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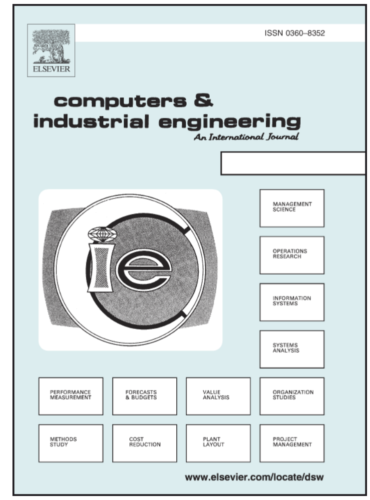

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Reliability evaluation of a multistate flight network under time and stopover constraints

Abstract

From the management perspective, network reliability is a crucial indicator to evaluate the service performance of many real-life networks. This research, therefore, focused on investigating network reliability in the airline industry and proposed an algorithm of evaluating flight network reliability of a multistate flight network (MSFN) considering the time and the number of stopovers. An MSFN is composed of nodes and arcs, in which each node denotes an airport and each arc represents a flight which connects a pair of nodes at fixed departure and arrival time. Flight network reliability is defined as the probability of successfully carrying a certain number of passengers from the origin to the final destination under the constraints of time and the number of stopovers. We first model the flight system into an MSFN and then generate all minimal capacity vectors that can satisfy demand under the time and stopovers constraints. Significantly, this study developed a searching method instead of giving all minimal flightpaths in advance and computed flight network reliability efficiently based on all the minimal capacity vectors. A case study from Ho Chi Minh City to Taipei is presented to demonstrate the solution procedure. The findings from this study are convinced to contribute equivalent information to policymakers and airlines executives for their strategic decision-making regarding flight network and thereby contribute to advancement towards sustainability.

Keywords: *flight network reliability; time constraint; stopovers constraint; minimal flightpaths; backtracking.*

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1. Introduction

In recent decades, along with rapid industrialization and economic growth, individual spending power has increased noticeably. In addition, people prefer travelling by air to save time (Strategyand, 2015), leading to a much faster increase in demand for air transport services than most other goods and services in the world economy. Indeed, the total revenue of the aviation industry has doubled from US$369 billion in 2004 to US$746 billion in 2014, and since 1970, air travel demand measured by revenue passenger kilometers flown has risen tenfold, compared to a three-to-fourfold expansion of the world economy (IATA, 2014). With such significant growth of the global aviation industry, airlines tend to design more efficient operations with a high level of reliability. To support for management in the aviation sector, scholars have proposed efforts for answering the following questions: "Is the total cost minimized?", "How long has the aircraft rotation been delayed?" and "Has the passenger's satisfaction been guaranteed?". Particularly, these scholars have mainly focused on optimizing the flight scheduling problem (Ovacikt & Uzsoy, 1994; Wu, 2006; Atkin Jason, 2007; Papadakos 2009, Clare & Richards 2011; Lee et al., 2011; Bae et al., 2013; Dong et al., 2016), fleet assignment problem (Sandhu & Klabjan, 2007; Bae et al., 2013; Dong et al., 2016), aircraft routing problem (Wu and Caves, 2002; Weideet al., 2010; Awasthi et al., 2013; Tjahjono et al., 2014; Zhang et al., 2015), and crew management (Sandhu & Klabjan, 2007; Weide et al., 2010; Hu et al., 2015; Zhang et al., 2015). Especially, apart from providing new optimal solutions, Wu and Caves (2002), Wu (2006) and Sohoni et al. (2011) evaluated airline performance based on the reliability of flight scheduling and aircraft routing; however, these studies did not take meeting specific demands of a flight network with constrained conditions into performance consideration.

Since network reliability is usually employed as an important index to evaluate the service performance of many real-life systems such as transportation, logistics, and distribution systems

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(Niu, 2012), this research investigates network reliability in the airline industry. Furthermore, to complete the research of Wu and Caves (2002), Wu (2006) and Sohoni et al. (2011), this study concentrates on how a flight system can carry a specific number d of passengers with constraints of time T and number o of stopovers through the concept of network reliability which is called flight network reliability $R_{d,o,T}$ herein. In general, a flight system includes a range of airports and flights, in which a flight transports passengers between a pair of airports with scheduled departure time to arrival times. Such a flight system can be modeled as a flow network consisting of arcs representing flights and nodes representing airports. In fact, most flights are set up to transfer passengers from one place to another in a range of different time schedules, and travel demand will vary with season, destination's attractions, lifestyle, and timing. Therefore, the number of empty seats on different flights will be different depending on the diversity of demand. The flow network characterized by such flights should be multistate owing to the various capacities of each flight. Such a network is a typical stochastic flow network and is called a multistate flight network (MSFN) herein. In general, network reliability is defined as the probability that a stochastic flow network can satisfy a specified demand under various constraints, namely, Lin et al. (2016) estimated the network reliability to meet the required orders composed with different types of products in a flow-shop manufacturing system under time constraint while Chang et al. (2017) focused on reliability evaluation for a multi-state manufacturing network with joint buffer stations and Niu and Xu (2012) defined the reliability of a multi-state system as the probability that amount of items can be transmitted under cost consideration. However, such existed algorithms cannot be applied to the MSFN with stopovers and time constraints. In particular, the MSFN is unique when several flights share only one non-stop route. Flight network reliability is defined as the probability that the MSFN can successfully transport a requested number of passengers from the origin to the final destination under constraints of time and number of

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stopovers. To evaluate flight network reliability, this work develops a new algorithm for generating all minimal capacity vectors that can satisfy demand under the time and stopovers constraints. These minimal capacity vectors are labeled (d,o,T)-LBs herein. With the recursive sum of disjoint products (RSDP) algorithm (Zuo et al., 2007), flight network reliability is computed based on (d,o,T)-LBs. Especially, this work proposes a method for searching all minimal flightpaths. The proposed algorithm in this study, consequently, can be applied even though all minimal flightpaths are not given in advance. Figure 1 summarizes the procedure for evaluating flight network reliability. Additionally, a case study is given to illuminate the solution procedure and a sensitivity analysis is accordingly adopted to discuss the management implication of flight network reliability.

< Insert Figure 1 >

The remainder of this study is organized as follows. In section 2, the terminologies, notation and the MSFN model are provided. The MSFN model consists of assumptions, constraints, object functions and the backtracking method. Section 3 presents a complete description of the proposed algorithm to evaluate flight reliability network. An illustrative example built up from one real MSFN is presented in section 4. Finally, Section 5 states some concluding remarks.

2. Multistate flight network model

In this section, we define the notation, the research problem and assumptions used in this study, backtracking method, and present the MSFN model.

2.1. Notation

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d	demand
N	set of airports (nodes)
s; t	an origin; a final destination
n	number of non-stop route in the route network
a_i	the i^{th} non-stop route connecting a pair of airports
A	$\{a_i \mid i = 1, 2,..., n\}$: set of non-stop routes
t_i	flying time on non-stop route a_i
L	$\{t_i \mid i = 1, 2,..., n\}$: set of flying time on non-stop routes
(**N**, **A, L**):	route network
μ	transit time
o	number of stopovers
T	limited time
b	number of minimal routes satisfying o and T in the route network
P^{β}	the β^{th} minimal route satisfying o and T in the route network
T^{β}	total time on P^{β}
e^{β}	length of P^{β}
b	number of minimal routes satisfying o and T in the route network
w	total number of flights
ω_i^k	k^{th} flight belonging to non-stop route a_i in the flight network for $k = 1, 2, \ldots, w; i \in \{1, 2, ..., n\}$
W	$\{\omega_i^k \mid k = 1, 2, \ldots, w;\ i \in \{1, 2,..., n\}\}$: set of flights
g_i^k	departure time of flight ω_i^k
v_i^k	arrival time of flight ω_i^k

