Int. J. Production Economics 166 (2015) 85–89

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A note on budget allocation for market research and advertising

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ARTICLE INFO

Article history:
Received 25 August 2014
Accepted 16 April 2015
Available online 14 May 2015

Keywords:
Marketing-operations interface
Advertising
Market research
Newsvendor model

ABSTRACT

Firms that introduce new products often conduct market research to reduce the substantial uncertainty in demand. When a fixed budget is assigned to marketing-oriented activity, investments in market research must be balanced against other advertising expenses. We characterize a firm's optimal marketing and production decisions for a new product. The larger a firm's production cost, the higher is the cost associated with unsold products. Market research increases the forecast accuracy and thus reduces the risk of overage. As a consequence, one might expect that a firm's investment in market research should be higher if it faces higher production costs. Interestingly we find that an increase in the production cost may sometimes lead to a decrease in the optimal investment in market research, even when the marketing budget is not restrictive.

1. Introduction

The nature of new products implies that their demand is highly uncertain. BlackBerry overestimated the popularity of its PlayBook handset, which was released in April 2011. BlackBerry Internet Service also suffered from a massive outage in September 2011. This outage coincided with the announcement of the launch of Apple's iPhone 4S. As a consequence, especially due to the introduction of Apple's iPhone, BlackBerry lost substantial market share and took a $485 million writedown against unsold stock (Arthur, 2012). There are numerous factors, such as the availability of product substitutes/complements, changes in consumer demographics, and the state of the economy, that can affect demand and thus increase uncertainty (Raju and Roy, 2000).

One approach to reduce demand uncertainty is to conduct market research. For example, Walmart, Procter & Gamble, Heineken USA and Levi Strauss & Co. all invest significantly in market research to obtain more precise demand predictions (Aviv, 2001). However, with limited financial resources, firms often have a fixed overall budget for marketing activities (Weber, 2002; Lachowetz et al., 2009; Fischer et al., 2011). Therefore investments in market research (to reduce demand uncertainty) have to be balanced against other marketing activities, such as advertising (to support the scale of market demand).

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http://dx.doi.org/10.1016/j.ijpe.2015.04.013
0925-5273/

Decisions regarding the marketing mix have direct implications on a firm's production decisions, and failure to coordinate the decisions along these dimensions can have severe financial consequences. For example, HP invested in advertisements to sell its Touch Pad which then was more popular than expected and stocked out quickly, leaving customers complaining (Phones Review, 2011). Similarly, after extensive advertisements for its products, Apple was surprised to see demand surpass its rosiest sales predictions, leading to shortages (Bertolucci, 2010).

Due to the complex interactions between marketing and operations, determining optimal marketing and production decisions is anything but simple. Managers often refer to simple heuristics (Bigne, 1995); however, empirical evidence shows that the results are far from optimal (Weber, 2002; Fischer et al., 2011). Therefore, research is needed on how marketing spending should be allocated at the strategic level (Marketing Science Institute, 2010).

