{(F,D)^*}$, and (ii) $\pi_2^{(D,D)^*}>\pi_2^{(D,F)^*}$. According to Table 1, the two conditions are respectively equal to $(\alpha^2/(2+\lambda)^2)>(\alpha^2(13-5\lambda)/36(1+\lambda))-f_1^F$ and $(a^2/(2+\lambda)^2)>(\alpha^2(13-5\lambda)/36(1+\lambda))-f_2^F$, i.e. $f_1^F>(\alpha^2(13-5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)=\bar{f}_2$ and $f_2^F>(\alpha^2(13-5\lambda)/36(1+\lambda))-(\alpha^2/(2+\lambda)^2)=\bar{f}_2$.

(*F*, *D*) is an equilibrium when (i) $\pi_1^{(F,D)^*}>\pi_1^{(D,D)^*}$, and (ii) $\pi_2^{(F,D)^*}>\pi_2^{(F,F)^*}$. According to Table 1, condition (i) is equal to $(\alpha^2(13-5\lambda)/36(1+\lambda))-f_1^F>(\alpha^2/(2+\lambda)^2)$, i.e. $f_1^F<\bar{f}_2$; condition (ii) is equal to $(\alpha^2/9)>(2\alpha^2/9(1+\lambda))-f_2^F$, i.e. $f_2^F>\bar{f}_1$.

(*D*, *F*) is an equilibrium when (i) $\pi_1^{(D,F)^*}>\pi_1^{(F,F)^*}$, and (ii) $\pi_2^{(D,F)^*}>\pi_2^{(D,D)^*}$. According to Table 1, condition (i) is equal to $(\alpha^2/9)>(2\alpha^2/9(1+\lambda))-f_1^F$, i.e. $f_1^F>\bar{f}_1$; condition (ii) is equal to $(\alpha^2(13-5\lambda)/36(1+\lambda))-f_2^F>(\alpha^2/(2+\lambda)^2)$, i.e. $f_2^F<\bar{f}_2$.

If $-1\le\lambda<1$, it is easy to verify that $\bar{f}_1=(\alpha^2(1-\lambda)/9(1+\lambda))>0$, $\bar{f}_2=(\alpha^2[4(1-\lambda)+7(1-\lambda^2)+5(1-\lambda^3)]/36(1+\lambda)(2+\lambda)^2)>0$. Because $\bar{f}_2-\bar{f}_1=(\alpha^2\lambda(\lambda-4)(1-\lambda)/36(1+\lambda)(2+\lambda)^2)$, then if $-1<\lambda<0$, $\bar{f}_2-\bar{f}_1>0$; if $0<\lambda<1$, $\bar{f}_2-\bar{f}_1<0$; if $\lambda=1$, $\bar{f}_2=\bar{f}_1=0$. Thus we have the following conclusions.

To summarize, the conditions for Nash equilibrium are as follows: (i) if $f_1^F<\bar{f}_1$ and $f_2^F<\bar{f}_1$, then (*F*, *F*) is an equilibrium; (ii) if $f_1^F>\bar{f}_2$ and $f_2^F>\bar{f}_2$, then (*D*, *D*) is an equilibrium; (iii) if $f_1^F<\bar{f}_2$ and $f_2^F>\bar{f}_1$, then (*F*, *D*) is an equilibrium; (iv) if $f_1^F>\bar{f}_1$ and $f_2^F<\bar{f}_2$, then (*D*, *F*) is an equilibrium.

Specially, if $\lambda=1$, then only (ii) will occur, which means (*D*, *D*) is the sole equilibrium.

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