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U	$\{(g_i^k, v_i^k) \mid k = 1, 2,\ldots, w;\ i \in \{1, 2,..., n\}\}$: set of departure and arrival time on flights
m	number of minimal flightpaths satisfying o and T
Q^j	j^{th} minimal flightpath satisfying o and T for $j = 1, 2,\ldots, m$
T_j	travel time on j^{th} minimal flightpath, $j = 1, 2,\ldots, m$
f_j	flow capacity of j^{th} minimal flightpath, $j = 1, 2,\ldots, m$
F	$(f_1, f_2, \ldots, f_m)$: flow vector
$\mathbf{H} = (\mathbf{N}, \mathbf{W}, \mathbf{U}, M)$	multistate flight network
M_i^k	maximum capacity on the flight ω_i^k
x_i^k	$x_i^k \in \{0,1,..., M_i^k\}$: current capacity on the flight ω_i^k
X	$(x_i^k \mid k = 1, 2,\ldots, w;\ i \in \{1, 2,..., n\})$: capacity vector in the MSFN
M	$(M_i^k \mid k = 1, 2,\ldots, w;\ i \in \{1, 2,..., n\})$: maximal capacity vector of the MSFN
(d,o,T)-LB	minimal capacity vector in the MSFN satisfying the demand d under limited number of stopovers o and limited time T
$R_{d,o,T}$	flight network reliability: the probability that the MSFN satisfies the demand d under limited number of stopovers o and limited time T
Ω	set of (d,o,T)-LB candidates
φ	number of (d,o,T)-LB
$\Omega_{\min}$	set of all minimal vectors in Ω

2.2. Research problem description

Basically, this study focuses on evaluating flight network reliability in terms of how a specific number d of passengers travel through an MSFN with constraints of time T and number o of stopovers. First, we model a flight system as an MSFN by building a route network and

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transforming it into an MSFN. In this study, a route network is a network composed of airports, non-stop routes and flying times on such non-stop routes. The MSFN is the detailed format of the route network in which each non-stop route has several flights and each flight shares the same flying time but different departure and arrival times. Particularly, a non-stop route shows a pair of linked airports, while a flight is a journey by air providing carry service on that specific non-stop route. Figure 2 shows an example that indicates the difference between a route network and an MSFN. The route network consists of 3 nodes (1, 2, 3) and 3 non-stop routes (a_1, a_2, a_3). Meanwhile, the MSFN shares the same 3 nodes (1, 2, 3) and 2 flights (ω_1^1, ω_1^2) sharing the same non-stop route a_1; 3 flights ($\omega_2^3, \omega_2^4, \omega_2^5$) sharing the same non-stop route a_2; and 3 flights ($\omega_2^6, \omega_2^7, \omega_2^8$) sharing the same non-stop route a_3.

< Insert Figure 2 >

In the MSFN, the capacity of each flight is defined as the number of empty (available) seats in the flight and the probability of each capacity is calculated according to the history record. A stopover is defined as a short airport stay on a journey. In terms of time, flying time in this study is defined as the necessary time to fly between a pair of airports; in other words, flying time is the time to fly on the non-stop route (considered in the route network) and on the flight (considered in the MSFN). In addition, transit time μ is the period of time necessary to move from an arrival gate to a departure gate at a stopover (airport), and journey travel time is calculated from the departure time at the origin to the arrival time at the final destination.

We then compute flight network reliability via all (d,o,T)-LBs which satisfy the given constraints. To obtain all (d,o,T)-LBs, it is necessary to search all minimal flightpaths satisfying given constraints. Referring to literature (Chang et al., 2017; Lin et al, 2016; Niu, 2012; Yeh,

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2004; Yeh, 2005), the minimal path is defined as a sequence of arcs without cycles. In the route network, a minimal route is thus defined as a sequence of non-stop routes without any cycle. And, a minimal flightpath is defined as a sequence of flights without cycles, in which the departure time of the following flight is not earlier than the arrival time of the previous flight plus transit time in the MSFN.

2.3. Route network, MSFN, and assumptions

Let (**N**, **A**, **L**) denote a route network with **N** representing the set of airports, **A** = $\{a_i|i = 1, 2,..., n\}$ representing the set of non-stop routes where n is the number of non-stop routes, and **L** = $\{t_i|\ i = 1, 2, ..., n\}$is the set of flying times on the non-stop routes. The number t_i is the flying time on the route a_i and the flying time on all flights ω_i^k on the route a_i. Let **H** = (**N**, **W**, **U,** M) be an MSFN with **W** = $\{\omega_i^k \mid k = 1, 2,\dots, w;\ i \in \{1, 2,..., n\}\}$ representing the set of flight, where k is the order of flight and w is the total number of flights, **U** = $\{(g_i^k, v_i^k) \mid k = 1, 2,\dots, w;\ i \in \{1, 2,..., n\}\}$ representing the set of departure time g_i^k and arrival time v_i^k on flight ω_i^k, and $M = (M_i^k \mid k = 1, 2,..., w;\ i \in \{1, 2,..., n\})$ denoting the maximal capacity vector. In the route network (**N**, **A**, **L**), each non-stop route a_i links a pair of airports with no stop while in the MSFN **H** = (**N**, **W**, **U,** M) there are some flights ω_i^k linking the same pair of airports as non-stop route a_i. With the maximal capacity vector M, the capacity vector $X = (x_i^k \mid k = 1, 2,..., w;\ i \in \{1, 2,..., n\})$is defined as the current capacity state of the MSFN **H**, where x_i^k represents the current capacity on the flight ω_i^k with corresponding available capacity.

In this study, the following assumptions are made:

i. The flow satisfy the flow-conservation law (no lost or added passengers in the MSFN)

ii. The capacities of different flights are statistically independent.

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iii. There is no delay occurred in the MSFN.

iv. All flights on the same non-stop route share the same flying time.

Vector operation rules:

$X \le Y$ $(x_i^k \mid k = 1, 2, \ldots, w; i \in \{1, 2, ..., n\}) \le (y_i^k \mid k = 1, 2, \ldots, w; i \in \{1, 2, ..., n\})$ if $x_i^k \le y_i^k$ for all *i*, *k*

$X < Y$ $X \le Y$ and $x_i^k < y_i^k$ for at least one *i*, *k*

$X \ngeq Y$ neither $X \ge Y$ nor $X < Y$

2.4. Flight time

According to the assumption (iv), airline companies set the same flying time for different flights sharing the same non-stop route. Hence, the number t_i is the flying time on the route a_i and all flights ω_i^k on it as expressed in Eq. (1).

$$t_i = v_i^k - g_i^k, \text{ for } k = 1, 2, \ldots, w;\ i \in \{1, 2, ..., n\}. \tag{1}$$

Let $P^\beta = \{P[1], P[2], ..., P[e^\beta] \mid P[z] = a_i\}$ be a minimal route in the route network, which means that all elements $P[z] \in \mathbf{A}$ and $P[1] \ne P[2] \ne ... \ne P[e^\beta]$. The minimal route P^β, which contains no more than (o +1) non-stop routes, satisfies the o constraint.

$$e^\beta \le o + 1. \tag{2}$$

For the route network, there is no departure and arrival time on the route provided, and the total time T^β on minimal route P^β is therefore calculated as follows:

$$T^\beta = \sum_i \{t_i \mid a_i \in P^\beta\} + (e^\beta - 1) * \mu. \tag{3}$$

To satisfy the time-limited T, constraint (4) below is a necessary condition.

$$T^\beta \le T \text{ for any } \beta. \tag{4}$$

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Let $Q^j = \{Q[1], Q[2], ..., Q[e^\beta] \mid Q[z] = \omega_i^k\}$ be one of minimal flightpaths corresponding from the minimal route $P^\beta = \{P[1], P[2], ..., P[e^\beta] \mid P[z] = a_i\}$, in which $Q[z]$ is a flight belonging to $P[z]$ so that Q^j meets the stopover *o* constraint. In the MSFN **H**, each flight has planned and fixed departure and arrival times, passengers can be transported successfully if only if the departure time of $Q[z]$ is not earlier than the arrival time of $Q[z-1]$ added transit time. If $Q[z-1] = \omega_r^h$ and $Q[z] = \omega_i^k$ then:

$$v_r^h + \mu \le g_i^k. \tag{5}$$

In addition, journey travel time is calculated from the departure time at the origin to the arrival time at the final destination. Therefore, the travel time on minimal flightpath Q^j is counted from the departure time of the flight $Q[1]$ to the arrival time of the flight $Q[e^\beta]$. If $Q[1] = \omega_u^l$ and $Q[e^\beta] = \omega_i^k$ then:

$$T_j = v_i^k - g_u^l. \tag{6}$$

The minimal flightpath Q^j meets the time constraint if only if the travel time on Q^j does not exceed stipulated time *T*:

$$T_j \le T. \tag{7}$$

2.5. Backtracking for searching all minimal routes and minimal flightpaths

Based on the fact that computing the number of minimal paths between the two specified nodes in given graph or constrained shortest paths problem is NP-hard (Wu et al., 2017), generating all minimal flightpaths which connect two specified airports under given constraints belongs to the class of NP-hard problem. However, in the MSFN with the complexity given, we employ backtracking method to develop an algorithm generating all minimal flightpaths meeting given constraints. The backtracking algorithm enumerates a set of partial candidates that

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generally could be completed in various ways to give all possible solutions to the given problem (Muniswamy, 2009). The underlying concept is that the partial candidates are represented as the branches of a potential search tree. Each partial candidate is the parent of the candidates that differ from it by a single extension step; the leaves of the tree are the partial candidates that cannot be extended any further. (Muniswamy, 2009). In short, backtracking is applied to find all solutions (or some) to some searching or sorting problems. Particularly, the concept of backtracking method is presented in Figure 3.