There is wide research that investigates the interface between marketing and operations. Tang (2010) and Martínez-Costa et al. (2013) provide detailed classifications of mathematical models used in this research stream. In particular, our paper relates to the literature that studies how advertising increases demand under uncertainty, in a newsvendor setting. Khouja and Robbins (2003) and Lee and Hsu (2011) examine the impact of advertising when demand is an increasing and concave function of advertising expense. Ma et al. (2013) consider a setting where the supply chain consists of a manufacturer and a retailer, either of which can be the Stackelberg leader. Investigating the effort that these two firms exert under different strategies, they find that if one firm does not commit to an effort level, the other firm reduces its level of effort. Lee (2014) considers a setting where the decision maker uses an order-up-to policy when facing stationary and non-stationary demands which are auto-correlated. The firm sets inventory levels using a Bayesian averaging technique. Mesak et al. (2015) investigate the effects of two competing firms' marketing investments and order quantities in an asymmetric duopoly. They find that marketing investments have a larger impact than quantity decisions and that the optimal behavior is strongly affected by firm size. They also show that when the two competitors cooperate, the larger firm continues to invest in marketing, whereas the smaller firm reduces its marketing investment. Our paper also relates to the literature that studies how market research can reduce demand uncertainty. Raju and Roy (2000) considered a scenario where competing firms forecast demand using market information-gathering techniques. They find that, when demand uncertainty is large, improvements in a firm's forecast precision have a large positive effect on the firm's profit. Hess and Lucas (2004) analyzed how market research affects decisions that are made when only prior information is used. They modeled this process using the current level of knowledge and the expected outcomes of market research. Gal-Or et al. (2008) studied whether a manufacturer should share demand information with its retailers and how this decision affects prices when various firms in the system have differing degrees of accuracy concerning their demand signals. They found that when transmitting information is costly, the manufacturer may share information only with the less-informed retailer. When budgets are low, Christen et al. (2009) investigated whether a manufacturer should learn a great deal about a narrow range of markets or learn a little about many markets. They considered a scenario where a firm has an a priori belief about market demand and conducts market research to update these beliefs using a Bayesian approach. They found that focusing the resources on a few markets is optimal when processing information is costly. Concentrating on a few markets can also be optimal when the firm has accurate prior knowledge about the unknown parameters and when it is efficient in processing information.

In this paper, we consider a firm that is selling a product with uncertain demand. The firm has an initial demand estimate and it needs to decide how to allocate its marketing budget between two investments. It can invest in market research to increase the accuracy of its forecast, or in advertising to increase the expected demand of the product. In addition, the firm must decide how many units to produce for the selling season.

We find that the optimal investments in the two marketing activities are strongly related to the profitability of the product and the financial consequences of failing to match supply and demand. Specifically, the optimal investment in advertising should be small when stockouts are costly, whereas investments in market research are less valuable when the product profit margin is large. A firm should invest less in market research when the production cost is small, since then a large production quantity is more effective in mitigating the consequences of demand uncertainty. However, even when a firm's marketing budget is not restrictive, we find that the investment in market research can decrease in the production cost. This is because market research minimizes the consequences of demand-supply mismatch. When the production cost is large, a firm incurs more cost due to any unsold products, but at the same time, since the profit margin is smaller, the financial consequences of stockouts are small. Therefore market research may at times be less valuable if production costs are higher.

2. Mathematical model

A manufacturer produces products at a per-unit production cost c for sale at a unit retail price r. Demand for the product, D, is uncertain and the firm needs to decide the number of units to produce for the selling season, q. In case of a shortage, the firm faces a unit penalty cost g. Any products unsold at the end of the season can be salvaged at a per-unit value v. To avoid trivial or nonsensical scenarios, we assume $r > c > v \geq 0$ and $g \geq 0$.

The demand D is distributed following a distribution $F_{\mu,\sigma}(D)$, with mean μ and standard deviation σ. We use $f_{\mu,\sigma}$ and $F^{-1}_{\mu,\sigma}$ to denote the corresponding density and inverse function, respectively. Similar to Zhang (2005) and Yan and Zhao (2011), we make the following assumption.

Assumption 1. $F_{\mu,\sigma}(D)$ belongs to the location-scale family.

The location-scale family is a family of distributions parametrized by a location parameter (e.g., mean) and a scale parameter (e.g., standard deviation). Many distributions, such as uniform, beta, triangular, normal, lognormal, weibull and gamma distributions, belong to the location-scale family.

Assumption 2. $\lim_{p\to 0} pF^{-1}_{\mu,\sigma}(p) = \lim_{p\to 1}(1-p)F^{-1}_{\mu,\sigma}(p) = 0$.

Assumption 2 requires that the two tails of the distribution are sufficiently thin; this assumption can be shown to hold for many different distributions (for example, uniform, beta and symmetrical triangular distributions).