<Insert Figure 3>

The searching algorithm is divided into Step 1 and Step 2. In particular, Step 1 aims to search all minimal routes in (**N**, **A**, **L**), which meet o and T constraints, by utilizing the following backtracking method 1.

The backtracking method 1: Search minimal routes meeting o and T constraints in (**N**, **A**, **L**)

Step 1	Set $\beta = 0$, $e^{\beta} = 0$ (β is stock which stores the indexes of minimal route P^{β})
Step 2	Algorithm Check (e^{β})
	For each $P[e^{\beta}] \in P^{\beta} = \{P[1], P[2], ..., P[e^{\beta}-1] \mid P[z] = a_i\}$ do
Step 3	If $P[1] \neq P[2] \neq ... \neq P[e^{\beta}]$ then // without any cycle
Step 4	If $(s \to P[1]) \wedge (P[1] \to P[2]) \wedge ... \wedge (P[e^{\beta}-1] \to P[e^{\beta}]) \wedge (P[e^{\beta}] \to t)$ then
Step 5	If $T^{\beta} \leq T$ where $T^{\beta} = \sum_i \{t_i \mid a_i \in P^{\beta}\} + (e^{\beta}-1) * \mu$ // constraint (4)
	Then $\beta = \beta + 1$ write $P^{\beta} = \{P[1], P[2], ..., P[e^{\beta}]\}$
	If $e^{\beta} < o+1$ then Check ($e^{\beta}+1$) // constraint (2)
Step 6	Backtrack

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Step 7 Backtrack

Step 8 End

Suppose that there are b minimal routes satisfying o and T which are $P^1, P^2, ..., P^b$. From such minimal routes, Step 2 focuses on searching all minimal flightpath Q^j satisfying o and T constraints in the MSFN by applying backtracking method as follows:

The backtracking method 2: Search minimal flightpaths satisfying o and T constraints in MSFN **H**

Step 1 For $\beta = 1$, $P^\beta = \{P[1], P[2], ..., P[e^\beta]|\}$; $j = 0$

Step 2 Algorithm Place (z)

For each $Q[z] \in Q^j = \{Q[1], Q[2], ..., Q[z-1]\}$ for $Q[z-1] = \omega_r^h$ and $Q[1] = \omega_u^l$ do

Step 3 If $P[z] = a_i$ then $Q[z] = \omega_i^k$ //according to **A** and **W**

Step 4 If $v_r^h + \mu \le g_i^k$. for $Q[z-1] = \omega_r^h$ then $Q^j = \{Q[1], Q[2], ..., Q[z]\}$ // constraint (5)

Step 5 If $z = e^\beta$

If $T^j \le T$; $T^j = v_i^k - g_u^l$

then $(j = j+1) \wedge (Q^j = \{Q[1], Q[2], ..., Q[z]\})$ // constraint (7)

Step 6 If $z < e^\beta$ then Place (z +1)

Step 7 Backtrack

Step 8 Backtrack

Step 9 End

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Let Q^1, Q^2,..., Q^m be all minimal flightpaths Q^j satisfying o and T. Obviously, such minimal flightpath Q^j meets constraint (5) and the travel time T_j calculated via Eq. (6) satisfies constraint (7).

2.6. Flow vector and capacity vector

Let $F = (f_1, f_2,..., f_m)$ be a flow vector in the MSFN **H**, where f_j is the flow through the minimal flightpath Q^j =, j = 1, 2,..., m. When the minimal flightpath Q^j fulfills the constraints limited time T and number o of stopovers, the remaining problem is to consider how the flow vector satisfies the demand. Under the assumption (i), no flow is lost in the MSFN and the total flow-in must be equal to the total flow-out for any node (airport) except for the origin and destination airport. Therefore, any flow vector F that satisfies the constraint below meets the demand d.

$$\sum_{j=1}^{m} f_j = d. \tag{8}$$

Moreover, any flow vector F satisfying the following constraint is said to be feasible under M.

$$\sum_j \{ f_j \mid \omega_{i,z}^k \in Q^j \} \le M_i^k. \tag{9}$$

Let $\mathbf{F} = \{F \mid F$ satisfies constraints (8) and (9)$\}$. Any capacity vector $X = (x_i^k \mid k = 1, 2, \ldots, w;$ $i \in \{1, 2, ..., n\})$ is said to fulfill the demand d, limited time T and number o of stopovers if there exists at least one $F \in \mathbf{F}$ such that.

$$x_{i,z}^k \ge \sum_j \{ f_j \mid \omega_{i,z}^k \in Q^j \}. \tag{10}$$

Let $\mathbf{X}$ be the set of such X fulfilling (d, o, T). Flight network reliability $R_{d,o,T}$, defined as the probability that the MSFN sends d passengers under the number o of stopovers and limited time T constraints, is thus $R_{d,o,T} = \Pr\{X \mid X \in \mathbf{X}\}$. Finding all capacity vectors $X \in \mathbf{X}$, then calculating

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the corresponding probability of these vectors, and summing up them will yield the flight network reliability.

2.7. Flight network reliability and (*d*,*o*,*T*)-LB

However, it is inefficient to enumerate all of such X. Instead, we employ the concept of minimal capacity vectors. Let (d,o,T)-LB denote a minimal capacity vector from **X** then flight network reliability becomes $R_{d,o,T} = \Pr\{X : X \geq (d,o,T) - \text{LBs}\}$.

Definition 1: A (d,o,T)-LB X is a minimal capacity vector in **X** if any capacity vector Y such that $Y < X$ does not belong to **X**.

Therefore, **X** = $\{X \mid X$ is greater than or equal to at least one (d,o,T)-LB$\}$ and the flight network reliability is revised to become $\Pr\{X \mid X$ is greater than or equal to at least one (d,o,T)-LB$\}$. We have the following Lemma describing the relationship between a flow vector $F\in$ **F** and a (d,o,T)-LB.

Lemma 1: Let X be a (d,o,T)-LB then there exists an $F\in$ **F** such that

$$x_{i,z}^{k} = \sum_{j} \{ f_j \mid \omega_{i,z}^{k} \in Q^j \}. \tag{11}$$

Proof: Let $X = (x_i^k \mid k = 1, 2, \ldots, w; i \in \{1, 2, ..., n\})$ be a (d,o,T)-LB. Suppose that X has at least one component x_i^k such that $x_i^k > \sum_j \{ f_j \mid \omega_i^k \in Q^j \}$. Set $Y = (y_i^k \mid k = 1, 2, \ldots, w; i \in \{1, 2, ..., n\})$ with $y_i^k = \sum_j \{ f_j \mid \omega_i^k \in Q^j \}$, then $Y \in$ **X**. Base on the vector operation rule $X > Y$, which contradicts to the definition of (d,o,T)-LB.

According to Lemma 1, with any given $F\in$ **F**, we can transform a capacity vector X via Eq. (11) then the set Ω contains all of them. Each such X may be a (d,o,T)-LB - in other words, such X is taken as a (d,o,T)-LB candidate, Ω is thus the set of (d,o,T)-LB candidates. Let $\Omega_{\min}$ denote

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the set of all minimal vectors in Ω. The following two critical results further prove that Ω_{min} is exactly the set of (d,o,T)-LBs.

Corollary 1: Any (d,o,T)-LB X belongs to Ω

Proof: It is trivial from Lemma 1.

Theorem 1: Ω_{min} is exactly the set of (d,o,T)-LBs.

Proof: Suppose that X is a (d,o,T)-LB but $X \notin \Omega_{min}$, then $X \in \Omega$ according to Corollary 1 and there will exist a $Y \in \Omega_{min}$ such that $Y < X$. Then $Y \in \mathbf{X}$ which contradicts to the fact that a (d,o,T)-LB is the minimal vector from $\mathbf{X}$. On the other hand, suppose $X \in \Omega_{min}$ but is not a (d,o,T)-LB, then $X \in \mathbf{X}$ and there exists a (d,o,T)-LB Y such that $Y < X$. That implies $Y \in \Omega$ according to Corollary 1 and thus conflicts that $X \in \Omega_{min}$. Therefore, Ω_{min} is exactly the set of $(d,o,T)-\text{LBs}$.

Suppose that Ω_{min} contains φ capacity vectors in total: X^1, X^2,..., X^φ. Since Ω_{min} is exactly the set of (d,o,T)-LBs based on Theorem 1, such capacity vectors X^1, X^2,..., X^φ are (d,o,T)-LBs. Flight network reliability, thus, becomes $R_{d,o,T} = \Pr\{\bigcup_{j=1}^{\varphi}\{X : X \geq X^j\}\}$. In order to derive $\Pr\{\bigcup_{j=1}^{\varphi}\{X : X \geq X^j\}\}$, there are several methods such as state-space decomposition (Alexopoulos, 1995), inclusion-exclusion (Huisheng & Jingyu, 2012) and RSDP (Zuo et al., 2007) can be utilized, in which the RSDP has higher efficiency for large multistate networks (Zuo et al., 2007). Hence, this paper adopts RSDP to calculate $R_{d,o,T} = \Pr\{\bigcup_{j=1}^{\varphi}\{X : X \geq X^j\}\}$.

3. Algorithm to evaluate flight network reliability

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The following algorithm is developed to generate all (d,o,T)-LBs and then evaluate flight network reliability.

Input: (**N**, **A**), **H** = (**N**, **W**, **U**, M), d, o, and T.

Step 1: (To satisfy o and T constraints) Search all minimal routes P^{β} in the route network.

According to the set of non-stop routes **A** in the route network, we find all possible minimal routes by utilizing the backtracking method 1. Minimal routes P^{β} will be accepted if and only if the following constraints are fulfilled.

$$e^{\beta} \le (o+1), \tag{12}$$

$$T^{\beta} \le T \text{ for any } \beta. \tag{13}$$

Suppose that there are b minimal routes P^{β} satisfying o and T constraints in this step.

Step 2: (To satisfy o and T constraints) Search all minimal flightpath Q^j in the MSFN.

Accepted minimal flightpath Q^j must satisfy constraint (15) and qualify flow rule (14).

$$v_r^h + \mu \le g_i^k. \tag{14}$$

$$T_j \le T \text{ where } T_j = v_i^k - g_u^l. \tag{15}$$

Conduct the backtracking method 2 for each minimal route in turn to gain all m satisfied minimal flightpaths in the MSFN.

Step 3: (To satisfy demand d) Find all feasible flow vectors F.

Find all $F = (f_1, f_2,..., f_m)$ satisfying demand constraint (16)

$$\sum_{j=1}^{m} f_j = d. \tag{16}$$

and check whether each F is a feasible or not based on constraint (17) below:

$$\sum_j \{ f_j \mid \omega_{i,z}^k \in Q^j \} \le M_{i,z}^k. \tag{17}$$

Step 4: Transform each feasible flow vector F into capacity vector X via the following equation.

$$x_{i,z}^k = \sum_j \{ f_j \mid \omega_{i,z}^k \in Q^j \}. \tag{18}$$

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Capacity vector X is called a (d,o,T)-LB candidate.

Step 5: $\Omega = \{X^1, X^2,..., X^{\alpha}\}$ Use the following comparison procedure to fitter out the set $\Omega_{\min}$ from the set Ω.

(5.1) Set $\boldsymbol{\psi} = \emptyset$ ($\boldsymbol{\psi}$ is the stock which stores the indexes of (d,o,T)-LBs, initially, $\boldsymbol{\psi}$ is empty)

(5.2) For $i=1$ to $\alpha \notin \boldsymbol{\psi}$

(5.3) For $j=i+1$ to $\alpha \notin \boldsymbol{\psi}$

(5.4) If $X^i \leq X^j$ then X^j is not a lower capacity vector, $\boldsymbol{\psi} = \boldsymbol{\psi} \cup \{j\}$

Else $X^i > X^j$ then X^i is a lower capacity vector, $\boldsymbol{\psi} = \boldsymbol{\psi} \cup \{i\}$ and go to (5.7).

(5.5) Next j

(5.6) $\Omega_{\min} \leftarrow \Omega_{\min} \cup \{X^i\}$

(5.7) Next i

Step 6: $\Omega_{\min} = \{X^1, X^2,..., X^{\varphi}\}$ is the set of all (d,o,T)-LBs. Use RSDP method to calculate flight network reliability $R_{d,o,T} = \Pr\{\bigcup_{j=1}^{\varphi}\{X : X \geq X^j\}\}$.

Output: $R_{d,o,T}$

4. Case study of flight network between Vietnam and Taiwan

To illustrate the solution process, a real flight case from Ho Chi Minh (SGN) to Taipei (TPE) is demonstrated. Figure 4 shows the route network with a range of stopovers including Hanoi (HAN), Da Nang (DAN), Nha Trang (CRX), Hong Kong (HKG), Kaohsiung (KHH) and Taichung (RMQ). Figure 5 is the transformed MSFN, in which there exist various flights sharing the same non-stop route. For instance, there are two flights ω_4^6 and ω_4^7 on the non-stop route a_4

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from the airport HAN to HKG. The data on capacity with probability, departure time and arrival time of each flight are presented in Table 1. For the case in which the demand of traveling is d = 4 under stopover o = 3 and limited time T = 8, flight network reliability $R_{4,3,8}$ can be obtained by the following steps.

< Insert Figure 4 >

< Insert Figure 5 >

< Insert Table 1 >

4.1. Solution procedure

Input: (**N**, **A**, **L**), **H** = (**N**, **W**, **L**, M), d = 4, o = 3, T = 8, μ = 0.5 hour.

Step 1: Search all minimal routes P^{β} which meet o and T constraints in the route network.

We utilize the backtracking method 1 to search all minimal routes such that the total time and the number of routes satisfy the following requirements. Figure 6 and Table 2 illustrate such generation process.

$$e^{\beta} \leq (3+1), \tag{19}$$

$$T^{\beta} \leq 8 \text{ for any} \beta \text{ where } T^{\beta} = \sum_i \{t_i \mid a_i \in P^{\beta}\} + (e^{\beta} - 1) * 0.5. \tag{20}$$

< Insert Figure 6 >

< Insert Table 2 >

After skimming, we obtain all qualified minimal routes $P^1, P^2, P^3, P^4, P^5, P^6, P^7$.

Step 2: Search all minimal flightpath Q^j satisfying o and T constraints in the MSFN.

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In this step, the backtracking method 2 is applied to find all minimal flightpaths satisfying *o* and *T* constraints. Figure 7 indicates a backtracking tree for minimal flightpaths on the minimal route P^7. When any minimal flightpath conflicts with the rule (21) or constraint (22) or is accepted, we will backtrack.

$$v_i^k + 0.5 \le g_z^h, \tag{21}$$

$$T_j \le 8 \text{ where } T_j = v_i^k - g_u^l. \tag{22}$$

Particularly, we have $v_3^5 + 0.5 = 10:00$ and $g_8^{11} = 9:30$, hence, $v_3^5 + 0.5 \ge g_8^{11}$, which conflicts the flow rule (21) therefore reject and backtrack at ω_8^{11}. With the minimal flightpath $\{\omega_3^1, \omega_8^{12}, \omega_{13}^{20}\}$, we obtain $T_j = v_{13}^{20} - g_3^4 = 17:00 - 7:00 = 10$ (hours) then $T_j > 8$, resulting in the rejection and backtracking. After accepting Q^8, we backtrack to find the remaining minimal flightpaths (Q^9 and Q^{10}).