The firm decides to invest in advertising, $i_a \geq 0$, to increase the expected demand $\mu(i_a)$ and in market research, $i_{mr} \geq 0$, to reduce the standard deviation $\sigma(i_{mr})$. Specifically, we assume investment in advertising increases the expected demand ($\mu'(i_a) > 0$), and investment in market research reduces the standard deviation ($\sigma'(i_{mr}) < 0$), and that both investments have decreasing marginal effects ($\mu''(i_a) < 0$ and $\sigma''(i_{mr}) > 0$). Consistent with the related literature (e.g., Raju and Roy, 2000; Ataman et al., 2008; Gal-Or et al., 2008; Christen et al., 2009), we assume that advertising only changes the expected demand, that the firm's forecast is unbiased, and that market research does not change the expected value. The firm decides the marketing investments i_a and i_{mr}, given a constrained budget B. It also decides the production quantity q. Hence, the firm faces the optimization problem:

$$\max_{i_a, i_{mr}, q} \pi = rE[\min(D,q)] - cq - gE[D-q]^+ + vE[q-D]^+ - i_a - i_{mr} \tag{1}$$

$$\text{s.t. } i_a + i_{mr} \leq B$$

where E is the expectation over demand.

3. Analysis

For any given marketing investments, we have the standard newsvendor problem, so

$$q = F^{-1}_{\mu(i_a),\sigma(i_{mr})}(k) \quad \text{and}$$

$$\pi = -g\mu(i_a) + (g+r-v)\int_{-\infty}^{F^{-1}_{\mu(i_a),\sigma(i_{mr})}(k)} y f_{\mu(i_a),\sigma(i_{mr})}(y)\, dy - i_{mr} - i_a,$$

where $k = (g+r-c)/(g+r-v)$.

In order to analyze the firm's optimal marketing strategy, we standardize the distribution to one with a mean of zero and a standard deviation of one. Lemma 1 provides the production quantity and the expected profit based on such a standardized distribution.

Lemma 1. *Optimal production quantity and expected profit.*

1. $q = \mu(i_a) + F^{-1}_{0,1}(k)\sigma(i_{mr})$
2. $\pi = (r-c)\mu(i_a) + (g+r-v)\left(\int_{-\infty}^{F^{-1}_{0,1}(k)} y f_{0,1}(y)\, dy\right)\sigma(i_{mr}) - i_a - i_{mr}$

The term $F_{0,1}^{-1}(k)$ is the well-known safety factor, that is, the number of standard deviations by which the production quantity differs from expected demand (cf. Theorem 3 of Petruzzi and Dada (1999)).

The first term in the profit function is the profit associated with expected demand, whereas the second term reflects the cost of uncertainty (cf. Theorem 5 of Petruzzi and Dada (1999)). The integral $\int_{-\infty}^{F_{0,1}^{-1}(k)} y f_{0,1}(y)\, dy$ is a non-positive convex function in k with a minimum at $F_{0,1}(0)$ (cf. Lemma 2 in the Appendix), so the cost of demand uncertainty is largest at a critical fractile $k = F_{0,1}(0)$, which equals 50% for all symmetric distributions.

Using superscripts N and B to denote the optimal investments for the case where the budget is not binding and where it is binding, respectively, Propositions 1 and 2 characterize the firm's optimal marketing investment strategy.

Proposition 1. *The optimal marketing mix is* $i_a^* = \min(i_a^N, i_a^B)$ *and* $i_{mr}^* = \min(i_{mr}^N, i_{mr}^B)$, *where* i_a^N, i_{mr}^N, i_a^B *and* i_{mr}^B *are uniquely defined by*

$$(r-c)\mu'(i_a^N) = 1, \tag{2}$$

$$(g+r-v)\sigma'(i_{mr}^N) \int_{-\infty}^{F_{0,1}^{-1}(k)} y f_{0,1}(y)\, dy = 1, \tag{3}$$

$$(r-c)\mu'(i_a^B) = (g+r-v)\sigma'(B-i_a^B) \int_{-\infty}^{F_{0,1}^{-1}(k)} y f_{0,1}(y)\, dy, \text{ and} \tag{4}$$

$$i_{mr}^B = B - i_a^B. \tag{5}$$

Proposition 2. *Sensitivity analysis of the optimal investments.*

1. $\frac{\partial i_a^*}{\partial r} > 0$ *and* $\frac{\partial i_{mr}^*}{\partial r} > 0 \Leftrightarrow i_a^N + i_{mr}^N < B \Leftrightarrow r < \hat{r}$
2. $\frac{\partial i_a^*}{\partial c} < 0$ *and* $\frac{\partial i_{mr}^*}{\partial c} < 0$ *or* $\frac{\partial i_{mr}^*}{\partial c} > 0$
3. $\frac{\partial i_a^*}{\partial g} \le 0$ *and* $\frac{\partial i_{mr}^*}{\partial g} > 0$
4. $\frac{\partial i_a^*}{\partial v} \ge 0$ *and* $\frac{\partial i_{mr}^*}{\partial v} < 0$

A firm's optimal advertising expenditures are larger when the retail price is high or the production cost is low; for higher profit margins, scaling demand simply is more profitable. The parameters that determine the cost of uncertainty, the stockout penalty cost g and the salvage value v, do not affect the optimal investment in advertising, unless the marketing budget is restrictive. When the marketing budget is limited, the firm would be better off allocating a larger share of its budget to market research, especially when the penalty cost g is high or when the salvage value v is low.

On the other hand, market research is relatively less valuable than advertising when the product sells at a high retail price r; a firm then should allocate less of a small budget to market research.

The impact of unit production cost on the optimal marketing mix cannot unambiguously be determined. In Fig. 1, we show optimal investment decisions in advertising and market research to illustrate the counteracting effects underlying this observation. Assume $r=5, g=3, v=1, B=1$. Moreover, assume that the demand follows a gamma distribution, where

$$\mu(i_a) = \mu_0 + \frac{i_a}{1+i_a} \text{ and}$$
$$\sigma(i_{mr}) = \sigma_0 - \frac{i_{mr}}{1+i_{mr}}.$$

When the production cost is relatively low ($c < 1.5$), the higher unit profit margin induces a larger investment into advertising to increase expected demand. Even though the marketing budget is not restrictive, the firm should only invest a small amount, or even not invest at all, in market research. This is because low production cost here implies low cost of overage, thus producing a larger quantity is a more efficient way to avoid stockouts. The low cost of demand uncertainty does not warrant significant market research. When the production cost is intermediate ($1.5 < c < 3$), both optimal investments are large, and the marketing budget constraint is binding. For even higher production cost ($c > 3$), the profit margin is small, so the firm should limit its advertising.

When the production cost is high, the firm incurs a larger cost for any unsold product, so the firm has incentive to invest more into market research to reduce demand uncertainty. When the firm has sufficient marketing budget, intuition suggests that the optimal investment in market research should increase in the production cost to improve forecast accuracy and reduce overage. Indeed, when the production cost is small, we find that a firm should invest less into market research, since increasing the production quantity is a cost-effective way of mitigating the consequences of demand uncertainty. However, interestingly, we find that the optimal investment in market research can also be small when the production cost is high, even when the firm has sufficient marketing budget. This is because a firm performs market research to reduce the consequences of demand-supply mismatch. When the production cost increases, unsold products imply a higher overage cost, but the financial consequences of stockouts decrease because of the smaller profit margin. Therefore the total benefit of market research may at times decrease as production costs grow.

4. Robustness of results

In the main model, we assumed that advertising only changes the expected demand, that the firm's forecast is unbiased, and that market research does not affect expected demand. While these assumptions are commonly made in the literature (e.g., Raju and Roy, 2000; Ataman et al., 2008; Gal-Or et al., 2008; Christen et al., 2009), to examine the robustness of our results with respect to these assumptions, we also considered the following more general model, where advertising and market research each have effects on both the scale of demand and its uncertainty (standard deviation).

$$\mu(i_{mr}, i_a) = \mu_0 + \frac{i_{mr}}{1+i_{mr}}\mu_{mr} + \frac{i_a}{1+i_a} \quad \text{and}$$
$$\sigma(i_{mr}, i_a) = \sigma_0 - \left(\frac{i_{mr}}{1+i_{mr}} + \frac{i_a}{1+i_a}\right)\sigma_a.$$