< Insert Figure 7 >

Repeat the same procedure for the remaining minimal routes; we obtain ten minimal flightpaths satisfying *o* and *T* in total:

$Q^1 = \{\omega_1^1; \omega_4^6; \omega_{10}^{14}\}$; $Q^2 = \{\omega_1^2; \omega_4^7; \omega_{10}^{15}\}$;

$Q^3 = \{\omega_1^1; \omega_4^6; \omega_9^{13}; \omega_{12}^{17}\}$; $Q^4 = \{\omega_2^3; \omega_5^8; \omega_{10}^{14}\}$;

$Q^5 = \{\omega_2^3; \omega_5^8; \omega_9^{13}; \omega_{12}^{17}\}$; $Q^6 = \{\omega_2^3; \omega_6^9; \omega_{12}^{17}\}$;

$Q^7 = \{\omega_2^3; \omega_7^{10}; \omega_{13}^{18}\}$; $Q^8 = \{\omega_3^4; \omega_8^{11}; \omega_{13}^{18}\}$

$Q^9 = \{\omega_3^4; \omega_8^{12}; \omega_{13}^{18}\}$; $Q^{10} = \{\omega_3^5; \omega_8^{12}; \omega_{13}^{18}\}$.

Step 3: All flow vectors *F* need to satisfy the following constraints:

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$$\sum_{j=1}^{10} f_j = 4. \tag{23}$$

$$\begin{aligned}
&\sum_j \{f_j \mid \omega_1^1 \in Q^j\} = f_1 + f_3 \le 2, && \sum_j \{f_j \mid \omega_7^{10} \in Q^j\} = f_7 \le 2,\\
&\sum_j \{f_j \mid \omega_1^2 \in Q^j\} = f_2 \le 2, && \sum_j \{f_j \mid \omega_8^{11} \in Q^j\} = f_8 \le 2,\\
&\sum_j \{f_j \mid \omega_2^3 \in Q^j\} = f_4 + f_5 + f_6 + f_7 \le 4, && \sum_j \{f_j \mid \omega_8^{12} \in Q^j\} = f_9 + f_{10} \le 2,\\
&\sum_j \{f_j \mid \omega_3^4 \in Q^j\} = f_8 + f_9 \le 2, && \sum_j \{f_j \mid \omega_9^{13} \in Q^j\} = f_3 + f_5 \le 2,\\
&\sum_j \{f_j \mid \omega_3^5 \in Q^j\} = f_{10} \le 2, && \sum_j \{f_j \mid \omega_{10}^{14} \in Q^j\} = f_1 + f_4 \le 2,\\
&\sum_j \{f_j \mid \omega_4^6 \in Q^j\} = f_1 + f_3 \le 2, && \sum_j \{f_j \mid \omega_{10}^{15} \in Q^j\} = f_2 \le 2,\\
&\sum_j \{f_j \mid \omega_4^7 \in Q^j\} = f_2 \le 2, && \sum_j \{f_j \mid \omega_{12}^{17} \in Q^j\} = f_3 + f_5 + f_6 \le 2,\\
&\sum_j \{f_j \mid \omega_5^8 \in Q^j\} = f_4 + f_5 \le 2, && \sum_j \{f_j \mid \omega_{13}^{18} \in Q^j\} = f_7 + f_8 + f_9 + f_{10} \le 3,\\
&\sum_j \{f_j \mid \omega_6^9 \in Q^j\} = f_6 \le 2.
\end{aligned} \tag{24}$$

We thus obtain 466 flow vectors: $F^1, F^2, \ldots, F^{466}$.

Step 4: Transform all 466 feasible flow vectors F into capacity vectors via the following equations to obtain 466 (d,o,T)-LB candidates presented as $X^1, X^2, \ldots, X^{466}$ in Table 3.

$$\begin{aligned}
&x_1^1 = \sum_j \{f_j \mid \omega_1^1 \in Q^j\} = f_1 + f_3, && x_7^{10} = \sum_j \{f_j \mid \omega_7^{10} \in Q^j\} = f_7,\\
&x_1^2 = \sum_j \{f_j \mid \omega_1^2 \in Q^j\} = f_2, && x_8^{11} = \sum_j \{f_j \mid \omega_8^{11} \in Q^j\} = f_8,\\
&x_2^3 = \sum_j \{f_j \mid \omega_2^3 \in Q^j\} = f_4 + f_5 + f_6 + f_7, && x_8^{12} = \sum_j \{f_j \mid \omega_8^{12} \in Q^j\} = f_9 + f_{10},\\
&x_3^4 = \sum_j \{f_j \mid \omega_3^4 \in Q^j\} = f_8 + f_9, && x_9^{13} = \sum_j \{f_j \mid \omega_9^{13} \in Q^j\} = f_3 + f_5,\\
&x_3^5 = \sum_j \{f_j \mid \omega_3^5 \in Q^j\} = f_{10}, && x_{10}^{14} = \sum_j \{f_j \mid \omega_{10}^{14} \in Q^j\} = f_1 + f_4,\\
&x_4^6 = \sum_j \{f_j \mid \omega_4^6 \in Q^j\} = f_1 + f_3, && x_{10}^{15} = \sum_j \{f_j \mid \omega_{10}^{15} \in Q^j\} = f_2,\\
&x_4^7 = \sum_j \{f_j \mid \omega_4^7 \in Q^j\} = f_2, && x_{12}^{17} = \sum_j \{f_j \mid \omega_{12}^{17} \in Q^j\} = f_3 + f_5 + f_6,\\
&x_5^8 = \sum_j \{f_j \mid \omega_5^8 \in Q^j\} = f_4 + f_5, && x_{13}^{18} = \sum_j \{f_j \mid \omega_{13}^{18} \in Q^j\} = f_7 + f_8 + f_9 + f_{10},\\
&x_6^9 = \sum_j \{f_j \mid \omega_6^9 \in Q^j\} = f_6.
\end{aligned} \tag{25}$$

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Step 5: Apply the comparison procedure, we eliminate 38 (d,o,T)-LB candidates. The set $\Omega_{\min}$ consists of 428 (d,o,T)-LBs as shown in Table 3.

< Insert Table 3 >

Step 6: Utilize RSDP method to calculate flight network reliability from $X^1, X^2, \ldots, X^{428}$

$$R_{4,3,8} = \Pr\{\bigcup_{j=1}^{428}\{X : X \geq X^j\}\} = \Pr\{\{X \geq X^1\} \cup \{X \geq X^2\} \cup \{X \geq X^{428}\}\} = 0.989$$

Output: $R_{4,3,8} = 0.989$

4.2. Sensitivity analysis of flight network reliability

By utilizing the proposed algorithm, airlines have an equivalent evidence to estimate their ability to provide services. For instance, with flight network reliability $R_{4,3,8} = 0.989$, the airline in the case study has high chance to successfully transport 8 passengers within 4 hours. Moreover, the proposed algorithm is a useful tool to assess flight network reliability by executing sensitivity analysis, which is also known as importance measure (Zhang et al., 2015), because the aim of that analysis is to identify the contribution of the uncertainty in model inputs to the uncertainty in the model output (Liu & Homma, 2010). A decision-maker can execute sensitivity analysis in serval approaches (Iooss & Lemaître, 2015) such as ranking the importance of a component which is measured by the relative network efficiency drop after removing that component from the network (Qiang & Nagurney, 2008); evaluating the influence of the external factors on network performance (Dui et al., 2017); figuring out the component maintenance priority based the extent of the change in the network reliability resulted from a change in the reliability of a component (Wu et al., 2016); quantifying the influence of making the component perfectly reliable on the network mean time to failure (Borgonovo et al., 2016) or identifying the

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most important component state where the transaction rate of a component degrading from one state to another state is a time-dependent function under Weibull distribution (Dui et al., 2017). However, the sensitivity analysis provided in this study aims to evaluate the influence of given constraints on flight network reliability. In particular, the sensitivity analysis maps the flight network reliability $R_{d,o,T}$ under various levels of demand and time limitation. First, given that passengers prefer two or fewer stopovers, we test flight network reliabilities $R_{d,2,T}$ with several demands from 1 to 11 under four levels at limited-time. We also carry out that analysis to test all flight network reliabilities when passengers can accept up to three stopovers ($R_{d,3,T}$). The obtained flight network reliabilities ($R_{d,2,T}$ and $R_{d,3,T}$) are shown in Table 4. The fluctuation under different levels of demand d and limited time T of such flight network reliabilities ($R_{d,2,T}$ and $R_{d,3,T}$) are separately presented in Figure 8 and Figure 9.