Note that this functional form satisfies the assumed properties that investments in advertising increase expected demand ($\mu'(i_a) > 0$)of demand ($\sigma'(i_{mr}) < 0$), and both investments have decreasing marginal effects ($\mu''(i_a) < 0$ and $\sigma''(i_{mr}) > 0$). We also considered several other functional forms to describe the impact of marketing activities on demand and made very similar observations.[1] Based on a base case with $\mu_{mr} = 0.1, \sigma_a = 0.1, g = 3, r = 5, v = 1$ and $B = 1$, we varied the parameters $\mu_{mr} \in \{0.05, 0.1, 0.15\}$, $\sigma_a \in \{0.05, 0.1, 0.15\}, g \in \{1, 3, 5\}, r \in \{4, 5, 6\}, v \in \{0, 1, 2\}$ and $B \in \{0.75, 1, 1.25\}$, one at a time. We also considered three demand distributions: uniform, normal and gamma. Across the resulting scenarios,[2] we observed that a firm's optimal investment in market research generally is large when the production cost is large. However, when the consequences of stockouts are much smaller than those of overage, or when market research is not every effective, the optimal investment in market research can be small even when the production cost is large (cf. Fig. 1).

[1] For example, we found that our insights are robust to the following two functional forms: (1) $\mu(i_{mr}, i_a) = \mu_0 + \mu_{mr}\sqrt{i_{mr}} + \sqrt{i_a}$; $\sigma(i_{mr}, i_a) = \sigma_0 - \left(\sqrt{i_{mr}} + \sigma_a\sqrt{i_a}\right)$, (2) $\mu(i_{mr}, i_a) = \mu_0 + \mu_{mr} \log i_{mr} + \log i_a$; $\sigma(i_{mr}, i_a) = \sigma_0 - (\log i_{mr} + \sigma_a \log i_a)$.

[2] The results of these studies are available from the authors upon request.

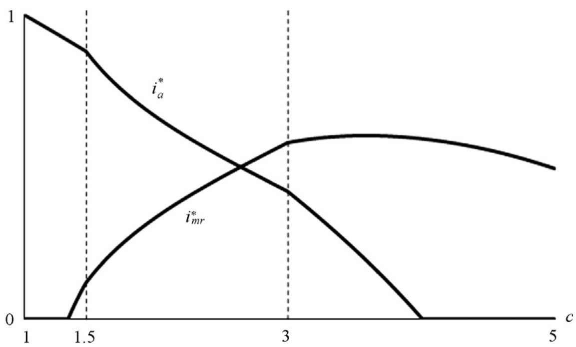

Fig. 1. Sensitivity of the investments with respect to the unit production cost (c).

5. Conclusion

We consider a firm's marketing budget allocation and production quantity decisions. We find that investments in market research should be large when failing to match supply and demand has significant financial consequences. However, when the product has a large profit margin, the firm should allocate more of its marketing budget to advertising. Interestingly, with a low production cost, a firm should not necessarily invest heavily in market research, but it should rather increase its production quantity to avoid underage. With a low overage cost, a strategy of matching supply and demand through such a production quantity adjustment might often be more cost efficient than investments in market research initiatives.

Appendix

Proof of Lemma 1. $f_{\mu(i_a),\sigma(i_{mr})}(y)=\frac{1}{\sigma(i_{mr})}f_{0,1}\left(\frac{z-\mu(i_a)}{\sigma(i_{mr})}\right)$, so $\int_{-\infty}^{z} yf_{\mu(i_a),\sigma(i_{mr})}(y)\,dy=\int_{-\infty}^{\frac{z-\mu(i_a)}{\sigma(i_{mr})}}[\sigma(i_{mr})y+\mu(i_a)]f_{0,1}(y)\,dy=\sigma(i_{mr})\int_{-\infty}^{\frac{z-\mu(i_a)}{\sigma(i_{mr})}} yf_{0,1}(y)dy+\mu(i_a)F_{0,1}\left(\frac{z-\mu(i_a)}{\sigma(i_{mr})}\right)$. $F^{-1}_{\mu(i_a),\sigma(i_{mr})}(k)=\mu(i_a)+F_{0,1}^{-1}(k)\sigma(i_{mr})$, so $\int_{-\infty}^{F^{-1}_{\mu(i_a),\sigma(i_{mr})}(k)} yf_{\mu(i_a),\sigma(i_{mr})}(y)\,dy=\sigma(i_{mr})\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy+\mu(i_a)k$. The result then follows.□

Lemma 2. *$\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy$ is a non-positive, convex function with $\int_{-\infty}^{F_{0,1}^{-1}(0)} yf_{0,1}(y)\,dy=\int_{-\infty}^{F_{0,1}^{-1}(1)} yf_{0,1}(y)\,dy=0$, $\frac{\partial}{\partial k}\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\right)=F_{0,1}^{-1}(k)$ and minimum at $k=F_{0,1}(0)$.*

Proof of Lemma 1. $\int_{-\infty}^{z} yf_{0,1}(y)\,dy\le 0$, because $\int_{-\infty}^{\infty} yf_{0,1}(y)\,dy$ is the mean of $F_{0,1}$, which is zero. $\int_{-\infty}^{F_{0,1}^{-1}(0)} yf_{0,}\ 1(y)\,dy=\int_{-\infty}^{-\infty} yf_{0,}\ 1(y)\,dy=0$. $\int_{-\infty}^{F_{0,1}^{-1}(1)} yf_{0,1}(y)\,dy=\int_{-\infty}^{\infty} yf_{0,1}(y)\,dy=0$. Using the chain rule, $\frac{\partial}{\partial k}\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\right)=F_{0,1}^{-1}(k)$, so the minimum is at $k=F_{0,1}(0)$. $\frac{\partial^2}{\partial k^2}\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\right)=\frac{1}{f_{0,1}[F_{0,1}^{-1}(k)]}>0$.

Proof of Proposition 1. The profit function is jointly concave in i_a and i_{mr} because $\frac{\partial^2\pi}{\partial i_a^2}=(r-c)\mu''(i_a)<0$ and $\frac{\partial^2\pi}{\partial i_a^2}\frac{\partial^2\pi}{\partial i_{mr}^2}-\left(\frac{\partial^2\pi}{\partial i_a\partial i_{mr}}\right)^2=(r-c)(g+r-v)\left(\int_{-\infty}^{F_{0,1}^{-1}(0)} yf_{0,1}(y)\,dy\right)\sigma''(i_{mr})\mu''(i_a)>0$. If the budget constraint is binding, the function is still concave since $i_{mr}=B-i_a$ and $\frac{\partial i_{mr}}{\partial i_a}=-1$. The solution at the constraint always gives a (weakly) smaller profit than interior solution. There are four possible orderings: (1) $i_a^N<i_a^B$ and $i_{mr}^N<i_{mr}^B$. The interior investments are feasible and therefore optimal. (2) $i_a^N<i_a^B$ and $i_{mr}^N>i_{mr}^B$. This ordering cannot be part of an optimal solution, since $\frac{\partial^2\pi}{\partial i_a^2}<0$ and $\frac{\partial^2\pi}{\partial i_{mr}^2}<0$, so the firm could increase profit by reducing i_a from i_a^B to $i_a^B-\epsilon$ and increasing i_{mr} from i_{mr}^B to $i_{mr}^B+\epsilon$, where ϵ is a small positive number. (3) $i_a^N>i_a^B$ and $i_{mr}^N>i_{mr}^B$. based on the same logic as in (2), this ordering cannot be part of an optimal solution. (4) $i_a^N>i_a^B$ and $i_{mr}^N>i_{mr}^B$. Since the interior optima are infeasible, the optimal solution is the solution at the constraint.