< Insert Table 4 >

< Insert Figure 8 >

< Insert Figure 9 >

It is clear that with the limited time T = 7.5 hours, this MSFN can provide service for up to only 6 passengers, and flight network reliability declines markedly from the lowest to highest levels of demand. Additionally, there is no difference in flight network reliability between the cases of two and three stopovers. In addition, for every demand, the longer the travel times and the greater the number of stopovers on the journey passengers can accept, the higher the flight network reliability is; this is reasonable because increasing the number of stopovers and the time on the journey may increase the number of qualified minimal routes/minimal flightpaths and (d,o,T)-LB solutions. Another finding from this sensitivity analysis is that the longer the journey

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is the more demand the flight network can satisfy at a high level of reliability. However, a long journey is normally at a low level of attractiveness. Therefore, to fulfill their supply capacity and increase their revenue, airline managers can consider launching promotion campaigns towards price-oriented customers who accept longer flying time and prefer services at a lower price. It is obvious that the proposed algorithm can be applied to sensitivity analysis, which helps airlines managers have an overview of their flight network, thereby develop adequate business campaigns.

4.3. Experiments of computational time complexity

In this section, the results of several computational experiments are presented to verify the computational efficiency of the proposed algorithm. The proposed algorithm is run for three levels of demand d = 10, d = 15 and d = 20, respectively. Each case is executed under four different time constraints and stopover constraints o = 1 and o = 2. An experimental MSFN is a random case with a different number of airports and flights. The computational time of using the proposed algorithm is listed in Table 5. The CPU time of each combination is executed on a personal computer with Core^{TM} 3.4 i7-4770M CPU 3.4 and 8GB of RAM with programmed in MATLAB programming algorithm.

<Insert Table 5>

As shown in the results, in the majority of experiments, the algorithm can be completed within 1 minute, especially, it takes only 0.01 second for the case (6, 30). The proposed algorithm can be applied to evaluate the flight network reliability in a reasonable time. Obviously, the time constraint has a significant effect on the efficiency of the algorithm. A few experiments take

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longer time, approximately 5.6 mins, than the others because the limited time constraint on such experiments is quite close to the journey's length that passengers are willing to accept, resulting in too many candidates for searching.

5. Conclusions and future work

In this work, we have studied a flight system with single origin and destination under travel time and the number of stopovers constraints. We first modeled the flight system as an MSFN consisting of nodes and arcs, in which each node represents an airport and each arc representing a flight which connects a pair of nodes at specified departure and arrival time and then computed flight network reliability. In contrast to most of the literature, network reliability is evaluated under the assumption that all minimal flightpaths are given in advance, this study develops the searching method to determine all minimal flightpaths. Thus, this study's algorithm can extend to apply for flight networks without giving all minimal flightpaths in advance. The proposed algorithm requires to input only an original data of the MSFN and constraints to give results. Obviously, the provided algorithm in this study is more completed than previous ones and its utility is demonstrated by the case study and the experiments of computational time complexity.

According to the flight network reliability gained from this study, managers can determine such a relevant service offer that it has high ability to meet specific constraints of time and the number of stopovers thereby can reach customer satisfaction. Moreover, the proposed algorithm can provide an overview of flight network reliability for decision-makers via sensitivity analysis which records the variation of flight network reliability in a range of demand, stopovers and time constraints. In short, this study provides a useful tool which supports for executives in observing and assessing their flight network and thereby contributes to the advancement towards sustainability.

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In the future, for more realistic findings, we will attempt to factor in delay situations and extend research object from a flight network with single origin and destination to multiple ones. In other words, the future research may consider flight network with multiple origins – single destination or single origin – multiple destinations flight systems and evaluate flight network reliability using the algorithm proposed in this study. In addition, extending the proposed algorithm for importance measure of flight network reliability may be a potential and valuable study.

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Tables:

Table 1

The probability distribution of each arc capacity.

a_i	t_i	ω_i^k	g_i^k	v_i^k	x_i^k	$\Pr(x_i^k)$	a_i	t_i	ω_i^k	g_i^k	v_i^k	x_i^k	$\Pr(x_i^k)$
a_1	2	ω_1^1	7:00	9:00	0	0.05	a_8	2.5	ω_8^{11}	9:30	12:00	0	0.05
					1	0.1						1	0.1
					2	0.85						2	0.85
		ω_1^2	8:30	10:30	0	0.05			ω_8^{12}	10:00	12:30	0	0.05
					1	0.1						1	0.1
					2	0.85						2	0.85
a_2	2	ω_2^3	7:00	9:00	0	0.05	a_9	1	ω_9^{13}	12:30	13:30	0	0.05
					1	0.05						1	0.05
					2	0.1						2	0.9
					3	0.05	a_{10}	2	ω_{10}^{14}	12:30	14:30	0	0.05
					4	0.75						1	0.1
a_3	2	ω_3^4	7:00	9:00	0	0.05						2	0.85
					1	0.1			ω_{10}^{15}	14:00	16:00	0	0.05
					2	0.85						1	0.1
		ω_3^5	7:30	9:30	0	0.05						2	0.85
					1	0.1	a_{11}	1	ω_{11}^{16}	12:30	13:30	0	0.05
					2	0.85						1	0.05
a_4	2.5	ω_4^6	9:30	12:00	0	0.05						2	0.9
					1	0.1	a_{12}	1	ω_{12}^{17}	14:00	15:00	0	0.05
					2	0.85						1	0.1
		ω_4^7	11:00	13:30	0	0.05						2	0.85
					1	0.1	a_{13}	2	ω_{13}^{18}	13:00	15:00	0	0.05
					2	0.85						1	0.05
a_5	2.5	ω_5^8	9:30	12:00	0	0.05						2	0.05
					1	0.1						3	0.85
					2	0.85			ω_{13}^{19}	14:00	16:00	0	0.05
a_6	2.5	ω_6^9	9:30	12:00	0	0.05						1	0.1
					1	0.1						2	0.85
					2	0.85			ω_{13}^{20}	15:00	17:00	0	0.05
a_7	2.5	ω_7^{10}	9:30	12:00	0	0.05						1	0.1
					1	0.1						2	0.85
					2	0.85							

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Table 2

List of all possible minimal routes.

Minimal route (P^β)	Total time (T^β)	The length (e^β)	Accepted minimal route
$\{ a_1; a_4; a_{10} \}$	2+2.5+2+(3-1)*0.5=7.5	3	P^1
$\{ a_1; a_4; a_9; a_{12} \}$	2+2.5+1+1+(4-1)*0.5=8	4	P^2
$\{ a_1; a_4; a_9; a_{11}; \ldots \}$	2+2.5+1+1+2+(5-1)*0.5=10.5	> 4	rejected
$\{ a_2; a_5; a_{10} \}$	2+2.5+2+(3-1)*0.5=7.5	3	P^3
$\{ a_2; a_5; a_9; a_{11}; \ldots \}$	2+2.5+1+1+2+(5-1)*0.5=10.5	> 4	rejected
$\{ a_2; a_5; a_9; a_{12} \}$	2+2.5+1+1+(4-1)*0.5=8	4	P^4
$\{ a_2; a_6; a_{11}; a_{13} \}$	2+2.5+1+2+(4-1)*0.5=9 ($T^\beta > 8$)	4	rejected
$\{ a_2; a_6; a_{12} \}$	2+2.5+1+(3-1)*0.5=6.5	3	P^5
$\{ a_2; a_7; a_{13} \}$	2+2.5+2+(3-1)*0.5=7.5	3	P^6
$\{ a_3; a_8; a_{13} \}$	2+2.5+2+(3-1)*0.5=7.5	3	P^7

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Table 3

The (4,3,8)-LB candidates and (4,3,8)-LBs.