Proof of Proposition 2. Part 1: $\frac{\partial\pi}{\partial i_a}=(r-c)\mu'(i_a)-1$, and $\frac{\partial^2\pi}{\partial i_a\partial r}=\mu'(i_a)>0$, so $\frac{\partial i_a^N}{\partial r}>0$. $\frac{\partial\pi}{\partial i_{mr}}=(g+r-v)\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}\ (y)\,dy\right)\sigma'(i_{mr})-1$. From Lemma 2 we have $\frac{\partial}{\partial r}\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}\ (y)\,dy\right)=\frac{\partial}{\partial k}\left(\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\right)\frac{\partial k}{\partial r}=\frac{c-v}{(g+r-v)^2}F_{0,1}^{-1}(k)$, so $\frac{\partial^2\pi}{\partial i_{mr}\partial r}=X_1\sigma'(i_{mr})$, where $X_1=\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy+(1-k)F_{0,1}^{-1}(k)$. Since $\sigma'(i_{mr})<0$, we have $\frac{\partial^2\pi}{\partial i_{mr}\partial r}>0\Leftrightarrow\frac{\partial i_{mr}^N}{\partial r}>0\Leftrightarrow X_1<0$. Using $\frac{\partial F_{0,1}^{-1}(k)}{\partial k}=\frac{1}{f_{0,1}[F_{0,1}^{-1}(k)]}>0$ from Lemma 2, we have $\frac{\partial X_1}{\partial r}=\frac{(c-v)^2}{(g+r-v)^3}\frac{1}{f_{0,1}[F_{0,1}^{-1}(k)}\}$, which is positive. Since X_1 is increasing in r, there is at most one $\bar{r}$ such that $\frac{\partial i_{mr}^N}{\partial r}<0\Leftrightarrow r>\bar{r}$. When r is small, k is small and $(1-k)F_{0,1}^{-1}(k)$ is negative. Since $\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\le 0$ (cf. Lemma 2), $X_1<0$ when r is sufficiently small. Note that $\lim_{r\to\infty}k=1$ and $\int_{-\infty}^{F_{0,1}^{-1}(1)} yf_{0,1}(y)\,dy=0$ (cf. Lemma 2). Since $F_{0,1}^{-1}(k)>0$, the lower bound of $\lim_{k\to 1}(1-k)F_{0,1}^{-1}(k)$ is zero. Hence, for any distribution that satisfies Assumption 1, we have $X_1=0$. Thus, r does not exist (i.e., $\bar{r}=\infty$) and $\frac{\partial i_{mr}^N}{\partial r}>0$. Since $\frac{\partial i_a^N}{\partial r}>0$ and $\frac{\partial i_{mr}^N}{\partial r}>0$, there exists a unique $\hat{r}$ such that $i_a^N+i_{mr}^N<B\Leftrightarrow r<\hat{r}$. Consider the constrained solution. $\frac{\partial^2\pi}{\partial i_a\partial r}=\mu'(i_a)-X_1\sigma'(B-i_a)$, so $\frac{\partial^2\pi}{\partial i_a\partial r}>0\Leftrightarrow\frac{\partial i_a^B}{\partial r}>0\Leftrightarrow X_1>\frac{\mu'(i_a)}{\sigma'(B-i_a)}$, because $\sigma'(B-i_a)<0$. Proof of non-existence: Recall from 4 that $\frac{\mu'(i_a)}{\sigma'(B-i_a)}=\frac{g+r-v}{r-c}\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy$. Therefore, we need to show that $\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy+(1-k)F_{0,1}^{-1}(k)>\frac{g+r-v}{r-c}\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy\Leftrightarrow$

$$(1-k)F_{0,1}^{-1}(k)>\frac{g+c-v}{r-c}\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)dy \tag{6}$$

When $g=0$, 6 $\Leftrightarrow kF_{0,1}^{-1}(k)>\int_{-\infty}^{F_{0,1}^{-}} 1(k)yf_{0,1}(y)\,dy\Leftrightarrow xF_{0,}\ 1(x)>\int_{-\infty}^{x} yf_{0,1}(y)\,dy\Leftrightarrow\int_{-\infty}^{x}(x-y)f_{0,1}(y)\,dy>0$ g297 , which is true because $f_{0,1}>0$. When $g>0$, 6 becomes

$$(c-v)(r-c)F_{0,1}^{-1}(k)>(g+c-v)(g+r-v)\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy \tag{7}$$

Since 6 is true when $g=0$, 7 is also true when $g=0$. The left-hand side of 7 increases in g because $\frac{\partial F_{0,1}^{-1}(k)}{\partial g}=\frac{c-v}{(g+r-v)^2}\frac{1}{f_{0,1}[F_{0,1}^{-1}(k)]}>0$. The first term of the right-hand side of 7, $g+c-v$, is positive and increasing in g. The second term, $(g+r-v)\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy$, is non-positive and decreasing: the derivative with respect to g *is* $X_2=\int_{-\infty}^{F_{0,1}^{-1}(k)} yf_{0,1}(y)\,dy+(1-k)F_{0,1}^{-1}(k)$ and $\lim_{k\to 1}X_2=0$ by Assumption 2; since $\frac{\partial X_2}{\partial k}=\frac{1-k}{f_{0,1}[F_{0,1}^{-1}(k)]}>0, X_2<0$.As a product of a positive increasing function and a negative decreasing function, the RHS of 7 decreases in g, and 6 is true for all g.

Part 2: $\frac{\partial^2\pi}{\partial i_a\partial c}=-\mu'(i_a)<0$, so $\frac{\partial i_a^N}{\partial c}<0$. $\frac{\partial i_{mr}^N}{\partial c}>0\Leftrightarrow\frac{\partial^2\pi}{\partial i_{mr}\partial c}>0\Leftrightarrow F_{0,1}^{-1}(k)>0\Leftrightarrow c<g+r-(g+r-v)F_{0,1}(0)$. $g+r-(g+r-v)F_{0,1}\ (0)>v.g+\ r-(g+r-v)F_{0,1}(0)>r\Leftrightarrow F_{0,1}(0)<\frac{g}{g+r-v}$. Therefore, $\frac{\partial i_{mr}^*}{\partial c}$ can either be positive or negative.

Part 3: $\frac{\partial^2\pi}{\partial i_a\partial g}=0$, so $\frac{\partial i_a^N}{\partial g}=0$. The proof for $\frac{\partial i_{mr}^N}{\partial g}>0$ follows the same logic as the proof in part a and thus is omitted. Since $\frac{\partial i_a^N}{\partial g}=0$ and $\frac{\partial i_{mr}^N}{\partial g}>0$, under a tight budget constraint we have $\frac{\partial i_a^B}{\partial g}<0$ and $\frac{\partial i_{mr}^B}{\partial g}>0$.

Part 4: $\frac{\partial^2 \pi}{\partial i_a \partial v} = 0$, so $\frac{\partial i_a^N}{\partial v} = 0$. $\frac{\partial i_{mr}^N}{\partial v} > 0 \Leftrightarrow \frac{\partial^2 \pi}{\partial i_{mr} \partial v} > 0 \Leftrightarrow X_3 < 0$ where $X_3 = -\int_{-\infty}^{F_{0,1}^{-1}(k)} y f_{0,1}(y)\, dy + k F_{0,1}^{-1}(k)$. Since $\frac{\partial X_3}{\partial v} = \frac{1}{g+r-v} \frac{k^2}{f_{0,1}[F_{0,1}^{-1}(k)]} > 0$, there is at most one $\overline{v}$ such that $\frac{\partial i_{mr}^N}{\partial v} < 0 \Leftrightarrow v > \overline{v}$. However, $X_3 = F_{0,1}^{-1}(1) > 0$ when $v = c$ and $\lim_{v \to -\infty} X_3 = 0$ by Assumption 2, so $\frac{\partial i_a^N}{\partial v} = 0$ and $\frac{\partial i_{mr}^N}{\partial v} < 0$, and under a tight budget constraint, we have $\frac{\partial i_a^B}{\partial v} > 0$ and $\frac{\partial i_{mr}^B}{\partial v} < 0$.

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