Step 4 (X^j)	Step 5 (Is X^j a (4,3,8)-LB?)
X^1=(0,0,2,0,2,0,0,2,0,0,0,2,2,0,0,0,2,2,0,0)	Yes
X^2=(0,0,2,0,2,0,0,2,0,0,0,2,2,0,0,0,2,2,0,0)	Yes
…	…
X^{160}=(1,0,2,0,1,1,0,2,0,0,0,1,1,2,0,0,1,1,0,0)	Yes
…	…
X^{378}=(1,0,2,0,1,1,0,2,0,0,0,1,1,2,0,0,1,1,0,0)	No, $X^{378} = X^{160}$
…	…
X^{391}=(2,0,2,0,0,2,0,1,0,1,0,0,1,2,0,0,1,1,0,0)	Yes
…	…
X^{431}=(2,1,1,0,0,2,1,1,0,0,0,0,1,2,1,0,1,0,0,0)	Yes
…	…
X^{455}=(2,0,2,0,0,2,0,1,0,1,0,0,1,2,0,0,1,1,0,0)	No, $X^{455} = X^{391}$
…	…
X^{466}=(2,1,1,0,0,2,1,1,0,0,0,0,1,2,1,0,1,0,0,0)	No, $X^{466} = X^{431}$

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Table 4

Flight network reliabilities $R_{d,2,T}$ and $R_{d,3,T}$

$R_{d,2,T}$					$R_{d,3,T}$				
	Time limited (T)					Time limited (T)			
Demand (d)	T = 7.5	T = 8.0	T = 8.5	T = 9.0	Demand (d)	T = 7.5	T = 8.0	T = 8.5	T = 9.0
1	0.9988	0.9999	1	1	1	0.9988	0.9999	1	1
2	0.9925	0.9993	0.9998	0.9999	2	0.9925	0.9995	0.9999	0.9999
3	0.9542	0.9963	0.9989	0.9994	3	0.9542	0.9972	0.9992	0.9995
4	0.856	0.9859	0.995	0.9973	4	0.856	0.989	0.9962	0.9977
5	0.6085	0.9557	0.9819	0.9887	5	0.6085	0.9635	0.9854	0.9906
6	0.3257	0.8834	0.9483	0.9652	6	0.3257	0.9006	0.9567	0.9709
7		0.7466	0.8742	0.9076	7		0.7765	0.8903	0.9217
8		0.5085	0.7414	0.7988	8		0.5467	0.7668	0.8266
9		0.2568	0.5432	0.6147	9		0.2911	0.5746	0.6581
10			0.3079	0.374	10			0.3362	0.4206
11			0.1118	0.1473	11			0.1278	0.1782

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Table 5

The CPU time of using the proposed algorithm

No. stopover (o = 1)		T = 6.5 (hours)	T = 7 (hours)	T = 7.5 (hours)	T = 8 (hours)
(Airports, flights)	Demand (passengers)	CPU time (unit: seconds)			
(6, 30)	d = 10	0.002440	0.002618	0.009773	0.010766
	d = 15	0.001838	0.011046	0.003452	0.004645
	d = 20	0.001784	0.002852	0.003097	0.015575
(10, 45)	d = 10	0.005209	0.059114	2.509691	2.610417
	d = 15	0.005098	0.039788	3.207511	3.279391
	d = 20	0.005912	0.033675	0.373143	0.463491
(15, 60)	d = 10	0.007759	48.005164	99.361902	189.617310
	d = 15	0.045298	31.403715	227.016340	248.706134
	d = 20	0.005105	84.164224	252.798275	305.434726
No. stopover (o = 2)		T = 6.5 (hours)	T = 7 (hours)	T = 7.5 (hours)	T = 8 (hours)
(Airports, flights)	Demand (passengers)	CPU time (unit: seconds)			
(6, 30)	d = 10	0.004106	0.002882	0.003966	0.004373
	d = 15	0.003552	0.001940	0.005211	0.006544
	d = 20	0.002662	0.002890	0.004193	0.005826
(10, 45)	d = 10	0.015861	0.066925	2.466449	2.439209
	d = 15	0.006514	0.036914	3.164539	3.093451
	d = 20	0.005164	0.033591	0.372483	0.370426
(15, 60)	d = 10	0.010573	4.401048	109.630834	203.733962
	d = 15	0.033840	13.214591	247.610095	262.844551
	d = 20	0.006552	4.135587	299.540200	331.565498

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Figures:

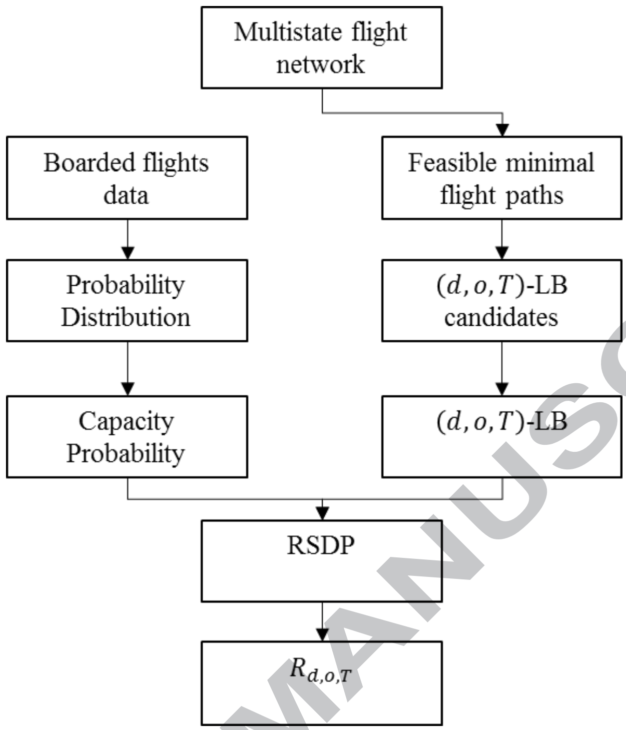

Figure 1 Flight network reliability evaluation procedure

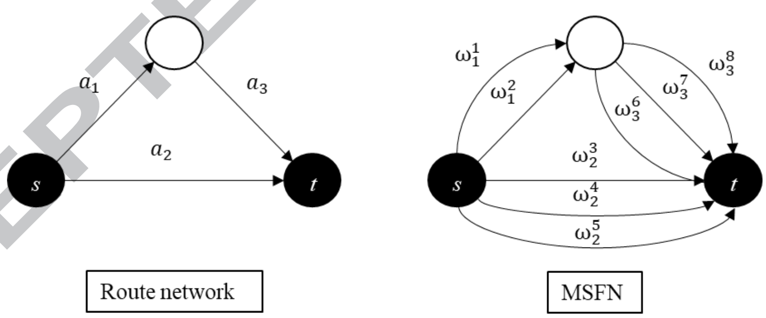

Figure 2 An example of a route network and an MSFN

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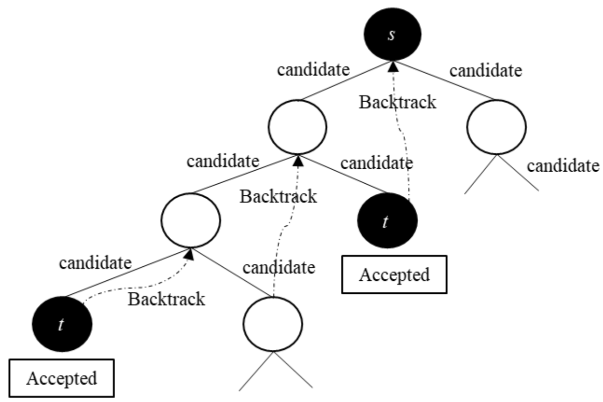

Figure 3 A sample of backtracking tree

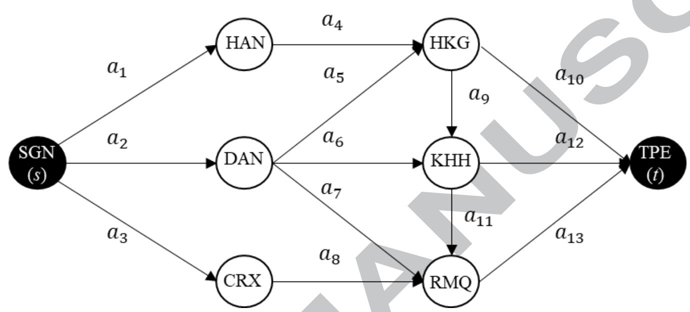

Figure 4 A route network

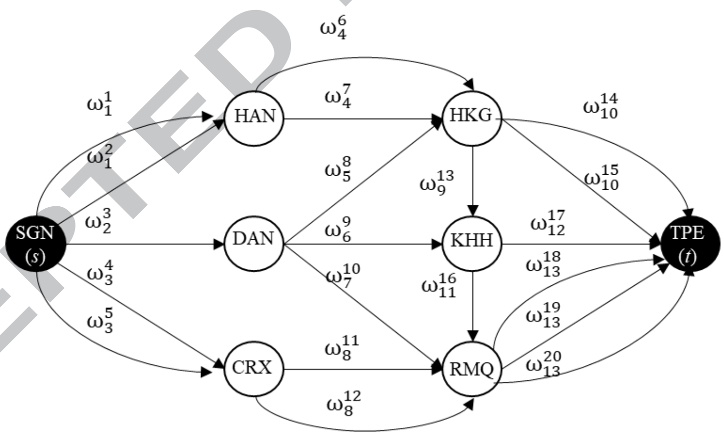

Figure 5 An MSFN

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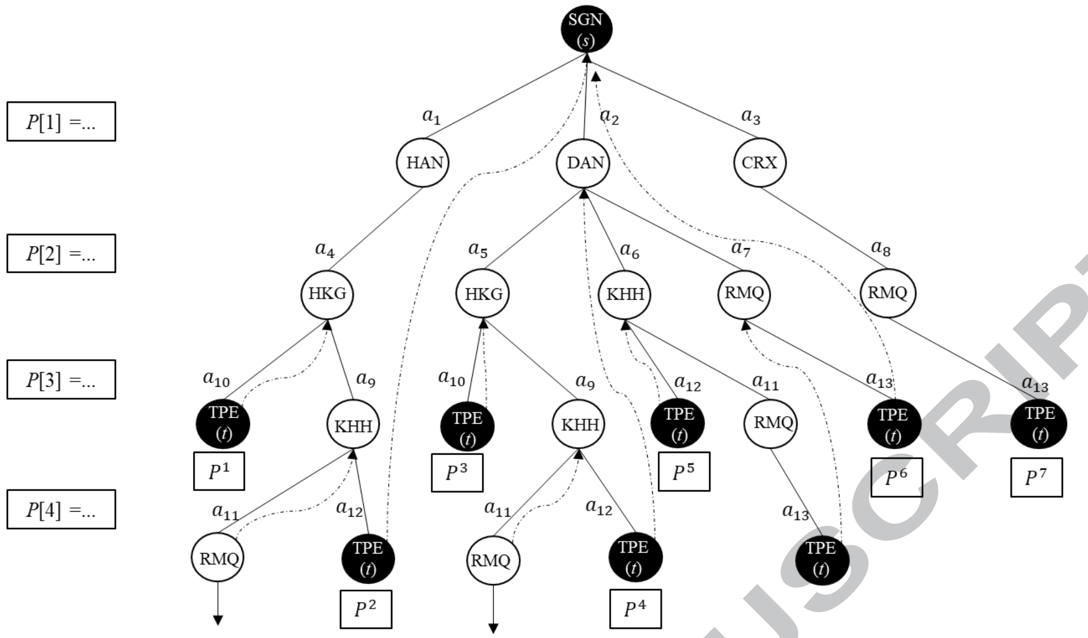

Figure 6 A backtracking tree for minimal routes

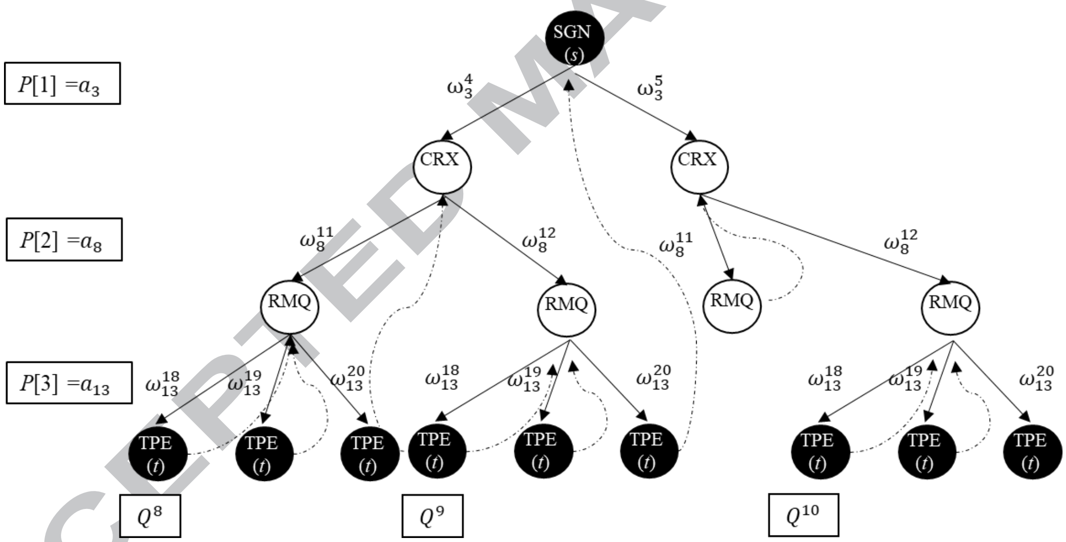

Figure 7 A backtracking tree for minimal flightpaths based on the minimal route $\boldsymbol{P}^{\mathbf{7}}$

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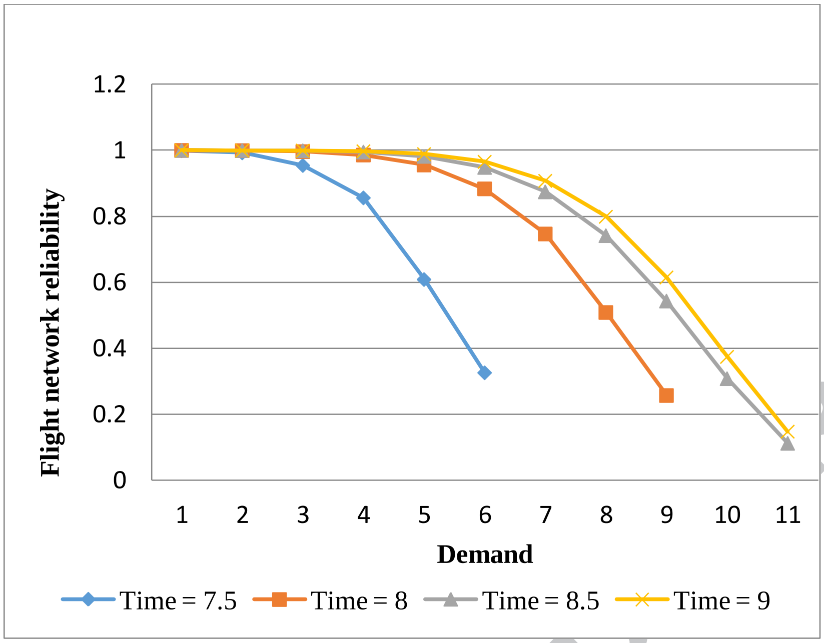

Figure 8 Flight network reliability under number of stopover o = 2 constraint

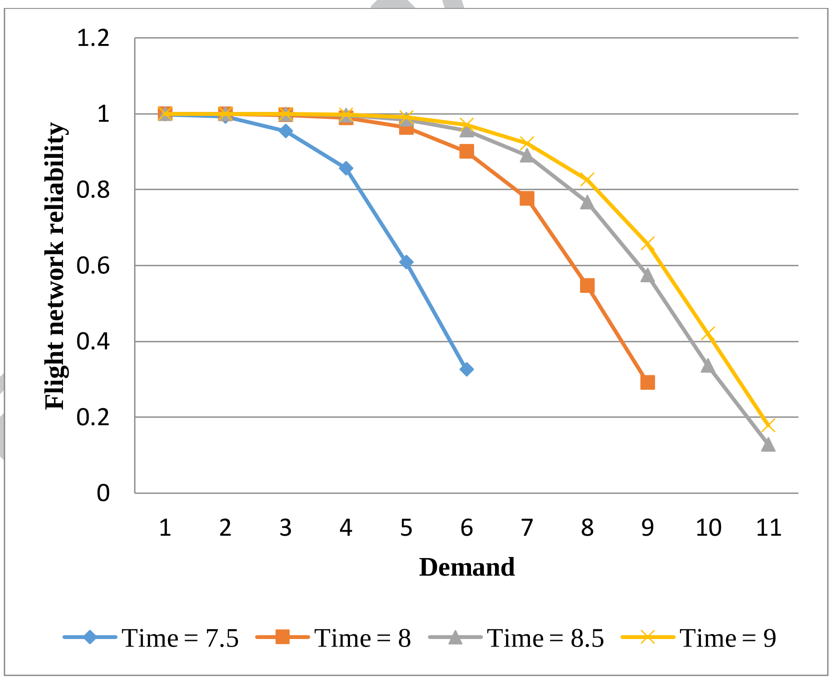

Figure 9 Flight network reliability under number of stopover o = 3 constraint

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Highlights

- A flight system with multistate capacity is studied.
- A multistate flight network (MSFN) is constructed to model the flight system.
- Propose an index, flight network reliability, to measure the MSFN.
- An algorithm with backtracking technique is proposed to evaluate flight network reliability.
- An illustrative example demonstrates how flight network reliability is evaluated.

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